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EDGE PROCESSES OF ONE DIMENSIONAL STOCHASTIC GROWTH MODELS

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Summary. *The following results are proven for one dimensional Stochastic Growth Models:*

1)The infinite system converges exponentially fast to the empty configuration if and only if the edge speed α is strictly negative (which is equivalent to $\alpha = -\infty$).

2)The corresponding system seen from the edge has at least one invariant probability measure if and only if $\alpha \geq 0$.

3)For the supercritical basic Contact Process the unique invariant probability for the system seen from the edge can be coupled to the upper invariant measure of the infinite system in such a way that they only differ at finitely many sites (to the left of the origin). At the critical point the asymptotic density of any invariant probability measure for the system seen from the edge is zero.

Résumé. On demontre les résultats suivants, pour les processus stochastiques de croissance unidimensionels:

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*Partially supported by CNPq

1) Le systeme infini converge exponentiellement vers le vide si et seulement si la vitesse du bord α est strictement negative (ceci est equivalent à $\alpha = -\infty$).

2) Le systeme correspondant vu du bord a au moins une probabilité invariante si et seulement si $\alpha \geq 0$.

3) Pour le processus de contact surcritique, l'unique mesure invariante du processus vu du bord peut être couplée avec la mesure invariante superieure du processus infini, de telle façon qu'elles different en un nombre fini de points (à gauche de l'origine). Au point critique la densité asymptotique de toute probabilité invariante pour le système vu du bord est nulle.

1. Introduction and statement of results

A one dimensional Stochastic Growth Model is a Markov process whose state at time t, ξ_t is in $\{0,1\}^Z$. If $\xi_t(x) = 1$ this means that we have a particle on $x \in Z$, if $\xi_t(x) = 0$ this means that the site x is empty. The process ξ_t evolves according to the following rules:

particles die at rate 1, i.e, if $\xi_t(x) = 1$ then

(1)
$$lim_{s\to 0}\frac{1}{s}P(\xi_{t+s}(x) = 0|\xi_t) = 1$$

and particles are born at rate $\lambda b_x(\xi)$, i.e, if $\xi_t(x) = 0$ then

(2)
$$lim_{s\to 0}\frac{1}{s}P(\xi_{t+s}(x) = 1|\xi_t) = \lambda b_x(\xi_t)$$

where $\lambda > 0$ is a real parameter.

Throughout this paper we will assume that there is no spontaneous generation of particles, that the process is attractive and invariant under translations and symmetries, and that the interaction between particles has a finite range. This means that the birth rates will satisfy the conditions (3) - (7), stated below.

(3)
$$b_x(\eta_0) = 0,$$

where η_0 is the configuration with no particles,

(4)
$$b_x(\eta) = b_0(\tau_x\eta),$$

where $(\tau_x\eta)(y) = \eta(y+x)$

(5) $$b_0(\xi^-) = b_0(\xi),$$

where $\xi^-(x) = \xi(-x)$ for all x,

(6) $$b_0(\eta) \leq b_0(\xi) \text{ if } \eta \leq \xi,$$

where we say that $\eta \leq \xi$ if $\eta(x) \leq \xi(x)$ for all x,

(7) for some positive integer R, $b_0(\eta) = b_0(\xi)$ if $\eta(x) = \xi(x) \quad \forall x \in [-R, R]$.

When $R = 1$ we say that we have a Nearest-Neighbor model. The Basic Contact process is a special Nearest-Neighbor model where:

$$b_x(\xi) = \xi(x-1) + \xi(x+1)$$

These processes can be constructed by Harris' graphical method as was indicated for instance in Durrett (1985). For this purpose, for each $x \in Z$ let $(S_n^x)_{n\geq 1}$ and $(T_n^x)_{n\geq 1}$ be the arrival times of two independent Poisson processes having rates 1 and $b = \lambda b_0(Z)$ respectively (Z is the configuration with one particle on every site of Z), and let $(U_n^x)_{n\geq 1}$ be a sequence of independent random variables each one with an uniform distribution on [0;1]. At times S_n^x, $n \geq 1$ we kill a particle at x if one is present there. At times T_n^x, $n \geq 1$ there will be a birth at x if it is not already occupied and if the configuration satisfies $b_x(\xi_{T_n^x-}) \geq \frac{b}{\lambda} U_n^x$, where $\xi_{T_n^x-}$ is the limit of ξ_t as t increases to T_n^x.

Using properties of the Poisson process one can show that the construction above for any initial configuration gives rise to a Markov process which evolves according to the rules given before.

We will denote the Stochastic Growth model by $\xi_{t,\lambda}^\eta$, with initial configuration η and birth rates: $\lambda b_x(\xi)$.

We also define $*$, the configuration with only one particle which is on site 0 and

$$\tau^*(\lambda) = \inf\{t > 0 : \sum_{x\in Z} \xi_{t,\lambda}^*(x) = 0\}$$

When no confusion is possible we will omit λ in all our notation. We define now two well known critical parameters, first the critical parameter for finite systems:

$$\lambda_f = \inf\{\lambda > 0 : P(\tau^*(\lambda) = \infty) > 0\}$$

It follows from the attractiveness assumption that $\xi^Z_{t,\lambda}$ converges in law, as $t \to \infty$, to a probability μ_λ. Let δ_0 be the Dirac probability which concentrates on the empty configuration. The critical parameter for infinite systems is:

$$\lambda_i = \inf\{\lambda > 0 : \mu(\lambda) \neq \delta_0\}$$

We can now state our results. Let 1 be the configuration with a particle on every site on the left of the origin and no particle on the right of the origin. Let r^1_t be the position of the right edge of the process with the initial configuration 1. From an appropriate version of the Subadditive Ergodic Theorem (Liggett (1985) Chap. VI Th. 2.6 and Th. 2.19) it is easy to see that there exists a real constant $\alpha(\lambda)$ in $[-\infty; +\infty[$ such that:

$$lim_{t\to\infty}\frac{r^1_t}{t} = \lim_{t\to\infty} E\left(\frac{r^1_t}{t}\right) = \alpha(\lambda) = \inf_{t>0} E\left(\frac{r^1_t}{t}\right) \quad a.s.$$

From the last expression above and monotonicity, $\alpha(.)$ is a right-continuous function.

THEOREM 1. *a) If $\alpha(\lambda) < 0$, then there exist strictly positive real numbers C and γ such that*

$$P(\xi^Z_{t,\lambda}(0) = 1) \leq Ce^{-\gamma t}.$$

b) If $\alpha(\lambda) > -\infty$, then

$$\lim_{t\to\infty} tP(\xi^Z_{t,\lambda}(0) = 1) = +\infty.$$

Both parts of this theorem were already known for the contact process (See Th. 3.4 and Th 3.10 in Chap. VI of Liggett (1985)), but their proofs relied on properties as self duality and additivity which are not satisfied by all the processes we consider here. As an immediate consequence of this theorem we obtain:

COROLLARY 1. *If $\alpha(\lambda) < 0$, then $\alpha(\lambda) = -\infty$.*

Because of Theorem 1, it seems natural to define the following critical values of λ:

$$\lambda_\alpha = \inf\{\lambda > 0 : \alpha(\lambda) \geq 0\},$$

$$\lambda_{f,e} = \sup\{\lambda > 0: \text{ there exist } C, \gamma \in]0, +\infty[\text{ such that } P(\tau^*(\lambda) > t) \leq Ce^{-\gamma t}\}$$

and

$$\lambda_{i,e} = \sup\{\lambda > 0: \text{ there exist } C, \gamma \in]0, +\infty[\text{ such that } P(\xi^Z_{t,\lambda}(0) = 1) \leq Ce^{-\gamma t}\}$$

Our next corollary concerns these critical values.

COROLLARY 2. *a)* $\lambda_\alpha = \lambda_{i,e} \leq \lambda_{f,e}$,
b) $\lim_{t\to\infty} tP(\xi^{Z}_{t,\lambda_{i,e}}(0) = 1) = \infty$.

Note that the equality in part a) follows immediately from Theorem 1, and that part b) is an easy consequence of part a), the right continuity of α and part b) of Theorem 1. The inequality in part a) will be proved at the end of Section 2. Finally, observe that part b) implies that if $P(\xi^{Z}_{t,\lambda_{i,e}}(0) = 1)$ behaves as $t^{-\kappa}$ when $t \to \infty$, then we must have $\kappa < 1$.

We consider now the process $\xi_{t,\lambda}$ seen from the edge with an initial configuration with no particles on the right of the origin. This means that when the rightmost particle dies or when a particle is born on the right of the rightmost particle then we make a translation to have at all times the rightmost particle at the origin. Let us call the process seen from the edge: $\tilde{\xi}_{t,\lambda}$. We have:

THEOREM 2. *The process* $\tilde{\xi}_{t,\lambda}$ *has an invariant probability measure* $\tilde{\mu}(\lambda)$ *if and only if* $\alpha(\lambda) > -\infty$.

For the contact process in discrete time, this result is not new; Durrett (1984) showed that the invariant measure exists if $\alpha(\lambda) > -\infty$ and Schonmann (1987) proved the converse. The proof we give simplifies Durrett's for the "if" part, and uses the ideas in Schonmann (1987), but is not a straightforward generalization of his proof, for the "only if" part.

We are now going to state some results which we proved only for the Contact Process. We will say that $\mu_1 \leq \mu_2$ where μ_1 and μ_2 are two probability measures, if for any continuous increasing function f we have:

$$\int f(\eta)d\mu_1(\eta) \leq \int f(\eta)d\mu_2(\eta)$$

Let us recall that μ_λ is the upper invariant probability for the system not seen from the edge, and let $\tilde{\mu}_\lambda$ be an invariant measure for the process seen form the edge. In the following two theorems we will compare $\tilde{\mu}_\lambda$ with the restriction of μ_λ to the non-positive integers. For this purpose, to any configuration η we associate $\hat{\eta}$, defined by: $\hat{\eta}(x) = 0$ if $x > 0$ and $\hat{\eta}(x) = \eta(x)$ if $x \leq 0$. We define $\hat{\mu}_\lambda$ by

$$\hat{\mu}_\lambda(A) = \mu_\lambda(\eta : \hat{\eta} \in A)$$

for every cylinder set A. Recall that for the contact process $\lambda_\alpha = \lambda_i = \lambda_f$ and denote by λ_c their common value.

Theorem 3. *Let $\lambda \geq \lambda_c$, then: $\tilde{\mu}_\lambda \geq \hat{\mu}_\lambda$.*

Theorem 4. *Let $\lambda \geq \lambda_c$, then there exist constants $C, \gamma, a \in]0, \infty[$ such that any invariant measure $\tilde{\mu}_\lambda$ satisfies:*

$$0 \leq \tilde{\mu}_\lambda(\eta: \eta(-x) = 1) - \hat{\mu}_\lambda(\eta: \eta(-x) = 1) \leq Ce^{-\gamma x} + P(x/a < \tau^* < \infty) \quad \forall x > 0.$$

Galves and Presutti (1987) proved that if $\lambda > \lambda_c$ then there is a unique invariant measure for the contact process seen from the edge. Since in this case we have the following estimate due to Durrett and Griffeath (see Liggett (1985), Chap. VI, Th. 3.23):

$$P(t < \tau^* < \infty) \leq Ce^{-\gamma t}$$

we obtain

Corollary 3. *Let $\lambda > \lambda_c$, then there exist constants $C, \gamma \in]0, \infty[$ such that the invariant measure $\tilde{\mu}_\lambda$ for the process seen from the edge satisfies:*

$$0 \leq \tilde{\mu}_\lambda(\eta: \eta(-x) = 1) - \hat{\mu}_\lambda(\eta: \eta(-x) = 1) \leq Ce^{-\gamma x} \quad \forall x > 0.$$

From Theorem 3, Corollary 3 and the Borel-Cantelli Lemma we conclude that for $\lambda > \lambda_c$, there exists a probability measure ν on $\{0,1\}^{Z_-} \times \{0,1\}^{Z_-}$ such that:

a) The first marginal of ν is $\tilde{\mu}_\lambda$,

b) The second marginal of ν is $\hat{\mu}_\lambda$,

c) $\nu((\eta, \xi): \eta \geq \xi) = 1$,

d) $\nu((\eta, \xi): \Sigma |\eta(x) - \xi(x)| < \infty) = 1$.

From Theorem 4 we can also conclude that

$$\text{(S)} \qquad \lim_{x \to \infty} \tilde{\mu}_{\lambda_c}(\eta: \eta(-x) = 1) = \hat{\mu}_{\lambda_c}(\eta: \eta(0) = 1) = 0,$$

where the last equality has been recently proved by Bezuidenhout and Grimmett (1989). This last result also implies that there is no measure ν satisfying a) - d) above at the critical point.

Since for $\lambda > \lambda_c$ μ_λ is ergodic and satisfies a Central Limit Theorem, (see Liggett (1985) (Chap. III, Prop. 2.16) and Griffeath (1981)), the comment following Corollary 3 implies that $\tilde{\mu}_\lambda$ satisfies a strong law of large numbers and a Central Limit Theorem, i.e. that for $\lambda > \lambda_c$

$$\lim_{n\to\infty} \frac{1}{n}\sum_{x=0}^{n-1} \eta(-x) = \rho_\lambda \quad \tilde{\mu}_\lambda \text{ a.s.},$$

where $\rho_\lambda = \mu_\lambda(\eta: \eta(0) = 1)$, and

$$\lim_{n\to\infty} \tilde{\mu}_\lambda(\eta: \frac{1}{\sqrt{n}}\sum_{x=0}^{n-1}(\eta(-x) - \rho_\lambda) \leq a) = F(a)$$

where ρ_λ is as above and F is the distribution function of a normal random variable with mean 0 and positive variance (which depends on λ).

2. Proofs of Theorem 1 and Corollary 2

We will prove part a) of Theorem 1 using a restart argument from Durrett (1984), Sect. 12. Let us define an auxiliary process ξ_t^r with one particle on every site at the initial time 0 and which evolves like the Stochastic Growth Model we are considering until time:

$$\tau_1 = \inf\{t > 0 : g_t \leq -2R \text{ and } d_t \geq 2R\}$$

where

$$g_t = \sup\{x \leq 0 : \xi_t^r(x) = 1\}$$

$$d_t = \inf\{x \geq 0 : \xi_t^r(x) = 1\}$$

At time τ_1 we put a particle on every site on the left of $-2R$ and on the right of $2R$. We let all the sites between $-2R$ and $2R$ empty. Recall that R is the range of the interaction. After τ_1 we let again the process ξ_t^r evolve like the Stochastic Growth Model until time

$$\sigma_1 = \inf\{t > \tau_1 : g_t \geq -R \text{ or } d_t \leq R\}$$

Note that τ_1 is finite with probability 1 but σ_1 is infinite with stictly positive probability. At time σ_1 (if it is finite) we put a particle on every site of Z and so ξ_t^r begins again from the initial configuration. We define by induction two sequences of random variables τ_k and σ_k, for $k \geq 2$

$$\tau_k = \inf\{t > \sigma_{k-1} : g_t \leq -2R \text{ and } d_t \geq 2R\}$$

$$\sigma_k = \inf\{t > \tau_k : g_t \geq -R \text{ or } d_t \leq R\}$$

At time τ_k we put a particle on every site on the left of $-2R$ and on the right of $2R$, and we let the sites between $-2R$ and $2R$ empty. At time σ_k we put a particle on every site of Z and so the system "restarts". Between these random times the system ξ_t^r evolves like the Stochastic Growth Model. Let us define the random variable:

$$K = \sup\{k \geq 1 : \sigma_k < \infty\}$$

We fix λ such that $\alpha(\lambda) < 0$ and will omit it in the rest of the proof. Let ξ_t^Z be the Stochastic Growth Model with initial configuration Z, by the attractiveness assumption (6), we have:

$$P(\xi_t^Z(0) = 1) \leq P(\xi_t^r(0) = 1) \leq P(\tau_{K+1} > t) \tag{2.1}$$

since after time τ_{K+1} we will not have any particle between the sites $-R$ and R. Let us define $X_k = \sigma_k - \tau_k$ for $1 \leq k \leq K$, $Y_1 = \tau_1$ and $Y_k = \tau_k - \sigma_{k-1}$ for $2 \leq k \leq K+1$. We can write:

$$\tau_{K+1} = \sum_{i=1}^{K} X_i + \sum_{i=1}^{K+1} Y_i \tag{2.2}$$

From (2.1) and (2.2) we have that:

$$P(\xi_t^Z(0) = 1) \leq P(\sum_{i=1}^{K} X_i \geq t/2) + P(\sum_{i=1}^{K+1} Y_i \geq t/2) \tag{2.3}$$

But conditioned on $\{K = k\}$ we have that $(X_i)_{1 \leq i \leq k}$ are i.i.d. random variables with the same distribution as X_1 conditioned on $\{X_1 < \infty\}$. Let $(Z_i)_{i \geq 1}$ be an infinite sequence of i.i.d random variables with this same distribution. For every $\epsilon > 0$ we have:

$$P(\sum_{i=1}^{K} X_i \geq t/2) = \sum_{k=1}^{\infty} P(\sum_{i=1}^{K} X_i \geq t/2 | K = k) P(K = k) \tag{2.4}$$

$$\leq P(K \geq [\epsilon t]) + P(\sum_{i=1}^{[\epsilon t]} Z_i \geq t/2)$$

where $[a]$ is the largest integer less than or equal to a. Our next task is to show that Z_1 has an exponentially decaying tail, i.e,

$$P(Z_1 > s) \leq Ce^{-\gamma s} \tag{2.5}$$

Here and in the sequel C and γ are strictly positive constants whose values may change from line to line. Since the range of the interaction is R, between times τ_1 and σ_1, the system to the left of the origin does not interact with the system to its right. So by symmetry,

$$P(s < X_1 < \infty) \leq 2P(r_u^1 > 0 \text{ for some } u > s) \tag{2.6}$$

And since $\alpha < 0$, we have (see the proof of Th.1 Sect.4.b of Durrett (1988))

$$P(r_u^1 > 0 \text{ for some } u > s) \leq Ce^{-\gamma s} \tag{2.7}$$

Now (2.5) follows from (2.6), (2.7) and the fact that $P(X_1 < \infty) > 0$.

Let m be the mean value of Z_1. Due to (2.5), m is finite and the Cramer-Chernoff Theorem implies that if ϵ is chosen so that $1/2\epsilon > m$ then

$$P(\sum_{i=1}^{[\epsilon t]} Z_i \geq t/2) \leq Ce^{-\gamma t} \tag{2.8}$$

Since K has a geometric distribution, for every $\epsilon > 0$

$$P(K > [\epsilon t]) \leq Ce^{-\gamma t} \tag{2.9}$$

From (2.4), (2.8) and (2.9) we have that

$$P(\sum_{i=1}^{K} X_i \geq t/2) \leq Ce^{-\gamma t}$$

Comparing ξ_t^r with a system where every site on the left of $-R$ and every site on the right of R has a particle which cannot die (so we have a finite Markov Process), it is easy to see that:

$$P(Y_i > t) \leq Ce^{-\gamma t}$$

This shows that we can treat the second term on the r.h.s of (2.3) exactly as the first term and this concludes the proof of part a).

Let us now prove part b). For every $\epsilon > 0$,

(2.10) $P(r_t^1 \in [t(\alpha(\lambda)-\epsilon); t(\alpha(\lambda)+\epsilon)]) \leq P(\exists x \in [t(\alpha(\lambda)-\epsilon); t(\alpha(\lambda)+\epsilon)]: \xi_t^1(x) = 1)$.

Using attractiveness and translation invariance of the model, we have that the r.h.s of (2.10) is less than:

$$(2\epsilon t + 1)P(\xi_t^Z(0) = 1)$$

And so

$$\liminf_{t\to\infty} tP(\xi_t^Z(0) = 1) \geq \frac{1}{2\epsilon} \liminf_{t\to\infty} P(r_t^1 \in [t(\alpha(\lambda)-\epsilon); t(\alpha(\lambda)+\epsilon)]) = \frac{1}{2\epsilon}$$

Since ϵ can be taken arbitrarly small this concludes the proof of Theorem 1.

We will now prove the inequality in part a) of Corollary 2. Let $\lambda < \lambda_{i,e}$ and with this fixed λ we have for every $c > 0$,

(2.11) $P(\tau^* > t) \leq P(\exists x: |x| < ct \text{ and } \xi_t^*(x) = 1) + P(\exists x: |x| \geq ct \text{ and } \xi_t^*(x) = 1)$

By attractiveness and translation invariance the first term in (6.1) is smaller than:

$$2ctP(\xi_t^Z(0) = 1)$$

For the second term we use the fact that each edge of ξ_t^* increases at most (when there are no deaths) like a Poisson process multiplied by R, with parameter λB. So (2.11) is smaller than:

$$2ctP(\xi_t^Z(0) = 1) + 2P(R\mathcal{P}(\lambda Bt) \geq ct)$$

where $\mathcal{P}(m)$ is a random variable with a Poisson distribution of parameter m. Since these two terms go to 0 exponentially fast as t goes to infinity, provided c is chosen large enough, there are two strictly positive constants C and γ such that:

$$P(\tau^* > t) \leq Ce^{-\gamma t}$$

and so $\lambda \leq \lambda_{i,e}$. This completes the proof of Corollary 2.

3. Proof of Theorem 2

First suppose that $\alpha(\lambda) > -\infty$, then set

$$X = \{\eta : \eta(0) = 1, \eta(x) = 0 \text{ for all } x > 0\}$$

$$\tilde{X} = \{\eta \in X : \sum_{x \leq 0} \eta(x) = \infty\}$$

For $\eta \in \tilde{X}$ the process seen from the edge $\tilde{\xi}_t^\eta$ is well defined by

$$\tilde{\xi}_t^\eta(x) = \xi_t^\eta(x + \sup\{y \in Z : \xi_t^\eta(y) = 1\})$$

Clearly $\tilde{\xi}_t^\eta \in \tilde{X}$ for every $t \geq 0$. Let νS_t be the distribution of $\tilde{\xi}_t^\nu$. Recall that $\mathbf{1}$ is the configuration with a particle on every site on the left of the origin and no particle on the right of the origin, δ_1 is the probability measure that concentrates on this configuration. And set $\tilde{\nu}_t := \delta_1 S_t$.

Since X is a compact space, there is a sequence t_n of real numbers going to infinity such that the sequence of probability measures:

$$\tilde{\mu}_n = \frac{1}{t_n} \int_0^{t_n} \tilde{\nu}_s ds$$

converges weakly to some probability $\tilde{\mu}$ on X when n goes to infinity. We will show now that $\tilde{\mu}$ concentrates on $\tilde{X}$. Let us define:

$$A_{i,j} \doteq \{\eta : \sum_{x=-i}^{0} \eta(x) < j\}$$

Set

$$B = Rb_0(Z)$$

A simple comparison shows that:

$$\text{(3.1)} \qquad E(r_{t+1}^1 - r_t^1)^+ \leq E(RP(\lambda B)) = \lambda RB$$

where $P(m)$ has a Poisson distribution with parameter m. The important inequality is the following one:

(3.2) $$E(r^1_{t+1} - r^1_t)^- \geq ip(j)\tilde{\nu}_t(A_{i,j}) \text{ where } p(j) > 0.$$

This comes from the fact that if the process is at time t on $A_{i,j}$ and if all the particles which are between 0 and $-i$ at time t die before time $t+1$ and if there are no births on the right of $-i$ during the interval $[t, t+1]$ then the rightmost particle goes backwards by at least i. The event we just described has a probability which is bounded below by a strictly positive constant which depends only on j. It is this constant that we have denoted by $p(j)$. From (3.1) and (3.2) we obtain

(3.3) $$\int_0^t E(r^1_{s+1} - r^1_s)ds \leq RB\lambda t - ip(j)\int_0^t \tilde{\nu}_s(A_{i,j})ds$$

The term on the left of (3.3) is also equal to:

$$\int_t^{t+1} Er^1_s ds - \int_0^1 Er^1_s ds$$

But

$$\lim_{t\to\infty} \frac{1}{t}\int_t^{t+1} Er^1_s ds = \alpha(\lambda) > -\infty$$

$$\lim_{t\to\infty} \frac{1}{t}\int_0^1 Er^1_s ds = 0$$

We substitute t_n for t and let n go to infinity in (3.3), so that:

$$\tilde{\mu}(A_{i,j}) \leq \frac{RB\lambda - \alpha(\lambda)}{ip(j)}$$

We let i go to infinity to conclude that:

$$\tilde{\mu}\left(\bigcap_{i=1}^{\infty} A_{i,j}\right) = 0 \quad \forall j, \quad \text{therefore}$$

$$\tilde{\mu}(\text{ configurations with finite particles }) = 0$$

Now we know that $\tilde{\mu}_n$ and $\tilde{\mu}$ concentrate on $\tilde{X}$ the subset of X of infinite configurations. But $\tilde{X}$ is a dense borelian of X and so any uniformly continuous function on $\tilde{X}$ may be extended to a uniformly continuous function on all X and since $\tilde{\mu}_n$ converges to $\tilde{\mu}$ on X,

$$\lim_{n\to\infty}\int f d\tilde{\mu}_n = \int f d\tilde{\mu}$$

for any f uniformly continuous on $\tilde{X}$. This implies that $\tilde{\mu}_n$ converges to $\tilde{\mu}$ on $\tilde{X}$. (See Theorem 3.3.1 in Ethier and Kurtz (1986)). For any function f on $\tilde{X}$ which depends only on finitely many sites and any $t > 0$ it is not difficult to see that the function: $\eta \to E(f(\tilde{\xi}_t^\eta))$ is continuous on $\tilde{X}$. So the process seen from the edge has the Feller property on $\tilde{X}$. To show now that $\tilde{\mu}$ is invariant it is enough to use a standard argument, since from the remarks above

$$\tilde{\mu}S_t = \lim_{n\to\infty}\tilde{\mu}_n S_t = \lim_{n\to\infty}\frac{1}{t_n}\int_t^{t+t_n}\tilde{\nu}_s ds = \tilde{\mu}$$

The key of the proof of the converse is the following lemma which gives a relation between the process seen from the edge and the process not seen from the edge. This lemma will also be very useful for the proof of Theorem 4. Recall that $\tilde{X}$ is the set of infinite configurations of X.

Lemma 1: *Given $\lambda > 0$, there exist constants C, γ and a in $]0,\infty[$ such that:*

$$\sup_{\eta\in\tilde{X}} P(\tilde{\xi}_t^\eta([-at]) = 1) \leq P(\xi_t^Z(0) = 1) + Ce^{-\gamma t}.$$

where [ ] denotes integer part of.

Proof of the Lemma:

We define a process ζ_t which is roughly going to play the role of the dual of ξ_t. To do so we start constructing for each $t > 0$ and each $x \in Z$ a process ${}_t\zeta_s^x$, for which the time parameter s runs in the interval $[0,t]$, in the following way: for each realization of $(\xi_u)_{0\leq u\leq t}$ we have a corresponding realization of Poisson marks (deaths and births) as we indicated in the construction of the process. The time for ${}_t\zeta_s$ is thought of as running backwards: $s = t - u$. Now we define ${}_t\zeta_s^x$ as follows: ${}_t\zeta_0^x = x$ and for all $s \leq t$, ${}_t\zeta_s^x$ is an interval which increases with s in the following way: if at time s there is a birth mark at a site y of ${}_t\zeta_{s-}^x$ then

$${}_t\zeta_s^x = {}_t\zeta_{s-}^x \cup [y - R; y + R]$$

For the purpose of this construction it is understood that there is a birth mark at a site y at time s if and only if $T_n^y = t - s$ for some $n \geq 1$. Now we define ζ_t^x as ${}_t\zeta_t^x$. It is easy to see that ξ_t^x, has the following property:

(3.4) $$\text{If } \eta_1(y) = \eta_2(y) \text{ for all } y \in \zeta_t^x \text{ then } \xi_t^{\eta_1}(x) = \xi_t^{\eta_2}(x)$$

In particular from (3) we have

(3.5) $$\{\xi_t^\eta(y) = 1\} \subset \{\exists x \in \zeta_t^y \text{ such that } \eta(x) = 1\}$$

Recall that:

$$B = Rb_0(Z)$$

and let $l({}_t\zeta_s^x)$ and $r({}_t\zeta_s^x)$ be the right and left edge respectively of ${}_t\zeta_s^x$. Since $l({}_t\zeta_s^x)$ only decreases if there is a birth mark at a distance at most equal to R to the right of $l({}_t\zeta_s^x)$ we have for every $c > 0$:

(3.6) $$P(l(\zeta_t^x) < x - c) \leq P(RP(\lambda Bt) > c)$$

where $P(m)$ is a variable having a Poisson distribution with parameter m. By symmetry a similar inequality is also true for the right edge. We have for any initial configuration η in X and any real number a:

(3.7) $$P(\xi_t^\eta(-at) = 1) = \sum_{x \in Z} P(r_t^\eta = x; \xi_t^\eta(x - at) = 1)$$

(In this proof when a real number appears as a site, we are referring to its integer part) Let a and b be positive constants such that $a > 2b$ and define the two events:

$$B_1(x,t) = \{l(\zeta_t^y) \geq x - bt, \text{ for all } y \geq x\} \text{ and } B_2(x,t) = \{r(\zeta_t^{x-at}) \leq x - at + bt\}$$

and define $B(x,t) = B_1(x,t) \cap B_2(x,t)$. Each term of the sum in (3.7) is less than:

(3.8) $$P(r_t^\eta = x; B_1(x,t); \xi_t^\eta(x - at) = 1; B_2(x,t)) + P(r_t^\eta = x; B^c(x,t))$$

By attractiveness the first term in (3.8) is less than:

(3.9) $$P(r_t^\eta = x; B_1(x,t); \xi_t^Z(x - at) = 1; B_2(x,t))$$

From (3.4) and the inequality $a > 2b$, the Poisson marks defining the event $\{r_t^y = x; B_1(x,t)\}$ are in a space time region which has no intersection with the region in which the Poisson marks defining the event $\{\xi_t^Z(x-at) = 1; B_2(x,t)\}$ lie, so these two events are independent and using translation invariance, (3.9) is less than

(3.10)
$$P(\xi_t^Z(0) = 1)P(r_t^y = x)$$

We now consider the second term of (3.8). We estimate first $P(B^c(x,t))$,

$$P(B^c(x,t)) \le \sum_{y \ge x} P(l(\zeta_t^y) < x - bt) + P(r(\zeta_t^{x-at}) > x - at + bt)$$

But $-l(\zeta_t^y)$ increases less than R times a Poisson process of parameter λB, therefore

$$P(l(\zeta_t^y) \le x - bt) \le P(R\mathcal{P}(\lambda Bt) \ge y - x + bt)$$

For $c \ge \lambda$ we have:

$$P(\mathcal{P}(\lambda) \ge c) = \lambda^c e^{-\lambda} \sum_{n \ge c} \frac{\lambda^{n-c}}{n!} \le e^{-\lambda}\lambda^c \sum_{n=0}^{\infty} \frac{c^{n-c}}{n!} =$$
$$\left(\frac{\lambda}{c}\right)^c e^{c-\lambda} = e^{-(c \ln \frac{c}{\lambda} - c + \lambda)}.$$

Therefore for c such that $\ln \frac{c}{\lambda} \ge 2$ we obtain:

(3.11)
$$P(\mathcal{P}(\lambda) \ge c) \le e^{-c}$$

Hence, for $b \ge \lambda BRe^2$

$$P(l(\zeta_t^y) \le x - bt) \le e^{-\frac{bt+y-x}{R}}$$

and since we can give an upper bound for $P(r(\zeta_t^{x-at}) \ge x - at + bt)$ exactly in the same way, we have:

(3.12)
$$P(B^c(x,t)) \le Ce^{-\frac{bt}{R}}$$

where C here depends only on R. Let $z_1 = 0$ and $z_{i+1} = \sup\{x < z_i : \eta(x) = 1\}$. We define $A_\eta(t)$ as:

$$A_\eta(t) = \{z_i, 1 \le i \le [te']\}, \text{ where } [te'] = \inf\{k \in \mathbf{Z}: k \ge te'\}.$$

Observe that if $\{r_t^\eta < z_{[tc']}\}$ occurs, then this means that all the $[tc']$ rightmost particles died before time t and since the death rate is 1, we have that:

$$P(r_t^\eta < z_{[tc']}) \leq (1 - e^{-t})^{[tc']} \leq e^{-t} \tag{3.13}$$

Let c be a constant and

$$C_\eta(t) = \{z \in Z : |z - x| < ct \text{ for some } x \in A_\eta(t)\}$$

We now consider

$$\sum_{x \in Z} P(r_t^\eta = x; B^c(x,t)) = \sum_{x \in C_\eta(t)} P(r_t^\eta = x; B^c(x,t)) + \sum_{x \notin C_\eta(t)} P(r_t^\eta = x; B^c(x,t)) \tag{3.14}$$

The first term on the r.h.s. of (3.14) is less than:

$$|C_\eta(t)| P(B^c(x,t))$$

and using (3.12) this is less than:

$$(2ct + 1)[tc']Ce^{-\frac{M}{R}} \tag{3.15}$$

The second term on the r.h.s. of (3.14) is less than:

$$P(r_t^\eta \notin C_\eta(t); r_t^\eta \geq z_{[tc']}) + P(r_t^\eta < z_{[tc']}) \tag{3.16}$$

For the second term in (3.16) we will use (3.13), the first term in (3.16) is less than:

$$\sum_{x \notin C_\eta(t); x \geq z_{[tc']}} P(\xi_t^\eta(x) = 1)$$

but using (3.5) this is less than:

$$\sum_{x \notin C_\eta(t); x \geq z_{[tc']}} P(\exists y \in \zeta_t^x, \eta(y) = 1)$$

but ζ_t^x must contain a z_i, $1 \leq i \leq [tc']$. So the preceeding term is less than

$$\sum_{1 \leq i \leq [tc']} \sum_{x \notin C_\eta(t)} P(z_i \in \zeta_t^x) \leq \sum_{1 \leq i \leq [tc']} 2 \sum_{y \geq 0} P(\mathcal{P}(\lambda Bt) \geq \frac{ct + y}{R}) \tag{3.17}$$

The last inequality comes from the fact that $-l(\zeta_t^x)$ and $r(\zeta_t^x)$ increase less than R times a Poisson process of parameter λB. For $c \geq \lambda BRc^2$, we use (3.11) and the r.h.s. of (3.17) is less than:

$$[tc^t]Ce^{-\frac{ct}{R}} \tag{3.18}$$

From (3.10), (3.15), (3.13) and (3.18) we have that:

$$P(\hat{\xi}(-at) = 1) \leq P(\xi_t^Z(0) = 1) + (2ct + 1)[tc^t]Ce^{-\frac{ct}{R}} + c^{-t} + [tc^t]Ce^{-\frac{ct}{R}}$$

To complete the proof of Lemma 1 just pick $b = c \geq \max(c^2\lambda BR, 2R)$ and $a > 2b$.

Now to complete the proof of Theorem 2, suppose that $\alpha(\lambda) = -\infty$ and that $\tilde{\mu}$ is an invariant probability for the process seen from the edge. From Lemma 1 we have:

$$\tilde{\mu}(\eta : \eta(-at) = 1) \leq P(\xi_t^Z(0) = 1) + Ce^{-\gamma t}$$

but from Theorem 1, we have

$$P(\xi_t^Z(0) = 1) \leq Ce^{-\gamma t}$$

Now, by Borel-Cantelli Lemma, $\tilde{\mu}$ concentrates on finite configurations, which is absurd.

4. Proofs of Theorem 3 and Theorem 4

We start proving Theorem 3. Since the critical contact process is ergodic (see Bezuidenhout and Grimmett (1989)), we only need to consider the case in which $\lambda > \lambda_c$. To treat this case we will explore the relation between the contact process and oriented percolation, as in Durrett and Schonmann (1988). Given $\epsilon > 0$ we define the lattice

$$\mathcal{L}_\epsilon = \{(j, k\epsilon/2) : j, k \in Z\}$$

And we make bonds open independently with the following probabilities (here m and n are in Z):

$$(m, n\epsilon) \to (m-1, (n+1/2)\epsilon), \text{ with probability } \lambda\epsilon$$

$$(m, n\epsilon) \to (m, (n+1/2)\epsilon), \text{ with probability } 1 - \epsilon/2$$

$$(m,(n+1/2)\epsilon) \to (m+1,(n+1)\epsilon), \text{ with probability } \lambda\epsilon$$

$$(m,(n+1/2)\epsilon) \to (m,(n+1)\epsilon), \text{ with probability } 1-\epsilon/2$$

An active path from $(j,k\epsilon/2)$ to $(j',k'\epsilon/2)$ in $\mathcal{L}_\epsilon$ is a finite sequence of adjacent open bonds leading from $(j,k\epsilon/2)$ to $(j',k'\epsilon/2)$ (by adjacent we mean that the endpoint of each bond is the starting point of the next one in the sequence). This percolation structure defines the following process $(\xi^{\eta}_{\epsilon,t})_{t\geq 0}$:

$\xi^{\eta}_{\epsilon,t}(x) = 1$ if there is an active path in $\mathcal{L}_\epsilon$ from $(y,0)$ to $(x,[2t/\epsilon]\epsilon/2)$ for some y such that $\eta(y)=1$, (here $[\quad]$ denotes integer part of)

$\xi^{\eta}_{\epsilon,t}(x) = 0$ otherwise.

We define the process seen from the edge and starting from $\eta \in \tilde{X}$:

$$\tilde{\xi}^{\eta}_{\epsilon,t}(x) = \xi^{\eta}_{\epsilon,t}(x + sup\{y \in Z : \xi^{\eta}_{\epsilon,t}(y) = 1\})$$

If $\eta \in \tilde{X}$ then $\tilde{\xi}^{\eta}_{\epsilon,t} \in \tilde{X}$ for every $t \geq 0$ almost surely. Let $\tilde{\nu}_{\epsilon,t}$ be the law of $\tilde{\xi}^{1}_{\epsilon,t}$ and $\mu_{\epsilon,t}$ be the law of $\xi^{Z}_{\epsilon,t}$.

We mark the closed bonds $(m,k\epsilon/2) \to (m,(k+1)\epsilon/2)$ with δ's and look at the open bonds $(m,k\epsilon/2) \to (m+1,(k+1)\epsilon/2)$ and $(m,k\epsilon/2) \to (m-1,(k+1)\epsilon/2)$ as arrows. Then as ϵ goes to 0 the percolation structure defined above gives rise for every $x \in Z$ to three independent Poisson processes. One of arrows to the right at rate λ, one of arrows to the left at rate λ and one of δ's at rate 1. So the percolation structure converges to the well known graphical construction of the contact process (see for instance Section 4a in Durrett (1988)). In particular it follows that in the weak sense

$$\lim_{\epsilon \to 0} \tilde{\nu}_{\epsilon,t} = \tilde{\nu}_t \tag{4.1}$$

$$\lim_{\epsilon \to 0} \mu_{\epsilon,t} = \mu_t \tag{4.2}$$

where $\tilde{\nu}_t$ is the law of $\tilde{\xi}^1_t$ and μ_t is the law of ξ^Z_t. For fixed $\epsilon > 0$, let $\Gamma(t)$ be the rightmost active path in $\mathcal{L}_\epsilon$ from $\{\ldots,-2,-1,0\} \times \{0\}$ to $Z \times \{[2t/\epsilon]\epsilon/2\}$. Given now a nondecreasing function $f: \{0,1\}^Z \to R$ which depends only on finitely many sites of $\{\cdots,-2,-1,0\}$ we have:

$$E(f(\tilde{\xi}^1_{\epsilon,t})) = \sum_{\gamma} E(f(\tilde{\xi}^1_{\epsilon,t})|\Gamma(t)=\gamma)P(\Gamma(t)=\gamma) \tag{4.3}$$

But for every fixed γ, conditioning on $\Gamma(t) = \gamma$, does not affect the percolation structure to the left of γ inside $Z \times [0, [2t/\epsilon]\epsilon/2]$. Since the interaction is among nearest neighbors a standard coupling argument implies, for nondecreasing f

(4.4) $$E(f(\tilde{\xi}^{1}_{\epsilon,t})|\Gamma(t) = \gamma) \geq E(f(\xi^{Z}_{\epsilon,t}))$$

From (4.3) and (4.4) it follows that

(4.5) $$\tilde{\nu}_{\epsilon,t} \geq \hat{\mu}_{\epsilon,t}$$

where $\hat{\mu}_{\epsilon,t}$ is defined by:

$$\hat{\mu}_{\epsilon,t}(A) = \mu_{\epsilon,t}(\eta: \hat{\eta} \in A)$$

Theorem 3 follows from (4.1), (4.2), (4.5), the proof of the existence of $\tilde{\mu}_\lambda$ and its uniqueness.

Theorem 4 is an almost immediate consequence of Theorem 3 and Lemma 1. From these results we have:

(4.6) $$0 \leq \tilde{\mu}_\lambda(\eta: \eta(-x) = 1) - \mu_\lambda(\eta: \eta(-x) = 1)$$

$$\leq Ce^{-\gamma x} + P(\xi^{Z}_{x/a}(0) = 1) - \mu_\lambda(\eta: \eta(-x) = 1)$$

From the fact that the Contact Process is self-dual we have:

$$P(\xi^{Z}_{x/a}(0) = 1) - \mu_\lambda(\eta: \eta(-x) = 1) = P(x/a < \tau^* < \infty).$$

The result follows immediatly form this equality and (4.6).

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