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Commutativity theorems for division rings and domains

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Abstract

We show that noncommutative rings which satisfy some funny conditions are commutative, a fact which makes the conditions even funnier.

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It is shown that if a division ring is such that for every element a there exist comonic integer polynomials f_a, g_a of different orders so that $f_a(a)/g_a(a)$ is central then the ring is commutative. Extending the condition to prime rings, and substituting the center by the extend centroid one still achieve commutativity for domains. This breaks down slightly for primitive rings, where the exceptions are characterized. This extends some earlier theorems of Herstein and Faith.

1 Introduction

Division rings seem to have some wild inner structure which precludes almost any strong conditioning hypothesis from defining an interesting subclass. One way of expressing this is "commutativity theorems", that is, statements like "the only division rings satisfying such and such (usually contrived) conditions are commutative fields". As for what such a wild structure would be, there is at least one conjecture: the multiplicative group contains lots of free subgroups. This has been investigated elsewhere ([6], [12], [13], [14]), and appears to be supported by strong evidence. That, if proved, would explain almost everything that is known about multiplicative groups of division rings, including many, but not all, commutativity theorems. On the other hand, when it comes to theorems about algebraic conditions implying commutativity, there is not even a counterpart for that conjecture ([14]

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speculates a bit). So, it looks like one must gather more information about what is going on, and that will take the form of commutativity theorems here. Indeed, as is often the case, we will be able to say something about a wider class of rings.

A basic commutativity theorem due to Kaplansky states that a division ring in which some positive power of each element lies in the center is commutative. This has been generalized and extended in several directions, and here we tackle two of these, further extending both along a similar theme.

Following Herstein [7], we will evaluate integer polynomials on rings. We do not assume that rings have 1 (except, of course, division rings); for rings without 1, these integer coefficients should be read as living in the centroid. For a nonzero polynomial $f \in \mathbb{Z}[x]$, we denote its degree by ∂f and its *order* (least degree of a monomial with nonzero coefficient) by ωf; f is *monic* (*co-monic*) provided its leading (trailing) coefficient is 1. In that paper, Herstein proved that a division ring for which every element is sent into the center by a non-constant, co-monic, integer polynomial must be a field. On positive characteristic, one has as a corollary that Herstein's result remains true if *co-monic* is replaced by *monic*; Richoux [16] completed the proof "on the other side", that is, monic polynomials in characteristic 0. We unify all this with a more general statement in which the center is substituted, so to speak, by the extended centroid, and the ring is what is usually called *reduced*:

Theorem 5 *Let R be a ring with no nonzero nilpotent elements. Suppose that for each a in R there are two co-monic polynomials $f, g \in \mathbb{Z}[x]$, of different orders, such that $f(a)xg(a) = g(a)xf(a)$ for every $x \in R$. Then R is commutative.*

There is another direction in which this special case of this theorem for division rings, extends - but here commutativity is lost:

Theorem 6 *For a primitive ring R, the following three conditions are equivalent:*

i) *For every element a there are two integer co-monic polynomials $f, g \in \mathbb{Z}[x]$, of different orders, such that $f(a)xg(a) = g(a)xf(a)$ for every $x \in R$.*

ii) *For every element a there are two monic integer polynomials $f, g \in \mathbb{Z}[x]$, of different degrees, such that $f(a)xg(a) = g(a)xf(a)$ for every $x \in R$.*

iii) *Either R is commutative, hence a field, or its multiplicative semigroup is torsion.-*

Further, in the noncommutative case, the commuting ring of R is an absolutely algebraic field, and every commutative subring of R is locally finite.

A semigroup is *torsion* if each of its cyclic subsemigroups is finite. We are going to call a ring or algebra torsion, in case its multiplicative semigroup is so.

Notice that condition (iii) implies that the former two hold in a quite trivial way. For division rings, the theorem states that (i) or (ii) are commutativity conditions. Actually, in this case, it is an easy exercise to show that the "distinct degrees" and "distinct orders" conditions are equivalent (derive the polynomials of one type for an element from the polynomials of the other type for its inverse), so one gets Herstein's and Richoux's theorems as specializations of either of the above. On the other hand, we do not see such a simple equivalence between (i) and (ii) for primitive rings. Indeed, primitivity and torsion are unrelated as far as other local conditions go, even in the presence of a unit element:

Theorem 1 *Any torsion algebra over an absolutely algebraic field can be embedded into a unital primitive torsion algebra.*

The combination of "co-monic" and "different orders" (or "monic" and "different degrees") appearing above are not just a restriction of the proof technique. Indeed, in an algebraic algebra over the rationals, one can always find for each element a distinct integer polynomials f, g such that $f(a) = g(a)$, by trivial manipulations of an integer polynomial having a as zero: it is also easy to get them both co-monic or having them with different orders.

Another extension of Kaplanski's theorem is in the work of Faith [3], [4] about radical extensions of rings. In particular, he showed that if a division ring D contains a proper subring which includes some power of each element, then D is a field. We show:

Theorem 11 *Let R be a proper subring of the division ring D, and suppose that for each $a \in D$ there are monic polynomials $f, g \in \mathbb{Z}[x]$ such that $\partial f > \partial g$ and $f(a)/g(a) \in R$. Then D is commutative.*

2 Division Rings and domains

Here we set up the basis for proving Theorem 5 and Theorem 6.

Lemma 2 *Let A be a noncommutative K-algebra without nilpotent elements, K a field of characteristic $p > 0$. Then A is not purely inseparable over K.*

Proof. Suppose that A is purely inseparable over K. Then, for each $a \in A$, the K-linear map $T_a(x) = [a, x]$ on A is nilpotent. Thus, starting with a noncentral element d, we can find $a, b \in A$ such that $T_d(a) = b \neq 0$, and $T_d(b) = 0$. Since b is purely inseparable over K and not nilpotent, we have $\alpha = b^{p^n} \in K^*$. Set $w = -db^{p^n-1}a$; then $T_w(d) = -db^{p^n-1}[a, d] = db^{p^n} = d\alpha$. Since $\alpha \in K^*$, T_w cannot be nilpotent, a contradiction. □

The next theorem is a basic tool for what follows, and is a minor modification of a theorem due to Nagata, Nakayama and Tuzuku[15].

Theorem 3 *Let R be an integral domain (possibly without 1) whose field of fractions K is not absolutely algebraic. Let L be a finite separable extension of K. Then there exists a pair of distinct exponential valuations ν_1 and ν_2 on L, and an element $r \in R$ such that*

$$\nu_1|_K = \nu_2|_K$$

and

$$\nu_1(r) = \nu_2(r) > 0$$

.

Now we prove 5 for the special case of division rings, but in an alternate form which we will user later. Just notice that in this case, the hypothesis imply that $f(a)/g(a)$ lies in the center of the ring. Let us denote the center of a ring R by ζR.

Lemma 4 *Let D be a division ring and let A be a subring of D. If for each $a \in A$ there are two co-monic polynomials $f, g \in \mathbf{Z}[X]$, of different orders, and two elements $z_1, z_2 \in \zeta A$, not both zero, such that*

$$f(a)z_1 = g(a)z_2,$$

then A is commutative.

Proof. Denote by K the field of fractions of ζA, P its prime field and p its characteristic. By hypothesis, A is algebraic over K. If K is absolutely algebraic, then every element of A would be algebraic over P, hence a root of 1 if nonzero; by a well known result of Jacobson, it follows that A is commutative. Thus, if the theorem fails to hold, K is not absolutely algebraic, and, from Lemma 2, there is a noncentral $a \in A$ separable over K. We show that this leads to a contradiction.

Indeed, in this case Theorem 3 yields distinct valuations ν_1, ν_2 on $K(a)$ coinciding on K and $r \in \zeta A$ on which they are both positive. Let $x \in K(a)$ be such that $\nu_1(x) \neq \nu_2(x)$. Getting rid of denominators, if necessary, we can assume that x belongs to $\zeta A[a]$. Substituting x by $r^m x$, where m is big enough, we can suppose that $\nu_1(x) > \nu_2(x) > 0$. So, if

$$h(X) = a_0 X^m + a_1 X^{m-1} + \cdots + a_{m-s-1} X^{s+1} + X^s \in \mathbf{Z}[X],$$

it follows that for $i = 1, 2$ and for $s + 1 \leq j \leq m$, since $\nu_i(a_j) \geq 0$,

$$\nu_i(x^s) = s\nu_i(x) < \nu_i(a_{m-j}x^j) = \nu_i(a_{m-j}) + j\nu_i(x).$$

Hence

$$\nu_i(h(x)) = s\nu_i(x).$$

Now, there exist $f, g \in \mathbb{Z}[X]$ of orders $m \neq n$, and $z_1, z_2 \in \zeta A$ such that $f(x)z_1 = g(x)z_2$. So, computing as above:

$$m\nu_i(x) + \nu_i(z_1) = \nu_i(f(x)) + \nu_i(z_1) = \nu_i(f(x)z_1) = \nu_i(g(x)z_2) = n\nu_i(x) + \nu_i(z_2)$$

Since both ν_1 and ν_2 both coincide on z_1 and z_2, and $m \neq n$, it follows that $\nu_1(x) = \nu_2(x)$, a contradiction. □

We are ready now to prove Theorem 5.

Theorem 5 *Let R be a ring with no nonzero nilpotent elements. Suppose that for each a in R there are two co-monic polynomials $f, g \in \mathbb{Z}[x]$, of different orders, such that $f(a)xg(a) = g(a)xf(a)$ for every $x \in R$. Then R is commutative.*

Proof. By the main theorem of [2], R is a subdirect product of domains. Since the hypothesis is preserved under homomorphic images, it is enough to consider the case where R is a domain.

Suppose we can show that

$$\zeta R \neq \{0\} \quad \text{and} \quad R \quad \text{is algebraic over} \quad \zeta R. \qquad (*)$$

Then, if K is the field of fractions of ζR and $D = \{rz^{-1} | r \in R, z \in \zeta R^*\}$ is the central localization of R, it follows that D is a division ring. Without loss one can assume that always $g(a) \neq 0$, hence the hypothesis can be read as $f(a)/g(a) \in \zeta D = K$, and we fall back on the conditions of 4, which yields commutativity of R.

We proceed then to prove $(*)$. Consider first the possibility that for every $a \in R$ there is a co-monic integer polynomial f of positive order (thus nonzero) such that $f(a) \in \zeta R$. This implies that $\zeta R \neq \{0\}$: if for some $a \neq 0$, $f(a) = 0$, then, since R is a domain one can assume $\omega f = 1$ and write $f(X) = X\varphi(X) - X$, where φ is an integer polynomial of order at least 1, hence $a\varphi(a) = a$, and $\varphi(a) = 1 \in R$. Thus, in this case $(*)$ holds.

Before completing the proof, we note that by Posner's Theorem [10, 1.4.3], every PI prime ring satisfies $(*)$, hence, every PI subring of R is commutative. The remaining possibility is that for some $a \in R$, $h(a)$ is not central in R for any co-monic integer polynomial h. Now, if f and g are the polynomials the hypothesis asserts exist for a, then $f(a)Xg(a) - g(a)Xf(a)$ is a generalized polynomial identity for R. Since R is prime (a domain), it follow from a theorem of Smith [18] that either R is PI, or R contains PI subrings of arbitrarily large PI degree. But we have just observed that a PI subring of R must be commutative, therefore the second alternative cannot hold, and we have finished the proof. □

3 Primitive Rings

Having settled the division ring case, we can prove now:

Theorem 6 *For a primitive ring R, the following three conditions are equivalent:*

i) *For every element a there are two integer co-monic polynomials $f, g \in \mathbb{Z}[x]$, of different orders, such that $f(a)xg(a) = g(a)xf(a)$ for every $x \in R$.*

ii) *For every element a there are two monic integer polynomials $f, g \in \mathbb{Z}[x]$, of different degrees, such that $f(a)xg(a) = g(a)xf(a)$ for every $x \in R$.*

iii) *Either R is commutative, hence a field, or its multiplicative semigroup is torsion.*

Further, in the noncommutative case, the commuting ring of R is an absolutely algebraic field.

Proof. We will prove that either of conditions (i) or (ii) implies (iii); both proofs go together, separating for just a little while.

We must consider now the case where R is a dense ring of linear transformations of a vector space of dimension ≥ 2 over a division ring D. Therefore, both D and $M_2(D)$ are homomorphic images of subrings of R, hence they inherit properties (i) or (ii), whichever we are assuming. The last section has already provided us the proof that D is a field. We show now that if $M_2(D)$ satisfies (i) or (ii), then D is absolutely algebraic.

If not, we can find on D a nontrivial nonarchimedean valuation ν. At this point we separate slightly our argument: we choose an element $d \in D$ whose value is positive in the "orders" case, negative in the "degrees" case. Let $a = \begin{pmatrix} d & 0 \\ 0 & d^2 \end{pmatrix}$ and let f, g be the appropriate polynomials such that $f(a)xg(a) = g(a)xf(a)$ for every $x \in R$. Applying this condition to $x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ we conclude that $f(d)g(d^2) = f(d^2)g(d)$. Noticing that ν is nonegative on the coefficients of the polynomials, and actually zero if D is of positive characteristic, we see that $\nu(f(d)) = \omega f \cdot \nu(d)$ if $\nu(d) > 0$, $= \partial f \cdot \nu(d)$ if $\nu(d) < 0$. In either case, the equality $\nu(f(d)) + \nu(g(d^2)) = \nu(f(d^2)) + \nu(g(d))$ gives us a contradiction.

Also, from he primitivity of R and from Lemma 8 below, we conclude that for every $a \in R$ there are integer polynomials f, g (of distinct orders or degrees), and $\alpha \in D$ such that $f(a) = \alpha g(a)$: thus a is a zero of the nonzero polynomial $f - \alpha g$ over D. In combination with the former paragraph, the result follows from the next Lemma. □

Proposition 7 *An algebraic algebra over an absolute field is torsion.*

Proof. This seems to be well known; we repeat the argument in [17]. Let a be an element of the algebra, F be the field, and $f \in F[X]$ be a nonzero polynomial such that $f(a) = 0$. The splitting field of f over the prime field of F is finite, with q elements. Thus, if m is larger than the multiplicity of any root of f, it follows that f divides $(X^q - X)^m$, hence, if m is a large enough power of the characteristic, $a^{qm} = a^m$. □

We still depend on the following Lemma, due to Amitsur [1, Lemma 6a] and proved here with a slight modification.

Lemma 8 *Let R be a dense set of endomorphisms of a (left) vector space V over a division ring D. If $\mathbf{f}, \mathbf{g}$ are nonzero endomorphisms of V such that for every $\mathbf{r} \in R$, $\mathbf{frg} = \mathbf{grf}$, then $\mathbf{f} = \alpha\mathbf{g}$ for some α in the center of D.*

Proof. Choose $v \in V$ such that $\mathbf{g}v \neq 0$. If $\mathbf{f}v$ and $\mathbf{g}v$ are linearly independent, then, by density, there exists an $\mathbf{r} \in R$ such that $\mathbf{rg}v = v = \mathbf{rf}v$, hence $\mathbf{f}v = \mathbf{frg}v = \mathbf{grf}v = \mathbf{g}v$, a contradiction. Thus, there is an $\alpha \in D$ such that $\mathbf{f}v = \alpha\mathbf{g}v$. Now, if $w \in V$, there is an $\mathbf{r} \in R$ such that $\mathbf{rg}v = w$, hence $\mathbf{f}w = \mathbf{frg}v = \mathbf{grf}v = \mathbf{gr}\alpha\mathbf{g}v = \alpha\mathbf{grg}v = \alpha\mathbf{g}w$, therefore $\mathbf{f} = \alpha\mathbf{g}$. Finally, if $d \in D$, $d\alpha\mathbf{g}v = d\mathbf{f}v = \mathbf{f}dv = \alpha\mathbf{g}dv = \alpha d\mathbf{g}v$, hence α is central in D. □

We have passed by the last remark in the statement of Theorem 6, that is, under the hypothesis of the theorem, if R is noncommutative, then every commutative subring is locally finite. This follows from two facts, both easily verified: a finitely generated, commutative, torsion semigroup is finite — and a finite multiplicative semigroup of an algebra of positive characteristic generates a finite subring (and a finite-dimensional subalgebra). Its presence in the theorem is mostly to draw attention to the fact that there is no other local property implied by either of conditions (i) or (ii) specifically for primitive rings. For instance, a primitive torsion algebra may contain a copy of a Golod-Shafarevitch [5] nil algebra, so it is not necessarilly locally finite. This assertion follows from Theorem 1; this one, in turn, in view of Proposition 7, follows from the more general:

Theorem 9 *Any algebraic algebra over a field may be embedded into a primitive, unital, algebraic algebra.*

Proof. Let R be an algebraic algebra over the field F. We may suppose that R is already embdded into the endomorphism ring of a vector space V over F. Let $\mathcal{F}$ be the set of endomorphisms of V of finite rank. If $f \in \mathcal{F}$, then, as its image fV is finite dimensional, there is a nonzero polynomial $p \in F[X]$ such that $p(f) \cdot fV = 0$, which shows that $p(f)f = 0$, hence f is algebraic. It is clear that $\mathcal{F}$ is an ideal of End(V), hence the set $S = \{f + r | f \in \mathcal{F}, r \in R\}$ is a subring of End(V), which is dense for containing $\mathcal{F}$. Let us show that S is algebraic. If $s \in S$, there is an $r \in R$ such that $s \equiv r \pmod{\mathcal{F}}$; as r is torsion, there exist

$0 \leq n < m$ such that $r^m - r^n = 0$, hence $s^m - s^n \in \mathcal{F}$. As $\mathcal{F}$ is algebraic, it follows that s, hence S, is algebraic too. To get a ring with a unit, we consider all sums of a member of S and one of F, viewed as a scalar multiplication in End(V); this is clearly an algebraic subalgebra of End(V). □

4 Rings rational over subrings

Let us extend the notion of order from polynomials to rational functions: if f, g are polynomials, let $\omega(f/g) = \omega f - \omega g$. Also we will say that a rational function is *co-monic* provided it can be expressed as the quotient of two co-monic polynomials with integer coefficients. Notice that if φ, ψ are rational functions, and $\omega\varphi > 0$, then, $\omega(\varphi \circ \psi) = \omega\varphi \cdot \omega\psi$, and if both are co-monic, so is the composition. In particular, co-monic rational functions of positive order are closed under composition (not surprising, if one thinks of power series).

Small subrings, as required below, are, for instance, proper subfields. The proof is vaguely reminiscent of that of [9].

Proposition 10 *Let R be a subring of the division ring D, such that the subring $R^{-1}R$ generated by the elements of R and its inverses is not D. Then, if for every element $x \in D$ there is a co-monic rational function of positive order φ such that $\varphi(x) \in R$, then D is commutative.*

Proof. For this proof, we shorten "co-monic rational function of positive order" simply to *function*. Let us show first that, given any elements $x, y \in D$, there exists a function φ such that $\varphi(y)$ commutes with x.

For a given y, consider first the case $x \in D \setminus R^{-1}R$. Let ψ be a function such that $\alpha = \psi(y) \in R$. Choose a function ψ_1 such that $\psi_1(x\alpha x^{-1}) = s \in R$, and let $\beta = \psi_1(\alpha) = x^{-1}sx$. Choose also a function ψ_2 such that $\psi_2((1+x)\beta(1+x)^{-1}) = r \in R$. Writing $\psi_2 = f/g$, where f, g are integer polynomials, we now observe that since

$$(1+x)\psi_2(\beta) = r(1+x)$$

it follows that

$$(1+x)f(\beta) = r(1+x)g(\beta)$$

and, as $x\beta = sx$,

$$f(\beta) - rg(\beta) = -(f(s) - rg(s))x.$$

Now, this equation says that $x \in R^{-1}R$, unless $f(s) - rg(s) = 0$. This implies that $\psi_2(\beta) = r$, whence it commutes with x. It follows that $\psi_2 \circ \psi_1 \circ \psi(y)$ commutes with x.

Now, given $x \in R^{-1}R$, choose $w \in D \backslash R^{-1}R$. Then, there is a function ψ such that $a = \psi(y)$ commutes with w, and, since $x + w \notin R^{-1}R$, there is a function ψ_1 such that $\psi_1(a)$ commutes with $x + w$. Since this clearly commutes with w, it also commutes with x. Therefore, $\psi_1 \psi(y)$ commutes with x.

We can finish the proof now. Given $x, y \in D$, let A be the subring of D they generate. Then, for every $a \in A$, there exist a function φ such that $\varphi(a)$ commutes with x, and a function ψ such that $\psi(\varphi(a))$ commutes with y. Since it clearly commutes with x also, $\psi\varphi(a) \zeta A$. It follows from Lemma 4 that A is commutative. That is, x and y commute. □

Theorem 11 *Let R be a proper subring of the division ring D, and suppose that for every $a \in D$ there exist monic polynomials $f, g \in \mathbb{Z}[X]$ such that $\partial f > \partial g$ and $f(a)/g(a) \in R$. Then D is commutative and R is a subfield.*

Proof. Pick a nonzero $a \in D$, and let f, g be as stated for a^{-1}. Let $n = \partial f, m = \partial g$, and define $\bar{f}(X) = X^n f(X^{-1}), \bar{g}(X) = X^m g(X^{-1})$. Note that $\bar{f}$ and $\bar{g}$ are both co-monic of order 0, and it follows that

$$\frac{\bar{f}(a)}{a^{m-n}\bar{g}(a)} \in R.$$

If $a \in R$, this shows, as $m > n$, that $a^{-1} \in R$, whence R is a (proper) division subring. But then, it also follows that for a general $a \in D$, $a^{m-n}\bar{g}(a)/\bar{f}(a) \in R$ hence the hypothesis of the last proposition are fulfilled (the possibility that $\bar{f}(a) = 0$ does not hinder this conclusion). So, the result follows. □

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