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# A numerical and experimental study of stiffened plates in bending

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## Abstract

Although in the analysis of building slabs the eccentricities of the beams play a very important role in the results of stresses, strains and displacements they are not usually considered in the practice of building slab analysis. In order to evaluate the error when these eccentricities are not considered an analytical and numerical study of the interaction of beams and plates in bending is presented. For the experimental part of the work an acrylic plate of 90cm X 90cm X 0.88 cm stiffened with beams of 0.82cm X 4 cm cross section and submitted to uniform loading was tested. In the numerical part of this study, the plate was analysed with a FEM code initially as a 3D problem, using shell elements to model both, the plate and the beams. The structure was also analysed with plates and beams elements with and without considering the eccentricities of the beams related to the plates. Finally the numerical and experimental results are compared, allowing the error of the numerical analysis to be evaluated.

## 1 Introduction

It is a well known fact that, in the analysis of building slabs, beams eccentricities in relation to the flat plate play a very important role in the final results of stresses, strains and displacements. However, codes such as Eurocode 2<sup>1</sup> and NRB 6118<sup>2</sup>

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(Brazil 1978) allow for analysis of these structures without considering these eccentricities. If the number of beams is too great and they are very close together, other simplifications can be made, such as analysing the slab as a flat plate with an equivalent stiffness to represent the contribution of the beams (Jil et al.<sup>3</sup>, Ajdukiewicz & Starosolki<sup>4</sup>). Slabs can also be analysed by the Equivalent Strip Method, as presented by Aalami<sup>5</sup>.

The best way to model these structures is using a Finite Element code and considering them as three-dimensional ones composed of plates in traction and in bending to represent both the flat plate and the beams, using shell finite elements to model them. Due to the fact that this analysis implies much additional work, it is not generally used in the practice of building slab analysis. An alternative to this is to analyse the slab as a plate coupled with beams, considering the beams to plate eccentricities, Mukhopadhyay<sup>6</sup>, Miller<sup>7</sup>, Gupta & Ma<sup>8</sup>. In this case the flat plate works in traction and in bending and, at least for the plate, shell finite elements must be used to represent its behaviour. Again, this type of analysis is not commonly employed. The building slab is generally analysed without considering beams eccentricities and the results are obviously very poor.

In order to evaluate the error when these eccentricities are not considered, an analytical and numerical study of plate-beam interaction in bending is presented. For the experimental part of the work, a 90cm X 90cm X 0.88cm acrylic plate stiffened with beams with 0.82cm X 4cm cross section and subjected to uniform loading was tested. In the numerical part of this study, this plate was analysed by FEM, initially as a 3D problem, using shell finite elements to model both plate and beams. The structure was also analysed with plate and beam finite elements, with and without taking into consideration the eccentricities of the beams in relation to the plate. Finally, the numerical and experimental results are compared, allowing for evaluation of the numerical analysis error.

This work is part of a project that aims to evaluate the discrepancies between results from the usual simplified way of analysing building slab structures and those from a more sophisticated approach, in order to define guidelines for structural designers.

## 2 Experimental and numerical models

The acrylic slab tested is 90cm x 90cm, 0.88cm thick, and is stiffened with eight beams with 0.88cm x 4cm cross section, as shown in figure 1. The Young modulus of the acrylic was obtained from samples of the material used in the model. In the samples' traction test, the stress-stroke relations showed a practically elastic-linear behaviour until their collapse. From these results, the Young modulus was assumed to be 250kN/cm and the Poisson rate was obtained from the literature and assumed to be 0.36. The model was supported at its corners and subjected to a uniformly distributed load of 0.000096kN/cm<sup>2</sup>.

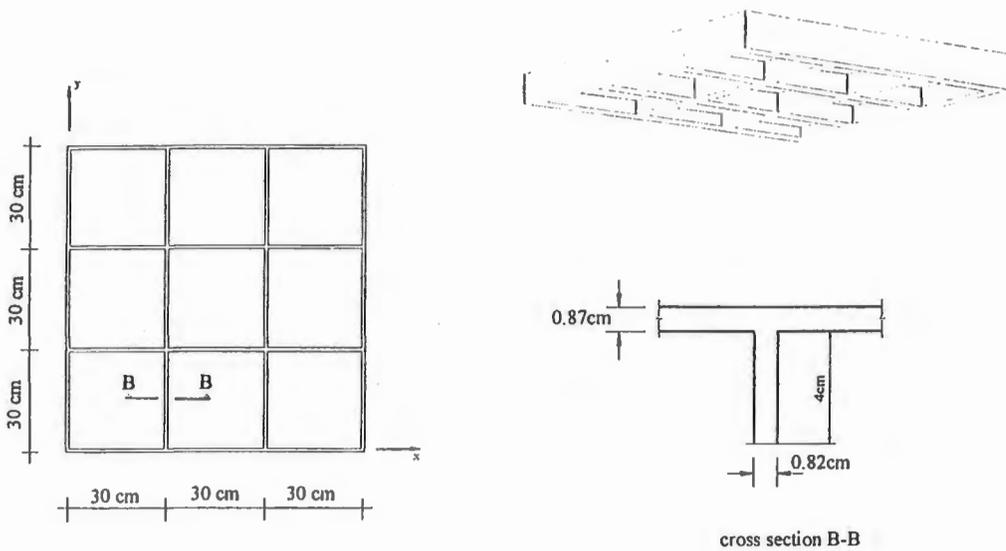


Figure 1: Slab model

In the numerical analysis, initially, the model was analysed as a three-dimensional structure with shell finite elements and a massive mesh of elements. Later, the model was analysed as stiffened plate in bending, with and without considering beams eccentricities in relation to the plate. The software used in the analysis was FEA. The modelling assumed are:

- TRID: The slab was analysed as a three-dimensional structure with shell elements. The finite element used was QSI4 with six nodal values,  $u$ ,  $v$ ,  $w$ ,  $x$ ,  $y$ ,  $z$ .
- GRIL1: The slab was analysed as a flat plate coupled with beams, without considering beams eccentricities. The finite element used for the plate was RPI4, with five nodal values,  $u$ ,  $v$ ,  $w$ ,  $x$ ,  $y$ . The finite element used for the beams was BRP2, with the same nodal values as RPI4. This beam element permits the beam eccentricity to be taken into consideration. In this analysis, it was assumed that there were no eccentricities.
- GRIL2: The slab was analysed as in GRIL1, this time considering the eccentricities of the beams.

As for the beams, two models were used to define their eccentricity and inertia. The first model considers the height of the beam as the distance from the bottom of the plate to the bottom of the beam (GRIL2\_V1), while the second considers it from the top of the plate to the bottom of the beam (GRIL2-V2).

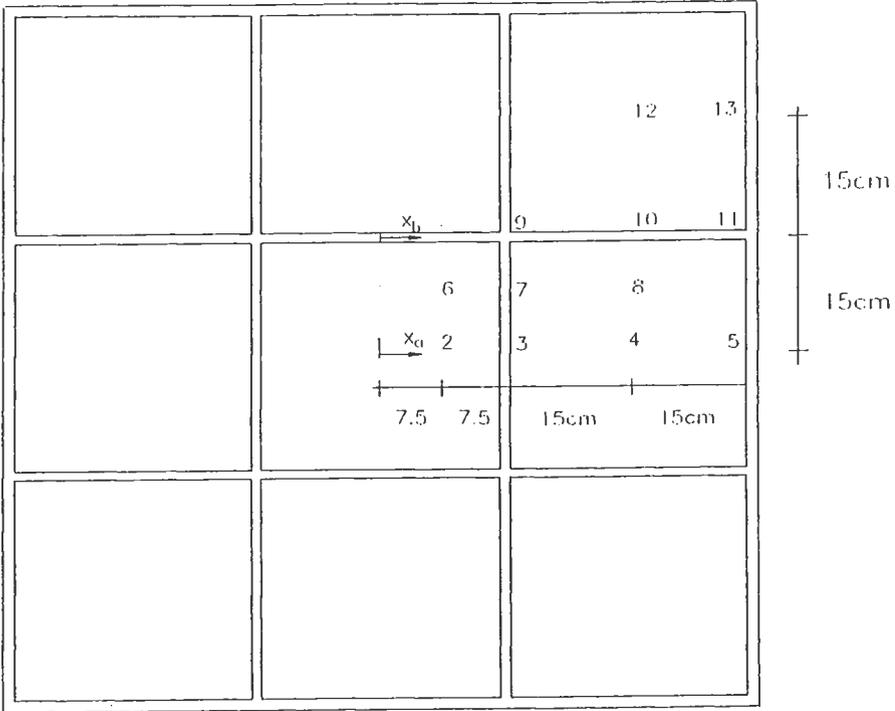


Figure 2 : Positions of the displacement transducers

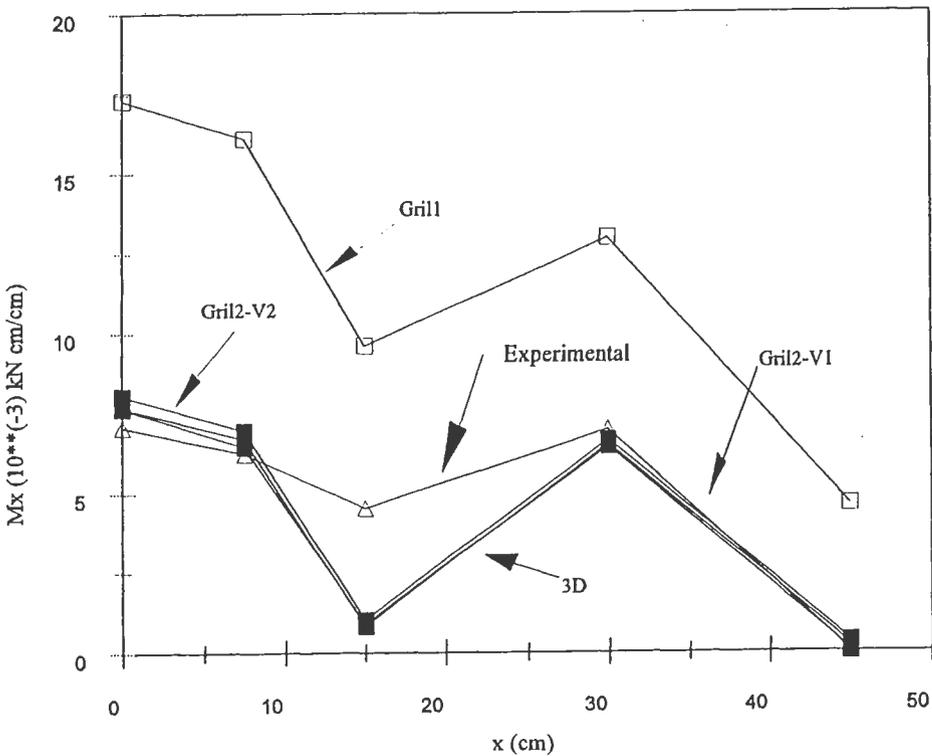


Figure 3: Numerical and experimental plate bending moments  $m_x$  along the central line  $x_a$

Figure 2 shows the points where the displacements were measured using displacement transducers. Table 1 contains the results of the experiment and those of the numerical analysis

It can be seen that 3D analysis provides very good results when compared with experimental ones. It can also be seen that there is a very good agreement among the experimental results, the 3D analysis and those considering beams eccentricities . It should be pointed out that the results that disregard beams eccentricities are very poor, with errors up to 129%, showing clearly that, with this type of modelling, the slab displays a very flexible behaviour and is not suited to slab analysis.

Table 1: Experimental and numerical displacements (mm)

Point	Experimental	3D	GRIL1	GRIL2-V1	GRIL2-V2
1	6.22	6.24	14.26	6.02	6.0
2	6.13	6.14	14.04	5.92	5.9
3	5.86	5.89	13.42	5.66	5.63
4	4.8	5.19	11.43	4.9	4.9
5	3.15	4.09	8.53	3.81	3.71
6	6.08	6.05	13.82	5.83	5.8
7	5.87	5.8	13.21	5.58	5.54
8	4.67	5.08	11.17	4.83	4.77
9	5.72	5.57	12.6	5.35	5.3
10	4.54	4.73	10.42	4.5	4.44
11	2.77	3.56	7.4	3.31	3.22
12	3.72	3.67	7.87	3.47	3.41
13	1.55	2.05	4.26	1.9	1.85

Figures 3, 4, 5 and 6 show the bending moments along the central line of the slab and along the plate-beam connection in the experimental and numerical analysis, once again showing that to disregard eccentricities leads to very poor results. As can be seen, there is a very good agreement between the 3D analysis and those that take beams eccentricities into account. These numerical results also show a very good agreement with the experimental results, with the exception of the plate-beam connection point. One possible answer to this is that, in the numerical analysis, the compatibility of displacements between the plate and the beams is imposed only on the finite element nodes, while in the experimental model there is displacement and strain compatibility along the plate-beam connection, which causes the experimental model to be stiffer than the numerical ones.

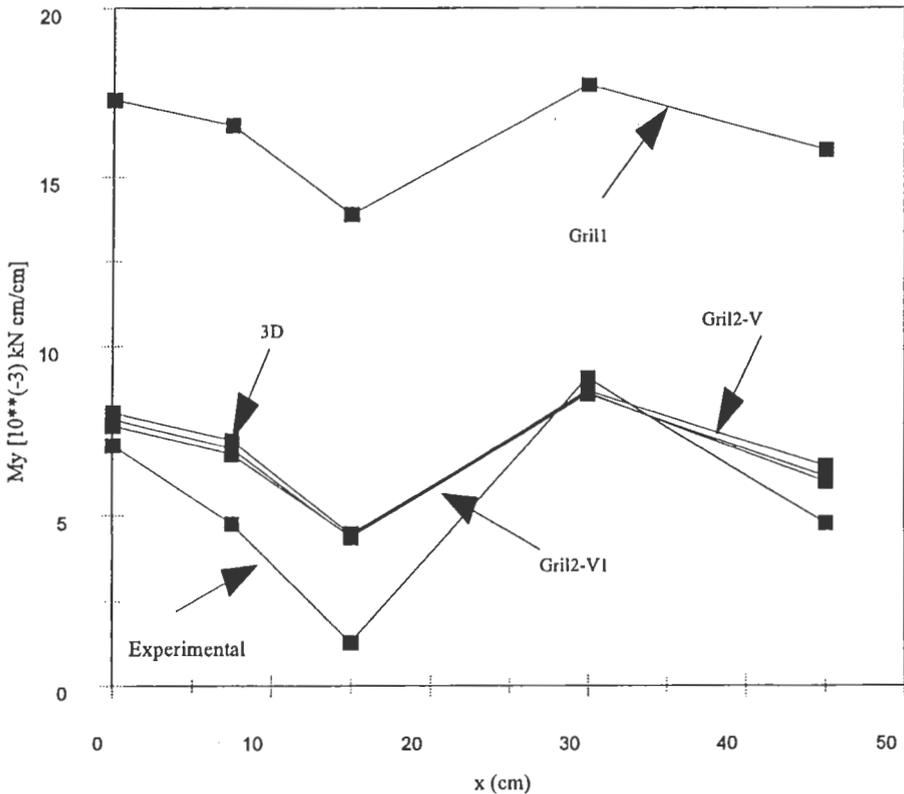


Figure 4: Numerical and experimental bending moments  $m_y$  along the central line  $x_a$

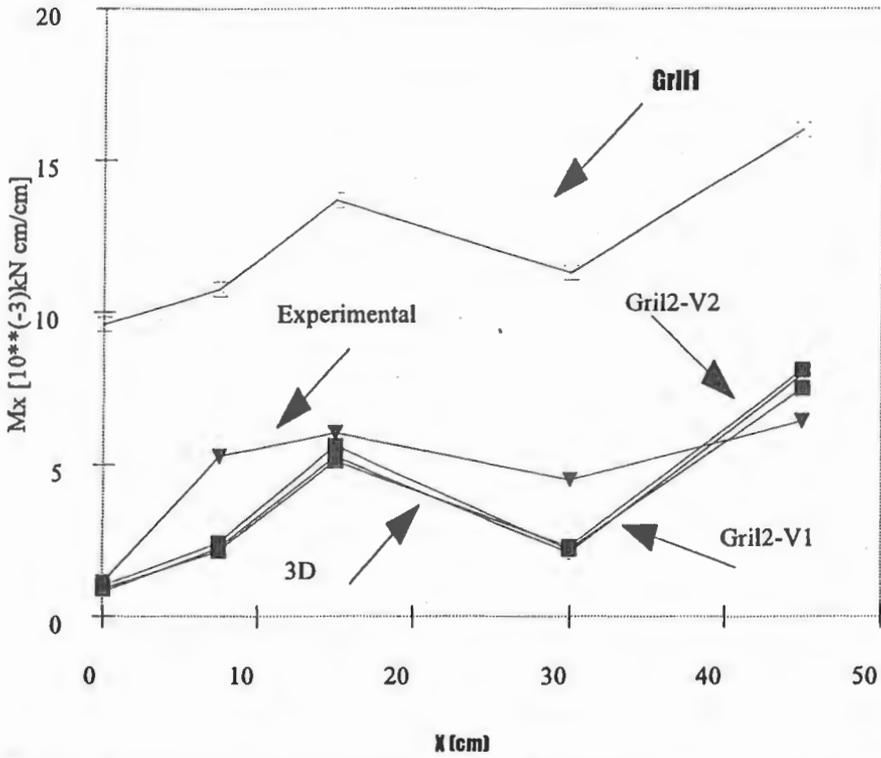


Figure 5: Numerical and experimental bending moment  $m_y$  along  $x_b$

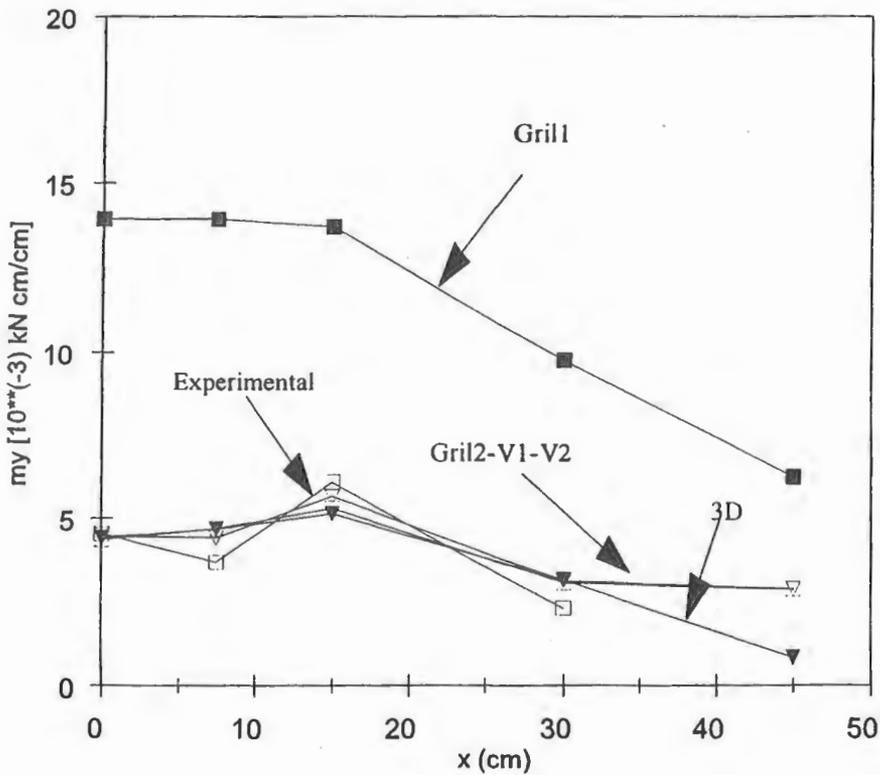


Figure 6: Numerical and experimental bending moment  $m_x$  along  $x_b$

### 3 Conclusions

The results presented herein demonstrate that, although the best way to analyse a slab is as a 3D structure, analysing it as a ribbed structure also gives very good results. The results also show that there is no significant difference if the height of the beam is taken as either the distance between the bottom of the beam to the lower or to the upper surface of the plate. As a guideline, it can be taken as the distance to the mid-surface of the plate. Moreover, the results show that disregarding beams eccentricities to the plate gives very poor results and should be avoided.

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