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by

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INFLUENCE MEASURE FOR THE MINIMUM SUM OF ABSOLUTE ERRORS REGRESSION

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Abstract

Because the least squares regression is unduly affected by outliers and leverage observations, the identification of influential observations is considered an important and integral part of the analysis. However, very few techniques have been developed for the residual analysis and diagnostics for the minimum sum of absolute errors, MSAE regression. Although the MSAE regression is more resistant to the outliers than the least squares regression, it appears that it may be affected by outliers (leverage) in the predictor variables. In this paper, our objective is to develop an influence measure for the MSAE regression based on the likelihood displacement function.

Keywords: influential observation; leverage point; likelihood displacement; likelihood function; outlier.

1. INTRODUCTION

Let y be an $n \times 1$ vector of the values of the response variable corresponding to X, an $n \times k$ matrix of the values of the k regressor (predictor) variables (which may include an intercept term), then

$$y = X\beta + \varepsilon \qquad (1)$$

is the multiple linear regression model where ε is an $n \times 1$ vector of the unobservable random errors and β is an $k \times 1$ vector of the unknown parameters of the model. The minimum sum of absolute errors, MSAE estimator of β minimizes

$$\sum_{i=1}^{n} |y_i - x_i \beta|,$$

where y_i is the i-th element of vector y and x_i is the i-th row of matrix X. The predicted values of the response variable are given by

$$\hat{y} = X\hat{\beta},$$

where $\hat{\beta}$ denotes the MSAE estimator of β. The residual for the i-th observation, e_i is given by $y_i - \hat{y}_i$.

It is well known that the MSAE regression hyperplane passes through at least k observations, Appa and Smith (1973). These observations with zero residuals are called defining (or basic) observations and the ones with nonzero residuals are called nondefining (or nonbasic) observations. That is, the MSAE estimate of β is completely determined by the defining observations. In particular, the system of equations corresponding to MSAE regression may be written as

$$\begin{bmatrix} y_{(1)} \\ y_{(2)} \end{bmatrix} = \begin{bmatrix} X_{(1)} & 0 \\ X_{(2)} & I^* \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ e_{(2)} \end{bmatrix},$$

where subscripts (1) and (2) refer to the observations with zero and nonzero residuals, respectively; vector $e_{(2)}$ is composed of the components of the vectors e^+ ($= y_i - \hat{y}_i$ if $y_i - \hat{y}_i \geq 0$, and zero, otherwise) and e^- ($= y_i - \hat{y}_i$ if $y_i - \hat{y}_i < 0$, and zero, otherwise); and I^* is an $(n - k) \times (n - k)$ diagonal matrix with diagonal elements that are either +1 or -1 depending upon whether $e^+ > 0$ or $e^- > 0$ for $i = k + 1, \ldots; n$. With this reordering, the first k observations [with subscript (1)] lie on the MSAE regression hyperplane. Furthermore, $X_{(1)}$ is $k \times k$ matrix, $X_{(2)}$ is an $(n - k) \times k$ matrix, and 0 is a $k \times (n - k)$ matrix of zeros. It follows that

$$\hat{\beta} = X_{(1)}^{-1} y_{(1)}, \text{ and } e_{(2)} = I^* y_{(2)} - I^* X_{(2)} X_{(1)}^{-1} y_{(1)}.$$

The hat matrix for the MSAE regression is:

$$\begin{bmatrix} \hat{y}_{(1)} \\ \hat{y}_{(2)} \end{bmatrix} = \begin{bmatrix} I \\ X_{(2)} X_{(1)}^{-1} \end{bmatrix} y_{(1)} = H y_{(1)},$$

where I is a $k \times k$ identity matrix, Narula and Wellington (1985). For the defining observations, the hat matrix is the identity matrix, i.e., $h_{ii} = 1$ for all i. The diagonal elements of the hat matrix for the defining observations are equal to one; so these are influential observations because each of these observations is used to predict itself; however, they are not influential in the same way as the observations associated with the large diagonal elements of hat matrix in the least squares analysis. Furthermore, as long as the observations stay on the same side of the MSAE regression hyperplane, the fitted MSAE regression model remains unchanged. However, it has been shown that the MSAE regression is more sensitive to leverage points, i.e., outliers in the direction of the predictor (or regressor) variable than the least squares regression, Rousseeuw and Leroy (1987).

Ellis and Morgenthaler (1992) point out that at present, *leverage* does not have a precise meaning. Vaguely stated, a design point far from the bulk of the others is called a leverage point. It is important to distinguish leverage points from *influential points*. An observation taken at a leverage point has the potential to influence the fit, but it does not necessarily do so.

Because the overall summary statistics (e.g., coefficient of determination, and the estimators of the parameters, etc.) arising from data analyses based on full rank linear regression models may present an incomplete picture, a number of diagnostic procedures have been proposed for the least squares regression. However, very few techniques have been developed for the residual analysis and diagnostics for the MSAE regression. Parker (1988) proposed a method using Box-Cox transformation to assess the need for transformation and the contribution of an individual observation to the evidence for a transformation for the MSAE regression.

Cook, Peña, and Weisberg (1988) proposed the likelihood displacement function as a unifying principle for influence measure. They point out that if desirable, this displacement can be transformed to a more familiar scale and compared to percentiles of a chi-squared distribution with k degrees of freedom. It is a general approach and does not use the special structure of the linear model. In this paper, our objective is to develop an influence measure for the MSAE regression based on the likelihood displacement function. The rest of the paper is organized as: In Section 2, we develop the likelihood displacement function for our model and illustrate it with two examples in Section 3. We conclude the paper with a few remarks in Section 4.

2. LIKELIHOOD DISPLACEMENT

It is well known that MSAE estimators are maximum likelihood estimators of the parameter β, when the errors ε_i's in (1) follow Laplace distribution with mean equal to zero and variance equal to $2\tau^2$, i. e., the probability density function of y_i is given by

$$f(y_i) = 1/(2\tau)\exp(-|y_i - x_i\beta|/\tau), \qquad -\infty < y_i < \infty.$$

Then the likelihood function *l* is

$$\ell(\beta,\tau|y) = 1/(2\tau)^n \exp(-\Sigma_{i=1}^n|y_i - x_i\beta|/\tau).$$

The log likelihood function L is

$$L(\beta,\tau) = -n\ln(2\tau) - \Sigma_{i=1}^n|y_i - x_i\beta|/\tau,$$

and it takes its maximum value when

$$\Sigma_{i=1}^n|y_i - x_i\beta|$$

is minimized, i. e., $\hat{\beta}$, the MSAE estimator, is the maximum likelihood estimator of β.

Differentiating the log likelihood function with respect to τ and setting it equal to zero, we obtain the maximum likelihood estimator $\hat{\tau}$ of τ as follows:

$$\partial L/\partial\tau = (-2n)/(2\tau) + (1/\tau^2)\Sigma|y_i - x_i\beta| = 0,$$

so

$$\hat{\tau} = \Sigma_{i=1}^n|y_i - x_i\hat{\beta}|/n = MSAE/n. \qquad (2)$$

To determine the influence of the i-th observation, $i = 1, \ldots, n$, Cook et al (1988) suggest the following likelihood displacement function

$$LD_i(\theta) = 2[L(\hat{\theta},y) - L(\hat{\theta}_{(i)},y)],$$

where $\hat{\theta}$ is the maximum likelihood estimator of θ based on all the observations and $\hat{\theta}_{(i)}$ is the maximum likelihood estimator of θ based on all the observations except observation i. They advocate that if this function is large, observation *i* is influential because deleting it may cause a substantial change in important conclusions. For our problem, $\theta = (\beta, \tau)$ and the likelihood displacement function is

$$LD_i(\beta,\tau) = 2[L(\hat{\beta},\hat{\tau}) - L(\hat{\beta}_{(i)},\hat{\tau}_{(i)})]$$

where $\hat{\beta}_{(i)}$ is the MSAE estimator of β based on all the observations except observation i and $\hat{\tau}_{(i)}$ is similarly given by

$$\hat{\tau}_{(i)} = \Sigma_{j=1,j\neq i}^{n} |y_j - x_j\hat{\beta}_{(i)}| / (n-1). \qquad (3)$$

Since

$$L(\beta,\tau) = -n\ln(2\tau) - \Sigma_{i=1}^{n} |y_i - x_i\beta| / \tau,$$

therefore,

$$L(\hat{\beta},\hat{\tau}) = -n\ln(2\hat{\tau}) - \Sigma_{i=1}^{n} |y_i - x_i\hat{\beta}| / \hat{\tau},$$

$$= -n\ln(2\hat{\tau}) - n,$$

and

$$L(\hat{\beta}_{(i)},\hat{\tau}_{(i)}) = -n\ln(2\hat{\tau}_{(i)}) - \Sigma_{j=1}^{n} |y_j - x_j\hat{\beta}_{(i)}| / \hat{\tau}_{(i)},$$

$$= -n\ln(2\hat{\tau}_{(i)}) - [(n-1)\hat{\tau}_{(i)} + |y_i - x_i\hat{\beta}_{(i)}|] / \hat{\tau}_{(i)},$$

$$= -n\ln(2\hat{\tau}_{(i)}) - (n-1) - |y_i - x_i\hat{\beta}_{(i)}| / \hat{\tau}_{(i)}.$$

Therefore, the likelihood displacement function for the i-th observation can be written as:

$$LD_i(\beta,\tau) = 2(n\ln(\hat{\tau}_{(i)} / \hat{\tau}) + |y_i - x_i\hat{\beta}_{(i)}| / \hat{\tau}_{(i)} - 1). \qquad (4)$$

The measure in (4) will be large if $|y_i - x_i\hat{\beta}_{(i)}| / \hat{\tau}_{(i)}$ is large, or $(\hat{\tau}_{(i)} / \hat{\tau})$ is large or both.

When the i-th observation is not influential, then the estimate $\hat{\tau}_{(i)}$, without the i-th observation , should be very similar to $\hat{\tau}$, i.e., $\hat{\tau}_{(i)} / \hat{\tau} \cong 1$, and $|y_i - x_i\hat{\beta}_{(i)}|$ should be close to the mean of the absolute errors, $\hat{\tau}_{(i)}$, i.e., $|y_i - x_i\hat{\beta}_{(i)}| \cong \hat{\tau}_{(i)}$, and so $LD_i \cong 0$. That is, when the i-th observation is not influential, the likelihood displacement function will be close to zero; large values of the function reflect the possibility that the observation might be influential.

Clearly, $LD_i(\beta,\tau)$ takes account of both β and τ, since both of their estimates are considered in expression (4). To determine if the i-th observation is influential only for the estimation of β, a measure based on the method proposed by Cook et al (1988) can be developed for the MSAE regression as follows: Consider

$$LD_i(\beta|\tau) = 2[L(\hat{\beta},\hat{\tau}) - \max_\tau(\hat{\beta}_{(i)},\tau)].$$

Since

$$L(\hat{\beta}_{(i)},\tau) = -n\ln(2\tau) - \sum_{j=1}^{n}|y_j - x_j\hat{\beta}_{(i)}|/\tau,$$

it is maximized for $\hat{\tau}_{max} = \sum_{i=1}^{n}|y_i - x_i\hat{\beta}_{(i)}|/n$.

Therefore,

$$\max_\tau L(\hat{\beta}_{(i)},\tau) = -n\ln(2\sum_{j=1}^{n}|y_j - x_j\hat{\beta}_{(i)}|/n) - n,$$

and

$$L(\hat{\beta},\hat{\tau}) = -n\ln(2\hat{\tau}) - n.$$

Thus,

$$LD_i(\beta|\tau) = 2n\ln(\frac{\sum_{j=1}^{n}|y_j - x_j\hat{\beta}_{(i)}|}{\sum_{j=1}^{n}|y_j - x_j\hat{\beta}|}). \qquad (5)$$

That is, we compare the sum of absolute value of the residuals when they are calculated using the MSAE estimator of β computed with and without the i-th observation. If the deletion of the i-th observation changes the estimates such that the sum of the absolute residuals does not change much then $LD_i(\beta|\tau)$ will be close to zero.

It is interesting to note that

$$LD_i(\beta|\tau) = 2n\ln(1+\lambda_i),$$

where $\lambda_i = \dfrac{\sum_{j=1}^{n}|y_j - x_j\hat{\beta}_{(i)}| - \sum_{j=1}^{n}|y_j - x_j\hat{\beta}|}{\sum_{j=1}^{n}|y_j - x_j\hat{\beta}|}$, which may be interpreted as relative increase in the minimum sum of absolute errors when $\hat{\beta}$ is substituted for $\hat{\beta}_{(i)}$.

Cook et al (1988) pointed that the values of $LD_i(\beta,\tau)$, and $LD_i(\beta|\tau)$ may be compared with the percentiles of the chi-squared distribution with degrees of freedom k and k - 1, respectively, to decide whether an observation is influential or not. However, for Laplace distribution, the necessary regularity conditions are not satisfied (Cox and Hinkley (1974)), and therefore, we do not use this approach here.

3. ILLUSTRATIVE EXAMPLES

All-but-one-point-on-a-line problem: Cook et al (1988) stated that Dempster and Green (1981) mentioned the all-but-one-point-on-a-line problem. The general idea is that diagnostic procedure will always find the point that lies off the lines to be most influential.

They use the following data as an illustration of the likelihood displacement function using the least squares estimators. Although this example is relatively simple, its essential characteristics are perfectly general.

Table 1: Data for the example, Cook, Peña, and Weisberg (1988).

Obs. No. I	x_i	y_i
1	0.0	0.0
2	0.2	0.2
3	0.2	-0.2
4	$\sqrt{0.92}$	$\sqrt{0.92}$

For these data, observations 1, 2, and 4 are collinear and lie on the line $y = x$. To observe how the MSAE estimates change with changes in the values of the response and predictor variables, we present the MSAE estimates of the intercept and the slope terms for different values for observation 3. Note that case 1 has the original value whereas case 2 has larger negative value for the response, and cases 3 through 6 have larger values of the regressor variable.

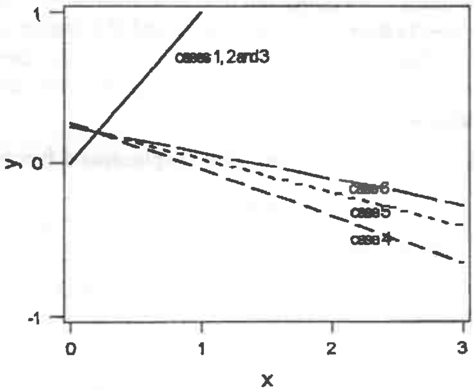
Table 2: MSAE estimates of the intercept and slope terms for different values of observation 3.

	Observation 3			
Case No.	x	y	Intercept	Slope
1*	0.2	-0.2	0.00000	1.00000
2	0.2	-2.0	0.00000	1.00000
3	1.0	-0.2	0.00000	1.00000
4	1.5	-0.2	0.26154	-0.30769
5	2.0	-0.2	0.24444	-0.22222
6	2.5	-0.2	0.23478	-0.17391

* denotes the original value of observation 3.

The intercept and the slope estimates are the same for the first three cases but change for the last three. These cases and the corresponding fitted MSAE models are shown in Figure 1.

Figure 1: The fitted MSAE lines for different values of observation 3



For different values of observation 3, Table 2, we give the values of $LD_3(\beta|\tau)$ in Table 3.

Table 3: The values of $LD_3(\beta|\tau)$ for different values of observation 3 in Table 2

Case No.	$LD_i(\beta\|\tau)$
1	0.000
2	0.000
3	0.000
4	3.630
5	3.910
6	6.998

As we can see in Figure 1, the MSAE line in case 6 differs the most from the line $y = x$. In this case the angle between the fitted line and the x axis is the most different from 45°, and the function $LD_i(\beta|\tau)$ identifies this fact.

If we substitute the observation number 3 by a new point (x_0, -0.2), with $x_0 > 2.5$, it is not very difficult to note that

$$LD_3(\beta|\tau) = 8\ln\left(\frac{x_0^2 - 0.04}{0.959x_0 + 0.192}\right).$$

Since this function is increasing with x_0, we observe that, as x_0 becomes larger and the point is more distant from the others three points, the measure increases.

Wood beam data: We now illustrate the use of the likelihood displacement function with the wood beams data, Draper and Stoneman (1966). The data has two regressor variables and ten observations. After a careful and thorough analysis of the data using least squares regression, Hoaglin and Welsch (1978, p. 21) concluded that "beam 1 and (to a lesser extent) beam 4 are still fairly damaging" in terms of their effect on the least squares estimates. Deleting one observation at a time, Narula and Wellington (1985) reported that for the MSAE regression, "beam 4 is more damaging" than other observations in terms of its effect on the estimators of the parameters. For these data the values of the proposed measures are given in Table 4.

Table 4: Values of the Likelihood Displacement functions

for the Wood Beam Data

Observation Number	$LD_i(\beta,\tau)$	$LD_i(\beta\|\tau)$
1	3.775	0.552
2	0.982	1.020
3	0.994	1.020
4	1,899	1.976
5	0.053	0.000
6	0.246	0.000
7	0.129	0.220
8	0.601	0.552
9	0.143	0.220
10	0.555	0.552

From Table 4, we observe that observation 4 has the largest value for $LD_i(\beta|\tau)$ and the second largest value for $LD_i(\beta,\tau)$; this implies that this observation may have the most influence on the estimates of β's. Observations 1 has the largest value for $LD_i(\beta,\tau)$, and therefore, may be influential for the estimation of β's and τ.

4. REMARKS

In this paper, we have proposed measures based on the likelihood displacement principle to identify influential observations for the MSAE regression analysis. Because the regularity conditions do not hold for the Laplace distribution it was not possible to get conclusions about the influence by comparing the value of the measure with percentiles of the chi-squared distribution. We have illustrated the use of this measure with examples.

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