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Nonextensivity of hadronic systems

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The predictions from a nonextensive self-consistent theory recently proposed are investigated. Transverse momentum (p_T) distribution for several hadrons obtained in $p + p$ collisions are analyzed to verify if there is evidence for a limiting effective temperature and a limiting entropic index. In addition, the hadron-mass spectrum proposed in that theory is confronted with available data. It turns out that all p_T distributions and the mass spectrum obtained in the theory are in good agreement with experiment with constant effective temperature and constant entropic index. The results confirm that the nonextensive statistics plays an important role in the description of the termodynamics of hadronic systems, and also that the self-consistent principle holds for energies as high as those achieved in the LHC. A discussion on the best p_T-distribution formula for fitting experimental data is presented.

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The Hagedorn's bootstrap idea based on a self-consistency requirement for the thermodynamics of fireballs predicted a limiting temperature for hadronic systems and also gave formulas for transverse momentum (p_T) distributions of secondaries and for the hadron-mass spectrum [1].

Experiments with $\sqrt{s} > 10$ GeV, however, have shown that the p_T-distribution formula fails to describe the data. An empirical formula was proposed [2] including non-extensive statistics and it results that the modified formula can fit all available data for p_T distributions. Although many works have been done on the subject, the use of nonextensive statistics in hadronic physics remains rather controversial.

Recently a nonextensive version of the self-consistency principle was proposed [3], leading to new formulas for mass spectrum and for transverse momentum distribution. The last one is similar to that proposed in Ref. [2]. In addition, the theory predicts a limiting effective temperature and a limiting entropic index for all hadronic systems. The limiting effective temperature was predicted also in a parton-gas model with Tsallis distribution [4]. These results establish a much more restrictive test to evaluate if the nonextensive statistics plays any role in the hot hadronic systems produced in high energy collisions.

In this work experimental data for p_T distributions from different experiments and for several hadrons produced in $p + p$ collisions at ultrarelativistic energies are analyzed in order to investigate the theoretical predictions given in Ref. [3]. Also, the theoretical mass spectrum is compared to experimental data. The experimental data for p_T distributions used in the present analysis are summarized in Table I. Since we are looking for the asymptotic limit of the temperature and of the entropic index, we consider data for high energy experiments only.

Before going into the details of the analysis, let us recall that the nonextensive self-consistent theory presents two main aspects [3]: (a) It assumes that the concept of fireball as defined by Hagedorn is valid even at the ultrarelativistic energies achieved at modern colliders, i.e., up to 7 TeV and that the self-consistency or bootstrap idea is also valid; (b) it assumes that the hadronic matter can be described as an ideal gas of fireballs obeying the Tsallis entropy [10]. In this regard we note that the nonextensive entropy for a gas of bosons and fermions was already obtained in Ref. [11].

The self-consistency is a consequence of the definition of fireball as proposed by Hagedorn [1], where it is proposed that a fireball is a thermodynamically equilibrated system composed by fireballs. This recursive definition leads to a constraint in the thermodynamical formulation of a fireball, where two forms of the partition function, one in terms of the density of states of a fireball and the other in terms of the masses of the constituent fireballs, must be asymptotically equivalent.

The nonextensivity is related to the use of the Tsallis entropy [10], and it was first suggested by Bediaga, Curado, and Miranda [2] in order to explain the experimental data for p_T distributions found in a high energy $e^+ - e^-$ collision. As a consequence, besides the temperature we have also the entropic index, q, as thermodynamical parameters. This parameter is a measure of the nonextensivity of the system, since the entropy of the system composed by two subsystems A and B is [10]

$$S_q(AB) = S_q(A) + S_q(B) + (q - 1)S_q(A)S_q(B), \quad (1)$$

where $S(A)$ and $S(B)$ are the entropies of the independent systems A and B, respectively, and $S(AB)$ is the entropy of the composed system. In the limit $q \to 1$ Tsallis statistics is identical to the Boltzmann statistics.

The nonextensive effects, which are introduced through the Tsallis entropy, are usually related to long-range interactions or to "memory effects." We observe that both effects may be present in the hot hadronic medium formed in ultrarelativistic collisions through quantum numbers

TABLE I. Set of experimental data for $p + p$ collisions.

Experiment	Particle	$\sqrt{s}$ (TeV)	Range p_T (GeV)	Reference
ALICE (LHC)	π^0	0.9	$0.4 \le p_T \le 7.0$	[5]
	π^0	7.0	$0.3 \le p_T \le 25.0$	
	η	7.0	$0.4 \le p_T \le 15.0$	
ALICE (LHC)	ϕ	7.0	$0.4 \le p_T \le 6.0$	[6]
	K^*	7.0	$0.0 \le p_T \le 6.0$	
ALICE (LHC)	$\pi^\pm$	0.9	$0.1 \le p_T \le 2.6$	[7]
	$K^\pm$	0.9	$0.2 \le p_T \le 2.4$	
	$P^\pm$	0.9	$0.35 \le p_T \le 2.4$	
CMS (LHC)	K_S^0	0.9, 7.0	$0.0 < p_T \le 10.0$	[8]
	Λ	0.9, 7.0	$0.0 < p_T \le 10.0$	
	Ξ^-	0.9, 7.0	$0.0 < p_T \le 6.0$	
ATLAS (LHC)	J/ψ	7.0	$7.0 \le p_T \le 70.0$	[9]

conservation and through feed-down mechanism of secondary production due to resonance decay. In fact the powerlike tail in p_T distributions, which is characteristic of nonextensive statistics at the high-p_T end, can be described by the Boltzmann thermodynamical description of fireball after the addition of the feed-down contributions through model dependent calculations [12,13]. Although most of the secondaries from resonance decay are in the low-p_T region, they still contribute to the appearance of a tail in the shape of the p_T distributions after normalization, as can be seen in [12].

Initially it is important to clarify that the p_T distribution given by

$$\frac{d^2N}{dp_T dy} = gV\frac{p_T m_T \cosh y}{(2\pi)^2}\left(1+(q-1)\frac{m_T\cosh y - \mu}{T}\right)^{-\frac{q}{q-1}} \quad (2)$$

can be directly obtained from the Tsallis entropy [10] through the usual thermodynamical relations [14]. Despite this fact, in many analyses other p_T distributions are used [5,7], as

$$\frac{d^2N}{dp_T dy} = p_T\frac{dN}{dy}\frac{(n-1)(n-1)}{nC[nC+m_o(n-2)]}\left(1+\frac{m_T-m_o}{nC}\right)^{-n}. \quad (3)$$

In the equations above, y is the hadron rapidity, μ is the chemical potential, $m_T = \sqrt{p_T^2 + m_o^2}$, with m_o being the hadron mass, n and C are constants, V is the volume and g is the degeneracy factor.

The p_T dependence in both formulas can be made quite similar by adopting [14]

$$n = \frac{q}{q-1} \quad (4)$$

and

$$nC = \frac{T}{q-1}, \quad (5)$$

but the factor m_T present in Eq. (2) and absent in Eq. (3) is sufficient to produce very different values for the parameters T and q when those equations are used to fit experimental data, even if quite good fittings are obtained with both equations.

For the fittings of experimental data for p_T distributions we follow Cleymans and Worku [14] and use $y = 0$ and $\mu = 0$ in Eq. (2), resulting in

$$\left.\frac{d^2N}{dp_T dy}\right|_{y=0} = gV\frac{p_T m_T}{(2\pi)^2}\left(1+(q-1)\frac{m_T}{T}\right)^{-\frac{q}{q-1}}. \quad (6)$$

Since experiments report data for relatively small ranges of rapidity this can be considered an appropriate approximation. In fact, according to [15], p_T distributions are independent of the rapidity, which can be considered approximately constant in the central region. The range of p_T considered in the fitting procedure depends on the experiment analyzed. For all cases we used the full range provided by the experiment.

In Fig. 1 the effective temperatures obtained from those fittings are presented. It is clear that the temperature obtained with Eq. (3) varies in a broad range, systematically increasing with the hadron mass. The results obtained with Eq. (6), on the other hand, give temperatures spread over a much narrower range around a constant value $T_o = (61.2 \pm 0.5)$ MeV (full lines in Fig. 1).

Comparing Eqs. (6) and (3), it is easy to understand that the absence of the m_T factor in the latter gives rise to the increasing temperature behavior observed in Fig. 1. Indeed, the effects of the increase in m_T due to the increase of p_T in Eq. (6) are reproduced in Eq. (3) by an increase of T.

The results of the entropic index obtained from the fittings are plotted in Fig. 2. In this case, for both equations the results are spread around an average value with no evidence of a systematic trend. But again the values are spread over a broader range in the case of Eq. (3), while they are limited to a narrower range around $q_o = 1.143 \pm 0.007$ when Eq. (6) is used.

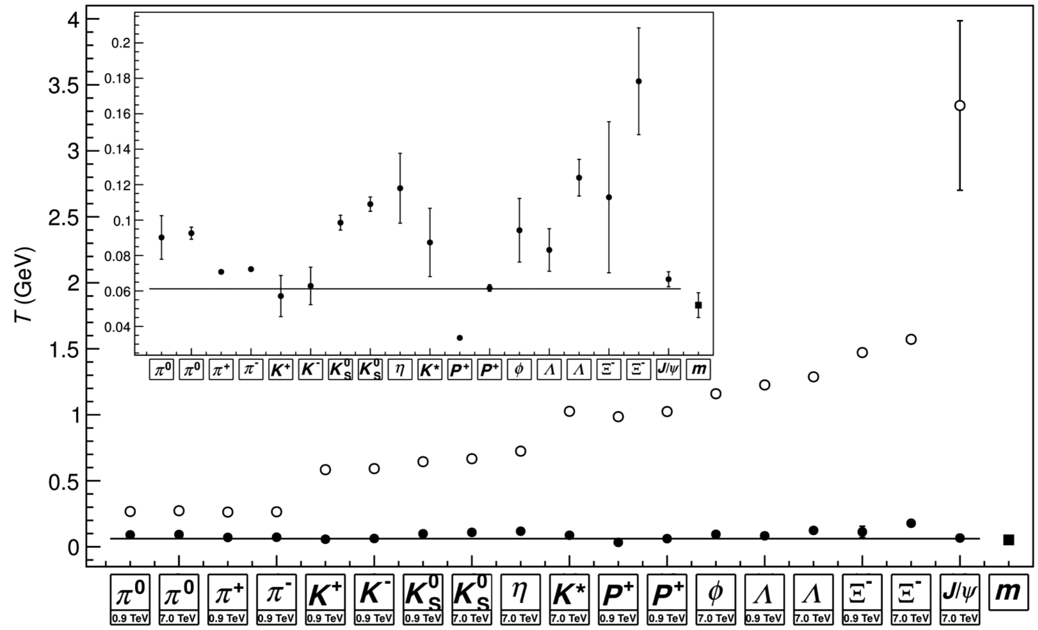

FIG. 1. Effective temperature, T, resulting from the fittings of Eq. (6) (full circles), assuming $y = 0$ and $\mu = 0$, and Eq. (3) (open circles). The full square indicates the result obtained from the mass spectrum analysis (see text). The inset shows the effective temperature obtained through the use of Eq. (6) in more details. Full lines indicate the constant value, T_o, which best fits the data (full symbols).

In principle, there is nothing wrong with an increasing temperature for increasing hadron mass. The relevant points here are:

(i) The p_T distribution obtained from Tsallis entropy by using the usual thermodynamical relations is Eq. (6), not Eq. (3).

(ii) If Tsallis statistics is the basis for a thermodynamical description of hadronic systems, then the self-consistency principle leads to a limiting effective temperature. Such limiting temperature is observed when Eq. (6) is used, as shown by the results in Fig. 1.

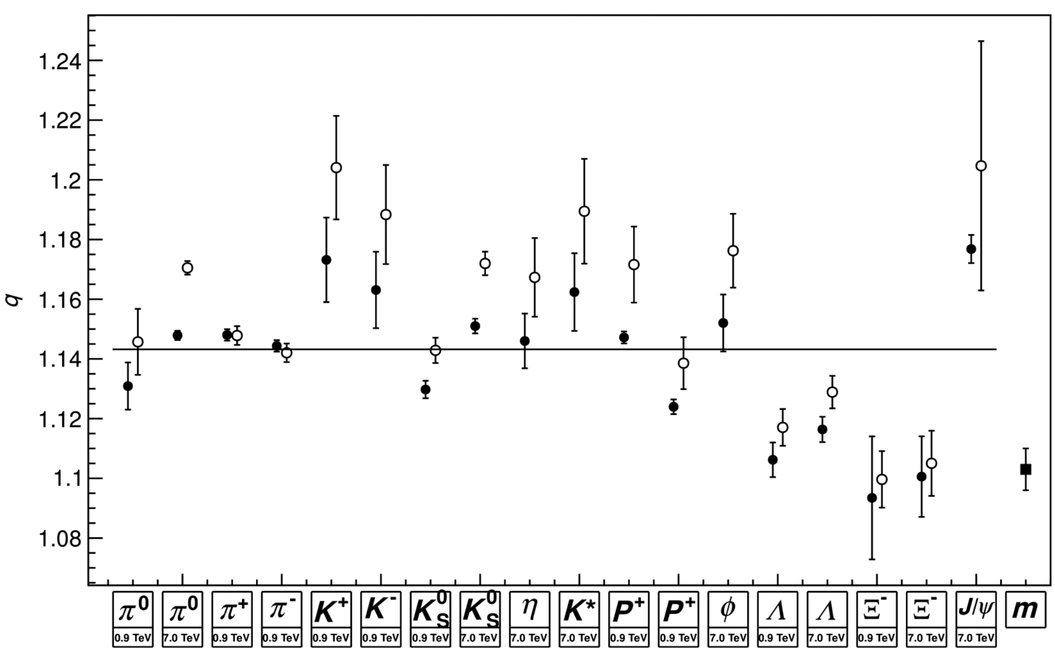

FIG. 2. Entropic factor, q, resulting from the fittings of Eq. (6) (full circles) and Eq. (3) (open circles). The full square indicates the result obtained from the mass spectrum analysis (see text). The full line indicates a constant q fitted to the data obtained with Eq. (6).

(iii) The theory predicts a limiting entropic index, which is also observed in the analysis of p_T distribution, as shown in Fig. 2.

These results are in agreement with recent analysis performed in Refs. [14,16,17], where constant temperature and entropic index were found with values similar to those obtained here. The present analysis extends those analyses by considering identified particles and by including $p + p$ collisions up to $\sqrt{s} = 7$ TeV. Regarding the parameter V in Eq. (6), which is also a free parameter, we cannot perform a systematic study of its values since we use in this analysis experimental yields obtained in different experiments and different laboratories. The analysis of correlation of the parameter V with T or q shows that the last two parameters are only slightly modified when V changes by 1 order of magnitude. Therefore the results obtained here will not be strongly affected by normalization.

Since the theory predicts a limiting temperature and a limiting entropic index, a precise determination of this parameters should consider their energy dependence and their asymptotic values. Some initial efforts in this direction are already done in Refs. [16–18], but in the present work we are interested in the confirmation that such limiting values indeed exist and are independent of the secondary particle analyzed. Therefore it is reasonable to expect some fluctuations around the mean value found for T and q. Also, the mean values found are to be considered just as approximations to the asymptotic values for those parameters.

The results discussed above show that there are strong evidences that the nonextensive statistics plays an important role in the thermodynamical description of hadronic systems and that the self-consistency conditions find support in the experimental data from ultrarelativistic collisions. It is worthwhile to stress the importance of using the p_T distribution formula which is consistently derived from Tsallis entropy by the use of thermodynamical relations.

A crucial verification of the theory is related to the mass spectrum. In fact, if Hagedorn's theory fails to describe p_T distributions for $\sqrt{s} > 10$ GeV, it also has problems to describe the data for hadron-mass spectrum. The Hagedorn temperature, T_H, varies from 141 MeV up to 340 MeV, depending on the parametrization used for the mass spectrum formula, especially for the multiplying factor [19–27]. For the most used parametrization, however, T_H is much higher than that expected from hadron-hadron collisions, where $T_H \approx 160$ MeV [19,22].

According to the nonextensive self-consistent theory, the hadron-mass spectrum is given by

$$\rho(m) = \gamma m^{-5/2} e_q^{\beta_o m}, \tag{7}$$

where e_q^x is the q-exponential function [3] given by

$$e_q^x = [1 + (q-1)x]^{1/(q-1)}. \tag{8}$$

It is important, therefore, to verify if this equation can describe the mass spectrum data with the same values T_o and q_o obtained in $p + p$ collisions. A power-law mass spectrum was already used in Ref. [4].

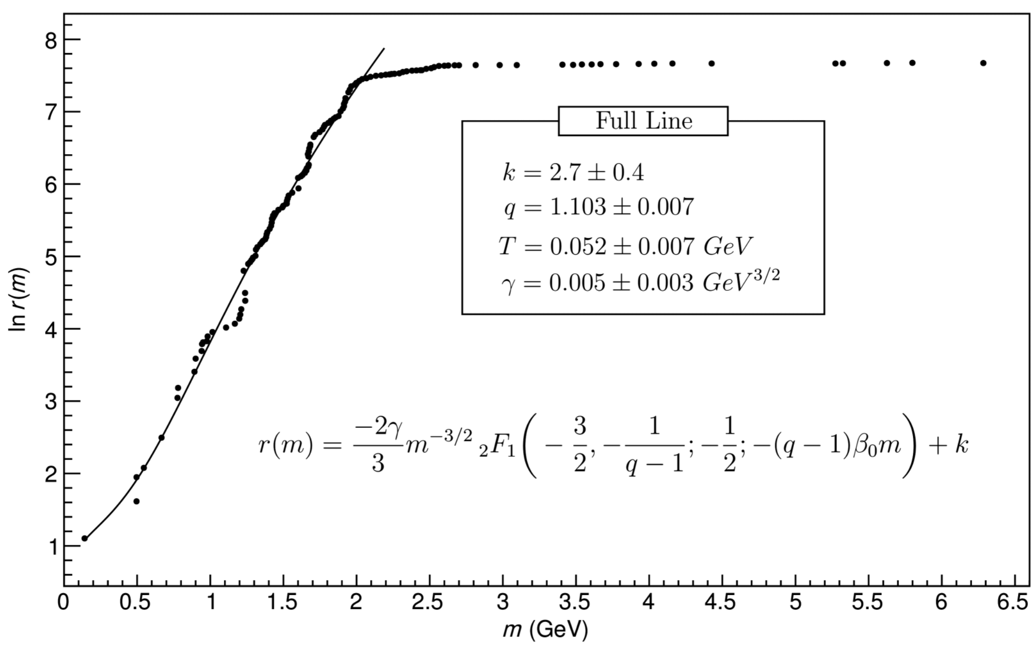

FIG. 3. Cumulative hadron-mass spectrum. The full line represents the calculation with Eq. (9) using $k = 2.7 \pm 0.4$, $\gamma = (5 \pm 3)10^{-3}$ GeV$^{3/2}$, $q = 1.103 \pm 0.007$ and $T = (52 \pm 7)$ MeV. Full circles represent the available data taken from Ref. [19].

The cumulative hadron-mass distribution is given by

$$r(m) = \int \rho(m)dm = \frac{-2\gamma}{3} m^{-3/2} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{q-1}; -\frac{1}{2}; -(q-1)\beta m\right) + k, \quad (9)$$

where k is a constant and ${}_2F_1(a, b; c; z)$ is the Gauss' hypergeometric function. This equation was fitted to the available data for cumulative mass spectrum [19].

In Fig. 3 the best fitted curve is shown, and it is possible to observe a good agreement between data and calculation. The fitting procedure does not take into account data above 2 GeV, since the information above this threshold is not considered reliable. This procedure is usual in the study of mass spectrum [19,22].

The curve in Fig. 3 is obtained from Eq. (9) with $T = (52 \pm 7)$ MeV and $q = 1.103 \pm 0.007$. These values fall in the same range of the corresponding ones obtained in $p + p$ analysis, as shown in Figs. 1 and 2. Therefore a good agreement is found between the results from p_T-distribution analysis and from mass spectrum analysis. It is possible to conclude that the nonextensive self-consistent theory proposed in Ref. [3] can describe simultaneously the p_T distribution and the hadron-mass spectrum with constant effective temperature and constant entropic index.

In conclusion, this work presents an extensive analysis of p_T distribution from $p + p$ collisions at ultrarelativistic energies in order to test the predictions of the nonextensive self-consistent theory proposed in Ref. [3]. The results show a limiting effective temperature $T_o = (61.2 \pm 0.5)$ MeV and a limiting entropic index $q_o = 1.143 \pm 0.007$.

Also the theoretical mass spectrum is compared with the available data resulting in good agreement between calculation and data for $T = (52 \pm 7)$ MeV and $q = 1.103 \pm 0.007$. These values are within the range of the values T_o and q_o found in p_T-distribution analysis.

With these results it is possible to observe that the nonextensive self-consistent thermodynamical approach can describe the main features of the hadronic system formed in high energy collisions. We do not claim that it describes all possible aspects of the problem, since this thermodynamical theory deals only with a system in its stationary state and therefore it is not supposed to explain what happens before this state is reached nor what happens after the freeze-out.

The results obtained here allow a complete thermodynamical description of dilute hadronic systems and indicates that microscopical mechanisms that lead to nonextensivity should be included in resonance hadron gas models.

Some hints on how to pursue this objective can be found in Refs. [4,28–31]. Borland [29] and Wilk and Wlodarczik [28] have shown that nonextensive statistics can emerge from Boltzmann statistics in a system in stationary equilibrium where some parameters may fluctuate, as for instance the temperature. A gas model has been proposed where the limiting temperature is also predicted, as shown by Biro and Peshier [4]. It has been shown that finite size effects in a Boltzmannian thermodynamical system can lead to nonextensive affects, as shown in [30,31].

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