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1D AGING

by

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$1D$ Aging

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Abstract

We derive exact expressions for a number of aging functions that are scaling limits of non-equilibrium correlations, $R(t_w, t_w + t)$ as $t_w \to \infty$, $t/t_w \to \theta$, in the $1D$ homogenous q-state Potts model for all q with $T = 0$ dynamics following a quench from $T = \infty$. One such quantity is $\langle \vec{\sigma}_0(t_w) \cdot \vec{\sigma}_n(t_w + t) \rangle$ when $n/\sqrt{t_w} \to z$. Exact, closed-form expressions are also obtained when one or more interludes of $T = \infty$ dynamics occur. Our derivations express the scaling limit via coalescing Brownian paths and a "Brownian space-time spanning tree," which also yields other aging functions, such as the persistence probability of no spin flip at 0 between t_w and $t_w + t$.

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Aging and related memory effects are a fundamental feature of nonequilibrium dynamics [1]. First observed in mechanical properties of glassy polymers and other amorphous materials [2], it was found to be a central property of spin glass dynamics as well [3]. Although originally thought to be a distinguishing feature of disordered systems, aging is now known to occur in both homogeneous and disordered systems following a quench to low temperature T (though debate persists over whether aging differs qualitatively in different systems [4]). Although many models and mechanisms for aging have been proposed, few exact (mostly, but not exclusively, for the $1D$ Ising chain) or rigorous results exist [5–9].

In this paper we present a general approach to aging in $1D$ discrete spin models (and equivalent systems, such as reaction-diffusion or voter models), in the continuum space-time scaling limit (e.g., lattice spacing $a \to 0$, with time scaled by a^2). We focus here on exact solutions for the scaling limit of the entire dynamical process, and thence for aging functions, in homogeneous ferromagnetic q-state Potts models (where $q = 2$ is the Ising model), but the approach is also applicable to inhomogeneous systems, as in [9]. The method uses earlier work by Arratia [10] to express the scaling limit via a "Brownian space-time spanning tree", depicting the histories of coalescing Brownian particles in one dimension starting from all possible locations and times.

In a typical aging experiment, a system is rapidly quenched from high to low T. After a time t_w following the quench, an external parameter (e.g., temperature or external field) is changed. The response $R(t_w, t_w + t)$ of the system (e.g., decay of thermoremanent magnetization) is then measured at time $t_w + t$. Aging can also be observed without a sudden parameter change; e.g., in the out-of-phase component χ'' of the ac susceptibility [1]. In either case, the essence of aging is that as $t_w \to \infty$, $t \to \infty$, the response depends only on the ratio t/t_w:

$$\lim_{\substack{t\to\infty, t_w\to\infty \\ t/t_w\to\theta}} R(t_w, t_w + t) = \mathcal{R}(\theta). \tag{1}$$

(Other scaling forms are discussed in [11].) Equilibrium responses are time-translation-invariant, so aging is a non-equilibrium, history-dependent phenomenon.

There already exist a few exact results for related quantities measuring coarsening [12] or persistence [13]. Exact results for persistence exponents (and fraction of persistent spins) in $1D$ Potts models appear in [14], and results on coarsening quantities (such as domain size distributions) appear in [15–17]. There are fewer exact results for aging quantities. An exception is the well understood Ising chain, for which two-time correlations have been derived [6] (see also [7]). However, the methods used seem specialized to the Ising case. For general Potts models, exact results have been obtained only for $F_q(t_w, t_w + t)$, the probability of no spin flip at the origin (in $1D$) between t_w and $t_w + t$. This was analyzed in [14] for a semi-infinite chain and, for $q = \infty$, in [5] on the full $1D$ lattice. In the following sections we present our general method and compute exact results for a variety of aging quantities in the continuum scaling limit, including as special cases rederivations of the results of [5,6].

Preliminaries. Consider the homogeneous q-state ferromagnetic Potts model on the $1D$ integer lattice, where the Potts spin variables σ_n, for $-\infty < n < \infty$, can take the values $1, \ldots, q$. We study $T = 0$ dynamics following a quench from $T = \infty$ — i.e., in the initial $\sigma(0)$, each site independently takes a random value uniformly from $1, \ldots, q$. (For $q = \infty$, each site takes its own unique value.) The standardly used (as in [14]) continuous time $T = 0$ dynamics, that of the $1D$ voter model (see, e.g., [18]), is given by independent Poisson "clock" processes at each site n, all of rate one, indicating when a flip at n is considered. When the clock at n rings, σ_n takes the value of one neighbor, chosen by a fair coin toss (regardless of whether it previously agreed with either or both neighbors). There are other natural $T = 0$ dynamics, but we defer their analysis to a later paper.

In studying this evolution, it is convenient to use a well-known mapping to a $1D$ reaction-diffusion system of "kinks" [19–21]. A kink corresponds to a site $n + 1/2$ in the *dual* lattice where $\sigma_n \neq \sigma_{n+1}$. The initial configuration is a random arrangement of kinks (with density $(q-1)/q$) that subsequently execute $1D$ random walks. For the Ising model ($q = 2$), the walks are purely annihilating, while for $q = \infty$ they are purely coalescing; for other q both annihilation and coalescence occur.

At time t_w, there will be a characteristic distribution, depending on q, of walkers (i.e., kinks). One aging quantity is the persistence probability $F_q(t_w, t_w + t)$, mentioned earlier, of no flip at the origin between t_w and $t_w + t$. A related quantity is $G_q(0, 0; t_w, t_w + t) = \langle \delta_{\sigma_0(t), \sigma_0(t_w+t)} \rangle$, the probability P that $\sigma_0(t_w + t) = \sigma_0(t)$ (regardless of intervening flips). More generally, $G_q(m, n; s, s')$ is $P(\sigma_n(s') = \sigma_m(s))$, and the spin-spin correlation $C_q = [q/(q-1)](G_q - 1/q)$, which is $\langle \vec{\sigma}_m(s') \cdot \vec{\sigma}_n(s) \rangle$ in the "tetrahedral" representation of Potts spins. Other aging quantities will be discussed later.

FIGURES

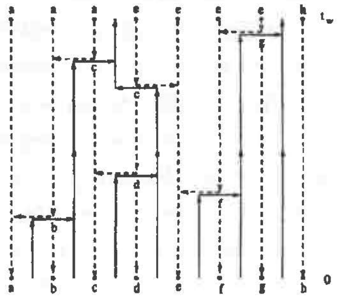

FIG. 1. An eight-site lattice for the $q = \infty$ model. At $t = 0$ all spins have distinct values, labeled a-h. Both forward in t coalescing walks representing domain boundary motion (solid lines), and backward in t coalescing walks representing "ancestry" (dashed lines) are shown; horizontal segments are at times of Poisson clock rings. This diagram can be used for any q; e.g., for $q = 2$, abdgh might all be $+1$ and cef -1.

Coalescing Brownian paths and spanning trees. The coalescing random walks of kinks correspond exactly to the motion of the boundaries between clusters of like spins for the $q = \infty$ model. Whether $q = \infty$ or not, to determine $\sigma_m(t_w)$, it is natural to trace backward in time to see successively from which neighbor the spin value came as various clocks rang. This leads to a dual process [18] of coalescing random walks on the *original* lattice such that all sites n whose (backward in time) walkers have coalesced and are located at ℓ at time zero have $\sigma_n(t_w) = \sigma_\ell(0)$ (see Fig. 1).

Thus, for $q = \infty$, the equal-time $G_\infty(m, n; t_w, t_w) = P(\sigma_m(t_w) = \sigma_n(t_w))$ is just the probability that two *independent* (backward in time) random walks starting at m and n meet within time t_w. But the dual process also works for unequal times, so $G_\infty(m, n; t_w, t_w + t)$ equals the probability that two walkers, one starting at m and the other starting t units of time "earlier" at n meet and hence coalesce between times 0 and t_w.

Not surprisingly, in the scaling limit, the walkers (both forward and backward) become particles doing Brownian motion. Concretely, when $t_w \to \infty$, one rescales the lattice by $a = 1/\sqrt{t_w}$ and time by a^2, so that backward walkers starting from sites 0 and $\sqrt{t_w}z$ become Brownian particles, one from 0 and the other starting $\theta = \lim t/t_w$ units of time "earlier" from z. This will be used below.

More surprisingly, a scaling limit is valid not just for a few walkers, but simultaneously for walkers starting from *every* lattice site at $t = 0$ [10]. The limit essentially has Brownian particles starting from every point on the continuous line at $t = 0$, but for any $t > 0$, coalescing has reduced them to a discrete set.

An extended limit, useful for understanding aging of persistence quantities, includes all starting times and simultaneously the (backward in time) dual particles with all *their* starting times. The collection of all forward (resp., backward) space-time paths forms a spanning tree of continuum space-time in the sense of [22].

Spin-spin correlation. As in the previous section, we express $G_\infty(0, \sqrt{t_w}z; t_w, t_w + t)$ in the scaling limit via coalescing dual Brownian paths. The limit of G_∞, denoted $g(z,\theta) = g_\infty(z,\theta)$, is the probability that the backward (i.e., dual) Brownian paths starting at the space-time points $(z, 1+\theta)$ and $(0,1)$ coalesce before time 0. Denote the location at time $t-s$ of the backward Brownian path starting at a generic (x,t) by $\tilde{B}_{x,t}(s), s \geq 0$. Conditioning on the value x of $\tilde{B}_{z,1+\theta}(\theta)$, we have

$$g(z,\theta) = \frac{1}{\sqrt{2\pi\theta}} \int_{-\infty}^{\infty} dx\, e^{-(x-z)^2/2\theta} g(x)\,, \tag{2}$$

with $g(x) = P(A_x)$, where A_x is the event that $\tilde{B}_{0,1}$ and $\tilde{B}_{x,1}$ coalesce at some $s \in [0,1]$.

Note from (2) that $g(z,\theta)$ satisfies the heat equation, $\partial g/\partial\theta = (1/2)\partial^2 g/\partial z^2$ (see, e.g., [6] for a corresponding result in the Ising case), with $g(z,0) = P(A_z)$. Since A_x is the event that $\tilde{B}_{0,1}(s) - \tilde{B}_{x,1}(s) = 0$ for some $s \in [0,1]$, and the difference of two independent (before coalescing) Brownian motions of rate 1 is a Brownian motion of rate 2, we can rewrite $P(A_x)$ as $P(B_x(s) = 0 \text{ for some } s \in [0,1]) = 1 - P(B_x \neq 0 \text{ during } [0,1])$, where $B_x(s)$ is a Brownian motion starting at x of rate 2. By a standard argument using the Reflection Principle [23] and the symmetry in x, the latter probability equals $P(B_{|x|}(1) > 0) - P(B_{-|x|}(1) > 0)$, and this equals $P(B_0(1) > -|x|) - P(B_0(1) > |x|) = 2P(0 < B_0(1) < |x|) = \phi(|x|/\sqrt{2})$, where $\phi(x) = \sqrt{2/\pi}\int_0^x dt\, e^{-t^2/2}$.

Substituting in (2) and rewriting again, we find that $1 - g(z,\theta)$ equals $(2\pi\theta)^{-1/2}[h(z) + h(-z)]$, where $h(z) = \int_{-z}^{\infty} dx\, e^{-x^2/2\theta}\phi\left((x+z)/\sqrt{2}\right)$. After further analysis,

$$g(z,\theta) = \psi(|z|/\sqrt{2+\theta}, \sqrt{2/\theta}), \tag{3}$$

where $\psi(a,b) = \sqrt{\frac{2}{\pi}}\int_a^\infty dt e^{-t^2/2}\phi(bt)$, and finally

$$g(z,\theta) = \frac{2}{\pi}\int_0^{\sqrt{2/\theta}} dt\, \frac{e^{-z^2(1+t^2)/(2(2+\theta))}}{1+t^2}. \tag{4}$$

Eqns. (3)-(4) simplify in particular cases, e.g.

$$g(0,\theta) = \tfrac{2}{\pi}\mathrm{arctg}\sqrt{2/\theta}, \tag{5}$$

$$g(z,0) = 1 - \phi(|z|/\sqrt{2}),\ g(z,2) = \tfrac{1}{2}[1 - \phi^2(|z|/2)]\,. \tag{6}$$

$g(0,\theta)$ gives the scaling limit probability that $\sigma_m(t_w+t) = \sigma_m(t_w)$, regardless of flips during (t_w, t_w+t). $g(z,0)$ is the scaling limit equal-time two-point correlation function and its exact formula in (6) is implicit or explicit in earlier work on inter-particle distributions [10,24,25].

The $q < \infty$ case of $g_q(z,\theta)$ is simply related to the $q = \infty$ case just discussed. Clearly, $\sigma_{\sqrt{t_w}z}(t_w + t) = \sigma_0(t_w)$ (in the scaling limit) if the backward Brownian paths starting at the space-time points $(0,1)$ and $(z, 1+\theta)$ coalesce before time 0. If not, there is still a $1/q$ probability that those paths end at time 0 on sites with the same spin value. Hence $g_q(z,\theta) = g(z,\theta) + \frac{1}{q}(1 - g(z,\theta))$ and so $c_q(z,\theta)$ is

$$\lim_{\substack{t\to\infty, t_w\to\infty \\ t/t_w\to\theta,\, m/\sqrt{t_w}\to z}} C_q(0,m;t_w,t_w+t) = g(z,\theta), \tag{7}$$

which in particular *does not depend on q*. Thus our exact results (2) and (5) reproduce, as a special case, the known Ising result for $c_2(z,\theta)$ (see [6,7]).

$T = \infty$ interludes. We now modify the dynamics by inserting an interval of duration Δ with $T = \infty$ dynamics. I.e., when the clock at n rings during such an interlude (which it still does at rate one), σ_n chooses a value uniformly at random from $\{1,\ldots,q\}$ (including the previous value); for $q = \infty$, the new value is chosen to be distinct from all other sites at that time. The entire interlude is inserted at a time t_I (of the unmodified dynamics), which may be either in $(0,t_w)$ or (t_w, t_w+t).

Using the backward random walks of the (unmodified) $q = \infty$ model, to analyze the aging quantities $G^I_\infty = C^I_\infty$ for the modified dynamics, we consider walkers starting from (m,t_w) and (n, t_w+t) and the events A that they coalesce at a time $\tau > 0$ and B, that for $\tau < t_I$, they pass through the $T = \infty$ interlude *with no clock ring*. Then $G^I_\infty = P(A \cap B)$. Furthermore, by the nature of the $T = \infty$ dynamics for finite q, it is clear that $G^I_q = G^I_\infty + \frac{1}{q}(1 - G^I_\infty)$, which yields $C^I_q = C^I_\infty = G^I_\infty$ for all q.

Next note that the probability that a walker passes through the interlude with no clock ring is $e^{-\Delta}$. For t_I in (t_w, t_w+t), $P(A)$ is just the unmodified G_∞, so $C^I_q = e^{-\Delta}C_q = e^{-\Delta}G_\infty$ ($= e^{-\Delta}g$ in the scaling limit); this is independent of the location of t_I within (t_w, t_w+t).

For $t_I < t_w$, we partition A into A_1, where $t_I > \tau$ and there are two independent walkers during the interlude (so $P(B) = e^{-2\Delta}$) and the remainder A_2. A_2 is the event that coalescence occurs at $\tau > t_I$, so $P(A_2) = G_\infty(m,n;t_w - t_I, t_w + t - t_I)$ and $C^I_q = P(A_2) + e^{-2\Delta}(P(A) - P(A_2))$. Taking the limit with Δ fixed, $(t_w - t_I)/t_w \to \rho > 0$, $t/t_w \to \theta > 0$ and $(n-m)/\sqrt{t_w} \to z$, the spin-spin correlation with one Δ-interlude is (see Fig. 2):

$$c^I_q(z,\theta,\rho) = g(z,\theta/\rho) + e^{-2\Delta}[g(z,\theta) - g(z,\theta/\rho)]. \tag{8}$$

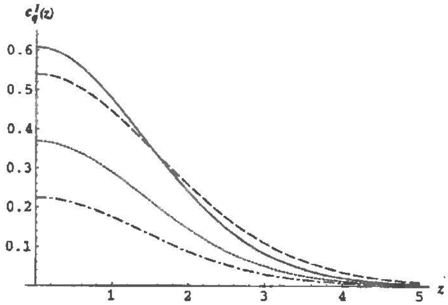

FIG. 2. One-interlude $c_q^I(z)$ curves with $\theta = 1$, $\Delta = \frac{1}{2}$: solid $(= c_q(z,1) = g(z,1))$ for $t_I = 0$, dashed for $t_I = \frac{1}{2}t_w$, dot-dashed for $t_I = t_w-$, and dotted for t_I in $(t_w, t_w + t)$.

Similar arguments for multiple Δ_j-interludes at times $(1-\rho_j)t_w$ $(0 < x\rho_1 < \ldots < \rho_\ell < 1)$ and $\tilde{\Delta}_j$-interludes at times $(1+\tilde{\rho}_k)t_w$ $(0 < \tilde{\rho}_k < \theta)$ yield for c_q^I:

$$e^{-\tilde{\Delta}} \sum_{j=0}^{\ell} \exp(-2\sum_{i=0}^{j} \Delta_i)[g(z,\theta/\rho_{j+1}) - g(z,\theta/\rho_j)], \tag{9}$$

where $\tilde{\Delta} = \sum_k \tilde{\Delta}_k$, $\Delta_0 = 0$, $\rho_0 = 0$ and $\rho_{\ell+1} = 1$.

Persistence. Let $\hat{N} = \hat{N}(t_w, t_w + t)$ be the number of distinct *backward* walkers remaining at time zero from all those starting at the origin during $(t_w, t_w + t)$. When $\hat{N} = k$ and $q < \infty$, the probability of no flips at 0 in this time interval is $(1/q)^{k-1}$ and so the persistence probability $F_q(t_w, t_w + t)$ is $\langle (1/q)^{\hat{N}-1} \rangle$ (for $q = \infty$, $F_\infty = P(\hat{N} = 1)$). In the scaling limit the distribution of $\hat{N}$ converges to that of $N(1, 1+\theta)$, the number of distinct particles, in the *dual* Brownian spanning tree, surviving at time zero from all particles starting at the origin at all times during $(1, 1+\theta)$. Writing $h_k(\theta)$ for $P(N(1,1+\theta) = k)$, we see that F_q converges to the aging function

$$f_q(\theta) = \begin{cases} \sum_{k=1}^{\infty} h_k(\theta)(1/q)^{k-1} & \text{if } q < \infty \\ h_1(\theta) = P(N(1,1+\theta) = 1) & \text{if } q = \infty \ . \end{cases} \tag{10}$$

The persistence function $f_\infty(\theta)$ (and hence $h_1(\theta)$) can be evaluated exactly, thus rederiving a result of [5] by quite different methods, as follows. Let $(-X, Y)$ denote the (random) spatial interval with the same $(q = \infty)$ spin value at time 1 as the origin.

The event $A_{x,y}$ that $X > x$ and $Y > y$ means that the backward Brownian paths starting at $(-x, 1)$ and $(y, 1)$ coalesce before time 0. Proceeding as in our analysis of (2), we see that $P(A_{x,y}) = 1 - \phi\left((x+y)/\sqrt{2}\right)$. The probability density of (X, Y) is then $\mu(x,y) = \frac{1}{2\sqrt{\pi}}(x+y)e^{-(x+y)^2/4}$ for $x, y > 0$. Now, given $(X, Y) = (x, y)$, $f_\infty(\theta)$ is the probability that the *forward* Brownian paths starting at $(-x, 1)$ and $(y, 1)$ do not touch the origin during $(1, 1+\theta)$, and thus

$$f_\infty(\theta) = \int_0^\infty\!\!\int_0^\infty dx\, dy\, \mu(x,y)\, \phi\left(\frac{x}{\sqrt{\theta}}\right) \phi\left(\frac{y}{\sqrt{\theta}}\right). \tag{11}$$

After further analysis of the same kind used to derive (3)-(4), one finds $f_\infty(\theta) = \frac{2}{\pi}\arcsin(1/(1+\theta))$ as in [5]. This formula is consistent with the $q = \infty$ persistence exponent value of one [14,26,27] (for t_w fixed and $t \to \infty$) since $f_\infty(\theta)$ is asymptotic to $1/\theta$.

Discussion. We presented a powerful and very general approach, based on coalescing random walks and Brownian paths run forward *and* backward in time, to nonequilibrium dynamics in $1D$. It yields exact, closed-form expressions in the scaling limit for a variety of aging (and persistence) quantities including the spin-spin correlation $\langle \vec{\sigma}_x(t_w) \cdot \vec{\sigma}_{x'}(t_w + t)\rangle$ for the q-state Potts model for all q, following a quench from $T = \infty$ to $T = 0$. This type of approach, based on an exact analysis of the space-time scaling limit for the entire dynamical process, should yield exact expressions for aging functions in a wide variety of $1D$ systems.

We also presented an exact, closed-form expression for the spin-spin correlation when the system undergoes a sequence of $T = \infty$ interludes. Perhaps surprisingly, we find that the effect of such interludes is *independent* of their timing *provided* they occur during the interval $(t_w, t_w + t)$. We believe our methods may work also for $T < \infty$ interludes, which represent a common experimental situation [1], and for other aging quantities of interest; these analyses will be deferred to a later paper.

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REFERENCES

[1] For recent reviews, see P. Nordblad and P. Svedlindh in *Spin Glasses and Random Fields*, ed. A.P. Young (World Scientific, Singapore, 1998), pp. 1 – 27; and J.-P. Bouchaud, L.F. Cugliandolo, J. Kurchan, and M. Mézard, *ibid.*, pp. 161 – 223.

[2] L.C.E. Struik, *Physical Aging in Amorphous Polymers and Other Materials* (Elsevier, Amsterdam, 1978).

[3] L. Lundgren, P. Svedlinh, P. Nordblad, and O. Beckmann, Phys. Rev. Lett. **51**, 911 (1983).

[4] E. Vincent, V. Dupuis, M. Alba, J. Hamann, and J.-P. Bouchaud, Europhys. Lett. **50**, 674 (2000).

[5] L. Frachebourg, P.L. Krapivsky, and S. Redner, Phys. Rev. E, **55**, 6684 (1997).

[6] A.J. Bray, in *Nonequilibrium Mechanics in One Dimension*, ed. V. Privman (Cambridge U. Press, Cambridge, 1997), pp. 143 – 165.

[7] C. Godrèche and J.M. Luck, *J. Phys. A* **33**, 1151 (2000).

[8] G. Ben Arous, A. Dembo, and A. Guionnet, "Aging of spherical spin glasses," Prob. Th. Rel. Fields, to appear.

[9] L.R.G. Fontes, M. Isopi, and C.M. Newman, math.PR/0009098 (2000).

[10] R. Arratia, Ann. Prob. **9**, 909 (1981) and unpublished partial manuscript (circa 1981) available from rarratia@math.usc.edu.

[11] B. Rinn, P. Maass, and J.-P. Bouchaud, Phys. Rev. Lett. **84**, 5403 (2000).

[12] For a review, see A.J. Bray, Adv. Phys. **43**, 357 (1994).

[13] See, for example, C.M. Newman and D.L. Stein, Phys. Rev. Lett. **82**, 3944 (1999), and references therein.

[14] B. Derrida, V. Hakim, and V. Pasquier, J. Stat. Phys. **85**, 763 (1996).

[15] C. Sire and S.N. Majumdar, Phys. Rev. E **52**, 244 (1995).

[16] P.L. Krapivsky and E. Ben-Naim, Phys. Rev. E **56**, 3788 (1997).

[17] D. ben-Avraham and T. Masser, Phys. Lett. A **275**, 382 (2000); cond-mat 0101212 (2001).

[18] See, e.g., Chap. V of T.M. Liggett, *Interacting Particle Systems* (Springer, New York, 1985).

[19] D. Schwartz, Ann. Prob. **6**, 398 (1978).

[20] Z. Rácz, Phys. Rev. Lett. **55**, 1707 (1985).

[21] G.J. Amar and F. Family, Phys. Rev. A **41**, 3258 (1990).

[22] M. Aizenman, A. Burchard, C.M. Newman, and D.B. Wilson, Ran. Struct. Algor. **15**, 319 (1999).

[23] See, e.g., W. Feller, *An Introduction to Probability Theory and Its Applications*, v. 1 (Wiley, NY, 1968), p. 72.

[24] B. Derrida and R. Zeitak, Phys. Rev. E **54**, 2513 (1996).

[25] C.R. Doering and D. ben-Avraham, Phys. Rev. A **38**, 3035 (1988).

[26] B. Derrida, A.J. Bray, and C. Godrèche, J. Phys. A **27**, L357 (1994).

[27] B. Derrida, J. Phys. A **28**, 1481 (1995).

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