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Equation of State of the Kappa-Distributed Solar Wind Particles in the Earth's Magnetopause

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Abstract
We provide a non-adiabatic equation of state of a space plasma based on the kappa distribution. Our formulation generalizes the polytropic gamma index. An analytic expression is deduced for gamma. Then, the equation of state is derived. The model is applied to describe the electron solar wind in the Earth's magnetopause. A relationship between the plasma pressure and electron concentration is fitted by our equation of state. It is found that: (i) the value of the kappa index is slightly larger than its lower limit, 3/2, thereby actually characterizing electrons in the ambient solar wind; (ii) the electron concentration varies between 2 and 4 particles per cubic centimeter at a null electric potential; (iii) the electron temperature varies between 1.1 and 1.5 million degrees kelvin. Further applications of our theory are briefly addressed.

Keywords Magnetosphere, geomagnetic disturbances · Solar wind, theory

1. Introduction

The uncovering of the topological structure and time evolution of the Earth's magnetosphere is of paramount importance to a satisfactory understanding of the interaction of the ambient solar wind with the magnetopause. Up to recent years, when in-situ measurements were generally available solely from a single spacecraft, one was usually compelled to analyze the outcoming data subjected to overly restrictive assumptions. Actually, one was often constrained to limit the analysis to one-dimensional structures that exhibit spatial variations only along the local normal to the magnetosphere surface. In current days, time series of measurements performed by spacecrafts reveal highly complex behavior, thereby suggesting that

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most of the observed structures are not strictly one-dimensional, but possesses relevant two-, or even three-, dimensional aspects.

The topology of the ambient magnetic field plays a key role in dynamical plasma phenomena involving transfers of mass, momentum, and energy. For example, we may cite the magnetic reconnection occurring in the magnetopause (Sonnerup et al., 1981), solar corona (Masuda et al., 1994), solar wind (Gosling et al., 2005), and magnetotail (Nagai et al., 2013), as well as magnetospheric substorms (Consolini and Chang, 2001) and solar or stellar flares (Kusano et al., 2012; Bamba et al., 2013). In order to appreciate the mechanism efficiency of those transfer processes across a current layer, it is necessary to unveil the nature of one-dimensional discontinuities and shocks, and the formation, location, and interplay of the O- and X-points in two-dimensions, and magnetic nulls and separators in three-dimensions (Wendel and Adrian, 2013).

A method of reconstruction of quasi-two-dimensional not time-dependent magnetic field structures from data measured along a single spacecraft orbit was put forward by Sonnerup and Guo (1996), and Hau and Sonnerup (1999). It has been commonly referred to as the Grad–Shafranov (G-S) reconstruction method (Sonnerup et al., 2006). The fundamental hypotheses which underly the technique are that the structure to be reconstructed shall be stationary, magnetohydrostatic, and moving at a constant velocity relative to the spacecraft (Hasegawa et al., 2015). Sonnerup and Hasegawa (2011) used the G-S method to reconstruct three-dimensional steady magnetohydrostatic structures based on plasma and magnetic field data recorded by two closely adjacent spacecrafts. The analysis consisted of integrating the Grad–Shafranov (G-S) equation as a spatial initial value problem, i.e., as a Cauchy problem, rather than as a boundary value problem. Once the measured signature of a structure satisfying the G-S equation at points along the spacecraft track is known, all properties of the configuration can be reconstructed on a strip surrounding the trajectory.

The mapping of the field structure of a magnetohydrostatic plasma in space may be improved by making use of a multi-spacecraft technique. In that case, the specification of data from a spacecraft cluster produces a single field map by ingesting the data from the individual spacecrafts. The electric potential has been measured by the cluster C1, C2, C3, and C4 satellites, using the Electric Fields and Waves device (EFW) (Gustafsson et al., 1997). Subsequently, the plasma pressure has been measured, with high-time resolution, by the cluster C1 and C3 satellites, using the Cluster Ion Spectrometer (CIS) (Réme et al., 2001). Thereafter, the plasma pressure has been estimated at the cluster C2 and C4 satellites via a numerical relationship between the plasma pressure and electron concentration, draw from the electric potential measured by the cluster C1 and C3 satellites (Hasegawa et al., 2005).

We now give the main motivation of this work. The above referred numerical relationship between plasma pressure and electron concentration has been fitted by a polynomial and exhibits a saddle point (see Figure 2(c) on page 976 in Hasegawa et al., 2005). However, it is widely known that the advent of critical points in the functional dependence of variables of state indicates the event of phenomena such as ionization processes (Hummer and Mihalas, 1988), long-range interactions (Latella and Pérez-Madrid, 2013), phase transitions (Thol et al., 2016), and even DNA-stretching of looped-DNA (Kulić et al., 2007). The latter is clearly ruled out, and none of the three former has been reported by Hasegawa et al. (2005). In this work, we propose an analytic equation of state of the solar wind in the Earth's magnetopause with basis on the so-called kappa density distribution, which fits the aforementioned numerical relationship and does not possess any saddle point.

The kappa density distribution was introduced by Vasyliunas (1968). It can be understood as a distribution of density with a tail deviation from the Maxwellian distribution, that is, it is a Maxwellian distribution with an elongated tail which follows a power law of the

κ-index (Summers and Thorne, 1991). Within this tail region, high energy particles are allowed. In space, binary collisions between particles are extremely rare events. Therefore, the kappa distribution becomes appropriate to describe stationary regimes of low density space plasmas out of thermodynamic equilibrium (Pierrard and Lazar, 2010). It has been employed in numerous investigations on planetary magnetospheres (Christon, 1987; Mauk et al., 2004; Schippers et al., 2008; Dialynas et al., 2009) and solar winds (Gloeckler and Geiss, 1998; Chotoo et al., 2000; Mann et al., 2002; Marsch, 2006). Data from the Voyager spacecraft have shown that ions in the outer heliosphere region can be described by the kappa distribution (Decker and Krimigis, 2003; Decker et al., 2005).

In a recent work, we have proposed a non-adiabatic equation of state of dense matter (based on the Thomas–Fermi density distribution) that gives a very good description of concentration discontinuities of alkali metals at high pressures (Silveira, Camargo, and Caldas, 2021). In this work, we follow a parallel path to provide another non-adiabatic equation of state, now of dilute matter (essentially, a space plasma) with basis on the above referred kappa density distribution. Our proposed formulation generalizes the polytropic (or adiabatic) gamma index. An analytic expression is deduced for gamma. Then the equation of state is derived. The model is applied to describe the electron solar wind in the Earth's magnetopause. The above referred numerical relationship between the plasma pressure and electron concentration is fitted by our equation of state. As expected, no saddle point is exhibited by the fitting of the data.

2. Non-adiabatic Regime

Consider a plasma composed of species with mass μ, charge q, and concentration ν. In the absence of a magnetic field, the time evolution of the flow $\boldsymbol{u}$ of those particles is determined by the gradients of the electrostatic potential Φ and isotropic pressure P in the medium through the equation of motion

$$\mu\left[\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u}\right]=-q\nabla\Phi-\frac{\nabla P}{\nu}. \tag{1}$$

On the assumption of an adiabatic equation of state, the pressure gradient is described in terms of the concentration gradient by

$$\nabla P=\gamma k_{\mathrm{B}}\Theta\nabla\nu, \tag{2}$$

where γ is the polytropic (or adiabatic) index (the ratio of specific heats at constant pressure to volume), k_{B} is the Boltzmann constant, and Θ is assumed to be a constant with the dimension of temperature (the interpretation of Θ adequate for our purposes will be given further). It is a well known fact that, for $\gamma=1$ (typically, an isothermal electron gas), the stationary state of equilibrium of the system (the left hand side of Equation 1 vanishes) recovers the Boltzmann relation (Chen, 2016)

$$\nu=\nu_{\infty}\exp\left(-\frac{q\Phi}{k_{\mathrm{B}}\Theta}\right), \tag{3}$$

where ν_{∞} is the concentration of particles at infinity (the magnitude of the electrostatic potential falls off very rapidly with the distance from the charges in a plasma).

Inspired by the above-mentioned remark, we have deduced a non-adiabatic equation of state of dense matter (based on the Thomas–Fermi distribution) that gives a very good description of concentration discontinuities of alkali metals at high pressures (Silveira, Camargo, and Caldas, 2021). Here, we follow a parallel path to derive another non-adiabatic equation of state, now of dilute matter (essentially, a space plasma) based on the kappa distribution (Vasyliunas, 1968),

$$\nu = \nu_\infty \left[1 + \frac{(q\Phi)/(k_\mathrm{B}\Theta)}{(\kappa - 3/2)}\right]^{-(\kappa-1/2)}, \tag{4}$$

where $\kappa > 3/2$. In our formulation, we replace Equation 2 with

$$\nabla P = k_\mathrm{B}\Theta\nabla(\gamma\nu), \tag{5}$$

where the γ-index is now a function of the concentration, $\gamma = \gamma(\nu)$, not the polytropic index. As a result, the stationary state of equilibrium of the system (again, the left hand side of Equation 1 vanishes) is now determined by

$$-\frac{\nabla(\gamma\nu)}{\nu} = \frac{q\nabla\Phi}{k_\mathrm{B}\Theta}. \tag{6}$$

In the next section, we derive the γ-index with basis on the kappa density distribution, as given by Equation 4.

3. Generalized Gamma Index

Equation 4 may be rearranged as

$$(\kappa - 3/2)\left[\left(\frac{\nu}{\nu_\infty}\right)^{-1/(\kappa-1/2)} - 1\right] = \frac{q\Phi}{k_\mathrm{B}\Theta}. \tag{7}$$

Taking the gradient of Equation 7, we get

$$-\left(\frac{\kappa-3/2}{\kappa-1/2}\right)\left(\frac{\nu}{\nu_\infty}\right)^{-1/(\kappa-1/2)-1}\nabla\left(\frac{\nu}{\nu_\infty}\right) = \frac{q\nabla\Phi}{k_\mathrm{B}\Theta}. \tag{8}$$

Combining Equation 6 with Equation 8, we have

$$\nabla\left[\gamma\left(\frac{\nu}{\nu_\infty}\right)\right] = \left(\frac{\kappa-3/2}{\kappa-1/2}\right)\left(\frac{\nu}{\nu_\infty}\right)^{-1/(\kappa-1/2)}\nabla\left(\frac{\nu}{\nu_\infty}\right). \tag{9}$$

Equation 9 may be promptly put in the more compact form

$$\nabla\left[\gamma\left(\frac{\nu}{\nu_\infty}\right)\right] = \nabla\left[\left(\frac{\nu}{\nu_\infty}\right)^{(\kappa-3/2)/(\kappa-1/2)}\right]. \tag{10}$$

Integrating Equation 10, we obtain

$$\gamma = \left(\frac{\nu}{\nu_\infty}\right)^{-1/(\kappa-1/2)} + C\left(\frac{\nu_\infty}{\nu}\right), \tag{11}$$

where C is a constant to be determined.

To determine the constant C, we note that γ achieves its maximum value, say γ_∞, at the minimum value, ν_∞, of ν, because $\kappa > 3/2$. Therefore, we first differentiate Equation 11 to get

$$\gamma' = -\frac{1}{(\kappa-1/2)}\left(\frac{\nu}{\nu_\infty}\right)^{-1/(\kappa-1/2)-1} - C\left(\frac{\nu_\infty}{\nu}\right)^2, \tag{12}$$

where the prime denotes the derivative of γ with respect to ν/ν_∞. Subsequently, we require that Equation 12 vanishes at $\nu = \nu_\infty$, thereby implying

$$C = -\frac{1}{(\kappa-1/2)}. \tag{13}$$

Substituting Equation 13 in Equation 11, we have

$$\gamma = \left(\frac{\nu}{\nu_\infty}\right)^{-1/(\kappa-1/2)} - \frac{1}{(\kappa-1/2)}\left(\frac{\nu_\infty}{\nu}\right). \tag{14}$$

But now, we may easily compute the maximum of γ by evaluating Equation 14 at $\nu = \nu_\infty$,

$$\gamma_\infty = \left(\frac{\kappa-3/2}{\kappa-1/2}\right). \tag{15}$$

Finally, dividing Equation 14 by Equation 15, we obtain

$$\frac{\gamma}{\gamma_\infty} = \left(\frac{\kappa-1/2}{\kappa-3/2}\right)\left(\frac{\nu}{\nu_\infty}\right)^{-1/(\kappa-1/2)} - \frac{1}{(\kappa-3/2)}\left(\frac{\nu_\infty}{\nu}\right). \tag{16}$$

In the next section, we derive a non-adiabatic equation of state of dilute plasmas with basis on the γ-index, as given by Equation 16.

4. Equation of State of Kappa-Distributed Particles

Equation 5 may be promptly put in the more convenient form

$$\nabla P = k_\mathrm{B}\Theta\nu_\infty\gamma_\infty\nabla\left[\frac{\gamma}{\gamma_\infty}\frac{\nu}{\nu_\infty}\right]. \tag{17}$$

Integrating Equation 17, we get

$$P = B + k_\mathrm{B}\Theta\nu_\infty\gamma_\infty\left[\frac{\gamma}{\gamma_\infty}\frac{\nu}{\nu_\infty}\right], \tag{18}$$

where B is a constant to be determined.

To determine the constant B, we first evaluate the pressure at infinity,

$$P_\infty = B + k_\mathrm{B}\Theta\nu_\infty\gamma_\infty. \tag{19}$$

Subsequently, we anticipate that P_∞ takes the value

$$P_\infty = k_\mathrm{B}\Theta\nu_\infty\gamma_\infty. \tag{20}$$

Therefore, substituting Equation 20 in Equation 19, we have

$$B = 0. \tag{21}$$

We will soon justify our conjecture in Equation 20.

We substitute Equation 21 in Equation 18 to obtain

$$P = k_B \Theta \nu_\infty \gamma_\infty \left[\frac{\gamma}{\gamma_\infty} \frac{\nu}{\nu_\infty} \right]. \tag{22}$$

Finally, we substitute Equation 15 and Equation 16 in Equation 22 to find the non-adiabatic equation of state of a dilute plasma whose charged species satisfy the kappa distribution 4,

$$P = k_B \Theta \nu_\infty \left[\left(\frac{\nu}{\nu_\infty} \right)^{(\kappa-3/2)/(\kappa-1/2)} - \frac{1}{(\kappa - 1/2)} \right]. \tag{23}$$

We may now justify our prediction in Equation 20. As expected, in the limit $\kappa \to \infty$, Equation 23 recovers the widely known equation of state

$$P = \nu k_B \Theta, \tag{24}$$

typically satisfied by isothermal electrons in ordinary plasma phenomena (Chen, 2016). In the next section, we fit the numerical relationship between plasma pressure and electron concentration, as given by Hasegawa et al. (2005), with our Equation 23.

5. Solar Wind in Earth's Magnetopause

Equation 23 may be read as

$$P = b + c\nu^a, \tag{25}$$

where we have introduced the quantities

$$a = \left(\frac{\kappa - 3/2}{\kappa - 1/2} \right), \qquad b = -\frac{k_B \Theta \nu_\infty}{(\kappa - 1/2)}, \qquad c = \frac{k_B \Theta}{\nu_\infty^{-1/(\kappa-1/2)}}. \tag{26}$$

We may now give the interpretation of Θ adequate for our purposes. Such a quantity determines the energy scale, $k_B \Theta$, in Equation 4. For observational purposes, it should be appropriately related to the average kinetic energy, say $k_B \theta$, of the dilute gas. As shown by Lazar, Fichtner, and Yoon (2016), in systems where only high-velocity enhancements occur, due to the existence of a strong external source of energy and lack of sufficient (effective) collisions between particles, as is the case of the ambient solar wind in the Earth's magnetopause, the Maxwellian temperature, θ, should be related to Θ through

$$\theta = \left(\frac{\kappa - 3/2}{\kappa} \right) \Theta, \tag{27}$$

in order to avoid obtaining unphysical results. Note that in the limit $\kappa \to \infty$, the Maxwellian temperature $\theta \to \Theta$, that is, the replacement of Θ with θ does not formally affect our previous results.

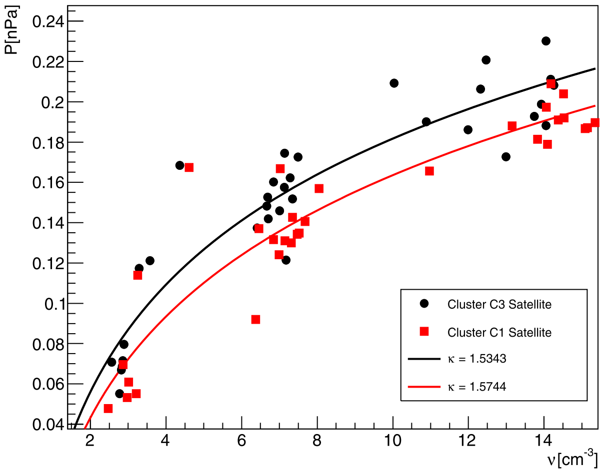

Figure 1 The fitting of the numerical relationship between plasma pressure, P [nPa], and electron concentration, ν [cm^{-3}], as given by Hasegawa et al. (2005), for the cluster C1 (squares) and C3 (circles) satellites, with our Equation 25. The curves intercept each other below the horizontal axis, and to the right of the vertical axis. The method of least squares together with Equations 26 have been used to obtain the values of the κ-index illustrated in the figure (see also Table 1). The κ-index is slightly larger than its lower limit, 3/2 (Vasyliunas, 1968), for both satellite data set, which actually characterizes electrons in the ambient solar wind (Livadiotis, 2015). As expected, no saddle point is exhibited.

Table 1 The geometrical parameters a, b, and c, introduced in Equation 25, obtained as a by-product of the application of the method of least squares to fit the curves exhibited in Figure 1 for the cluster C1 and C3 satellites. Such geometrical parameters provide the physical parameters κ, ν_∞ [cm^{-3}], and Θ [MK] via Equations 26, which we also show in the table. Finally, the κ-index together with Θ [MK] furnish the electron Maxwellian temperature, θ [MK], through making use of Equation 27, which again we show in the table.

Parameters	Cluster C3 Satellite	Cluster C1 Satellite
a	0.0332	0.0693
b	−2.2370	−0.9767
c	2.2407	0.9721
κ-index	1.5343	1.5744
ν_∞ [cm^{-3}]	2.6305	3.0175
Θ [MK]	63.738	25.200
θ [MK]	1.4265	1.1917

In Figure 1, we show the fitting of the numerical relationship between plasma pressure and electron concentration, as given by Hasegawa et al. (2005), for the cluster C1 and C3 satellites, with our Equation 25. The method of least squares has been used. As expected, no saddle point is exhibited.

In Table 1, we show the geometrical parameters a, b, and c, introduced in Equation 25, obtained as a by-product of the application of the method of least squares to fit the curves exhibited in Figure 1 for the cluster C1 and C3 satellites. Such geometrical parameters provide the physical parameters κ, ν_∞, and Θ via Equations 26, which we also show in Table 1. Finally, the κ-index together with Θ furnish the Maxwellian temperature, θ, through making use of Equation 27, which again we show in Table 1.

We obtain the following results. The κ-index is slightly larger than its lower limit, 3/2 (Vasyliunas, 1968), for both satellite data set, which characterizes electrons in the ambient solar wind (Livadiotis, 2015). The concentration of electrons at infinity (that is, at a null electric potential) $\nu_\infty \sim 2-4$ particles per cubic centimeter. The electron Maxwellian temperature $\theta \sim 1.1-1.5$ million degrees kelvin.

An average ion temperature of ~ 7 million degrees kelvin was estimated with basis on measurements effected by the CIS device at the magnetopause in the time interval analyzed by Hasegawa et al. (2005). Subsequently, numerical treatments of data from the Time History of Events and Macroscale Interactions during Substorms (THEMIS) mission (Angelopoulos, 2008) have indicated that the ion temperature could be slightly higher than $\sim 4-12$ times the electron temperature in the magnetosheath, at locations closer to the magnetopause (Wang et al., 2012). Following our above referred results, we conclude that the electron temperature is $\sim 4.9-5.9$ times lower than ~ 7 million degrees kelvin, which is slightly larger than the lower limit inferred by Wang et al. (2012).

Finally, it should be emphasized that our analytical formulation is consistent with a modeling of electron populations in the solar wind due to Lazar et al. (2017). Regarding the description of observational data, that theory supports a kappa-temperature depending on the κ-power of the distribution law as stated in Equation 27.

6. Conclusion

We have provided a non-adiabatic equation of state of a space plasma based on the kappa distribution. Our formulation has generalized the polytropic gamma index. An analytic expression has been deduced for gamma. Then the equation of state has been derived. The model has been applied to describe the electron solar wind in the Earth's magnetopause. A numerical relationship between the plasma pressure and electron concentration has been fitted by our equation of state. It has been found that: (i) the value of the kappa index is slightly larger than its lower limit, 3/2, thereby characterizing electrons in the ambient solar wind; (ii) the electron concentration varies between 2 and 4 particles per cubic centimeter at a null electric potential; (iii) the electron temperature varies between 1.1 and 1.5 million degrees kelvin.

The aforementioned G-S method of magnetic field reconstruction has been recently extended by Tian et al. (2020), in order to describe drift-mirror-like instabilities which drive a compressional wave that propagates sunward along with the background plasma retaining bottle-like structures (Burch et al., 2016). Such an anisotropic effect is a first issue that could be naturally approached by our analysis.

Our formulation is not restricted to the electron solar wind but could also be applied to investigate the flow of the ion species. Numerical treatments of satellite data have shown that the energy of the ion population is conserved in a frame of reference moving with the frozen-in interplanetary magnetic field, but the noise and temporal resolution of the data do not allow one to decide whether the total momentum is also conserved in this coordinate system (Němeček et al., 2020). The application of our model to the above referred ion streaming could help to settle that matter.

A set of empirical energy laws with two polytropic exponents inferred from observed mirror events has been recently used to numerically reconstruct the Earth's magnetosphere structure with temperature anisotropy (Hau, Chang, and Chen, 2020). The application of the theory proposed in this work to that problem could provide an explanation of those empirical polytropic exponents in terms of our non-adiabatic gamma index.

The above issues shall be addressed in forthcoming communications.

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Declarations

Declaration of Potential Conflicts of Interest The authors have no conflicts of interest to declare that are relevant to the content of this article.

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