Existence and Uniqueness for Differential Equations in Banach Spaces with State-Dependent Delay in Unbounded Phase Spaces

José Paulo Carvalho dos Santos, Michelle Pierre
Universidade Federal de Alfenas, Brazil

In this presentation, we investigate the existence and uniqueness of local and maximal solutions for a general class of abstract differential equations with state-dependent delays in phase spaces that allow for unbounded delays described by

$$\frac{du(t)}{dt} = Au(t) + F(t, u(t - \sigma(t, u_t))), \quad t \in [0, a], \tag{6}$$

$$u_0 = \varphi \in \mathcal{B}, \tag{7}$$

where $A\colon D(A) \subset Z \to Z$ is the generator of an analytic semigroup $(\mathcal{S}(t))_{t\geq 0}$, u_t denotes the history of $u(\cdot)$ at time t (that is, $u_t(\theta) = u(t+\theta)$), which belongs to an abstract phase space $\mathcal{B}$ defined axiomatically, and $F(\cdot), \sigma(\cdot)$ are suitable continuous functions. . We develop a framework for treating state-dependent delays via unbounded delay operators defined in space os Lipschitz functions, and establish our results without assumption that the nonlinear forçant terms are locally Lipschitz. To illustrate the applicability of the theory, we include an application to partial differential equations with state-dependent delay.

Atypical bifurcation for periodic solutions of nonlinear ODEs with indefinite weight

Juan Francisco Camasca Fernández, Pierluigi Benevieri
Universidade Estadual de Campinas, Brazil

We study the periodic boundary-value problem (BVP), associated with a nonlinear second order ordinary differential equation of the form

$$u'' + cu' + a(t)g(, u) = 0, \tag{8}$$

depending on a real parameter . We suppose that $a : \boldsymbol{R} \to \boldsymbol{R}$ is a locally integrable T-periodic function, $g : \boldsymbol{R} \times \boldsymbol{R}^n \to \boldsymbol{R}^n$ is a continuous function and c is an arbitrary real constant. We obtain a global bifurcation result for T-periodic pairs $(, u)$, such that u is a T-periodic solution of (8) for the corresponding .

Global existence and regularity results for a class quasilinear non-uniformly elliptic problems with fast diffusion

Lucas da Silva , Jefferson Abrantes dos Santos, Gustavo Ferron Madeira
Universidade Federal de Campina Grande, Brazil

In this work, we investigate the existence, nonexistence, and regularity of solutions for a class of quasilinear non-uniformly elliptic problems characterized by a fast diffusion term at infinity. We establish Schauder and Calderón-Zygmund type regularity results, providing new insights into the smoothness of weak solutions associated with the operator

$$\mathcal{L}_p u = -\operatorname{div}\Big(p|\nabla u|^{p-2} e^{|\nabla u|^p} \nabla u\Big)$$

for $p > 1$. By employing variational techniques, sub and supersolution methods, and advanced functional analysis in Orlicz-Sobolev spaces, we prove that weak solutions belong to $C^1(\Omega)$ and, under certain conditions, are strong solutions in $W^{2,2}_{\text{loc}}(\Omega)$. Moreover, we establish an optimal threshold Λ_p determining the existence and nonexistence of positive strong solutions for a class of quasilinear elliptic problems.