

tral. Pequena cápsula envolve-as, encontrando-se ao seu redor uma camada de melanóforos dendríticos.

Na ilha encontramos células alfa, beta e delta, em proporções variáveis, e sem disposição regular em relação aos capilares.

Nos animais em estivação ocorre ligeiro aumento do número de células beta, e nota-se maior volume insular comparado ao dos animais em fase aquática. — (23 de setembro de 1975).

GENERIC CRITICAL POINTS OF NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS — JOSÉ CARLOS FERNANDES DE OLIVEIRA, presented by CHAIM SAMUEL HÖNIG — *Instituto de Matemática e Estatística, Universidade de São Paulo, São Paulo, SP* — Let E^n denote the Euclidian space of real or complex n -vectors, r a fixed positive number, and let C be the space of continuous functions $\phi: [-r, 0] \rightarrow E^n$ with the sup-norm. If x is a continuous map of $[-r, \infty)$ into E^n and $t \geq 0$, let x_t denote the element of C given by $x_t(\theta) = x(t + \theta)$, $-r \leq \theta \leq 0$.

Let $\mathcal{D}$ be the space of continuous linear functional D from C into E^n such that: (i) the representation of D by Stieltjes integral is given by

$$D(\phi) = \phi(0) - \sum_{k=1}^{\infty} A_k \cdot \phi(-r_k) + \int_{-r}^0 A(\theta) \cdot \phi(\theta) d\theta \quad (\phi \in C)$$

for some $n \times n$ -matrices A_k and $A(\theta)$, and some numbers r_k with the properties $0 < r_k \leq r$ and

$$\sum_{k=1}^{\infty} |A_k| + \int_{-r}^0 |A(\theta)| d\theta < \infty;$$

(ii) we suppose that D is stable, that is there exist constants $K > 0$ and $a_0 < 0$ such that $|T_D(t)| \leq K \cdot \exp(a_0 \cdot t)$ for $t \geq 0$, where $(T_D(t), t \geq 0)$ is the dynamical system on $D^{-1}(0)$ given by $T_D(t) \cdot \phi = y_t$, $y: [-r, \infty) \rightarrow E^n$ being the solutions of the initial value problem: $D(y_t) = 0, t > 0$, $y_0 = \phi \in D^{-1}(0)$.

We note that $\mathcal{D}$ is an open subset of the Banach space of all linear continuous functionals from C into E^n .

Let $\mathcal{X}$ be the Banach space of all C^1 -functions $f: C \rightarrow E^n$ with $\|f\|_1 = \sup \{ \|f^{(j)}(\phi)\| : \phi \in C, j = 0, 1 \} < \infty$.

For each $(D, f) \in \mathcal{D} \times \mathcal{X}$ we have a neutral functional differential equation

$$\frac{d}{dt} D(x_t) = f(x_t), \quad t > 0.$$

Let $(T_{D,f}(t), t \geq 0)$ be the dynamical system defined on C by the NFDE (D, f) .

Any constant function $c \in E^n$ such that $f(c) = 0$ is called a critical point of (D, f) . We say that c is a simple critical point of (D, f) if $1 \notin \sigma(T_{D,f(c)}(t))$ for some, (hence for all) $t > 0$, c is called hyperbolic if $S^1 \cap \sigma(T_{D,f(c)}(t)) = \emptyset$ for some (hence for all) $t > 0$, S^1 being the unit circle of the complex plane.

For each $N = 1, 2, \dots$ let $\mathcal{G}_0(N)$ — resp. $\mathcal{G}_1(N)$ — be the set of all $(D, f) \in \mathcal{D} \times \mathcal{X}$ such that all critical points of (D, f) , contained in the closed ball of E^n with radius N , are

simple — resp. hyperbolic — and let $\mathcal{G}_0 = \bigcap_{N=1}^{\infty} \mathcal{G}_0(N)$ and $\mathcal{G}_1 = \bigcap_{N=1}^{\infty} \mathcal{G}_1(N)$.

THEOREM: $\mathcal{G}_0(N)$ is an open dense subset of $\mathcal{D} \times \mathcal{X}$, $\mathcal{G}_1^{(N)}$ is an open dense subset of $\mathcal{G}_0(N)$, $N = 1, 2, \dots$, and $\mathcal{G}_0$ and $\mathcal{G}_1$ are dense in $\mathcal{D} \times \mathcal{X}$.

The theorem says the property $\mathcal{G}_1 = \dots$ admits only hyperbolic critical points" is a generic property of the neutral functional differential equations of the class $\mathcal{D} \times \mathcal{X}$. It is an extension of the analogous result in [W. M. OLIVA, *J. Diff. Equations*, 5, (1969), 483-496] for retarded functional differential equations on a compact manifold.

The proof of the above uses Theorem 4.1 in [D. HENRY, *J. Diff. Eq.*, 15, (1974), 106-128] and Theorem 4.2 in [J. K. HALE, *Functional Differential Equations of Neutral Type, International Symposium on Dynamical Systems*, August 1974, Brown University].

It seems probable that the $\mathcal{G}_2$ -property of admitting only hyperbolic periodic solutions is also a generic property in $\mathcal{D} \times \mathcal{X}$. If this were true, it would extend the result of J. MALLET-PARET, in *Generic Periodic Solutions of Functional Differential Equations*, *J. Diff. Equations*, to appear. — (14 de outubro de 1975).

GENERIC PROPERTIES OF VECTOR FIELDS TANGENT TO A REEB FOLIATION

— ELVIA MUREB SALLUM, presented by CHAIM SAMUEL HÖNIG — *Instituto de Matemática e Estatística, Universidade de São Paulo, São Paulo, SP* — The present work was suggested to us by W. M. OLIVA and it is a generalization of some results contained in the last chapter of ALCILÉA, A. H. MELO's doctoral dissertation which we shall indicate by [A.N.]. See also: W. M. OLIVA, Propriedades genéricas dos campos de Appell, *An. Acad. brasil. Ciênc.*, 44 592-R, (1972).

Let σ denote the Reeb foliation of the sphere S^3 (see G. REEB, Sur certains propriétés topologiques de variétés feuilletées, *Actualités Sci. Ind.*, N.º 1183 Hermann, Paris, 1952) and let $\mathcal{X}_r^t(S^3)$ be the space of all vector fields in S^3 of class C^r with $r \geq 4$, which are tangent to σ . We shall denote by T^2 the compact leaf of σ , and assume $\mathcal{X}_r^t(S^3)$ with the usual topology.

DEFINITION 1. ([A.M.]) — Let $p \in S^3$ and $X \in \mathcal{X}_r^t(S^3)$ be such that $X(p) = 0$. Call p a σ -simple singular point if DX_p has rank 2.

As a particular case of some results in [A.M.] one has:

PROPOSITION — The set $G_0 = \{X \in \mathcal{X}_r^t(S^3) | X \text{ has only } \sigma\text{-simple singularities}\}$ is open and dense in $\mathcal{X}_r^t(S^3)$. Also if $X \in G_0$, then the set V_X of all singular points of X is a compact 1-dimensional submanifold of S^3 and V_X varies continuously in class C^r with X in a neighborhood of X .

DEFINITION 2. Denote by F_p the leaf of σ containing p . A singular point p of $X \in G_0$ is said to be σ -hyperbolic (Σ -quasi generic) if p is an hyperbolic singular point (quasi generic) for the restriction $X|_{F_p}$ (see J. SOTOMAYOR, Generic one parameter families of vector fields on two dimensional manifolds, *Publications Mathématiques*, N.º 43, IHES, 1974).

We will state now the main results:

THEOREM — Let G_1 be the set of vector fields $X \in G_0$ such that:

- i) $X|_{T^2}$ is Morse-Smale.
- ii) V_x contains only hyperbolic singular points except for a finite number which are σ -quasi generic.
- iii) Each leaf contains at most one σ -quasi generic singular point.

Then G_1 is open and dense in G_0 (thus in $\mathcal{X}^r(S^3)$).

Some examples show us the impossibility of deleting vector fields $X \in G_0$ with σ -quasi generic points.

The above theorem follows from the lemmas:

LEMMA 1 — Let J' be the manifold of 1-jets of vector fields $X \in \mathcal{X}^r(S^3)$ and let W be the submanifold $W = \{j_p X \in J' | X(p) = 0, DX|_{F_p}(p) \text{ has rank } 1\}$. Then $N_1 = \{X \in G_0 | j_p X \in W\}$ is open and dense in G_0 .

LEMMA 2 — If $X \in N_1$, $X(p) = 0$ and the eigenvalues of $DX|_{F_p}(p)$ are 0 and $\lambda \neq 0$, then p is a saddle-node for $X|_{F_p}$.

LEMMA 3 — The set $N_2 = \{X \in N_1 | j_p X \in W \text{ implies } p \text{ be a saddle-node for } X|_{F_p}\}$ is open and dense in N_1 .

LEMMA 4 — Let $\sigma(S^3)$ be the sub-bundle of $T(S^3)$ associated to σ , $O(S^3)$ the zero-section of $T(S^3)$ and $W = \{0\} \times \times R_{++} \times O(S^3)$. Let $\rho_x: N_2 \rightarrow C^{r-1}(M, R \times R \times \sigma(S^3))$ be given by $\rho_x(p) = (tr DX|_{F_p}(p), \det DX|_{F_p}(p), p, x(p))$ (see R. ABRAHAM, *Transversal mappings and flows*, New York, 1967, page 46). Then $N_3 = \{X \in N_2 | \rho_x \in W\}$ is open and dense in N_2 .

LEMMA 5 — The set $N_4 = \{X \in N_3 | p \text{ is a composed focus for } X|_{F_p} \text{ when } \rho_x(p) \in W\}$ is open and dense in N_3 .

LEMMA 6 — Given a Morse-Smale vector field $X \in \mathcal{X}^r(T^2)$ and $\varepsilon > 0$ there exists another vector field $Y \in N_4$ such that $Y|_{T^2}$ is Morse-Smale, $\|Y|_{T^2} - X\|_r < \varepsilon$ and $Y|_{T^2}$ is topologically equivalent to X (see J. PALIS and W. MELO, *Introdução aos Sistemas Dinâmicos*, 10.º Colóquio Brasileiro de Matemática, Poços de Caldas, 1975, page 50).

We remark that properties (ii) and (iii) in the above theorem hold for any compact manifold with a smooth foliation $\mathcal{F}$ with codimension 1. Property (i) also holds if $\mathcal{F}$ has at most a finite number of compact leaves.

The structural stability of σ -gradient vector fields ([A.M.]) in G_1 and of $X \in G_1$ which have only hyperbolic or quasi generic periodic orbits on the leaves will be considered in another paper. — (14 de outubro de 1975).

O CÁLCULO DE PREDICADOS COM "vbtos" — IOLE F. DRUCK e NEWTON C. A. DA COSTA, credenciados pelo Acadêmico CHAIM SAMUEL HÖNIG — Instituto de Matemática e Estatística, Universidade de São Paulo, São Paulo SP — Denomina-se vbto (de variable binding term operator) um operador que forma termos ligando

variáveis. Entre os vbts mais conhecidos encontram-se o símbolo ε de Hilbert, o descritor τ , o classificador $\{\dots\}$ e o abstrator $\hat{x}_1 \dots \hat{x}_n F(x_1, \dots, x_n)$. Neste trabalho consideramos apenas os vbts que ligam uma única variável, mas nossos resultados valem para o caso de vbts que se aplicam a várias variáveis simultaneamente. (Sobre tais operadores, ver: W. S. HATCHER, *Foundations of Mathematics*, (1968), e J. CORCORAN W. S. HATCHER e J. HERRING, *Zeitschr. f. math. Logik. u. Grundlagen d. Math.*, 18 (1972), 177-182.

Seja L_v uma linguagem de primeira ordem (cf. J. R. SHOENFIELD, *Mathematical Logic*, (1968), cuja terminologia adaptamos, de um modo evidente, às nossas necessidades), com o conjunto V de vbts. Uma estrutura para L_v é qualquer par $\langle a, d \rangle$, onde a é uma estrutura no sentido usual para a linguagem L_v sem os vbts e d é uma função de domínio V e tal que: 1) se $v \in V$, $d(v)$ é uma aplicação da forma $\mathcal{P}(\kappa) \rightarrow \kappa$; 2) Se F é fórmula de $L_v(a)$, com a única variável livre x , $a(vxF(x)) = d(v)(\{\bar{a} \in \kappa : a(F_x[\bar{a}]) = V \text{ (verdadeiro)}\})$, onde $\bar{a}$ é o nome de $a \in \kappa$. Definem-se, sem dificuldades, noções semânticas tais como as de modelo do conjunto de Γ de fórmulas e de consequência semântica do conjunto Γ . (Sentença é qualquer fórmula sem variáveis livres).

Os postulados lógicos de L_v são os do cálculo de predicados (de primeira ordem) com igualdade, juntamente com os seguintes, nos quais os símbolos têm significados patentes:

$$(I) \quad vx F(x) = vy F(y), \quad e$$

$$(II) \quad \forall x (F(x) \leftrightarrow G(x)) \rightarrow vx F(x) = vx G(x)$$

TEOREMA 1 — Seja $\Gamma \cup \{F\}$ um conjunto de sentenças de L_v . Tem-se, então: $\Gamma \models F$, se, e somente se, $\Gamma \vdash$ em L_v .

TEOREMA 2 — Se o conjunto $\Gamma \cup \{F\}$ de sentenças de L_v não contém vbts, então $\Gamma \vdash F$ em L_v , se, e somente se, $\Gamma \vdash F$ no cálculo de predicados sem vbts (e com igualdade).

Vários teoremas sobre o descritor e o símbolo de Hilbert são casos particulares das proposições anteriores.

Além disso, os conceitos usuais da teoria dos modelos (Γ -subestrutura de uma estrutura, subestrutura de uma estrutura, subestrutura elementar, etc.) e os seus resultados básicos podem ser estendidos a L_v (dentre esses resultados, mencionaremos os teoremas dos Łoś-Tarski, de Löwenheim-Skolem, de Craig, de Craig-Robinson, de Chang-Łoś-Suszco, de Beth, de Ehrenfeucht, de Ryll-Nardzewski e de Lyndon: cf. SHOENFIELD, obra citada). Por exemplo, tem-se:

TEOREMA 3 — Sejam Γ um conjunto de fórmulas de L_v nas quais não figuram quantificadores, e T uma teoria de L_v . Então, T é equivalente a uma teoria cujos axiomas não lógicos não contém quantificadores se, e somente se, toda Γ -subestrutura de um modelo de T é também modelo de T .