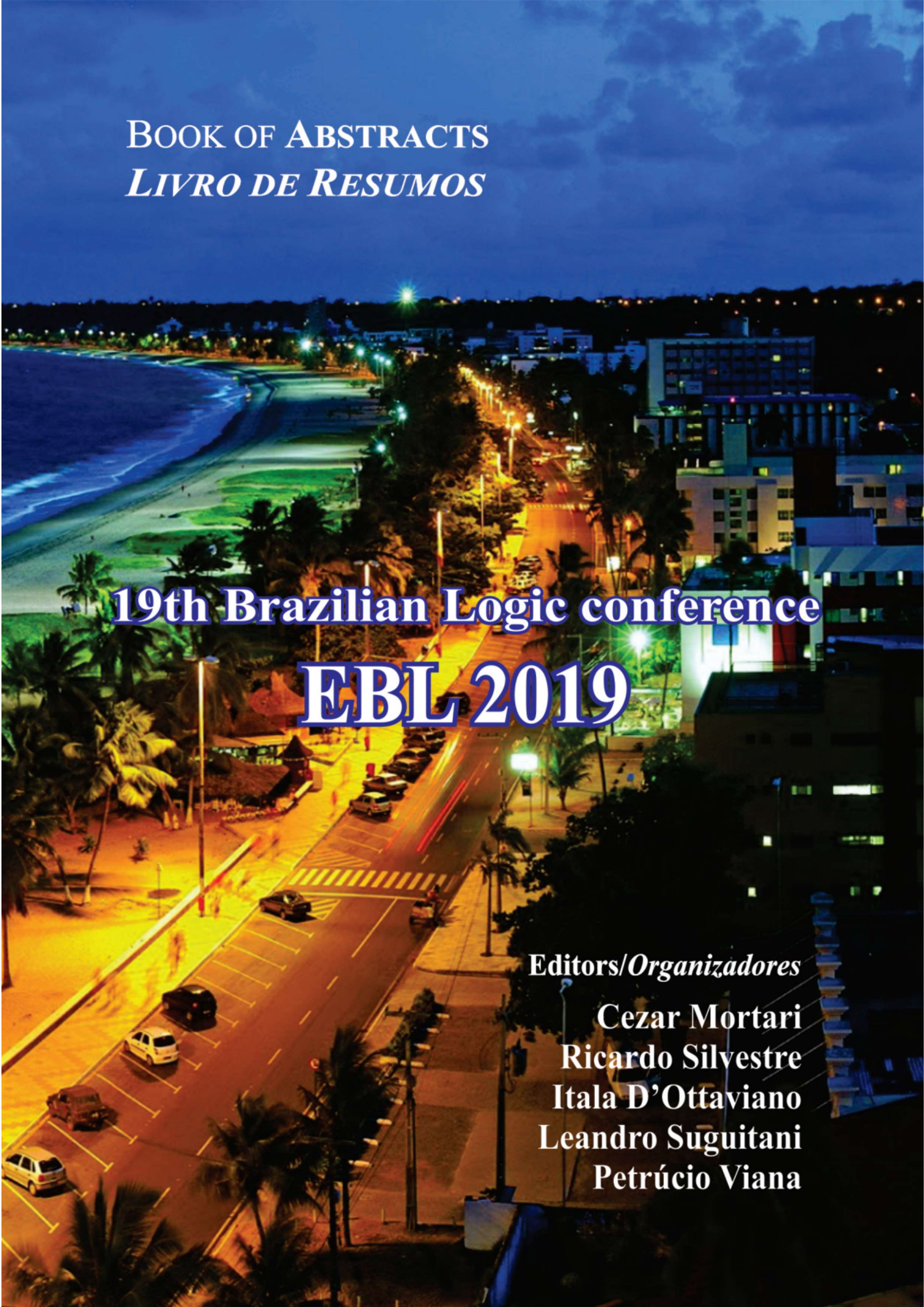

BOOK OF ABSTRACTS
LIVRO DE RESUMOS

19th Brazilian Logic conference
EBL 2019

Editors/*Organizadores*

Cezar Mortari
Ricardo Silvestre
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19TH BRAZILIAN LOGIC CONFERENCE

BOOK OF ABSTRACTS

XIX ENCONTRO BRASILEIRO DE LÓGICA

LIVRO DE RESUMOS

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Some contributions to Boolean-valued set theory regarding arrows induced by morphisms between complete Boolean algebras

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The expression "Boolean-valued set theory" in the title has two (related) meanings, both parametrized by a complete Boolean algebra $\mathbb{B}$:

(i) The canonical Boolean-valued models in set theory, $V^{\mathbb{B}}$, as introduced in the 1960s by D. Scott, P. Vopěnka and R. M. Solovay in an attempt to help understand the, then recently introduced, notion of *forcing* in ZF set theory developed by P. Cohen ([4], [5], [1]);

(ii) The (local) "set-like" behavior of categories called *topoi*, specially in the case of the (Boolean) topoi of the form $Sh(\mathbb{B})$ ([3], [2]).

The concept of a Boolean-valued model is nowadays a general model-theoretic notion, whose definition is independent from forcing in set theory: it is a generalization of the ordinary Tarskian notion of structure where the truth values of formulas are not limited to "true" and "false", but instead take values in some fixed complete Boolean algebra $\mathbb{B}$. More precisely, a $\mathbb{B}$-valued model M in a first-order language L consists of an underlying set M and an assignment $[\varphi]_{\mathbb{B}}$ of an element of $\mathbb{B}$ to each formula φ with parameters in M, satisfying convenient conditions.

The canonical Boolean-valued model in set theory associated to $\mathbb{B}$ is the pair $(V^{\mathbb{B}}, [\ ]_{\mathbb{B}})$, where both components are recursively defined. Explicitly, $V^{\mathbb{B}}$ is the proper class $V^{\mathbb{B}} := \bigcup_{\beta\in On} V^{\mathbb{B}}_{\beta}$, where $V^{\mathbb{B}}_{\beta}$ is the set of all functions f such that $dom(f) \subseteq V^{\mathbb{B}}_{\alpha}$, for some $\alpha < \beta$, and $range(f) \subseteq \mathbb{B}$. $(V^{\mathbb{B}}, [\ ]_{\mathbb{B}})$ is a model of ZFC in the sense that for each axiom σ of ZFC, $[\sigma]_{\mathbb{B}} = 1_{\mathbb{B}}$.

On the other hand, it is well known that $V^{\mathbb{B}}$ gives rise to a Boolean topos, $Set^{(\mathbb{B})}$, that is equivalent to the (Grothendieck) topos $Sh(\mathbb{B})$ of all sheaves over the complete Boolean algebra $\mathbb{B}$ ([1], [2]). The objects of $Set^{(\mathbb{B})}$ are equivalence classes of members of $V^{\mathbb{B}}$ and the arrows are (equivalence classes of) members f of $V^{\mathbb{B}}$ such that "$V^{\mathbb{B}}$ believes, with probability $1_{\mathbb{B}}$, that f is a function". A general topos encodes an internal (higher-order) intuitionistic logic, given by the "forcing-like" Kripke-Joyal semantics, and some form of (local) set-theory ([2], [3]); a Boolean Grothendieck topos is guided by a much more well behaved (Boolean) internal logic and set theory.

All the considerations above concern a fixed complete Boolean algebra $\mathbb{B}$. However, to the best of our knowledge, there are very few results on how Boolean-valued models are affected by the morphisms on the complete Boolean algebras that determine them: the only cases found are concerning automorphisms of complete Boolean algebras and complete embeddings (*i.e.*, injective Boolean algebra homomorphisms that preserves arbitrary suprema and arbitrary infima). In the present (ongoing) work, we consider and explore how more general kinds of morphisms between complete Boolean algebras $\mathbb{B}$ and $\mathbb{B}'$ induce arrows between $V^{\mathbb{B}}$ and $V^{\mathbb{B}'}$, and between their corresponding Boolean toposes $Set^{(\mathbb{B})}$ and $Set^{(\mathbb{B}')}$. In particular, we verify that these induced arrows are useful to understand and connect the corresponding Tarskian semantics, Boolean-valued semantics and Kripke-Joyal semantics.

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