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Language, Metaphor and Metaphysics: The Subjective Side of Science

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*"Why (it is) that my (the angel) name (you) ask!
And there (at the face of God) He blessed him."*
Genesis, XXXII, 30.

*"Metaphor is perhaps one of man's most fruitful potecialities.
Its efficacy verges on magic, and it seems a tool for creation
which God forgot inside His creatures when He made them."*
José Ortega y Gasset, The Dehumanization of Art, 1925.

1 Introduction

In Stern (2005a, 2006a,b), we analyzed questions concerning How objects (eigen-solutions) come to be, that is, How (eigen-solutions) emerge from a system's interaction with its environment. These questions had to do with laws, patterns, etc., expressed as sharp or precise hypotheses, and we argued that statistics had an important role to play in testing the validity of these sharp hypotheses.

It is natural then to ask, Why? Why do these objects are and interact the way they do? Why-questions claim for a causal nexus in a chains of events. Hence, the answers must be theoretical constructs based on interpretations of the laws used to describe these events. We will also analyze the interplay of the How and Why levels of inquire that, in the constructivist perspective, are not neatly stratified in separate hierarchical layers, but interact in complex (often circular) patterns. This article is dedicated to the investigation of these issues. As in the last articles, the discussion is exemplified by concrete mathematical models. In the process, we raise some interesting questions related to the practice of statistical modeling.

Section 2 begins the investigation of the importance of model interpretability discussing the rhetorical power of mathematical models, self-fulfilling prophecies and some related issues. This section presents a practical consulting case in Finance, concerning the detection of trading opportunities for intraday operations at the *BOVESPA* and *BM&F* financial markets. The statistical technique used in this example is the REAL classification tree algorithm, presented in Laureto et al. (1998).

The main issue studied in section 3 is language dependence. This section continues the investigation of model interpretability, and the eternal counterpoint of models for prediction vs. models for insight. Section 3 presents an example from Psychology, concerning dimensional personality models. These models are based on a dimension reduction technique known as Factor Analysis.

Sections 4 presents some simple examples from Physics related to the calculus of variations. Section 5 discusses efficient and final causal relations, teleological explanations, necessary and best world arguments, and the possibility or desirability of having multiple interpretations for the same model or multiple models for the same phenomenon. Section 6 addresses the role of metaphysical arguments in the construction of physical theories.

Section 7 presents some simple, but widely applicable, models based on "averages" computed over "all possible worlds", more specifically, as "path integrals" over "all possible trajectories" that a system could follow. The first example in this section relates to the Dirichlet problem, solved via Monte Carlo solution of linear systems. These techniques are based on a stochastic process known as Gaussian random walk, or Brownian motion. This section also comments on a generalization of this process known as Fractional Brownian Motion.

Section 8 discusses the relation of hypothetical models, mathematical equations, etc., to the "true nature" of "real objects". The importance of this relation in the history of science is illustrated with two cases: The Galileo affair, and the atomic or molecular hypothesis, as presented by L.Boltzmann, A.Einstein and J.Perrin. Section 9 presents our final remarks.

2 Rhetoric and Self-Fulfilling Prophecies

As a first example for discussion, we present a consulting case in finance. The goal of this project was the implementation of a model for the detection of trading opportunities for intraday operations at the *BOVESPA* and *BM&F* financial markets, for details, Lauretto et al. (1998). The first algorithms implemented in this project were based on Polynomial Networks, see Farlow (1984) and Madala and Ivakhnenko (1994), as well as some standard time series analysis pre-processing techniques, like de-trending, de-seasonalization, differencing, stabilization and linear transformations, see Box and Jenkins (1976) and Brockwell and Davis (1991). The predictive power of the Polynomial Network model was considered good enough to render a profitable return / risk performance.

Unfortunately, the Polynomial Network model was not well accepted, that is, it was seldomly used by the client for actual trading. The main complaint of the client was the model's lack of interpretability, that is, the model was perceived as being cryptic, a "black box" selecting strategic operations and computing predicted margins and success rates, but offering no understandable explanation of Why the selected strategies were recommended at those particular instances. This state of affairs was quite frustrating, for the client had never explicitly requested any explanatory functionality at the specification stage of the project, so the model was never conceived to offer such explanations. Moreover, with a fresh Ph.D. in Operations Research from Cornell University, I was well trained in the minutiae of Measure Theory and Hilbert spaces, but knew very little on how to build a model that could be understood by somebody else.

Nevertheless, since (good) costumers are always right, a second model was specified, developed and implemented. The first step was to find out what the client meant by an interpretable model. After a few brain storm sections with the client, we narrowed it down to two main conditions.

Understandable I/O and understandable rules:

-Understandable I/O: The first condition called for the model's input and output data to be already known, familiar or directly interpretable.

-Understandable rules: The second condition called for the transformation functions, re-presentation maps or derivation rules used in the model to be based in principles that were, again, already known, familiar or directly interpretable.

The second model used as input data some pre-processed price and volume trading data known as Technical Indicators. We will give further details on the nature of these technical indicators latter on in this section. For now, it is enough to know that these indicators are widely used at the financial markets, and that the client already had several experts in technical analysis. The statistical processing used in this model was based on a classification tree algorithm specially developed for this application. This algorithm, REAL or Real Attribute Learning Algorithm, was presented in Lauretto et al. (1998). For general classification tree algorithms, see Breiman (1993), Denison et al. (2002), Michie et al. (1994), and Mueller and Wysotzki (1994).

The REAL model turned out to be very successful. Its statistical performance, using the same performance metric specified in Lauretto et al. (1998), was almost as good as the Polynomial Network model. Moreover, the performance of the REAL model combined with a final interpretative analysis and go-ahead decision from the traders, surpassed the performance of the Polynomial Network model. Finally, the REAL model was perceived at interpretable and understandable, so it was actually put to use. Since a large part of our consulting fees depended on the results of actual trading, this was an important condition for getting fair economical compensation for all this intellectual endeavor.

As already mentioned, at the time we were (and still are) intrigued by many aspects related to model interpretation and understanding. In this section we begin to study this and similar situations. First, about the very need of explanations: Humans seem to be always thirsty for explanations. They need them in order to take action, and they want them to be based on already known schemata, as acknowledged in Damodaran (2003,ch.7,p.17):

"The Need for Anchors: When confronted with decisions, it is human nature to begin with the familiar and use it to make judgments. ...

The Power of the Story: For better or worse, human actions tend to be based not on quantitative factors but on story telling. People tend to look for simple reasons for their decisions, and will often base their decision on whether these reasons exist."

The rhetorical purpose and power of statistical models has been able to obtain in the statistical literature only a small fraction of its relative importance in the consulting practice, nevertheless, there are some remarkable exceptions, see for example, Abelson (1995, p.xiii):

" The purpose of statistics is to organize a useful argument from quantitative evidence, using a form of principled rhetoric. The word principled *is crucial. Just because rhetoric is unavoidable, indeed acceptable, in statistical presentations does not mean that you should say anything you please. "*

"Beyond the rhetorical function, statistical analysis also has a narrative role. Meaningful research tells a story with some point to it, and statistics can sharpen the story."

Let us now turn our attention to the REAL model's inputs, the Technical Indicators, also known as Charting Patterns, for a general description, see Damodaran (2003,ch.7), for some of the indicators used in the REAL project, see Colby (1988) and Murphy (1986). Technical indicators are primary interpreted as behavioral patterns in the markets or, more appropriately, as behavioral patterns of the market players. Damodaran defines five groups that categorize the indicators according to the dominant aspects of these behavioral patterns. A condensed description of these five groups of indicators is given in Damodaran (2003,ch.7,p.46-47):

1- External Forces / Large Scale Indicators: *"If you believe that there are* long-term *cycles in stock prices, your investment strategy may be driven by the cycle you subscribe to and where you believe you are in the cycle."*

2- Lead / Follow Indicators: *"If you believe that there are some traders who trade ahead of the market, either because they have better analysis tools or information, your indicators will follow these traders - specialist short sales and insider buying/selling, for instance - with the objective of piggy-backing on their trades."*

3- Persistence / Momentum Indicators: *"With momentum indicators, such as relative strength and trend lines, you are assuming that markets often learn slowly and that it takes time for prices to adjust to true values."*

4- Contrarian / Over Reaction Indicators: *"Contrarian indicators such as mutual fund holdings or odd lot ratios, where you track what investors are buying and selling with the intention of doing the opposite, are grounded in the belief that markets over react."*

5- Change of Mind / Price-Value Volatility Indicators: *"A number of technical indicators are built on the presumption that investors often change their views collectively, causing shifts in demand and prices, and that patterns in charts - support and resistance lines, price relative to a moving average- can predict these changes."*

At this point, it is important to emphasize the dual nature of technical indicators: They tell us something about what is going on with the trading market, but they also tell us something about what is going on with the traders themselves. They portray dynamical patterns of the market that reflect behavioral patterns of the traders.

The rhetorical and psychological aspects captured by the REAL model that have been commented so far should help to explain two characteristics of vital importance for the success of the consulting case presented:

- The importance of having both, good predictive power and good rhetorical power, in order to motivate the client to take action based on the analyses provide by the model.

- The importance of being able to combine and integrate the analyses provided by the model with expert opinion.

Technical indicators are often accused of being based in self-fulfilling prophecies, use over-simplified formulas, consider shallow and naive behavioral patterns, have no fundamental economic grounding, etc. From a pragmatical perspective, market analysts do not usually care about technical analysis compatibility with fundamental economic theories, the use (or lack there off) sophisticated mathematics, etc. What is important is its ability to detect trading opportunities. From a conceptual perspective, each of these analyses does tell a story about a cyclic reinforcement or correction (positive or negative feed-back) mechanism in the financial system. What is peculiar about self-fulfilling prophecies is that the collective story telling activity is a vital link in the feed-back mechanism. No wonder than that how good the story itself is perceived to be by the market players should play an important role in the prognostic that (if, how) the prophecy will be realized. In this way one can make sense of the comment stated in Murphy (1986, p.19):

"The self-fulfilling prophecy (argument) is generally listed as a criticism of charting. It might be more appropriate to label it as a compliment."

The importance of the psychological aspects of the models studied in this section motivate us to take a look, in the next section, on some psychometric statistical models.

3 Language, Metaphor and Insight

In Stern (2005a) we established the dual role played by Statistics in scientific research, namely, predicting experimental events, and testing hypotheses. We also established that, in the constructivist view, these hypotheses are often expressed as equations in some mathematical model. In the last section we began to investigate the importance of the interpretation of these models. The main goal of this section is to continue the investigation of the subjective aspects of, that is, the subject's understanding of, or insight provided by statistical or mathematical models. Perhaps the best way to start this section is with the celebrated motto of Richard Hamming, stated next, in several versions:

"The purpose of models is insight, not numbers."

"The purpose of computing is insight, not numbers."

"The purpose of numbers is insight, not numbers."

Dictionary definitions of Insight include:

- A penetrating, deep or clear perception of a complex situation;
- Grasping the inner or hidden nature of things;
- An intuitive or sudden understanding.

The illustrative case presented in this section is based on psychological models of personality. Many personality models rely on symmetric configurations known as "mandala" schemata, see for example Jung (1968). A good example of such a model is the five elements model of traditional Chinese alchemy. The Chinese traditional labels for the five elements, and its associated personality traits are:

1- Fire: Extroverted, emotional, emphatic, self-aware, sociable, talkative.
2- Earth: Caring, grounding, supportive, stable, protective, worried.
3- Metal: Analytical, controlling, logical, meticulous, precise, zealous.
4- Water: Anxious, deep, insecure, introspective, honest, nervous.
5- Wood: Angry, assertive, creative, decisive, frustrated, leading.

Interactions between elements is conceived as double feed-back cycle, represented by a pentagram inscribed in a pentagon. The pentagon arrows or external cycle represent the creation, stimulus or positive feed-back in the system, while the pentagram arrows or internal cycle represent the destruction, control or negative feed-back in the system. The traditional representation of these systemic generative mechanisms or causal relations are:

Pentagon: fire [*calcinates to* ⟩ earth [*harbors*⟩ metal [*condenses*⟩ water [*nourishes*⟩ wood [*fuels*⟩ fire.

Pentagram: fire [*melts*⟩ metal [*cuts*⟩ wood [*incorporates*⟩ earth [*absorbs*⟩ water [*extinguishes*⟩ fire.

This double feed-back system allows the representation of system with complex interconnections and nontrivial dynamical properties. In fact, the systemic interconnections are considered the key for the understanding a generic five-element model, rather than any superficial resemblance to the five elements' traditional labels. It is an entertaining exercise to compare and relate the five alchemical elements listed above with the five groups of technical indicators presented in the last section, or with the big-five personality factors presented next.

Modern psychometric models constitute testable scientific hypotheses and require a solid statistical analysis. Factor Analysis has been one of the tools of choice for building psychometric models, and it is the tool used in the examples discussed next. For a synthetic introduction to factor analysis, see Ghaharamani and Hilton (1997) and Everitt (1984). For some of the details, see Abadir and Magnus (2005), Harville (2000) and Russel (1998). For the technical issue of factor rotation, see Browne (1974, 2001), Jennrich (2001, 2002, 2004) and Bernaards and Jennrich (2005).

In Allport and Odbert (1936) the authors presented their Lexical Hypothesis, according to which, important aspects of human life correspond to words in the spoken language. Also the number of corresponding terms in the lexicon is supposed to reflect the importance of each of these aspects:

"Those individual differences that are most salient and socially relevant in peoples lives will eventually become encoded into their language; the more important such a difference, the more likely is it to become expressed as a single word."

Subsequent work converged on five main factors or personality traits. These are the "big-five" dimensions in the personality OCEAN. Further details on the meaning of these factors can be found in Shelder and Westen (2004), from the list of the most relevant factor loadings. The OCEAN labels, ordered according to the statistical relevance of the corresponding factors, are:

1- Extraversion, Energy, Enthusiasm;
2- Agreeableness, Altruism, Affection;
3- Conscientiousness, Control, Constraint;
4- Neuroticism, Negative Affectivity, Nervousness;
5- Openness, Originality, Open-mindedness.

Subsequent studies pointed to the "existence" of more factors, for a review of several of such models, see Widiger and Simonsen (2005). We focus our attention in the 12-factor model of Shelder and Westen. After the publication of the 12-factor model, one can follow a heated debate in the literature about the necessity (or not) for more than 5 factors. In the quotation following this paragraph, Shelder and Westen (2004, p.1752-1753) pinpoint the issue of language dependence in the description of reality. This is an issue of paramount importance in cognitive constructivism, and one of the main topics analyzed in this section.

"Applying the Lexical Hypothesis to Personality Disorders:

Ultimately, the five-factor model is a model of personality derived from the constructs and observations of lay-people, and it provides an excellent map of the domains of personality to which the average layperson attends. However, the present findings suggest that the five-factor model is not sufficiently comprehensive for describing personality disorders or sophisticated enough for clinical purposes.

In contrast to laypeople, practicing clinicians devote their professional lives to understanding the intricacies of personality. They develop intimate knowledge of others lives and inner experience in ways that may not be possible in everyday social interaction. Moreover, they treat patients with variants of personality pathology that laypeople encounter only infrequently (and are likely to avoid when they do encounter it). One would therefore expect expert clinicians to develop constructs more differentiated than those of lay observers.

Indeed, if this were not true, it would violate the lexical hypothesis on *which the five-factor model rests: that language evolves over time to reflect what is important. To the extent that mental health professionals observe personality with particular goals and expertise, and observe the more pathological end of the personality spectrum, the constructs they consider important should differ from those of the average layperson."*

The issue of language dependence is very important in cognitive constructivism. For further discussions, see Maturana (1988, 1991).

So far, we have stressed the lexical aspect of language, that is, the importance of the available vocabulary in our description of reality. In the remaining part of this section we focus on the symbolic or figurative use of the language constructs in these descriptions. We proceed examining in more detail the factor analysis model.

Factor analysis is a dimensionality reduction technique. A given factor analysis model renders a 'simpler' object, in the space of reduced dimension, as a representation of (similar or analogous to) a complex 'real' object, in the space of full dimension. When using dimension reduction techniques, one hopes to acquire a good understanding of the simple object, and that some of this understanding can be (validly) transferred to the complex object. The Greek word metaphor means transport or transfer. A linguistic metaphor transfers some of the characteristics of one object, called the source or vehicle, to a second, distinct object, called the target, tenant or topic, see Lakoff and Johnson (2003). Hence, the process of using and interpreting these models can be conceived as metaphorical.

For reasons similar to the ones studied in the last section, most users of a personality model demand the model to be statistically sound, and many users also demand the model to be interpretable, to give them good insights on their patient's personality and problems. A good model should not only be useful in predicting recovery rates or medication efficacy, but also help them giving good insights for their counseling or therapeutic work.

Paraphrasing Vega-Rodríguez (1998):

The metaphorical mechanism should provide an articulation point between the empirical and the hypothetical, the rational and the intuitive, between calculation and insight.

The main reason for our choice of factor analysis as the statistical technique used to illustrate this section, is its capability of, with efficiency and transparency, take care of these two goals: building sound statistical models that, at the same time, provide intuitive interpretations. In Factor Analysis, there is first set of "estimation and identification tools", like ML or MAP (maximum likelihood or a posteriori) optimization, hypothesis testing and model selection, that take care of the first goal; meanwhile, there is a second set of "representation tools", like orthogonal and oblique factor rotation techniques, that are devoted entirely to the second goal.

Factor rotation tools are capable of reconfiguring the structure of a given factor analysis model, so that its probabilistic explanation power is maintained, while its heuristic explanation power is maximized. Factor rotations are performed by optimization of some objective criteria, like maximum sparsity or maximum entropy. The optimal solution (for each criterion) is unique and, hopefully, greatly facilitates model interpretability.

How important are heuristic arguments in other areas of science? Do other application areas require a similar rhetorical role of statistical or mathematical models? We will try to answer these questions in sections 5 to 8, discussing the role played by similar heuristic (or meta-theoretical) arguments used in physics. In sections 2 and 3 we dealt with applications areas where the very spinning of the threads was a big part of fabricating the text(ure). Nevertheless, one can have the false idea that the constructivist approach in more appropriate to high level, soft science areas, than it is to low level, rock bottom Physics. This is certainly not the case, although it is a widely spread misconception. In section 5 to 8 we analyze the role played in science by a very special form of heuristic arguments known as metaphysics. We begin this journey in the next section, studding a simple and seemingly innocent mathematical puzzle.

4 Necessary and Best Worlds

Let us begin this section solving a simple mathematical puzzle known as the lifeguard's problem. We solve the problem directly, using only elementary calculus. The epistemological implications of this simple problem are far reaching, as discussed in the next section.

Consider a beach, where the shore line is represented by $x = a$. A lifeguard, at position $(x, y) = (0, 0)$, spots a drowning person, at position $(x, y) = (a + b, d)$. In the athletic track, the lifeguard car run at top speed c, on the sand he can run at speed c/ν_1, and on water he can only swim at speed c/ν_2, $1 < \nu_1 < \nu_2$. Let $(x, y) = (a, y(a))$ be the point where he enters the water. He wants to reach position $(a+b, d)$ as soon as possible, so what is the optimal value $y(a) = z$? The shortest path in an homogenous medium is a strait line, so the optimal trajectory is a broken line, from $(0, 0)$ to (a, z), and then from (a, z) to $(a + b, d)$. The total travel time is $J(z)/c$, where

$$J(z) = \nu_1 \sqrt{a^2 + z^2} + \nu_2 \sqrt{b^2 + (d - z)^2} \, .$$

Since we want $J(z)$ at a minimum, we set its derivative to zero,

$$\frac{dJ}{dz} = \nu_1 \frac{-2z}{2\sqrt{a^2 + z^2}} + \nu_2 \frac{-2(d-z)}{2\sqrt{b^2 + (d-z)^2}} = 0 \; .$$

Hence, we should have

$$\nu_1 \sin(\theta_1) = \nu_2 \sin(\theta_2) \; .$$

Professional lifeguards claim that this simple model can be improved dividing the sand in a dry band, ν_1, and a wet band, ν_2, and the water in a shallow band, ν_3, and a deep band, ν_4, where the different media 'resistance' indices obey $\nu_4 > \nu_3 > \nu_1 > \nu_2 > 1$. We could solve the improved model in a similar way, but there is a general formalism to solve this kind of 'variational' problems known as the Euler-Lagrange equation, for a didactic introduction see Krasnov et al. (1973), Leech (1963) and Marion (1970).

The trigonometric relation obtained in the last equation, $\nu(x)\sin(\theta) = K$, is known in optics as Snell's law, where ν is the medium refractive index. Snell's law explains the refraction (bending) of a light ray incident to the surface separating two distinct optic media.

The variational problem solved in the last section was proposed by Pierre de Fermat in 1662 to 'explain' Snell's law. Fermat's *principle of least time* states that a ray of light, from one point to another, follows the path which is traversed in the least time.

Notice that Fermat enunciated his principle *before* any measurement of the velocity of light. The first quantitative estimate of the speed of light, in sidereal space, was obtained by O.Romer in 1676. He measured the Doppler effect on the period of Io, a satellite of Jupiter discovered by Galileo in 1610, that is, he measured the violet and red shifts, i.e., the variation for shorter and longer in the observed periods of Io, as the Earth traveled in its orbit towards or away from Jupiter. Romer final estimate was $c = 1$ astronomical unit (earth's orbit radius) per 11 minutes; Today's value is around $1AU/8'20''$. The first direct measurements of the comparative speed of light in distinct material media (air and water) was obtained by L.J.B. Foucault, using a rotating mirror device, almost two centuries latter, in 1850, see Tobin (1993) and Jaffe (1960). See also Ronchi (1970) and Sabra (1981) for an historical perspective of several competing theories of light.

Snell's "law" is an example of mathematical model that dictates a "necessary world", stating, plain and simple, how things "have to be". In contrast, Fermat's "principle" is a theoretical construct that elects a "best world" according to some criterion used to compare "possible worlds".

Fermat's principle is formulated minimizing the integral of $ds = 1/dt$. Leibnitz, Euller, Mauperius, Lagrange, Jacobi, Hamilton, and many others were able to reformulate Newtonian mechanics in a similar way, minimizing the integral of a quantity $ds = L\,dt$, where the Lagrangian, L, is the difference between the kinetic energy (Leibnitz' vis viva), $(1/2)mv^2$, and the potential energy of the system (Leibnitz' vis morta). In mechanics the minimized quantity is called action, hence these formulations are called principles of minimum action or principles of least action.

5 Efficient and Final Causes

Necessary world vs. best world formulations of optics or mechanics discussed in the last section are historically connected to the discussion of the metaphysical concepts of efficient and final causes. This terminology goes back to Aristotle, that in his metaphysics distinguishes four forms of causation, that is, four types of answers that can be given to a Why-question. They are:

- Material cause: Because it is made of, or its constituent parts are ...
- Formal cause: Because it has the form of, or is shaped like ...
- Efficient cause: Because it is produced by, or by the agency of ...
- Final cause: Because it is intended to, or has the purpose of ...

For a general overview of the theme of efficient and final causes in the history of XVII and XVIII century Physics, see Brunet (1938), Dugas (1988), Pulte (1989), Goldstine (1980), Wiegel (1986) and Yourgrau and Mandelstam (1979). This is the subject of this section.

Newtonian mechanics is formulated using only efficient causes, that is, an existing force acts on a particle (or body) producing a movement described by the Newtonian differential equations. In contrast, Fermat's principle is formulated using a final cause, that is, the trajectory followed by the particle (or light ray) is the one that will optimize some characteristic, given the trajectory's extremes, i.e., its original and final positions. These formulations are also called teleological, from the Greek telos=purpose. A general discussion of teleological principles in this context was given by Leibniz in his Specimen Dynamicum of 1695, translated in Loemker (1969, p.442).

"In fact, as I have shown by the remarkable example of the principles of optics, ...(that) final causes may be introduced with great fruitfulness even into the special problems of physics, not merely to increase our admiration for the most beautiful works of the supreme Author, but also to help us make predictions by means of them which could not be as apparent, except perhaps

hypothetically, through the use of efficient cause... It must be maintained in general that all existent facts can be explained in two ways - through a kingdom of power or efficient causes and through a kingdom of wisdom or final causes... Thus these two kingdoms everywhere permeate each other, yet their laws are never confused and never disturbed, so the maximum in the kingdom of power, and the best in the kingdom of wisdom, take place together."

Euler and Maupertuis generalized Fermat and Leibniz arguments, deriving Newtonian mechanics from the least action principle. The Principle of Least Action, was stated in 1748 in Maupertuis (1756, IV, p.36) as:

"Lois du Mouvement, Principe Général:

Lorsqu'il arrive quelque changement dans la Nature, la quantité d'action nécessaire pour ce changement, est la plus petite qu'il soit possible.

La quantité d'action est le produit de la masse des corps, par leur vîtesse et par l'espace qu'ils percourent. Lorsqu'un corps est transporté d'un lieu dans un outre, l'action est d'autant plus grande que la masse est plus grosse, que la vîtesse est plus rapide, que l'espace par lequel il est transporté est plus long."

The translation, available at wikisource, reads:

"Laws of Movement, General Principle:

When a change occurs in Nature, the quantity of action necessary for that change is as small as possible.

The quantity of action is the product of the mass of the bodies times their speed and the distance they travel. When a body is transported from one place to another, the action is proportional to the mass of the body, to its speed and to the distance over which it is transported."

Maupertuis also used the same theological arguments as Leibniz, regarding the harmony between efficient and final causes. See for example his *Accord de Différents Lois de la Nature, qui avoient jusqu'ici paru incompatibles*, in Maupertuis (1756, IV, p.20-23):

"Accord Between Different Laws of Nature, that seemed incompatible. ...

I know the distaste that many mathematicians have for final causes applied to physics, a distaste that I share up to some point. I admit, it is risky to introduce such elements; their use is dangerous, as shown by the errors made by Fermat (and Leibniz(?)) in following them. Nevertheless, it is perhaps not the principle that is dangerous, but rather the hastiness in taking as a basic principle that which is merely a consequence of a basic principle.

One cannot doubt that everything is governed by a supreme Being who has imposed forces on material objects, forces that show his power, just as he has fated those objects to execute actions that demonstrate his wisdom. The *harmony between these two attributes is so perfect, that undoubtedly all the effects of Nature could be derived from each one taken separately. A blind and deterministic mechanics follows the plans of a perfectly clear and free Intellect. If our spirits were sufficiently vast, we would also see the causes of all physical effects, either by studying the properties of material bodies or by studying what would most suitable for them to do.*

The first type of studies is more within our power, but does not take us far. The second type may lead us stray, since we do not know enough of the goals of Nature and we can be mistaken about the quantity that is truly the expense of Nature in producing its effects.

To unify the certainty of our research with its breadth, it is necessary to use both types of study. Let us calculate the motion of bodies, but also consult the plans of the Intelligence that makes them move.

It seems that the ancient philosophers made the first attempts at this sort of science, in looking for metaphysical relationships between numbers and material bodies. When they said that God occupies himself with geometry, they surely meant that He unites in that science the works of His power with the perspectives of His wisdom.

From the all too few ancient geometers who undertook such studies, we have little that is intelligible or well-founded. The perfection which geometry has acquired since their time puts us in a better position to succeed, and may more than compensate for the advantages that those great minds had over us."

Some of the metaphysical explanation used by Leibniz and Maupertuis are based on theological arguments, and these arguments can be seen as late inheritances of medieval philosophy. This kind of metaphysical argument faded away from mainstream science after the XVIII century. Nevertheless, in the next century, the (many variation of the) least action principle developed more powerful formalisms and found several new applications in physics, see Goldstine (1980). As stated in Yourgrau and Mandelstam (1979, ch.14, The Significance of Variational Principles in Natural Philosophy),

"Towards the end of the (XIX) century, Helmholtz invoked, on purely scientific grounds, the principle of least action as a unifying scientific natural law, a 'leit-motif' dominating the whole of physics, Helmholtz (1887).

'From these facts we may even now draw the conclusion that the domain of validity of the principle of least action has reached far beyond the boundaries of the mechanics of ponderable bodies. Maupertuis' high hopes for the absolute general validity of his principle appear to be approaching their fulfillment, however slender the mechanical proofs and however contradictory the metaphysical speculation which the author himself could at the time ad-

duce in support of his new principle. Even at this stage, it can be considered as highly probable that it is the universal law pertaining to all processes in nature. ... In any case, the general validity of the principle of least action seems to me assured, since it may claim a higher place as a heuristic and guiding principle in our endeavor to formulate the laws governing new classes of phenomena. Helmoltz (1887).' "

Yourgrau and Mandelstam (1979) continue to stress the importance of teleological arguments in theoretical physicists, this time quoting Planck,

"Such a principle, he (Planck) asserts, suggests to a person free from prejudice the presence of a rational, purposive will governing nature, for a physical system must chose that route which directs it most easily towards its objective, Planck (1937):

'In fact, the least-action principle introduces an entirely new idea into the concept of causality: The 'causa efficiens', which operates from the present into the future and makes future situations appear as determined by earlier ones, is joined by the 'causa finalis' for which, inversely, the future -namely, a definite goal- serves as the premise from which there can be deduced the development of the processes which lead to this goal.' "

In Biology and related fields, teleological arguments are perceived to be problematic. This perception has good reasons, resulting from the frequent abuse of teleological arguments, in the form of crude fallacies or obvious tautologies, offering bogus arguments to explain whatever statement in needed of some support. Maupertuis himself, the proponent of the first general least action principle, is aware of such problems, as clearly stated in the text of his we just quoted. Why then did important theoretical physicist insist in keeping this kind of argument among the regular tools of the trade? In 1915 Max Planck offers the following answare in the encyclopedia Die Kultur der Gegenwart (1915, p.68):

"As long as there exists physical science, its highest desirable goal had been the solution of the problem to integrate all natural phenomena observed and still to be observed into a single simple principle which permits to calculate all past and, in particular, all future processes from the present ones. It is natural that this goal has not been reached to date, nor ever will it be reached entirely. It is well possible, however, to approach it more and more, and the history of theoretical physics demonstrates that on this way a rich number of important successes could already be gained; which clearly indicates that this ideal problem is not merely utopical, but eminently fertile. Among the more or less general laws which manifest the achievements of physical science in the course of the last centuries, the Principle of Least Action is probably the one which, as regards form and content, may claim

to come nearest to that final ideal goal of theoretical research."

See also Planck (1915, p.71-72):

"Who instead seeks for higher connections within the system of natural laws which are most easy to survey, in the interest of the aspired harmony will, from the outset, also admit those means, such as reference to the events at later instances of time, which are not utterly necessary for the complete description of natural processes, but which are easy to handle and can be interpreted intuitively."

6 Modern Metaphysics

In this section we continue to investigate the use and nature of metaphysical principles in theoretical Physics. The adjective metaphysical is a word that, like so many others, has acquired a positive (meliorative, eulogistic) and a negative (pejorative, derogatory, dyslogistic) connotation.

Logical positivism or logical empiricism was a mainstream school in the early XX century philosophy of science. One of the goals of the positivist school was to build science using only empirical (observable) concepts. According to this view all non-empirical or directly observable terms are cognitively meaningless, including all metaphysical concepts or principles.

Perhaps influence by this school, Yourgrau and Mandelstam (1979, p.10) downplay the role of metaphysics in the development of science, or give it a very humble definition:

"In conformity with the scope of our subject, the speculative facets of the thinkers under review have been emphasized. Historically by far more consequential were the positive contributions to natural science, contributions which transferred the emphasis from 'a priori' reasoning to theories based upon observation and experiment. Hence, while the future exponents of least principles may have been guided in their metaphysical outlook (1) by idealistic background we have described, they had nevertheless to present their formulations in such fashion that the data of experience would thus be explained. A systematic scrutiny of the individual chronological stages in the evolution of minimum principles can furnish us with profound insight into continuous transformation of a metaphysical canon to an exact natural law.

(1) By 'metaphysical outlook' we comprehend nothing but those general assumptions which are accepted by the scientist."

Maybe the definition of Metaphysics used by Yougrau is a bit too vague. We believe that a deeper understanding of the role played by metaphysics in modern theoretical physics can be found in Einstein (1950), also quoted in Chaitin (2002):

"There exists a passion for comprehension, just as there exists a passion for music. That passion is rather common in children, but gets lost in most people later on. Without this passion, there would be neither mathematics nor natural science. Time and again the passion for understanding has led to the illusion that man is able to comprehend the objective world rationally, by pure thought, without any empirical foundations-in short, by metaphysics. I believe that every true theorist is a kind of tamed metaphysicist, no matter how pure a 'positivist' he may fancy himself. The metaphysicist believes that the logically simple is also the real. The tamed metaphysicist believes that not all that is logically simple is embodied in experienced reality, but that the totality of all sensory experience can be 'comprehended' on the basis of a conceptual system built on premises of great simplicity. The skeptic will say that this is a 'miracle creed.' Admittedly so, but it is a miracle creed which has been borne out to an amazing extent by the development of science."

A tentative definition for metaphysics based on a similar understanding, that seems appropriate to this specific context, can be found in Muntean (2006, p.7):

"Metaphysics can be considered the meta-science of physics in the sense that it provides the set of conditions of its comprehensibility. If physics is a representation of the real world, the principles of this representation are meta-physical, i.e. non-physical conditions of physical representation."

"Principles are all logical and therefore metaphysical. Metaphysics is nothing more nor less than the science of the comprehensibility of physics, and logic is the mental instrument which mediates the process of cogitation."

So far we have been arguing that the laws of mechanics in integral form, given by the least action principle, and its associated teleological metaphysical concepts, should be accepted along side the "standard" formulation of mechanics in differential form, namely, the differential equations of Newtonian mechanics. However, Schlick (1979, V.1, p.297) proposes a complete inversion of the empirical / metaphysical status of the two formulations, see also Stöltzner (2003). According to Schlick's view, the integral or macro-law formulation is directly grounded in observable quantities, while the differential or micro-law formulation is based on non-empirical concepts:

"That the event at a point depends only on those processes occurring in its immediate temporal and special neighborhood, is expressed in the fact that space and time appear in the formulae of natural laws as infinitely small quantities; these formulae, that is, are differential equations. We can also describe them in a readily intelligible terminology as micro-laws. *Through the mathematical process of integration, there emerge from them the* macro-laws

(or integral laws), which now state natural dependencies in their extension over spatial and temporal distances. Only the latter fall within experience, for the infinitely small is not observable. The differential laws prevailing in nature can therefore be conjectured and inferred only from the integral laws, and these inferences are never, strictly speaking, univocal, since one can always account for the observed macro-laws by various hypotheses about the underlying micro-laws. Among the various possibilities we naturally choose that marked by the greatest simplicity. It is the final aim of exact science to reduce all events to the fewest and simplest possible differential laws."

From this and other examples presented in sections 5 to 8, we come to the conclusion that, no matter the formulation in use, metaphysical concepts are unavoidable. The positivist desire of a metaphysical free science, using only concepts with 'direct' empirical content, seems to be a chimera. Moreover, metaphysical arguments are essential to build our intuition. Without this intuition, physical reasoning would be downgraded to cranking the formalism, either by algebraic manipulation of the symbolic machinery or by sheer number crunching. Planck (1950, p.171-172), states:

"To be sure, it must be agreed that the positivistic outlook possesses a distinctive value; for it is instrumental to a conceptual clarification of the significance of physical laws, to a separation of that which is empirically proven from that which is not, to an elimination of emotional prejudices nurtured solely by customary views, and it thus helps to clear the road for the onward drive of research. But Positivism lacks the driving force for serving as a leader on this road. True, it is able to eliminate obstacles, but it cannot turn them into a productive factors. For its activity is essentially critical, its glace is directed backward. But progress, advancement requires new associations of ideas and new queries, not based on the results of measurements alone, but going beyond them, and toward such things the fundamental attitude of Positivism is one of aloofness.

Therefore, up to quite recently, positivists of all hues have also put up the strongest resistance to the introduction of atomic hypotheses ... "

The last example cited by Planck gives another excellent illustration for the point under discussion, and for other topics we want to discuss as well. The next section gives a short introduction to one of the most important models related to the debate about the atomic hypothesis, namely, the Brownian motion and other related models.

7 Averages over All Possible Worlds

We are interested in the Dirichlet problem, describing the steady state temperature at a two dimensional plate, given the temperature at its border. The partial differential equation that the temperatures, $u(x, y)$, must obey in the Dirichlet problem is known as the 2-dimensional Laplace equation, see Butkov (1968, Ch.8),

$$\text{div grad } u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \ .$$

From elementary calculus we have the forward and backward finite difference approximations for a partial derivative, see Demidovich (1976),

$$\frac{\partial u}{\partial x} \approx \frac{u(x+h, y) - u(x, y)}{h} \approx \frac{u(x, y) - u(x-h, y)}{h} \ .$$

Using these approximations twice, we obtain the symmetric or central finite difference approximation for the second derivatives,

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u(x+h, y) - 2u(x, y) + u(x-h, y)}{h^2} \ ,$$

$$\frac{\partial^2 u}{\partial y^2} \approx \frac{u(x, y+h) - 2u(x, y) + u(x, y-h)}{h^2} \ ,$$

substitution in the Laplace equation give us the "next neighbors' mean value" equation,

$$u(x, y) = \frac{1}{4} \left(u(x+h, y) + u(x-h, y) + u(x, y+h) + u(x, y-h) \right) \ .$$

We can use the last equation to form a linear system for the temperatures in a rectangular grid. The unknown variables correspond to the temperatures at the interior points of the grid, and the right hand side to the temperature known at the boundary points.

If we knew the temperature at the four neighbors of a given grid point, $[x, y]$, we could estimate its temperature, $u(x, y)$, as the expected value of the random variable $Z(x, y)$ that is uniformly sampled from the north, south, east and west neighbors,

$$\{u(x+h, y), u(x-h, y), u(x, y+h), u(x, y-h)\} \ .$$

If we also did not know the temperature at the neighbor point sampled, we could estimate the neighbor's temperature sampling one of neighbor's neighbors. Using the argument recursively, we could estimate the temperature $u(x, y)$ using the Monte Carlo algorithm as follows.

Consider a "particle" following a random walk or path, that is, a stochastic trajectory, $T = [T(1), \dots T(m)]$, starting at position $T(1) = [x(1), y(1)]$. From a given position at step k, the particle can jump, with uniform probability, to any neighboring position, that is, given $T(k) = [x(k), y(k)]$, the next step, $T(k+1)$, is sampled uniformly from its neighbors,

$$T(k+1) = [x(k+1), y(k+1)] \in$$

$$\{[x(k)+h, y(k)], [x(k)-h, y(k)], [x(k), y(k)+h], [x(k), y(k)-h]\} \ .$$

The path ends when it reaches a boundary point at $T(m) = [x(m), y(m)]$. Defining the random variable $Z(T) = u(x(m), y(m))$, it can be shown, see Demidovich (1976), that the expected value of $Z(T)$, for T starting at $T(1) = [x(1), y(1)]$, equals $u(x(1), y(1))$, the value of the solution to the Dirichlet problem at $[x(1), y(1)]$.

This algorithm is only a particular case of more general Monte Carlo algorithms for solving linear systems, see Demidovich (1976), Hammersley and Handscomb (1964), Halton (1970), Ripley (1987).

Notice that these Monte Carlo algorithms allow us to compute the solution of many continuous problems as expected (average) values in a stochastic flow of discrete particles. More generally, efficient Monte Carlo algorithms are available for solving linear systems, and many of the mathematical models used in Physics, or science in general, are (or can be approximated by) linear equations. Hence, one should not be surprised to find physical models whose interpretation is based directly on these particle flows.

Robert Brown, in 1827, observed the movement of plant spores (pollen) immersed in water. He observed that these spores were in perpetual movement, realizing an erratic or chaotic notion. He also quickly dismissed the hypothesis of live of self propelled motion, since: (a) The motion persisted over long periods of time, on different liquid media, and (b) Powder particles of inorganic minerals exhibit the same movements. This "Brownian motion" was the object of several subsequent studies, linking the intensity of this movement to the liquid medium temperature, see Brush (1968) and Haw (2002).

In 1905 Einstein published a paper explaining Brownian motion as a fluctuation phenomenon, caused by the collision of individual water molecules with the particle in suspension. Using a simplified argument we can model the possible positions of the particle in suspension by a rectangular grid, like the one used to solve the Dirichlet problem, and model the effect of each molecule collision as pushing the particle one position, to the north, south, east or west. The basic mathematical properties of this kind of stochastic process, known as random walk, are reviewed in Beran (1994) and Embrechts (2002). For an elementary introduction, see Berg (1993), Lemons (2002) and Mikosch (1998).

A basic assumption of the random walk model is that distinct collisions, or movements of the particle, are uncorrelated. Let us consider the one dimensional random walk process, where a particle, initially positioned at the origin, $y_0 = 0$, suffers incremental unitary steps, that is, $y_{t+1} = y_t + x_t$, and $x_t = \pm 1$. We assume that the steps are unbiased, and uncorrelated, that is, $E(x_t) = 0$ and $\text{Cov}(x_s, x_t) = 0$. We also have $\text{Var}(x_t) = 1$. From the linearity of the expectation operator, we conclude that $E(y_t) = 0$, that is, the expected distance from the origin, at any time, t, is zero. Nevertheless, the expected square distance from the origin is

$$E(y_t^2) = E\left(\sum_{j=1}^{t} x_j\right)^2 = E\sum_{j=1}^{t} x_j^2 + E\sum_{j\neq k} x_j x_k = t + 0 = t\ .$$

Hence, at time t, the standard deviation of the distance from the origin is,

$$\sqrt{E(y_t^2)} = t^H,\ \text{for } H = \frac{1}{2}\ .$$

From this simple model we can derive an important characteristic, expressed as a sharp statistical hypothesis, that can be the object of experimental verification: Brownian motion is a self-similar process, with scaling factor, or Hurst exponent, $H = 1/2$. The last statement means that, in other to make coherent observations of a Brownian motion, if one rescales the time scale by a factor ϕ, one should also rescale the space scale by a factor ϕ^H. One can generalize this process for $0 < H < 1$, this is the Fractional Brownian Motion, for more details, please refer to Beran (1994) and Embrechts (2002).

The sharp hypothesis $H = 1/2$ takes us back to the eternal underlying theme of system coupling / decoupling. The regular Brownian motion was build on the essential axiom of decoupling (no correlation) of increments on non-overlapping time intervals, whereas the relaxation of this condition, while keeping self-similarity, leads to long range correlations, see Beran (1994) and Embrechts (2002).

As seen in this section, the regular Brownian motion is very useful in modeling several low level processes, often found in unorganized physical systems. Nevertheless, many complex or (self) organized systems, like the ones found in soft matter science (like colloids or liquid crystals), biological signal analysis (like electro-cardiograms or encephalograms), development analysis (like embryology or social and urban systems), and many other phenomena related to organisms or living systems, exhibit long range correlations. Many of these areas can benefit from the techniques of fractional Brownian motion, see Addison (1997), Beran (1994), Bunde and Havlin (1994), Embrechts (2002) and Feder (1988). Some of the epistemological consequences of the mathematical and computational models introduced in this section are commented in the next section.

8 Hypothetical vs. Factual Models

The Monte Carlo algorithms introduced in the last section are based on the stochastic flow of particles. Yet, these particles can be regarded as nothing more then imaginary entities in a computational procedure. By the other hand, some models based on similar ideas, like the kinetic theories of gases, or the random walk model for the Brownian motion, seem to give a higher ontological status to these particles. Hence, it is worthwhile to discuss the epistemological or ontological status of an entity appearing in a computational procedure, like the example above.

This discussion is not as trivial, innocent and harmless, at it may appear at first glace. In 1632 Galileo Galilei published in Florence his Dialogue Concerning the Two Main World Systems. At that time it was necessary to have a license to publish a book, the *imprimatur*. Galileo had obtained the imprimatur from the ecclesiastical authorities two years earlier, under the explicit condition that some of the theses presented in the book, dangerously close to the heliocentric heretical ideas of Nicolas Copernicus, should be presented as an "hypothetical model" or as a "calculation expedient" as opposed to the "truthful" or "factual" description of "reality".

Galileo did not only fail to fulfill this condition, but ridiculed the official doctrine. He presented his theories in a dialogue form, where the character defending the orthodox geocentric ideas of Aristotle and Ptolemy, Simplicio, was constantly mocked by his opponent in the dialogues, Salviati, representing the views of Galileo himself. In 1633 Galileo was prosecuted by the Roman Inquisition, under the accusation of making heretical statements, as quoted from Santillana (1955, p.306-310):

"The proposition that the Sun is the center of the world and does not move from its place is absurd and false philosophically and formally heretical, because it is expressly contrary to Holy Scripture. The proposition that the Earth is not the center of the world and immovable but that it moves, and also with a diurnal motion, is equally absurd and false philosophically and

theologically considered at least erroneous in faith."

In the Italian renaissance, one of the most open and enlighted of its time, but sill a pre-modern society where subsystems were only incipient, and not clearly differentiated, the consequences of mixing scientific and religious arguments were daring. In 1633 Galileo is sentenced to prison for an indefinite term. After he abjures his heretical statements, the sentence is commuted to house-arrest at his villa. Legend has it that, after the formal abjuration, Galileo muttered the now celebrated phrase, *Eppur si mouve*, "But indeed it (the earth) moves (around the sun)".

Around 1610 Galileo built a telescope that he used for astronomical observations. Among his findings were four satellites to planet Jupiter, Io, Europa, Ganymedes and Callisto. He also observed phases (like the lunar phases) exhibited by planet Venus. Both facts are either compatible or explained by the Copernican heliocentric theory, but problematic or incompatible with the orthodox Ptolemaic geocentric theory. During the trial Galileo tried to use these observations to corroborate his theories, but the judges would not even, literally, 'look' at them. The chief church astronomer, Christopher Clavius, refused to look trough Galileo's telescope, stating that there was no point in 'seeing' some objects at an instrument that had been made just in order to 'create' them. Nevertheless, only a few ears after the trial, the same Clavius was building fine telescopes and using them to make new astronomical observations. Of course he took care of never upsetting his boss with "theologically incorrect" explanations for what he was observing.

From the late XIX century to 1905 the world saw an other trial, perhaps not so famous, but even more dramatic, namely, that of the atomistic ideas Ludwig Boltzmann. For a excellent biography of Boltzmann, intertwined (as it oght to be) with the history of his scientific ideas, see Cercignani (1998). The final verdict on this controversy was given by Albert Einstein in his *annus mirabilis* paper about Brownian Motion, together with the subsequent experimental work of Jean Perrin, see Einstein (1956) and Perrin (1950). A simplified version of these models was presented in the last section, including a "testable" sharp statistical hypothesis, $H = 1/2$, that, as already mentioned, can be used to empiricaly check the theory. In his Autobiographical Notes, Einstein states, as quoted in Brush (1968):

"The agreement of these considerations with experience together with Planck's determination of the true molecular size from the law of radiation (for high temperatures) convinced the skeptics, who were quite numerous at that time (Ostwald, Mach) of the reality of atoms. The antipathy of these scholars towards atomic theory can indubitably be traced back to their positivistic philosophical attitude. This is an interesting example of the fact

that even scholars of audacious spirit and fine instinct can be obscured in the interpretation of facts by philosophical prejudices. The prejudice - which has by no means died out in the meantime - consists in the faith that facts themselves can and should yield scientific knowledge without free conceptual construction.

Such misconception is possible only because one does not easily become aware of the free choice of such concepts, which through verification and long usage, appear to be immediately connected with the empirical material"

Let us follow Perrin's perception of the "empirical connection" of the concepts used in the melecular theory, in contrast to those of the rival energetic theory, during the first decade of the XX century. In 1903 Perrin was already an advocate for the molecular hypothesis, see Perrin (1903). As reported in Brush (1968, p.30-31), he refused the positivist demand for using only directly observable entities. Perrin referred to an analog situation in biology where,

"the germ theory of disease might have been developed and successfully tested before the invention of the microscope; the microbes would have been hypothetical entities, yet, as we know now, they could eventually be observed."

Only three years latter, Perrin (1906) was confident enough to reverse the attack, accusing the "energetic" view rivaling the "atomic" theory, of having "degenerated into a pseudo-religious cult". It was the energetic theory, Perrin claimed, that was making use of non-observable entities! To begin with, Classical thermodynamics had a differential formulation, where the functions describing the evolution of a system are supposed to be continuous and differentiable (notice the similarity of Perrin's argument with Schlick's argument presented in section 6). Perrin based his argument of the contemporary evolution of mathematical analysis where, until late in the XIX century, continuous functions were assumed to be naturally differentiable. Nevertheless, the development of mathematical analysis at the turn to the XX century showed this assumption to be rather naive. Referring to this background material, Perrin argues:

"But they still thought the only interesting functions were the ones that can be differentiated. Now, however, an important school, developing with rigor the notion of continuity, has created a new mathematics, within which the old theory of functions is only the study (profound, to be sure) of a group of singular cases. It is curves with derivatives that are now the exception; or, if one prefers the geometrical language, curves with no tangents at any point become the rule, while familiar regular curves become some kind of curiosities, doubtless interesting, but still very special."

In thee years more, even former opponents were joining the ranks of the atomic theory, as W.Nernst (1909, 6th.ed., p.212):

"In view of the ocular *confirmation of the picture which the kinetic theory provides us of the world of molecules, one must admit that this theory begins to lose its hypothetical character."*

Before presenting our final remarks, let us make a brief digression on the important them of the alternation of continuous and discrete solutions. In Stern (2005a) we discussed how objects emerge as discrete or sharp eigen-solutions in a continuum of possibilities. In the last section we have shown how continuous eigen-solutions can emerge as expected values in a stochastic flow of discrete particles. This issue could also be analyzed under the general framework of invariant measures and de Finetti type theorems discussed in Stern (2005a).

9 Magic, Miracles and Final Remarks

In several incidents analyzed in the last sections, one can find the repeated occurrence in the history of science of theoretical "phase transitions". In such transitions, we observe a dominant theory with strong support, being challenged by an alternative point of view. In a first moment, an alternative theory is proposed, but the cheerleaders of the dominant group present a variety of "disqualification arguments", showing why the underdog theory, plagued by bogus concepts and faulty constructions, should not even be considered as a serious contestant. In an intermediate period, the alternative theory is kept alive by a small minority, that is able to foster its progress. In a second moment, the alternative theory quite abruptly becomes the dominant view, and many wonder how is it that the old, now abandoned theory, could ever have had so much support. This process is captured by the following quotation, from the preface to the first edition of Schopenhauer (1818):

"To truth only a brief celebration of victory is allowed between the two long periods during which it is condemned as paradoxical, or disparaged as trivial."

Perhaps this is the basis for the much more somber statement found in Planck (1950, p.33-34):

"A new scientific truth does not triumph by convincing its opponents and by making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it."

As for the abrupt transition between the two phases, representing the two theoretical paradigms, this phenomenon have been extensively studied by Thomas Kuhn, from sociological, systemic and historical perspectives, see for example Kuhn (1996); see also Feyerabend (1993).

We finish this section with a quick and simple alternative explanation, or maybe it should be considered just as a hint, that I believe can give us a good insight on the nature of this phenomenon. This type of explanation was used many times by von Foerster who, among so many other things, was a skilful magician and illusionist, see Foerster (2003,b,e).

An Ambigram, or ambiguous picture, is a picture that can be seen in two or more different ways. A famous example is the Rubin's vase, see Rubin (1915), where looking at the white (black) part of the figure one sees a face (vase). Looking at an ambigram one can "switch" between the alternative interpretations, but at least until one becomes accustomed to "see" both alternatives, it takes some (or a big) effort to switch between them. This mechanism can explain either Nernst's readiness to accept his "ocular confirmation" or Clavius' stubborn "ocular blindness". After all, the satellites of Jupiter were quite palpable objects, ready to be seen at Galileo telescope, whereas the grains of colloidal suspension that could be observed at the lunette of Perrin apparatus gave a much more indirect evidence for the existence of molecules. Or maybe not, after all, it all depends on what one is ready or willing to see...

Yet, the existence of sharp, stable, separable and composable eigen-solution for the scientific system in its iteration with its environment, goes far beyond our individual or collective desire to have them there.

These eigen solutions are the basis upon which technology builds so much of the world we live in. How well the eigen-solutions used in these technological gadgets conform with von Foerster criteria? The machine I am using to write this article has an 2003 Intel Pentium CPU carved on a silicon waffle with a "precision" of 1E-7m (0.1 micron), and it is "composed" by about 5E7 (five followed by 7 zeros) transistors. This CPU has a clock of 1GHz, hence, each and every one of the transistors in this composition must operate synchronized to a fraction of a thousandth of a thousandth of a thousandth of a second!

And how well the eigen-solutions expressed as fundamental physical constants, upon which technological projects rely, conform with von Foerster criteria? Again, some of these constants are known with a precision (relative standard uncertainty) of 1E-9, or a thousandth of a thousandth of a thousandth! The world wide web site of the United States' National Institute of Standards and Technology, at `www.physics.nist.gov` , gives an encyclopaedic view of these constants and their inter-relations. Planck (1950, Ch.6) comments on their epistemological significance.

But far beyond their practical utility or even scientific interest, the existence of these eigen-solutions are not magical illusions, but true miracles. Why "true" miracles? Because the more they are explained and the better they are understood, even the more wonderful they become!!

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