RT-MAT 2000-01

Spatially periodic equilibria for a non local evolution equation

S. R. M. Barros, A. L. Pereira,
C. Possani and A. Simonis

Fevereiro 2000

Spatially periodic equilibria for a non local evolution equation

S.R.M. Barros, A.L. Pereira, C. Possani, A. Simonis *

February 22, 2000

Abstract

In this work we prove the existence of a global attractor for the non local evolution equation $\frac{\partial m(r,t)}{\partial t} = -m(r,t) + \tanh(\beta J * m(r,t))$ in the space of τ-periodic functions, for τ sufficiently large. We also show the existence of non constant (unstable) equilibria in these spaces.

1 Introduction

We consider here the non local evolution equation

$$\frac{\partial m(r,t)}{\partial t} = -m(r,t) + \tanh(\beta J * m(r,t)) \tag{1.1}$$

where $m(r,t)$ is a real function on $\mathbb{R} \times \mathbb{R}_+$; $\beta > 1, J \in C^2(\mathbb{R})$ is a non negative even function supported in the interval $[-1,1]$ and with integral equal to 1; the $*$ product denotes convolution, namely:

$$(J * m)(x) = \int_{\mathbb{R}} J(x-y)m(y)\,dy \quad .$$

This equation arises as a continuum limit of one-dimensional Ising spin systems with Glauber dynamics and Kac potentials [4]; m represents then a magnetization density and β^{-1} the temperature of the system.

*Instituto de Matemática e Estatística, USP - Brazil

Equation (1.1) clearly has the spatially homogeneous equilibria 0 and $\pm m_\beta$; m_β being the positive solution of the equation

$$m_\beta = tanh\beta m_\beta \ . \tag{1.2}$$

It also has been proved in [3] that, in the space of continuous bounded functions in $\mathbb{R}$, there exists a exponentially stable stationary solution whose asymptotic values at $\pm\infty$ are $\pm m_\beta$ (the instanton).

We consider here the same equation restricted to the subspace $\mathcal{P}_{2\tau}$ of functions periodic in space with a given period 2τ, $\tau > 1$. As we will see below this leads naturally to the consideration of a flow in $L^2(S^1)$, where S^1 denotes the one dimensional unit sphere. In this space, one can show existence of a global compact attractor and the existence of stationary solutions in addition to the ones mentioned above.

2 The flow in $L^2(S^1)$

We start by observing that, using uniqueness of solutions, it is easy to show that $\mathcal{P}_{2\tau}$ is invariant under the flow defined by (1.1). Now, if $\tau > 1$ is a given positive number, we define J^τ as the 2τ periodic extension of the restriction of J to $[-\tau,\ \tau]$.

Lemma 2.1 *If $u \in \mathcal{P}_{2\tau}$, then*

$$(J * u)(x) = \int_{-\tau}^{\tau} J^\tau(x-y)u(y)\,dy.$$

Proof: If $u \in \mathcal{P}_{2\tau}$, then

$$\begin{aligned}
(J * u)(x) &= \int_{\mathbb{R}} J(x-y)u(y)\,dy \\
&= \int_{x-\tau}^{x+\tau} J(x-y)u(y)\,dy \\
&= \int_{x-\tau}^{x+\tau} J^\tau(x-y)u(y)\,dy \\
&= \int_{-\tau}^{\tau} J^\tau(x-y)u(y)\,dy.
\end{aligned}$$

In view of this lemma, the problem (1.1), restricted to $\mathcal{P}_{2\tau}$, with $\tau > 1$, can be written as

$$\frac{\partial m(r,t)}{\partial t} = -m(r,t) + \tanh\left(\beta \int_{-\tau}^{\tau} J^\tau(x-y)m(y)\,dy\right)$$

Now, define $\varphi : [-\tau, \tau] \to S^1$ (the exponential map), by

$$\varphi(x) = e^{i\frac{\pi}{\tau}x}$$

and, for any $u \in \mathcal{P}_{2\tau}$, $v : S^1 \to \mathbb{R}$ by

$$v(\varphi(x)) = u(x).$$

In particular, we write $\tilde{J}(\varphi(x)) = J^\tau(x)$. Then, a simple computation shows that $u = u(x,t)$ is a $2\tau-$ *periodic* solution of 1.1 if and only if $v(y,t) = u(\varphi^{-1}(y), y)$ is a solution of

$$\frac{\partial m(y,t)}{\partial t} = -m(y,t) + \tanh\left(\beta \tilde{J} * m(y,t)\right) \tag{2.3}$$

where now $*$ denotes convolution in S^1, that is

$$\left(\tilde{J} * m\right)(y) = \int_{S^1} \tilde{J}(y \cdot z^{-1}) m(z)\, d\, z$$

and $d\,z = \frac{\tau}{\pi} d\,s$ where $d\,s$ denotes integration with respect to arclenght. This will be the measure adopted in S^1 in the sequel. From now on, we drop the ˜ sign in J for simplicity.

Equation (2.3) generates a C^1 semiflow $T(t)$ in $X = L^2(S^1)$ since its right-hand side is a Lipschitz continuous function in this space.

3 Existence of the global attractor

In this section we prove the existence of a global maximal invariant compact set $A \subset X$ for the semiflow T, which attracts the bounded sets of X (the global attractor) (see [1] or [5]).

We recall that a set $\mathbf{B} \subset X$ is an *absorbing set* for the semiflow T if, for any bounded set C in X, there is a $t_1 > 0$ such that $T(t)C \subset B$ for any $t \geq t_1$ (see [5]).

Lemma 3.1 *For any $\varepsilon > 0$, the ball of radius $\frac{\sqrt{2\tau}}{1-\varepsilon}$ is an absorbing set for the flow $T(t)$.*

Proof: For $u \in X$, we denote the norm of u by $||u||$. Let $u(t) = T(t)u$. Then, while $||u|| \geq \frac{\sqrt{2\tau}}{1-\varepsilon}$, we have

$$
\begin{aligned}
\frac{d}{dt}\int_{S^1}|u|^2\,dx &= -2\left(\int_{S^1}u^2\,dx-\int_{S^1}u\tanh(\beta J*u)\,dx\right)\\
&\le -2\int_{S^1}u^2\,dx+2\left(\int_{S^1}u^2\,dx\right)^{\frac{1}{2}}\left(\int_{S^1}(\tanh(\beta J*u))^2\,dx\right)^{\frac{1}{2}}\\
&\le -2\int_{S^1}u^2\,dx+2\sqrt{2\tau}\left(\int_{S^1}u^2\,dx\right)^{\frac{1}{2}}\\
&\le -2(||u||^2-\sqrt{2\tau}||u||)\\
&\le -2||u||^2(1-\frac{\sqrt{2\tau}}{||u||})\\
&\le -2\varepsilon||u||^2
\end{aligned}
$$

Therefore, while $||u||\geq\frac{\sqrt{2\tau}}{1-\varepsilon}$, we have

$$\frac{d}{dt}||u(t)||^2\leq-2\varepsilon||u||^2$$

Thus

$$||u(t)||\leq e^{-\varepsilon(t-t_0)}||u(t_0)||. \tag{3.4}$$

and the result follows imediatelly.

Remark 3.1 *The estimate (3.4) above actually shows uniform exponential decay with rate ε to the ball of radius $\frac{\sqrt{2\tau}}{1-\varepsilon}$.*

Theorem 3.2 *There exists a global attractor A for the semiflow $T(t)$ generated by 2.3 in X, which is contained in the ball of radius $\sqrt{2\tau}$.*

Proof: If $u(x,t)$ is a solution of 2.3, we have by the variation of constants formula

$$u(x,t)=e^{-t}u(x,0)+\int_0^t e^{t-s}\tanh\{\beta(J*u)(x,s)\}\,ds\ . \tag{3.5}$$

Write $T_1(t)u=e^{-t}u(x,0)$, $T_2(t)u=\int_0^t e^{t-s}\tanh\{\beta(J*u)(x,s)\}\,ds$ and suppose $u\in C$, where C is a bounded set in X. Then $||T_1(t)u||\to 0$ as $t\to\infty$, uniformly in u. Also, by lemma 3.1 there exists a t_0 such that, for any $u\in C$, $||u(t)||\leq\frac{\sqrt{2\tau}}{1-\varepsilon}$ for $t\geq t_0$. Therefore, for these values of t

$$
\begin{aligned}
|\frac{\partial}{\partial x}T_2u| &\leq |\int_0^t e^{t-s}\frac{\partial}{\partial x}\tanh\{\beta(J*u)(x,s)\}\,ds| \\
&\leq \int_0^t e^{t-s}|\frac{\partial}{\partial x}\tanh\{\beta(J*u)(x,s)\}|\,ds \\
&\leq \int_0^t e^{t-s}\beta\left\{|\int_{S^\tau}(J'(x\cdot y^{-1})u)(y,s)\,dy\right\}|\,ds \\
&\leq \int_0^t e^{t-s}\beta\left\{\int_{S^\tau}|(J'(x\cdot y^{-1})|u)(y,s)\,dy\right\}\,ds \\
&\leq \int_0^t e^{t-s}\beta||J'||\,||u(\cdot,s)||\,ds \\
&\leq (\frac{\sqrt{2\tau}}{1-\varepsilon})\beta\int_0^t e^{t-s}||J'||\,ds \\
&\leq (\frac{\sqrt{2\tau}}{1-\varepsilon})||J'||
\end{aligned}
$$

It follows that, for $t \geq t_0$ and any $u \in C$ $||\frac{\partial}{\partial x}T_2u||$ is bounded by a constant (independent of t and u). Thus $\bigcup_{t\geq t_0} T_2(t)C$ is relatively compact in X by Sobolev's imbedding theorem.

The result follows than imediately from Theorem 1.1 of [5].

Now, once estimates in L^2 for solutions in the global attractor have been proved, one can use a bootstrap argument to obtain more regularity for them.

Theorem 3.3 *The global attractor A is bounded in $H^2(S^1)$.*

Proof: If $u(x,t)$ is a solution of 1.1 in A, we have by the variation of constants formula

$$u(x,t) = e^{-(t-t_0)}u(x,t_0) + \int_{t_0}^t e^{t-s}\tanh\{\beta(J*u)(x,s)\}\,ds \tag{3.6}$$

Since $u(x,t_0)$ is bounded by $\sqrt{2\tau}$ for any choice of t_0, letting $t_0 \to -\infty$, we obtain

$$u(x,t) = \int_{-\infty}^t e^{t-s}\tanh\{\beta(J*u)(x,s)\}\,ds \tag{3.7}$$

(equality in L^2).

Therefore, proceeding as in the proof of theorem 3.2, we get

$$
\begin{aligned}
|\frac{\partial}{\partial x}u(x,t)| &\le |\int_{-\infty}^{t} e^{t-s}\frac{\partial}{\partial x}\tanh\{\beta(J*u)(x,s)\}\,ds| \\
&\le (\frac{\sqrt{2\tau}}{1-\varepsilon})\beta\int_{\infty}^{t} e^{t-s}||J'||\,ds \\
&\le (\frac{\sqrt{2\tau}}{1-\varepsilon})||J'||
\end{aligned}
$$

and, similarly

$$
|\frac{\partial^2}{\partial x^2}u(x,t)| \le (\frac{\sqrt{2\tau}}{1-\varepsilon})||J''||.
$$

The result is, therefore, proved.

4 Stationary solutions

The following functional $\mathcal{F}(m)$ is used in [3] to prove the existence of the instanton.

$$
\mathcal{F}(m) = \int [f(m(x)) - f(m_\beta)]\,dx + \frac{1}{4}\int\int J(x-y)[m(x)-m(y)]^2\,dx\,dy
$$

where $f(m)$ (the free energy density) is given by

$$
f(m) = -\frac{1}{2}m^2 - \beta^{-1}i(m)
$$

and $i(m)$ is the entropy density

$$
i(m) = -\frac{1+m}{2}log\{\frac{1+m}{2}\} - \frac{1-m}{2}log\{\frac{1-m}{2}\}
$$

A difficulty encountered with this functional in the space of continuous bounded functions in $\mathbb{R}$ is that it is not defined in the whole space. $\mathcal{F}(m) < \infty$ if, and only if $m(x)$ is close -in a certain sense- to $\pm m_\beta$ in a neighborhood of the infinity (see [3] for details).

In our setting though, we have a similar functional defined in the whole phase space, as follows

$$
\mathcal{F}(m) = \int_{S^1} [f(m(x)) - f(m_\beta]\,dx + \frac{1}{4}\int_{S^1}\int_{S^1} J(x\cdot y^{-1})[m(x)-m(y)]^2\,dx\,dy \ .
$$

The following result is an adaptation of proposition 2.8 of [3] to our context.

Lemma 4.1 *Let $m(\cdot,t)$ be a solution of 2.3 $m(\cdot,0) \leq 1$. Then $(\mathcal{F}(m(.,t))$ is well defined for all $t \geq 0$, it is differentiable with respect to t for $t > 0$ and*

$$\frac{d}{dt}\mathcal{F}(m(\cdot,t)) = -I(m(\cdot,t)) \leq 0$$

where, for any $h \in L^2(S^1)$, $||h||_\infty \leq 1$,

$$I(h(\cdot)) = \int_{S^1} [(J*h)(x) - \beta^{-1} arctanh(h(x))][tanh\beta(J*h)(x) - h(x)]\,dx$$

We are now ready to establish the existence of non trivial periodic equilibria for equation 1.1. For any $n \in \mathbf{N}^*$, we define the subspace A_n of X by

$$A_n = \left\{ v \in X | v(\varphi(\frac{\tau}{n} + y)) = -v(\varphi(y)) \right\}$$

It is easy to show, using uniqueness, that these subspaces are invariant under $T(t)$. Our result is given by

Theorem 4.2 *For any $n_0 \in \mathbf{N}$, there is a $\tau(n_0)$ such that, if $\tau \geq \tau(n_0)$, there exists a nontrivial stationary solution of 2.3 in A_n, for any $n \leq n_0$. Furthermore, these solutions are all unstable.*

Proof: Consider the function in A_n defined by $l(\varphi(x)) = m_\beta$ for $0 \leq x \leq \frac{\tau}{n}$, and $l(x,t)$ the solution of 1.1, with initial condition $l(\cdot,0) = l$. We then have, if $\tau > n$

$$\begin{aligned}
\mathcal{F}(l) &= \frac{1}{4}\int_{-\tau}^{\tau}\int_{-\tau}^{\tau} J^\tau(x-y)(l(x)-l(y))^2\,dy\,dx \\
&= \frac{1}{4}\sum_{-(n-1)}^{n-1}\int_{j\frac{\tau}{n}}^{j\frac{\tau}{n}+1}\int_{x-1}^{j\frac{\tau}{n}} J^\tau(x-y)4m_\beta^2\,dy\,dx \\
&+ \frac{1}{4}\sum_{-(n-1)}^{n-1}\int_{j\frac{\tau}{n}-1}^{j\frac{\tau}{n}}\int_{j\frac{\tau}{n}}^{x+1} J^\tau(x-y)4m_\beta^2\,dy\,dx \\
&= (2n-1)m_\beta^2\int_0^1\int_{x-1}^0 J^\tau(x-y)\,dy\,dx + (2n-1)m_\beta^2\int_{-1}^0\int_0^{x+1} J^\tau(x-y)\,dy\,dx \\
&\leq 2(2n-1)m_\beta^2\int_0^1\int_{x-1}^{x} J^\tau(x-y)\,dy\,dx \\
&= 2(2n-1)m_\beta^2\int_0^1\int_{-1}^{0} J^\tau(z)\,dz\,dx \\
&= (2n-1)m_\beta^2
\end{aligned}$$

On the other hand

$$\mathcal{F}(0) = \int_{-\tau}^{\tau} (f(0) - f(m_\beta))\, dx = 2\tau(f(0) - f(m_\beta)).$$

Therefore, if $\tau > \frac{(2n-1)m_\beta^2}{2(f(0)-f(m_\beta))}$, $\mathcal{F}(0) > \mathcal{F}(l)$ and the ω-limit of l does not contain the null stationary solution.

Now, the existence of a global compact attractor implies precompacity of the orbits of $T(t)$. It follows then by La Salle's invariance principle (see [2]) that $l(x,t) \to M$, where M is the maximal invariant subset of $E = \{m \in L^2(S^\tau) | \dot{\mathcal{F}} = 0\}$.

Since any point in E is a stationary point of 2.3, the first part of the theorem follows immediately.

Let $\bar{m}(x)$ be a nontrivial equilibrium in A_n. The linearization of the evolution equation 2.3 around $\bar{m}$ is

$$\partial_t v = -v + (1 - \bar{m}^2)\beta J * v \equiv \mathcal{L}v$$

The operator $Tv = (1 - \bar{m}^2)\beta J * v$ is compact and self-adjoint in L^2. Now, since $\mathcal{L}(\bar{m}') = 0$, $\bar{m}'$ is an eigenfunction of T with corresponding eigenvalue $\lambda = 1$, that is, $T\bar{m}' = \bar{m}'$. Let $\bar{v} = |\bar{m}'|$. From $T\bar{m}' = \bar{m}'$, we obtain immediately $T\bar{v} \geq \bar{v}$. Furthermore, since $\bar{m}'$ changes sign, there exists a point $\bar{x}$ in S^τ where $\bar{v}(\bar{x}) = 0$, and $\bar{v}$ does not vanish in any neighborhood of $\bar{x}$. It follows that $T\bar{v}(\bar{x}) > \bar{v}(\bar{x})$. But then, by the minimax principle, we obtain for the maximum eigenvalue λ_p of T

$$\begin{aligned} \lambda_p &= \max\left\{\frac{< Tv, v >}{< v, v >} | v \in L^2\right\} \\ &\geq \frac{< T\bar{v}, \bar{v} >}{< \bar{v}, \bar{v} >} \\ &> 1 \end{aligned}$$

Therefore $\mathcal{L} = -I + T$ has a positive eigenvalue and $\bar{m}$ is unstable.

Remark 4.1 *Since $m_\beta \to 0$ as $\beta \to 1$ and $\mathcal{F}(0) = \tau(1 - \frac{1}{\beta})m_\beta^2 + O(m_\beta^3)$ as $m_\beta \to 0$, it follows that $\mathcal{F}(0) < \mathcal{F}(l)$ if β is close to 1. Therefore, for any $n \geq 1$ and τ* fixed *the argument above cannot be used to show existence of solutions in A_n.*

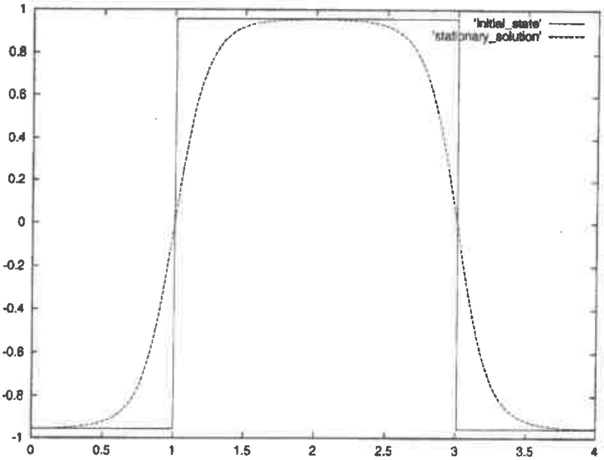

Figure 1: Stationary solution: $\tau = 2$ and $n = 1$

5 Numerical Simulations

We have implemented a numerical scheme for solving equation (2.3), based on a second order predictor-corrector method for the discretization of the differential equation. The convolution is computed through a multiple Simpson-rule, while the J function is chosen as a cubic spline, giving it the further property of being twice differentiable.

Numerical simulations helped on gathering evidence of the existence of the stationary solutions that we proved to exist. On the other hand, numerical experiments also provide means for illustrating the behaviour of the solutions in certain cases. In Figure 1 we plot the stationary solution obtained as in theorem 4.2 with $\tau = 2$ and $n = 1$, after the temporal evolution of the initial state. We used the value $\beta = 2$ in the simulations.

In Figure 2 we display how the shape of these stationary solutions vary with n, for a fixed value of τ. In this case we employ $\tau = 8$ and show in the same graph the cases $n = 4$, $n = 2$ and $n = 1$. We point out that the stationary function for $\tau = 8$ and $n = 4$ is actually the same as the one for $\tau = 2$ and $n = 1$, presented in Figure 1. All these stationary solutions present a similar shape, having almost flat parts close to $+m_\beta$ or $-m_\beta$ and transition zones between them. We also note that the value $n = 8$ in the case $\tau = 8$ does not lead to a stationary solution (the conditions for theorem 4.2 are not satisfied in this case).

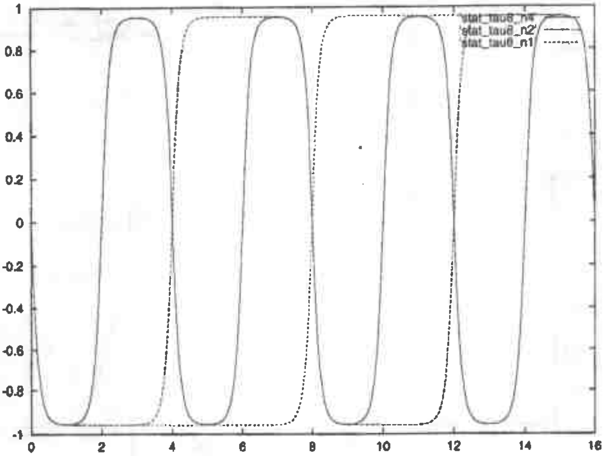

Figure 2: Stationary solutions: $\tau = 8$ and $n = 4$, $n = 2$, $n = 1$.

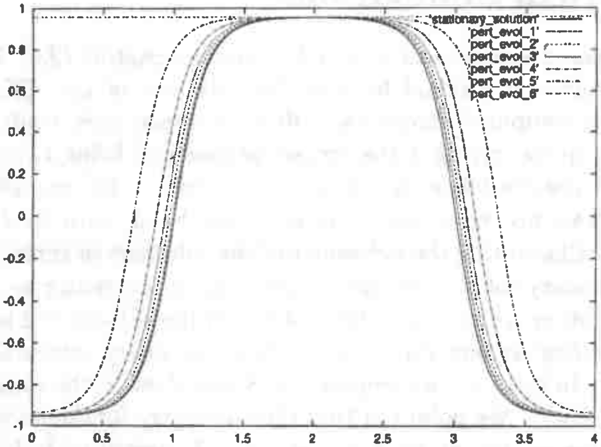

Figure 3: Unstable behaviour of perturbed stationary solution: $\tau = 2$ and $n = 1$

In Figure 3 we present results of the use of the numerical method as a mean for illustrating the unstable behaviour of the stationary solutions. In this case, the proof of theorem 4.2 provided the guidance for the choice of the perturbation we added to the stationary solution, in order to be able to observe the instability of the solution. We perturbed the stationary solution ($\tau = 2$, $n = 1$, $\beta = 2$) by one percent of the modulus of its derivative and computed the evolution of this initial state. The solution remains close to the stationary one for a long time, departing slowly from it, until reaching a point when it converges very fast to the stable stationary point given by the constant solution equal to m_β. This evolution is shown in Figure 3.

6 Acknowledgements

The authors would like to thank Profs. O. Lopes and K. Mischaikow for helpful comments and suggestions.

References

[1] J. K. Hale, *Asymptotic Behavior of Dissipative Systems,* American Surveys and Monographs, no. 25, (1988).

[2] D. Henry, *Geometric Theory of Semilinear Parabolic Equations*, Lecture Notes in Mathematics 840, Springer-Verlag, (1981).

[3] A. de Masi, E. Orlandi, E. Presutii, L. Triolo, *Uniqueness and global stability of the instanton in non local evolution equations,* Rendiconti di Matematica, Serie VII, Volume 14, 693-723, (1994).

[4] A. de Masi, E. Orlandi, E. Presutii, L. Triolo, *Glauber evolution with Kac potentials: I. Mesoscopic and macroscopic limits, interface dynamics*, Nonlinearity 7, 633-696, (1994).

[5] R. Teman, *Infinite Dimensional Dynamical Systems in Mechanics and Physics*, Springer Verlag (1988).

TRABALHOS DO DEPARTAMENTO DE MATEMÁTICA

TÍTULOS PUBLICADOS

1999-01 FERNANDES, J.D., GROISMAN, J. and MELO, S.T. Harnack inequality for a class of degenerate elliptc operators. 19p.

1999-02 GIULIANI, O. F. and PERESI, A.L., Minimal identities of algebras of rank 3. 9p.

1999-03 FARKAS, D. R., GEISS, C., GREEN, E.L., MARCOS, E.N. Diagonalizable Derivations of Finite-Dimensional Algebras I. 25p.

1999-04 FARKAS, D. R., GEISS, E.L., MARCOS, E.N. Diagonalizable Derivations of Finite-Dimensional Algebras II. 13p.

1999-05 LOBÃO, T. P. and MILIES, C. P. The normalizer property for integral group rings of Frobenius groups. 7p.

1999-06 PICCIONE, P. and TAUSK, D.V. A note on the Morse index theorem for geodesics between submanifolds in semi-Riemannian geometry. 15p.

1999-07 DOKUCHAEV, M., EXEL, R. and PICCIONE, P. Partial representations and partial group algebras. 32p.

1999-08 MERCURI, F., PICCIONE, P. and TAUSK, D.V. Stability of the focal and geometric index in semi-Riemannian geometry via the Maslov index. 72.

1999-09 BARBANTI, L. Periodic solution for Volterra-Stiltjes integral linear equations of type (K). 9p.

1999-10 GALINDO, P., LOURENÇO, M.L. and MORAES, L.A. Compact and weakly compact homomorphisms on Fréchet algebras of holomorphic functions. 10p.

1999-11 MARCOS, E.N, MERKLEN, H.A. and PLATZECK, M.I. The Grothendiek group of the category of modules of finite projective dimension over certain weakly triangular algebras. 18p.

1999-12 CHALOM, G. Vectorspace Categories Immersed in Directed Components. 32p.

1999-13 COELHO, F.U. Directing components for quasitilted algebras. 5p.

1999-14 GOODAIRE, E.G. and POLCINO MILIES, C. Alternative Loop Rings with Solvable Unit Loops. 13p.

1999-15 GOODAIRE, E.G. and POLCINO MILIES, C. A Normal Complement for an Ra Loop in its Integral Loop Ring. 9p.

1999-16 LOURENÇO, M. L. and MORAES, L.A. A class of polynomials. 9p.

1999-17 GRISHKOV, A. N. The automorphisms group of the multiplicative Cartan decomposition of Lie algebra E_8. 18p.

1999-19 GRISHKOV, A. N. Representations of Lie Algebras over rings. 14p.

1999-20 FUTORNY, V., KÖNIG, S. and MAZORCHUK, V. $\mathcal{S}$-subcategories in $\mathcal{O}$ 16p.

1999-21 BASSO, I., COSTA, R., GUTIÉRREZ, J. C. and GUZZO JR., H. Cubic algebras of exponent 2: basic properties. 14p.

1999-22 GORODSKI, C. Constant curvature 2-spheres in CP^2. 5p.

1999-23 CARDONA, F.S.P and WONG, P.N.S, On the computation of the relative Nielsen number. 15p

1999-24 GARCÍA, D., LOURENÇO, M.L., MAESTRE, M. and MORAES, L.A. de, The spectrum of analytic mappings of bounded type. 19p.

1999-25 ARAGONA, J. and JURIAANS, S.O. Some structural properties of the topological ring of Colombeau's generalized numbers. 35p.

1999-26 GIULIANI, M.L.M. and POLCINO MILIES, C. The smallest simple Moufang loop. 27p.

1999-27 ASPERTI, A. C. and COSTA, E. A. Vanishing of homology groups, Ricci estimate for submanifolds and applications. 21p.

1999-28 COELHO, F.U., MARTINS, M.I.R. and DE LA PEÑA, J.A. Quasitilted Extensions of Algebras I. 11p.

1999-29 COELHO, F.U., MARTINS, M.I.R. and DE LA PENA, J.A. Quasitilted Extensions of Algebras II. 17p.

1999-30 FARKAS, D. R., GEISS, C. and MARCOS, E. N. Smooth Automorphism Group Schemes. 23p.

1999-31 BOVDI, A. A. and POLCINO MILIES, C. Units in Group Rings of Torsion Groups. 15p.

1999-32 BARDZELL. M. J. and MARCOS, E. N. $H^1(A)$ and presentations of finite dimensional algebras. 8p.

1999-33 GRISHKOV, A. N. and SHESTAKOV, I. P. Speciality of Lie-Jordan algrbras. 17p.

1999-24 ANGELERI-HÜGEL, L. and COELHO, F. U. Infinitely generated tilting modules of finite projective dimension. 14p.

1999-25 AQUINO, R. M. and MARCOS, E. N. Koszul Tilted Algebras. 19p.

1999-26 CHALOM, G. and MERKLEN, H. Representation Type of One Point Extensions of Tilted Euclidean Algebras. 28p.

2000-01 BARROS, S.R.M., PEREIRA, A.L., POSSANI, C. and SIMONIS, A. Spatially periodic equilibria for a non local evolution equation. 11p.

Nota: Os títulos publicados nos Relatórios Técnicos dos anos de 1980 a 1998 estão à disposição no Departamento de Matemática do IME-USP.

Cidade Universitária "Armando de Salles Oliveira"
Rua do Matão, 1010 - Cidade Universitária
Caixa Postal 66281 - CEP 05315-970