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The Spectral Maximum of Synchrotron Radiation[1]

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Abstract—A three-level system is considered as a simplest example where a possibility of solving the spectral problem is observed. We thoroughly analyze the relation between the amount of radiation emitted by the particle during the transitions to the first excited state and to the ground state. Generalizing basic expressions we can follow the evolution of spectral maximum. It turns out there is a condition for radiation maximum to stay at highest harmonic.

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INTRODUCTION

The main reason we focus on the study of synchrotron radiation (SR) from weakly excited particles is the manifestation of quantum effects on low energy levels. It means that the characteristics of radiation can be adequately described only with the use of quantum theory methods. The transitions from the first excited state to the ground state were thoroughly studied in [1, 2], where it was found that quantum and classical theoretical results coincide in non-relativistic limit only. However, at such transitions just one frequency is radiated and therefore there is no way to formulate and solve problems concerning spectral properties of radiation. The simplest example of a system appropriate for answering these questions is a three level system which (with respect to radiation) is to be considered below.

THE SPECTRAL PROPERTIES OF SR IN CLASSICAL AND QUANTUM THEORIES

Let us consider SR from spinless particle (boson) of charge e_0, rest mass m_0, moving in a constant uniform magnetic field of intensity $\vec{H} = (0, 0, H)$, $H > 0$. The particle energy is $E = \gamma m_0 c^2 = m_0 c^2 (1 - \beta^2)^{-1/2}$, here c is the speed of light, γ is the relativistic factor, $\beta = \upsilon/c$, υ is particles velocity in classical terms. In this case the energy of boson has discrete values, which we label by $n = 0, 1, 2, 3...$ (the number of energy level).

$$\gamma^2 = 1 + (2n+1)b, \quad b = \frac{H}{H_0},$$
$$H_0 = \frac{m_0^2 c^3}{|e_0|\hbar} = \frac{Q\hbar}{e_0^2 |e_0|}, \quad Q = \frac{e^2 m_0^2 c^3}{\hbar^2}. \tag{1}$$

The number of harmonic radiated during the transition from level n to level s is ν, $\nu = n - s$.

For i-polarization component of SR spectral-angular distributions at the transitions to the first excited state ($\nu = n - 1$) and to the ground state ($\nu = n$) well-known are the expressions (the parameter θ describes the direction of photon emission) [3]

$$\frac{dW_i^b}{d\Omega} = \frac{Q\beta^6 \sum_{\nu=1}^{n} f_i^b(n; \nu, \beta, \theta)}{2(2n+1)^3(1-\beta^2)}, \quad d\Omega = \sin\theta d\theta,$$
$$x = x(n, \nu, \beta, \theta) = \nu \frac{1 - \sqrt{1 - \frac{2\nu}{2n+1}\beta^2 \sin^2\theta}}{1 + \sqrt{1 - \frac{2\nu}{2n+1}\beta^2 \sin^2\theta}},$$

$$f_i^b(n; n-1, \beta, \theta)$$
$$= \frac{(n-1+x)^3 x^{n-2}}{n!(n-1-x)}(l_2[n(n-1) - (2n+1)x + x^2]$$
$$+ l_3(n-x)(n-1+x)\cos\theta)^2 e^{-x}, \quad \nu = n-1, \quad n \geq 2,$$

$$f_i^b(n; n, \beta, \theta) = \frac{(n+x)^3 x^{n-1}}{n!(n-x)}$$
$$\times [l_2(n-x) + l_3(n+x)\cos\theta]^2 e^{-x}, \quad \nu = n \geq 1.$$

We enumerate the SR polarization components with $i = -1, 0, 1, 2, 3$ as follows: $i = g = \pm 1$ stays for right ($i = 1$) and left ($i = -1$) circular polarization, $i = 2$ stays for σ-component of linear polarization, $i = 3 - \pi$ describes the σ-component of linear polarization, finally, $i = 0$ corresponds to the total radiated power, which may be represented as a sum over linear or cir-

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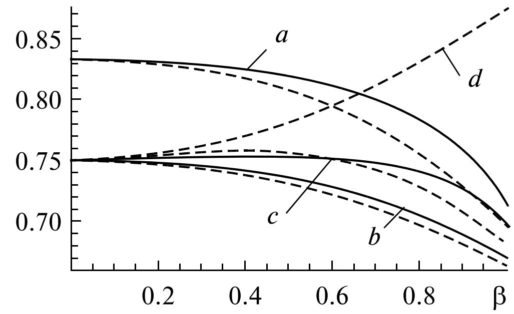

Fig. 1. The functions $q_2^b(\nu = 2, 1, \beta)$ (a, b) and $q_2^b(\beta)$ (c) (dashed lines stay for classical theory predictions, (d)-line corresponds to the sum over classical spectrum)

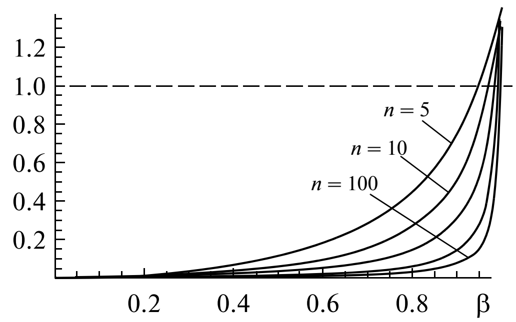

Fig. 2. Graphs of the functions $K(n; \beta)$, $n = 5, 10, 20, 50, 100$.

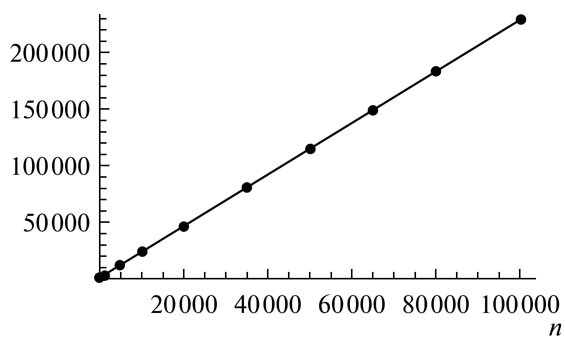

Fig. 3. The dots stay for the solutions of $K(n; \beta_n) = 1$.

cular polarization components. The parameters l_2 and l_3 for each i have to be chosen as represented below

$$l_2 = 1,\ \ l_3 = 0 \text{ for } i = 2;\ \ l_2 = 0,\ \ l_3 = 1 \text{ for } i = 3;$$

$$l_2 = l_3 = 1,\ \ l_2 l_3 = 0 \text{ for } i = 0;$$

$$l_2 = \pm l_3 = \frac{1}{\sqrt{2}} \text{ for } i = \pm 1.$$

For the translations from $n = 2$ the total power radiated in the upper half-plane is

$$W_i^b \sim F_i^b(\beta) = F_i^b(2;1,\beta) + F_i^b(2;2,\beta),$$

$$F_i^b(n;\nu,\beta) = \int_0^{\pi/2} f_i^b(n;\nu,\beta,\theta)\sin\theta d\theta.$$

The functions $q_i^b(\beta) = F_i^b(\beta)\big/F_0^b(\beta)$ and $q_i^b(\nu,\beta) = F_i^b(2;\nu,\beta)\big/F_0^b(2;\nu,\beta)$, whose graphs for $i = 2$ are shown in Fig. 1, characterize the polarization properties of radiation. One can see that the linear polarization contribution decreases with energy (the function $q_2^b(\beta)$ decreases). Moreover, we found that unlike linear polarization, the degree of circular polarization increases with β both for the total radiation and its spectral components. This contradicts results obtained within classical theory for total classical spectrum [3–5] but at the same time stays in good agreement with the characteristics of its separate harmonics. At $\beta = 0$ quantum and classical results coincide.

THE POSITION OF SPECTRAL MAXIMUM

According to classical SR theory, the maximum in radiation spectrum of non-relativistic particle stays at the first harmonic and shifts to higher harmonics with energy. In ultrarelativistic case the number of harmonic, corresponding to radiation maximum ν_{max} is proportional to energy cube $\nu_{max} \sim \gamma^3$ [6, 7]. In quantum theory for a non-relativistic particle $\nu_{max} = 1$, besides at any energy $\nu \le n$ and thus the question is when the equality $\nu_{max} = n$ becomes adequate.

Let us consider the functions $K(n;\ \beta) = F_0^b(n;n,\beta)\big/F_0^b(n;\,n-1,\beta)$ (Fig. 2). For a fixed n the condition $K(n;\ \beta_n) = 1$ defines such energy (such $\beta = \beta_n$, $\gamma = \gamma_n = (1 - \beta_n^2)^{-1/2}$), that the spectral maximum lies on the highest harmonic. It could be seen from Fig. 3 that for large n one can write an asymptotic formula $\gamma_n^2 = 2k_0 n$, where $k_0 \approx 1.146128792697$, which, according to (1), is not necessarily performed but in certain fields (at $b = b_0$). In the moment when the radiation maximum shifts from $\nu = n - 1$ harmonic to $\nu = n$ the following equality holds

$$\gamma_n^2 = 2k_0 n = 1 + (2n+1)b_0,\ \ k_0 = b_0 \text{ for } n \gg 1.$$

The value b_0 does not depend on n, which means that if the magnetic field satisfies $b_0 < k_0$, then whatever the initial state of the particle, the SR spectral maximum lies on $\nu_{max} \le n - 1$ harmonic. Otherwise ($b \ge k_0$) the spectral maximum will always stay at the last harmonic $\nu_{max} = n$.

The results obtained allow us to assume that there exists an ordered set of numbers $k_0 > k_1 > k_2 > \ldots > k_s$, such that if $b < k_s$, then the spectral maximum lies on the

$n > s$ harmonic whatever the initial state $\nu_{max} \leq n - s$ of the particle, and it shifts to higher harmonics with n (conserving the inequality $\nu_{max} \leq n - s$). If $k_m < b < k_{m-1}$ $(1 \leq m \leq s)$, then whatever the particle initial state $n > m$, the spectral maximum of SR lies on $\nu_{max} = n - m$. Finally, if $b > k_0$, at any n we have $\nu_{max} = n$.

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