Kriging With External Drift Applied to Evaluation of Mineral Resources of Limestone and Lateritic Ore

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Abstract

This paper is a work on applied geostatistics to evaluation of mineral resources. This application can be considered as the last step to such the task. The basic concepts are based on current geostatistics, with extensive use of informatics resources.
The first goal of this work was to test the technique named kriging with external drift in the evaluation of mineral resources, concerning to the resulting gain in the use of more than one variable, mainly whether the use of auxiliary variables can be done in a friendly way.
Two deposits were chosen with different geology, a deposit of base metal saprolitic ore and another deposit of limestone.
In each deposit two variables were selected, the principal and secondary variable, at lateritic deposit variables were respectively SiO2 and Fe and at the deposit of limestone were CaO and SiO2.
Estimates by kriging with external drift was compared to ordinary kriging, ones this comparison was done to measure the differences between a traditional method widely used to another underutilized, or even non widespread.
Results showed minor differences between the blocks estimated by both methods. But as in mining sub-sampling can occur from several factors, one can say that the kriging with external drift is a reliable alternative since it requires less effort to perform multivariate estimation than those, for example, to perform the ordinary cokriging.

Keywords: Applied geoestatistics, evaluation of mineral resources, multivariate geostatistics, kriging with external drift, residuals variograms

1. INTRODUCTION

The evaluation and classification of mineral resources is a key area in the analysis of mining projects. Geostatistics is inside in this context by providing a means to estimate values of the variables of interest in non-sampled locations of the deposit.

Among the geostatistical techniques are some that allow the simultaneous use of two or more variables, always treated as a main variable and the other as secondary. These techniques are multivariate kriging with external drift, which was used in this work. It is noteworthy that beyond this there are others, such as cokriging and co-located cokriging, both more widespread than the first.
Like all multivariate techniques, the kriging with external drift using an auxiliary variable, which has good linear correlation with the main variable to estimate the main variable, but there is a need for multicolocalization of the data.
Was analyzed two deposits with different variables with different forms of statistical distributions. The data have heterotopic partial sampling and target variables were SiO2, CaO and Fe. In the limestone deposit, SiO2 has negative asymmetric distribution and CaO positive skewness. In lateritic deposit the SiO2 and Fe despite being asymmetric approach is a normal distribution.
As a component of the external drift in the kriging with external drift needs to be known at all points to be estimated and the sampling points (colocalization data), secondary parameters were estimated by ordinary kriging, before the kriging with external drift for primary variable be held.

2. KRIKING WITH EXTERNAL DRIFT

Kriging with external drift is applied in case the main variable is correlated with the dependent variable external help.
This method uses auxiliary variables to predict a trend model. These should be known around the area where the main variable will be estimated, variable should be multicolocalize According to Deutsch & Journel (1998) to kriging with external drift kriging is an extension of the trend kriging.
According to Soares (2000) to kriging with external drift can be expressed in accordance with the functions:
Drift between the primary and secondary variable.
$m(x) = a_0(x) + a_1(x)\, y(x)$

In this function and the coefficients a_0 e a_1 are estimated by $Z(x_\alpha)$ and he drift $m(x_0)$ takes the value of the variable $Y(x_0)$. The estimator of x_0 is calculated using the n neighboring samples, as:

$$[Z_{KDE}(x_0)] = \sum_{\alpha=1}^{n} \lambda_\alpha^{KDE} Z(x_\alpha)$$

With λ_α^{KDE} being weight of the kriging and $Z(x_\alpha)$ the variable to be estimated.
Also according to Soares (2000), for which there is no bias is necessary to ensure that the difference between the actual and estimated to be equal to zero:

$$E\{[Z^*{}_{KDE}(x_0)] - Z(x_0)\} = 0$$

$$E\left\{\sum_{\alpha=1}^{n} \lambda_\alpha^{KDE} Z(x_\alpha) - Z(x_0)\right\} = \sum_{\alpha=1}^{n} \lambda_\alpha^{KDE} m(x_\alpha) - m(x_0)$$

$$\sum_{\alpha=1}^{n} \lambda_\alpha^{KDE} [a_0 + a_1 Y(x_\alpha)] - [a_0 + a_1 Y(x_0)] = 0$$

Where $m(x_0)$ is the drift of the point estimate and $m(x_\alpha)$ the drift of the samples.

It results a condition of universality, for each the P auxiliary variables of external drift, adding to the first condition that a second is the condition of universality of the ordinary kriging by:

$$\sum_{\alpha=1}^{N} \lambda_{\alpha}^{KDE} Y(x_{\alpha}) = Y(x_0)$$

$$\sum_{\alpha=1}^{N} \lambda_{\alpha}^{KDE} = 1$$

With $Y(x_0)$ known as the secondary variable at all points.

Luís (2004) suggests, to better understand the kriging with external drift, to decompose the function $Z(x)$ into a sum of a mean value with a residual:

$Z(x) = m(x) + R(x)$, which $R(x)$ zero mean and covariance C_R

Minimizing the variance is obtained by the Lagrange formalism, equating to zero the (n + 1 + P) of partial derivatives:

$$E\left\{ \left[[R(x_0)] - R(x_0)\right]^2 \right\} + 2\mu_0 \left[\sum_{\alpha} \lambda_{\alpha}^{KDE} - 1 \right] + 2\mu 1 \left[\sum_{\alpha} \lambda_{\alpha}^{KDE} Y(x\alpha) - Y(x_0) \right]$$

The weights λ_i^{KDE} are obtained by (n + 1 + P) linear equations:

$$\begin{cases} \sum_{\alpha=1}^{N} \lambda_{\alpha}^{KDE} C_R(x_{\alpha}, x_{\beta}) + \mu_0 + \sum_{k=1}^{N} \mu_K Y(x_{\beta}) = C_R(x_{\beta}, x_0), \beta = 1,\dots, N \\ \sum_{\alpha=1}^{N} \lambda_{\alpha}^{KDE} = 1 \\ \sum_{\alpha=1}^{N} \lambda_{\alpha}^{KDE} Y_P(x_{\alpha}) = Y_P(x_0) \end{cases}$$

with P=1,..., number of auxiliary variables

With C_R being the covariance of the residue.

According Bourennane et al (2000) kriging with external drift is more accurate than linear regression, and this precision is directly proportional to the increase in the number of samples used. And if the variogram of the variable of interest is a pure nugget effect, the kriging with external drift becomes equivalent to linear regression.

Snepvangers et al (2003) show that kriging with external drift as having some advantages over ordinary kriging in the spatial-temporal domain, mainly because it can control the trend model. But also report advantages of ordinary kriging over kriging with external drift especially when it comes to simplicity.

Luís (2004) has used kriging with external drift in the estimated grades of Au with auxiliary information such as lithology, lithological associations, grades of Ag, Ag content classes, among others, and concluded that the use of this information has helped improve the accuracy of estimates of the grades of Au.

Watanabe (2008) describes the results obtained with the kriging with external drift in relation to the correlation between variables, and concludes that even when the correlation is low, the results did not show significant distortions.

Rocha, Yamamoto & Fonteles (2009) compared kriging with external drift in relation to the application for ordinary cokriging estimate potentiometric levels in aquifers, the authors considered the results of kriging with external drift higher in relation to the results of ordinary cokriging.

3. CALCULATION OF RESIDUAL

To be able to use the Kriging with external drift is necessary to calculate the residual between the primary and secondary, this residue is necessary for the construction of the variogram, used in estimating the main variable.
According Bourennane (2003), determining the function of residue can be done by selecting items from the database within a predetermined search radius, use these points to test various degrees of polynomials using the least square error. Assuming that the best function of residue it shows on average the smallest error variance, the polynomial is chosen that result in these characteristics.
According to the same author one of the problems in this method is that it is susceptible to outliers, that produce variations which mask the differences between the various polynomials. Another criterion that can be used, is the classification of the various errors of least squares of several polynomials. The first category is attributed to the small error, and makes the classification as a "rank", so in the end it will have on average the lowest average score corresponding to an optimum drift.
In the software Isatis the method used for determining the residue is similar to that used for cross-validation, and is done according to the steps below (Geovariances, 2009):

(1) selection of a given number N of attempts to drift;

(2) definition of neighborhood;

(3) selection of a sample point (center) of the value Z;

(4) determination of neighbors corresponding to the selected sample;

(5) The selected point is taken from the sample and its value Z * is estimated from a model that corresponds to a pure nugget effect;

(6) for each test is derived, the experimental error ZZ * is calculated. No experimental errors are thus produced, which are classified according to their values (from smallest to largest).
The steps 3 to 6 are repeated for all points of the sample. For each test are displayed, the average rating, the average experimental error (mean error), and the error variance (error variance).
The number of selected samples in the neighborhood search is divided into two rings for the processing and recognition of covariance. Then each point of the two rings, compared to all points in the other ring will be an achievement. The average error of the outer ring, to the detriment of the average error of the inner ring, is used to define the best polynomial, the smaller is the better.
The syntax for the polynomials Isatis software available for testing is:
- 1 corresponds to the condition of universality;
- monomial x is the deviation that corresponds to the first coordinate;
- y is the deviation monomial that corresponds to the second coordinate;

- z is the deviation monomial that corresponds to the third coordinate;
- f1, ... f10 represent the standard for the different functions external drift.
- 1, x, y correspond to a linear drift;
- 1, x, y, x2, XY, y2 for a quadratic deviation;
- 1, f1 to a single external drift;
- 1, x, y, f1 to an external drift combined with a linear plane.

At the identification stage covariance for each combination, we conducted a test "Jackknife," resulting in the ratio between the experimental and theoretical variance. In the technique "Jackknife", each measure is formed by removing the sample point estimate, where each point estimate is seen as a random variable independent and identically distributed, leading to a very simple estimator of variance.

4. GEOLOGY OF DEPOSITS

In targeted areas already are operated mines, the classifications and lithologic divisions used primarily serve the needs of the mining operation and planning. The description of the geology of each deposit is sufficient to support the studies conducted geostatistical.

4.1. Lateritic ore deposit

The first deposit is Ni lateritic ore, Fe and SiO2 variables were chosen because they are important parameters in processing. Another reason for this choice are the frequency distributions presented by these variables.
Following less complex ultramafic rocks were differentiated as follows: basal gabbro, dunite massif, pyroxenites, dunites sequences, pyroxenite and gabbro sequences higher. The ultramafic complex is represented by the variation of cyclic units ultramafic in the base and gabbro to the top. Occur four to five main courses, ranging from decametric to hectametric of layers Dunite - Pyroxenite - Gabbro (Figure 1).

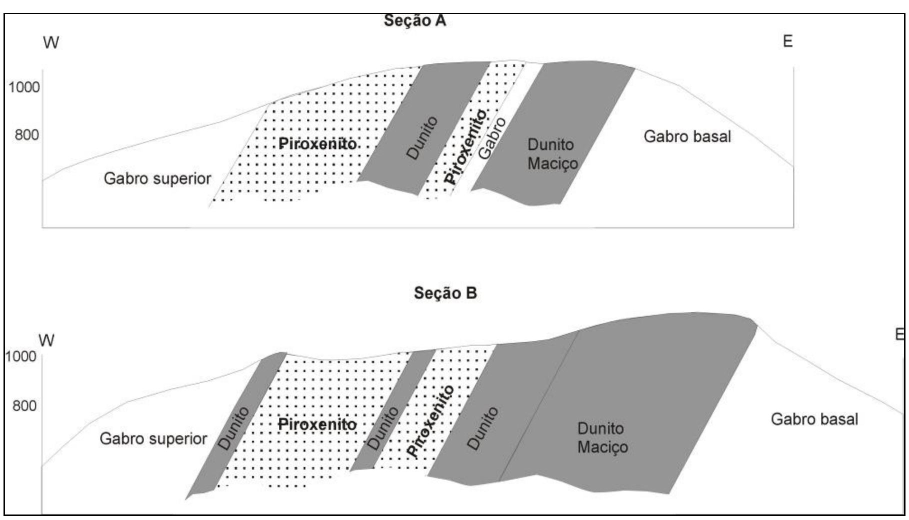

Fig.1. Schematic section of the ultramafic complex.

There are a two types of ore in the area, silicates and oxides, the silicate ore comes from the pyroxenite to dunite while oxides, like all laterite deposit, shows great variability that influence the structures and the weathering profile.

The data include lateritic deposit oxidized zones and silicate. Such areas vary SiO_2 and Fe, with stretches where SiO_2 is low and Fe is high, also occurs transition to areas where SiO_2 is high and the Fe is low. The excerpts of selected model to estimate, are portions mineralized.

Was chosen only part of the mining area, with about 1085 drilholes, the mean depth of this drilholes is only 20 meters.

The lateritic model was constructed from vertical sections, based on information from chemical and lithological description made in the holes.

With the vertical sections were constructed 3D solids (Figure 2), these were filled with solid blocks of dimensions in the X direction at 10m, 25m and 3m in the direction Y towards Z. The block models were constructed with sub-blocks, for the importation into Isatis ® was necessary regularize the blocks.

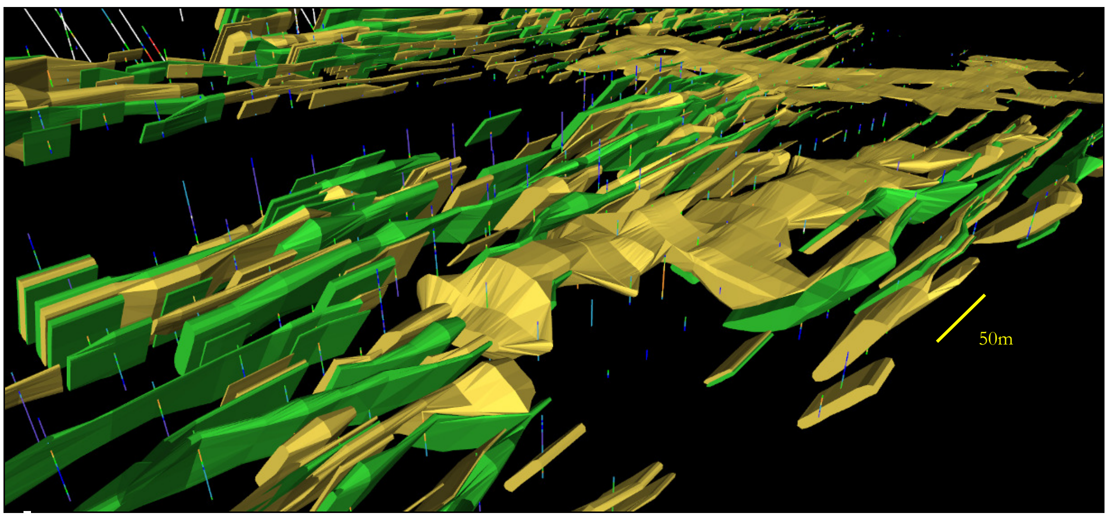

Fig. 2. Wireframes of mineralized zone, constructed from the vertical sections, showing green areas SiO_2 greater than Fe and yellow zones, SiO_2 less than Fe.

4.2. Limestone ore deposit

The limestone deposit studied is composed of metasedimentary rocks, alternating metalimestones calcitic, dolomitic and calc-silicates rocks with SiO_2 both high as low. The SiO_2 is a high negative correlation with CaO, and directly reflects the lithological variation. Was chosen throughout the mined area with approximately 115 holes (Figure 3), the average depth of the holes is approximately 200 meters. Only portions of the block model with mineralized zone was used to estimate.

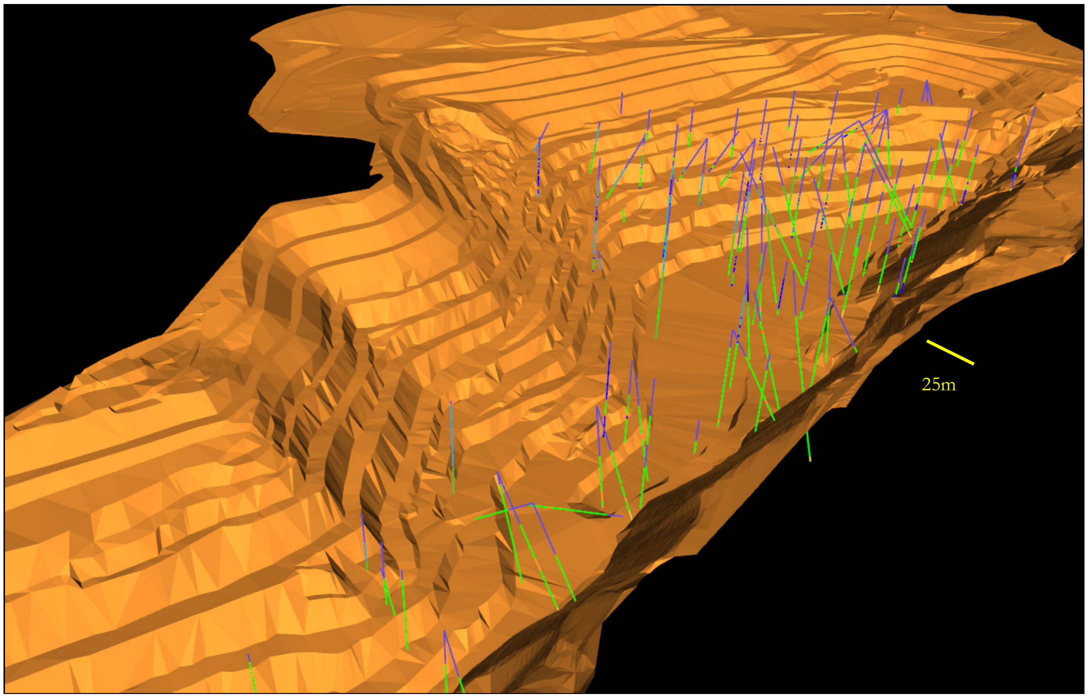

Fig. 3. Holes in the limestone deposit and the final pit configuration of mining.

The model of limestone was also built from vertical sections, based on information from chemical and lithological description made in the holes.
With the vertical sections were constructed by triangulation, 3D surfaces, these were filled with solid blocks of dimensions in the direction X 50m, 50m and 3m in the Y direction toward Z.
The block models were constructed with sub-blocks, for the importation into Isatis ® was necessary regularize the blocks.

5. Statistical analysis

Statistical analysis was done for two basic models, with the original holes and holes with the samples regularized, for comparison purposes. This step is important because part of the process of data validation, any inconsistency or problem can be detected in this phase.

5.1. Lateritic ore deposit

The drilholes were composite the samples in 1m of length, the modal length of the samples sent to the laboratory and the minimum height of bench mining.
Was made to basic statistics for the variables before and after composite, the results show consistency between the two databases because there is a lack of significant differences (Tables 01 and 02).

Were prepared base maps (Figure 4) to visualize the spatial distribution of samples, on this map can observe the heterotopy of the variables, with more samples of Fe than SiO2.

Table 01: Statistics of the samples before composite.

Univariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Norm.Mean	Norm.Variance
SIO2	5635	0.25	91.1	30.1	371.87	29.86	373.68
FE	20942	0.05	62.4	22.71	195.75	23.14	201.26

Bivariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Correlations	Norm.Mean	Norm.Variance
SIO2	5635	0.25	91.1	30.1	371.87	-0.75	29.86	373.68
FE	5635	0.53	62.4	25.77	208.76	-0.75	25.8	208.02

Table 02: Statistics of the samples after composite in 1m.

Univariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Norm.Mean	Norm.Variance
SIO2	5813	0.25	90.3	29.9	363.55	30.05	363.81
FE	18973	0.06	61.8	23.03	199.13	23.04	199.18

Bivariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Correlations	Norm.Mean	Norm.Variance
SIO2	5813	0.25	90.3	29.9	363.55	-0.75	30.05	363.81
FE	5813	0.53	61.8	25.53	206.29	-0.75	25.51	205.94

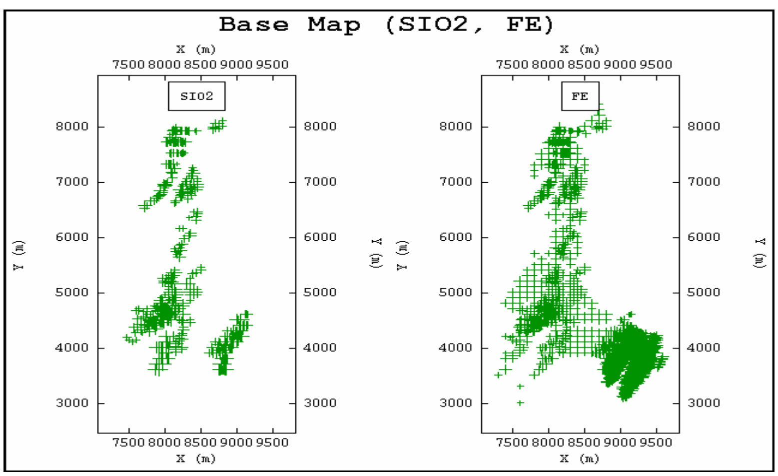

Fig. 4. Base map of the distribution of the samples, variables Fe and SiO2, showing heterotopy between variables.

5.2. Limestone ore deposit

The procedures used were the same model lateritic, samples prior to calculating the variograms had composite in 3m length.

To identify the behavior of the thicknesses of the samples were constructed histograms of the thickness before and after settlement, to identify the pattern of distribution of variables is also produced histograms, respectively before and after settlement.

The basic statistics of the samples (Tables 03 and 04) before and after settlement, shows consistency between the two databases. In addition to the histograms, were "plotted" base map (Figure 5) to visualize the spatial distribution of samples, based on the maps can be observed the heterotopy between variables CaO and SiO2, but smaller than the lateritic model.

Table 03: Statistics of the samples before composite.

Univariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Norm.Mean	Norm.Variance
CAO	1582	0.42	55.98	35.37	475.16	36.11	460.72
SIO2	1600	0.8	71	25.22	776.36	24.29	753.77

Bivariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Correlations	Norm.Mean	Norm.Variance
CAO	1582	0.42	55.98	35.37	475.16	-0.99	36.11	460.72
SIO2	1582	0.8	71	25.31	784.22	-0.99	24.37	761.37

Table 04: Statistics of the samples after composite in 3m.

Univariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Norm.Mean	Norm.Variance
CAO	726	0.42	54.94	23.88	545.01	35.17	475.07
SIO2	735	1.36	71	39.75	889.43	25.44	775.23

Bivariate Statistics

	Count	Minimum	Maximum	Mean	Variance	Correlations	Norm.Mean	Norm.Variance
CAO	726	0.42	54.94	23.88	545.01	-0.99	35.17	475.07
SIO2	726	1.36	71	40.03	893.68	-0.99	25.6	784.01

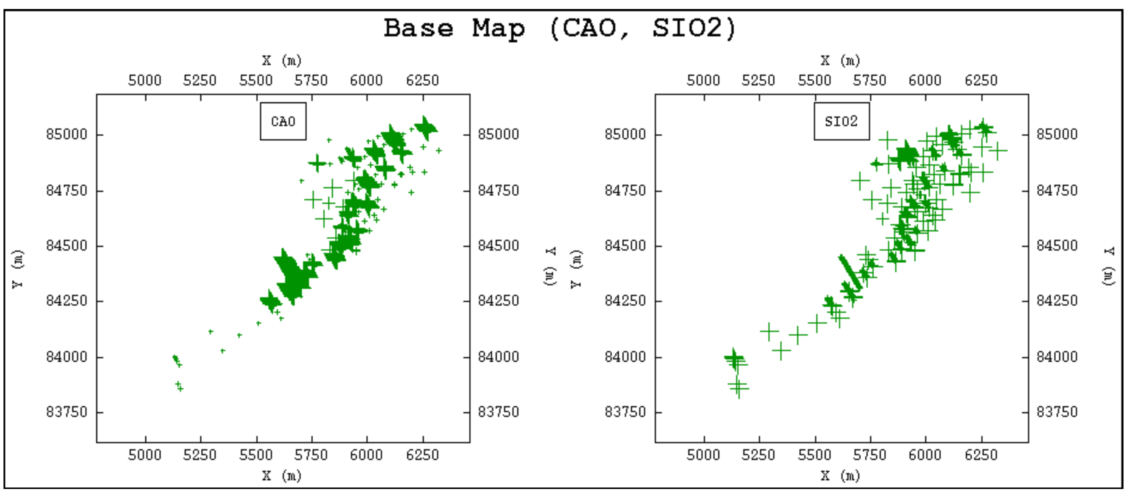

Fig. 5. Base map of the distribution of the samples, variables CaO and SiO2.

6. GEOSTATISTCS ANALYSIS

6.1. Lateritic ore deposit

The procedure of structural analysis was carried out in Isatis, with 12 different directions, totaling 180º. The analysis was conducted for the two variables of interest SiO2 and Fe

In the structural analysis concluded that the direction of greater continuity was N15 ° for both the variable SiO2 as for variable Fe.
With the directions of greater continuity defined, the variograms were calculated for variables SiO2, Fe and residue between Fe and SiO2. The azimuth of 15º, set from the variogram maps, match the information on the structural geology of the deposit.
Before calculating the variogram analysis was performed of the residue between the variable SiO2 and the variable Fe, were tested seven different functions and choose the one with the lowest average in the second "Ring", a function f1 xy (Table 05).

Table 05: Table showing the procedure for determining the best basis for calculating the residual between the variables SiO2 and Fe.

	Mean Error			Mean Squared Error			Mean Rank			Function
Trial	Ring1	Ring2	Total	Ring1	Ring2	Total	Ring1	Ring2	Total	
T5	-3.26E-01	-1.20E+00	-4.88E-01	1.93E+02	2.48E+02	2.03E+02	3.545	3.679	3.57	1 x y f1
T2	-1.07E+00	-2.86E-01	-9.27E-01	2.20E+02	2.18E+02	2.20E+02	4.107	3.832	4.056	1 x f1
T1	1.73E+00	-4.66E-01	1.32E+00	2.17E+02	1.17E+02	1.99E+02	4.197	3.934	4.148	1 f1
T7	-3.10E-02	-1.41E+00	-2.88E-01	6.94E+02	2.89E+02	6.18E+02	4.27	3.942	4.209	1 x y x2 xy y2 f1
T4	1.91E+00	-1.05E+00	1.36E+00	2.53E+02	1.15E+02	2.28E+02	4.033	4.073	4.041	1 z f1
T3	2.53E+00	-7.13E-01	1.93E+00	2.31E+02	1.41E+02	2.14E+02	3.967	4.197	4.009	1 y f1
T6	1.50E+00	-9.95E-01	1.04E+00	3.59E+02	3.14E+02	3.51E+02	3.882	4.343	3.967	1 x y z f1

Figure 6 shows the experimental variogram and its model of the variable residue between SiO2 and Fe.

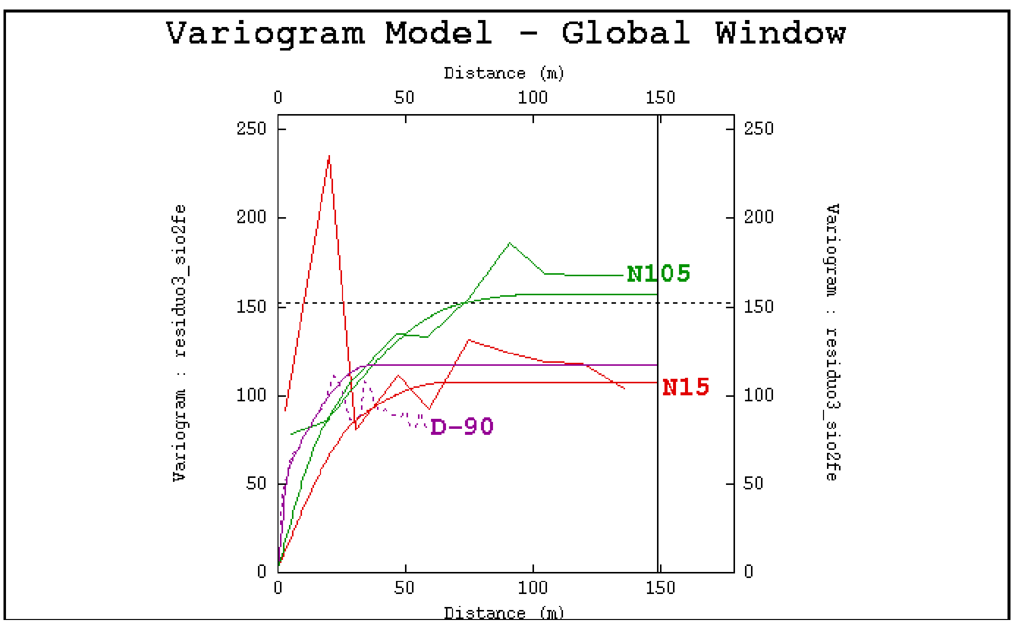

Fig. 6. Experimental variogram and its theoretical model, adjusted for the residual between the SiO2 and Fe.

For the Fe could be used as a secondary variable, it was necessary to estimate it in advance at all points where SiO2 will be estimated. The estimate of Fe was performed by ordinary kriging.
To estimate the models were considered blocks of ore, these models were settled in blocks of standard size and transferred to the software Datamine to the Isatis, after the estimate was

done the other way, the block model was transferred from Isatis for Datamine, so that they could work with the block model in 3D and build figures of analysis.
The neighborhood parameters used were octants division, in the local neighborhood with a minimum of 4 and maximum of16 samples to estimate a block as shown in Table 06. In this table are observed ranges also search used.
To facilitate comparison between the results of kriging with external drift with ordinary kriging, the same parameters were used to search for samples in both estimates, even the theoretical models of variograms have been different.

Tabela 06: Table with the search parameters of ordinary kriging and kriging with external drift variables for SiO2, Fe and residual SiO2/Fe.

Search parameters - Ordinary Kriging and Kriging with External Drift

	X (m)	Y (m)	Z (m)
Azimuth			15^{0}
Search radius	120	120	80
Method		octants	
Number of samples		4	
Samples by octants		2	
Minimum distance between samples		1	
Max. empty octants Consec.		4	

After the estimates, were made statistical analysis and the results analysis were compared with those from boreholes trying to verify the consistency of the estimate.
Looking at Table 07 we observe that the average estimates are similar to the samples and that the variances decreased, featuring an expected smoothing effect of kriging.

Tabela 07: Table comparing the model values estimated by kriging with external drift (kde_ SiO2) and ordinary kriging (K_ SiO2) with samples of the holes.

	MODEL		DRILHOLES
VARIABLE	**KDE_SiO2**	**KSiO2**	**SiO2**
TOTAL	16865	16865	3744
NSAMPLES	16865	14325	2317
MINIMUM	2.4	1.1	0.6
MAXIMUM	72.1	75.5	90.3
RANGE	69.7	74.4	89.7
MEAN	31.8	30.8	29.9
VARIANCE	142.2	128.5	316.4
STANDDEV	11.9	11.3	17.8
SKEWNESS	0.1	0.3	0.5
KURTOSIS	-0.5	0.2	-0.7
GEOMEAN	29.2	28.4	17.6
DIF. SAMPLES	107%	103%	

As already stated, together with the kriging with external drift the ordinary kriging was performed for variable SiO2, the choice of this technique is due to this being one of the most widespread methods of use and reliability which is quite acceptable. The search parameters for ordinary kriging were the same used for the kriging with external drift.

By analysis of histograms of both results (Figures 07 and 08), one can note that the results are closes, but the kriging with external drift shows greater dispersion in the distribution around the mean values, while the ordinary kriging shows a distribution indicating more smoothing.

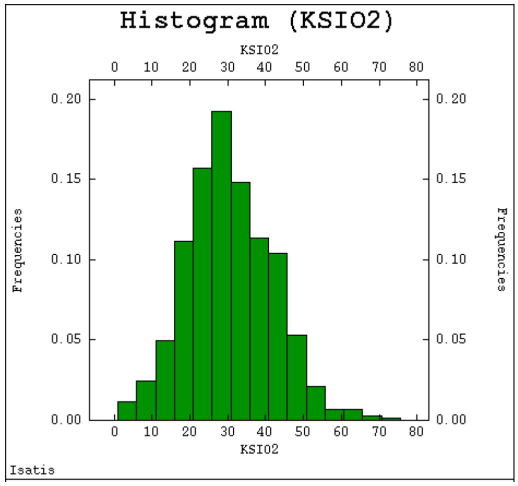

Fig. 07. Histogram of SiO2 grades estimated by Ordinary Kriging.

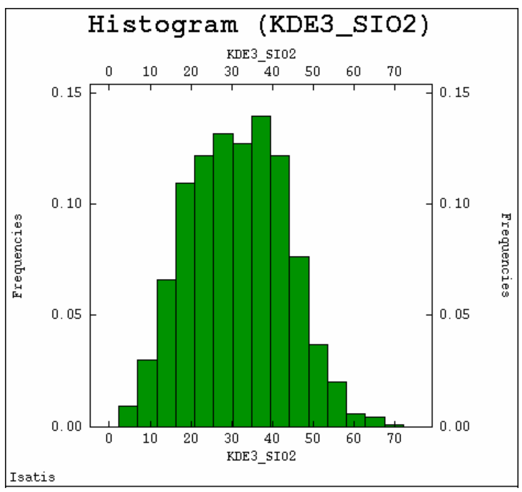

Fig. 08. Histogram of SiO2 grades estimated by kriging with external drift.

In the blocks models (Figures 9 and 10) can be noticed differences, but not too large discrepancies, the degree of difference of the variables estimated by the two methods is small. The kriging with external drift had a higher estimated number of blocks than ordinary kriging, this heterotopy due to sampling. As the number of samples Fe is greater than the number of samples of SiO2 and this was used as auxiliary variable by kriging with external drift, there was the estimate of the primary variable in more points than that by ordinary kriging.

FROM	TO	
1.07027	18.4206	
18.4206	23.0713	
23.0713	26.7667	
26.7667	29.7395	
29.7395	33.9660	
33.9660	38.8263	
38.8263	43.9230	
43.9230	75.5063	

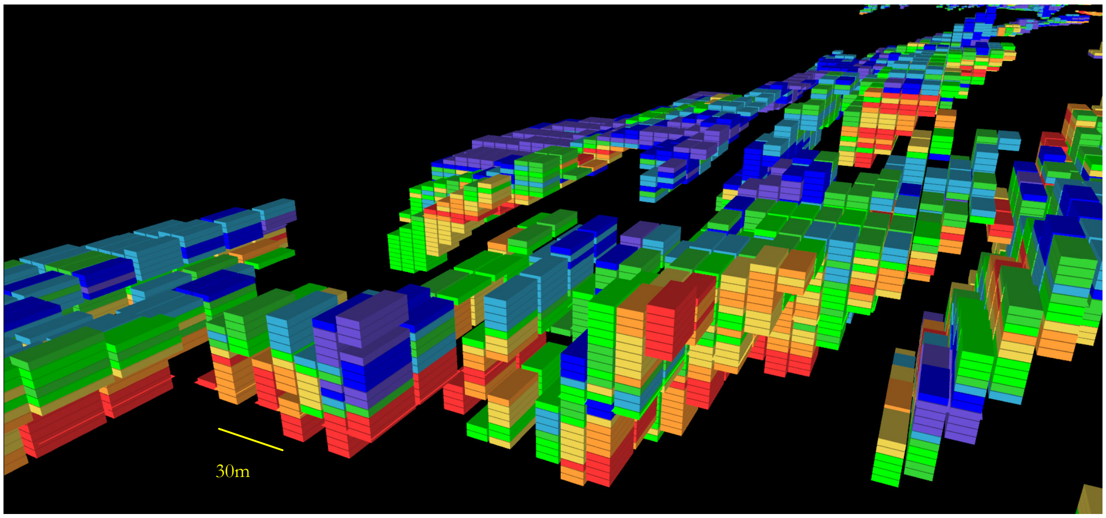

Fig. 09. The ore block model (3D) at grades estimated by kriging with external drift, legend on top.

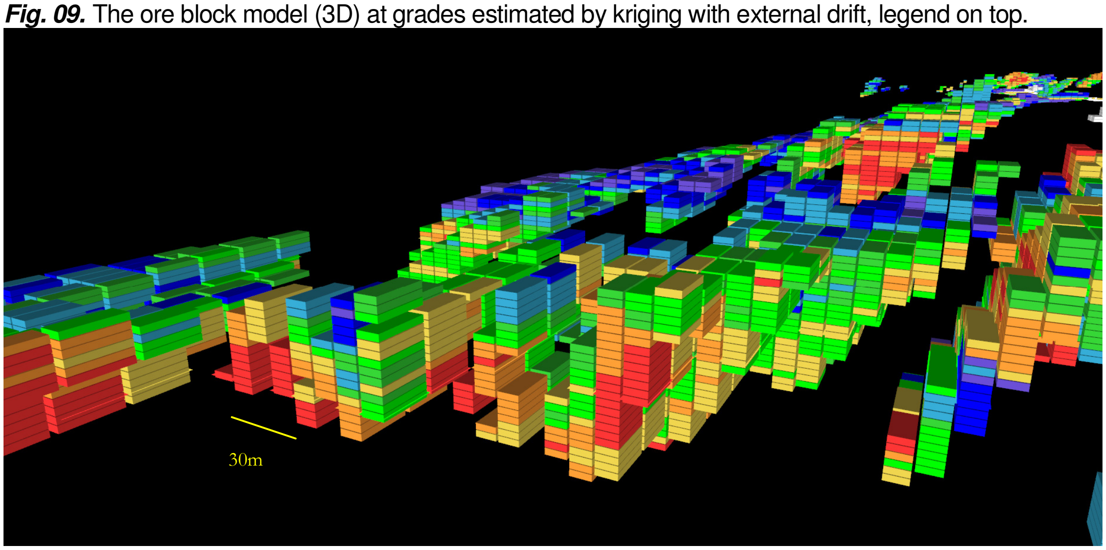

Fig. 10. The ore block model (3D) at levels estimated by Ordinary Kriging, legend on top.

To compare the means of local SiO2 estimated by kriging with external drift and ordinary kriging, averages were calculated on blocks of variables in the model continuously tracks every 10 meters in the vertical (Figure 11). Analyzing these results, it is observed that the average local SiO2 estimated by kriging with external drift is closer to the average estimate by ordinary kriging.

In addition to the estimated values of SiO2 were considered the values of SiO2 in the samples which were the basis for the estimate also the values of Fe estimated by ordinary kriging who served as auxiliary variable. The average of sample differs somewhat from the estimated average showing the smoothing of kriging, it can be noted also that the values of Fe are opposed to the values of SiO2 estimated by kriging with external drift.

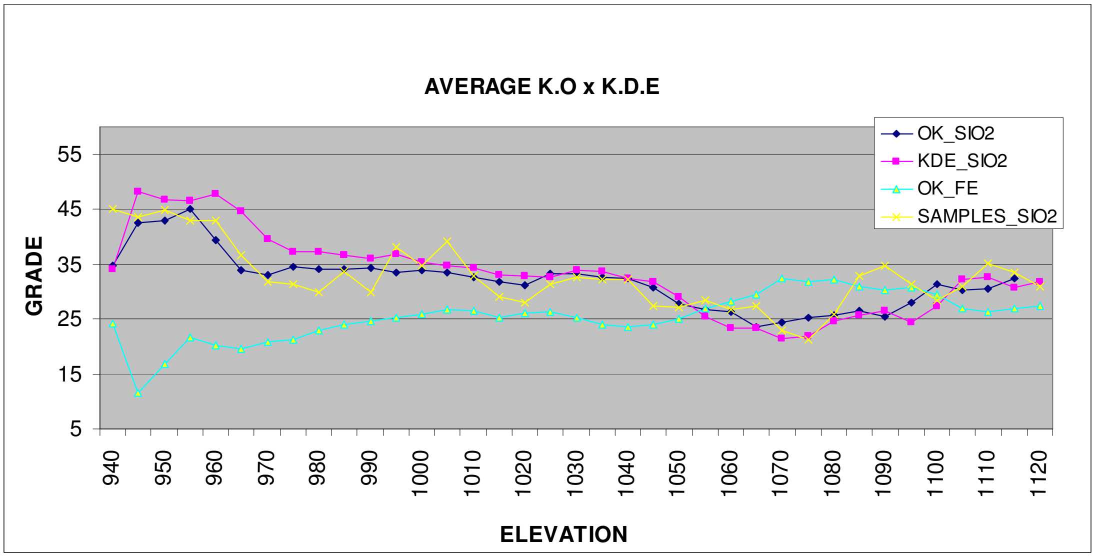

Fig. 11. Profile in the elevation with points average with external drift kriging and ordinary kriging, calculated every 10m.

6.2. Limestone ore deposit

The structural analysis for limestone followed the same manner as the model made for lateritic, with some variations. The map of variogram was calculated for 18 different directions, totaling 180, the parameters for calculating the variogram can be seen in Table 12. The analysis was conducted for the two variables of interest CaO and SiO2.

The structural analysis shows that the direction of greater continuity is N45°, both for the variable CaO to as the variable SiO2.

After defining the direction of greater continuity, the variograms were calculated for variables CaO, SiO2 and residual between CaO and SiO2. The main direction of 45° shown in the maps of variogram is compatible with the direction of the layers of the mineralized zone.

In order to calculate the variogram of the residual between the variables CaO and SiO2, was the analysis of the residue between a variable and another. Were tested 07 polynomials and the result was chosen according to the criteria presented in section 5.9. The polynomial that had the lowest average in the "rank" outer ring "Ring2", Table 08, was a function f1, which is chosen.

Table 08: Table showing the procedure for determining the best basis for calculating the residual between the variables SiO2 and CaO.

	Mean Error			Mean Squared Error			Mean Rank			Function
Trial	Ring1	Ring2	Total	Ring1	Ring2	Total	Ring1	Ring2	Total	
T1	1.16E-02	3.26E-02	1.68E-02	3.15E+00	3.17E+00	3.16E+00	3.674	3.486	3.627	1 f1
T7	-2.11E+00	-1.54E+01	-5.42E+00	2.69E+02	1.94E+03	6.87E+02	5.187	3.837	4.85	1 x y x2 xy y2 f1
T5	1.27E-01	1.68E+00	5.15E-01	3.75E+00	4.37E+01	1.37E+01	3.949	3.918	3.941	1 x y f1
T2	1.15E-01	1.08E-01	1.13E-01	3.75E+00	3.83E+00	3.77E+00	3.641	3.928	3.712	1 x f1
T3	2.25E-02	7.05E-01	1.93E-01	3.43E+00	1.03E+01	5.15E+00	3.792	3.957	3.833	1 y f1
T4	2.85E-02	1.48E-01	5.83E-02	3.67E+00	8.46E+00	4.86E+00	3.647	4.284	3.806	1 z f1
T6	1.42E-01	1.64E+00	5.17E-01	4.40E+00	6.12E+01	1.86E+01	4.11	4.591	4.23	1 x y z f1

With the experimental variograms calculated, it moved to the adjustment of theoretical models. The experimental variogram and model of residual between the variable CaO and variable SiO2, figure 12.

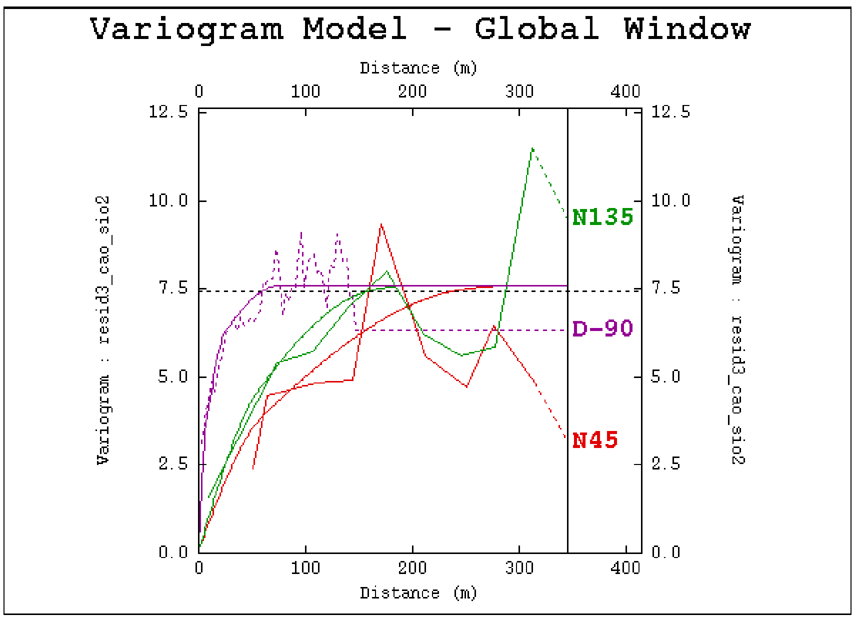

Fig. 12. Experimental variogram and respective model adjusted, for the residual between CaO and SiO2.

As the SiO2 was used as a secondary variable was necessary to estimate it in advance in all blocks where CaO was estimated. It was used as the SiO2 and variogram estimation method used was ordinary kriging.

For search method were used octants to estimate with the minimum of 4 samples and a maximum of 16 samples per block, according to the one presented in Table 09. The search ranges were used 300m in X, 200m in Y and 200m in Z.

The search parameters were the same for with external drift kriging and ordinary kriging, to facilitate comparative analysis between the two estimates, the variogram used were those of the respective variables.

Table 09: Table with the search parameters of ordinary kriging and kriging with external drift for variables CaO, SiO2 and CaO/SiO2 residue.

Search parameters - Ordinary Kriging and Kriging with External Drift

	X (m)	Y (m)	Z (m)
Azimuth			45 ^{0}
Search radius	300	200	50
Method		octants	
Number of samples		4	
Samples by octants		2	
Minimum distance between samples		3	
Max. empty octants Consec.		4	

Made the estimate, the statistics and the results were calculated to assess the quality of these statistics were compared to those of the drilholes.
The mean estimated blocks are similar to the averages of the samples, with the smoothing of the variance as expected in kriging (Table 10).

Table 10: Table comparing the model results estimated by kriging with external drift (kde_cao) and ordinary kriging (k_cao) with samples of the drilholes.

	MODEL		DRILHOLES
VARIABLE	**KDE_CAO**	**K_CAO**	**CAO**
TOTAL	3607834	3607834	5093
NSAMPLES	3399456	3399456	1071
MINIMUM	12.7	25.5	2.0
MAXIMUM	54.7	54.3	56.0
RANGE	42.1	28.7	53.9
MEAN	48.8	48.4	47.9
VARIANCE	26.9	26.5	82.8
STANDDEV	5.2	5.1	9.1
SKEWNESS	-2.4	-1.9	-2.2
KURTOSIS	6.8	3.4	4.5
GEOMEAN	48.5	48.1	46.4
DIF. SAMPLES	102%	101%	

Like, for the same reasons that the model laterite, in the limestone model the main variable was estimated by ordinary kriging. The search parameters used were the same for both techniques, only the theoretical variogram models were different.
The histograms of the two estimative (Figures 13 and 14) show similar results, but with some distinct points, the kriging with external drift presents less limit lower. For high values of CaO the behavior of both histograms are very similar, although the frequencies of each class are subtly different.

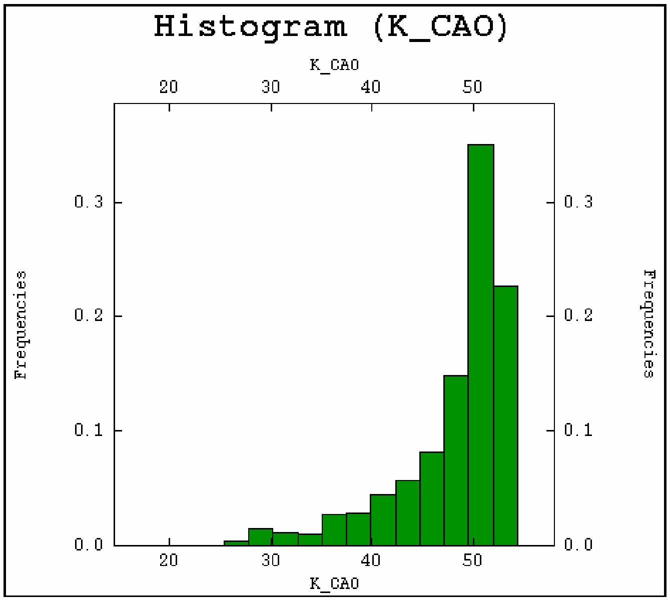

Fig. 13. Histogram of the grades of CaO estimated by Ordinary Kriging.

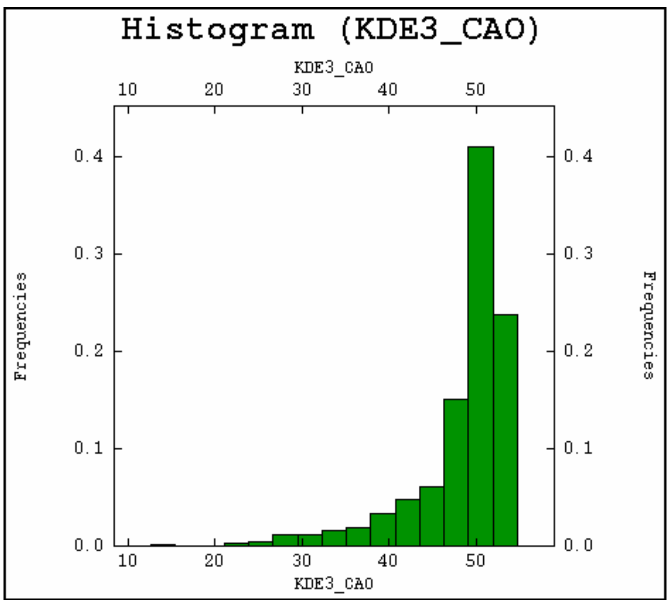

Fig. 14. Histogram of the grades of CaO estimated by kriging with external drift.

In the limestone deposit the visual analysis of 3D blocks (Figures 15 and 16) shows subtle differences between the two estimative, but with some variations, especially in regions where there are lower grades of the variable CaO.

Unlike the lateritic model the number of blocks estimated by kriging with external drift and ordinary kriging were the same, due to the fact that heterotopy is far less pronounced.

FROM	TO	
38.5930	47.8104	
47.8104	49.1722	
49.1722	49.8440	
49.8440	50.4985	
50.4985	51.1906	
51.1906	51.6420	
51.6420	52.2441	
52.2441	54.2779	

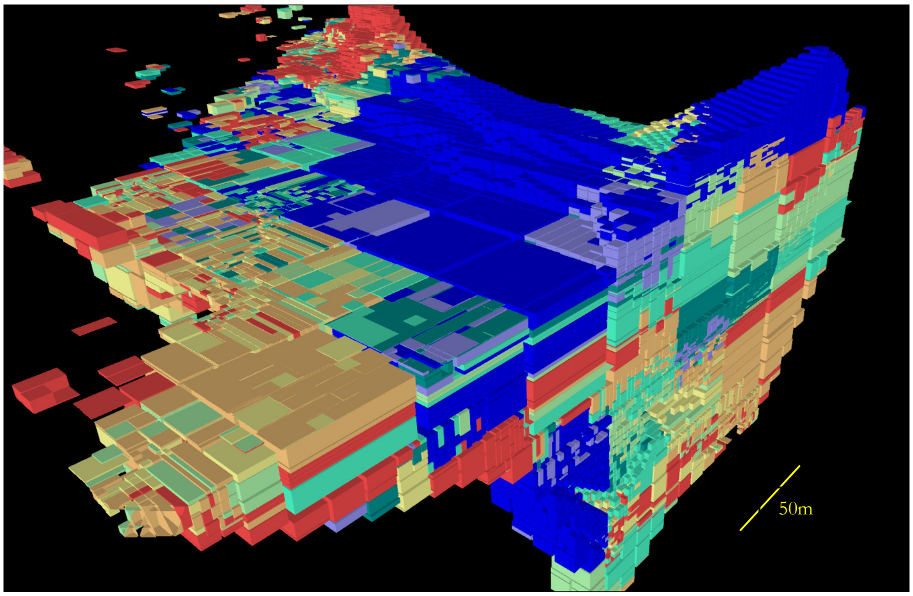

Fig. 15. Block model (3D) at grades estimated (CaO) with kriging with external drift, model cut in the east-west, legend on top.

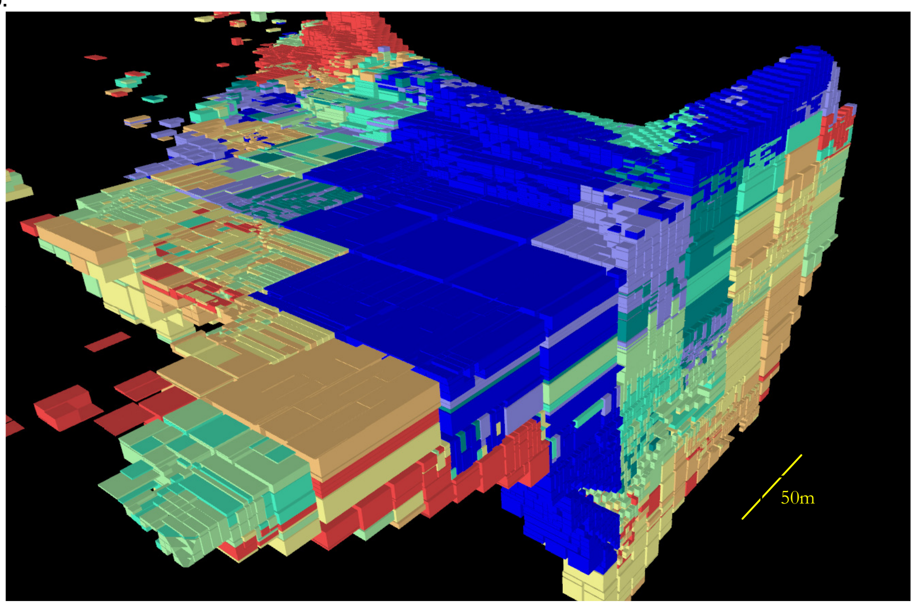

Fig. 16. Block model (3D) with the grades (CaO) estimated by ordinary kriging, model cut in the east-west, legend on top.

To compare, were built sections across block model, with the average grades, each 15 meters in Z (Figure 17). The averages were calculated for the variable CaO estimated by kriging with external drift and ordinary kriging; for SiO2 estimated by ordinary kriging and used as a secondary variable, and variable CaO present in the samples used in the estimative.
The local means of the samples are greater than the estimated means emphasizing the smoothing of kriging with external drift and ordinary kriging. The values of SiO2 used as auxiliary variable behave as "mirror" the values of CaO estimated by kriging with external drift, that because the two variables have high negative correlation. Watching the lines of the averages can be inferred that the use of auxiliary variable, SiO2, to estimate CaO declined the smoothing of the values for CaO obtained by ordinary kriging.

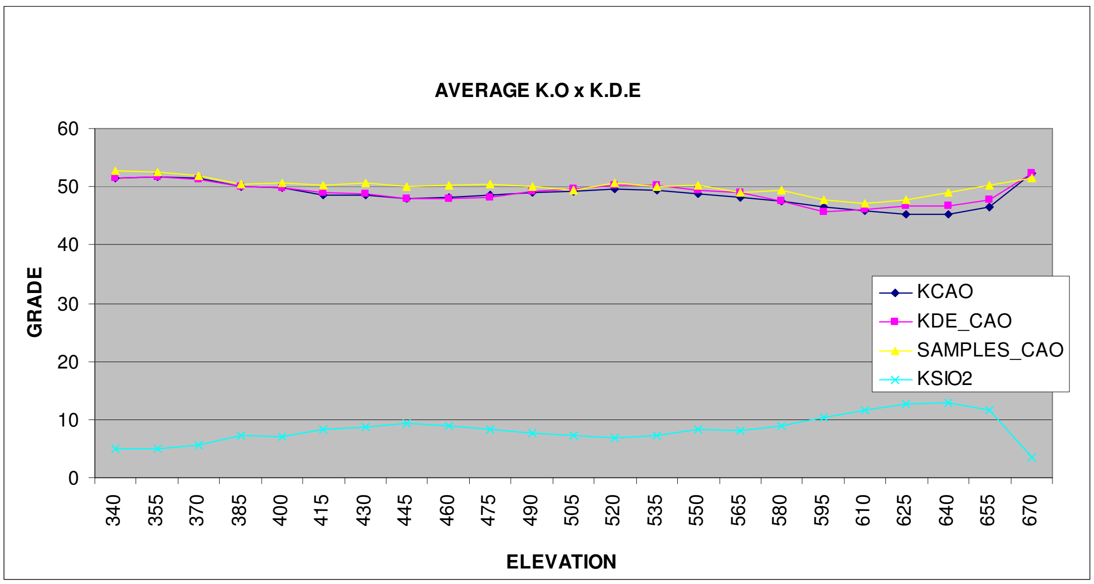

Fig. 17. Profile in the vertical direction with points averaged every 15m.

6. CONCLUSIONS

The application of alternative methods to estimative is valid, since some methods may have greater ease in application than others, reducing the workload in the preparation of highly complex models.
The variograms of all variables were well structured, allowing good fits of theoretical models of variograms.
Due to the high correlation variograms of waste produced results similar to the main variable, with similar structures, but with significant differences in the variances, because the nature of the variable and the waste is different.
The calculation of the residue is one of the most important steps and sensitive, since it directly influences the estimates. The tests performed during the study show that residues calculated with polynomials not suitable biasing the results of the estimation.
The well-distributed sampling allowed a good estimate of the blocks with virtually all of the models being estimated. In the case of model limestone both methods estimated the same number of blocks due to heterotopy between the variables is small, the model lateritic

differences occurred between the number of blocks estimated by two methods because the heterotopy between variables to be significant. Thus the heterotopy and the areas sampled directly affect the number of blocks estimated.

The results of the comparison tests performed, such as, mean, mean local histograms, curves content x tonnage showed that kriging with external drift presents results similar to many ordinary kriging, where the model was found in a limestone more apparent softening of the lowest average levels estimated by kriging with external drift for the average content of the samples and that the average content estimated by ordinary kriging.

These results confirm that the kriging with external drift has advantages in case of a variable sub-sampled, which has good correlation with another variable over-sampled. This scenario is not very common in mining, but occurs mainly in older projects and projects to analyze unique elements, and thus a plausible method to be used when these conditions are present.

3. REFERENCES

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