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GROUP IDENTITIES ON SYMMETRIC UNITS IN ALTERNATIVE LOOP ALGEBRAS

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EDGAR G. GOODAIRE AND CÉSAR POLCINO MILIES

ABSTRACT. If $\alpha = \sum \alpha_\ell \ell$ is an element of an alternative loop ring RL, we denote by $\alpha^\dagger$ the element $\sum \alpha_\ell \ell^{-1}$ and call α *symmetric* if $\alpha^\dagger = \alpha$. In previous work, the authors have considered the possibility that the unit loop of RL satisfies a group identity. Here, we assume merely that the symmetric units of RL (usually for R a field) satisfy a group identity.

1. INTRODUCTION

From a commutative, associative ring R with 1 and a loop L, one forms the loop algebra RL precisely as if L were a group. The algebra RL is alternative if it satisfies the alternative laws

$$(yx)x = yx^2 \quad \text{and} \quad x(xy) = x^2y.$$

In the early 1980s, the existence of loop algebras that are alternative but not associative was established [Goo83, CG86] and since that time there has been quite a bit of research into the problems associated with such algebras and the underlying *RA loops* which produce them. A good place to find a discussion of some of these problems is [GJM96], which is also the best reference for RA loops and alternative loop algebras. Of note is the fact that an RA loop is Moufang and hence *diassociative*: subloops generated by two elements are associative. In fact, if three elements of a Moufang loop associate in some order, then the subloop they generate is a group.

Just as with group rings, the set of units (that is, invertible elements) in an alternative loop ring RL is closed under products and inverses and hence forms a (Moufang) loop $\mathcal{U}(RL)$, the *unit loop* of RL, and this loop contains L. This observation makes it natural to ask how many properties of the loop L are inherited by $\mathcal{U}(RL)$. The answer is "not many." For example, L is solvable, nilpotent, FC, has the torsion product property and is torsion over its centre, but $\mathcal{U}(RL)$ rarely satisfies any of these conditions

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[GM96b, GM96c, GM97, GM02, GM01]. Generalizing all these concepts, the authors have recently considered the possibility that $\mathcal{U}(RL)$ satisfies a group identity [GM].

Let K denote the free group on a set of variables $x_1, x_2, \ldots$ and let $w(x_1, x_2, \ldots, x_n) \in K$ be a nonempty reduced word in K. We say that $w = 1$ is a *group identity* for a group G if $w(g_1, g_2, \ldots, g_n) = 1$ for all $g_1, g_2, \ldots, g_n \in G$. We extend the notion of group identity to Moufang loops with the following definition.

Definition 1.1. A Moufang loop M satisfies a group identity if and only if there is a nonempty reduced word $w = w(x_1, x_2)$ in the free group on two variables such that $w(\ell_1, \ell_2) = 1$ for all $\ell_1, \ell_2 \in M$.

The restriction to two generators here is actually artificial because of diassociativity and the fact that a free group on n generators can always be embedded in a free group on just two generators [Rob82, Theorem 6.1.1]. For example, any nilpotent Moufang loop satisfies a group identity in our sense. To see this, let $f = 1$ be an identity (in a finite number of nonassociating variables) satisfied by a nilpotent Moufang loop L. Now view f as a word in the free group on these same variables and express f as a word $w(x_1, x_2)$ in two variables. Suppose a and b are elements of L and apply w to the pair a, b. Since $\langle a, b\rangle$ is a group and it is nilpotent, $w(a, b) = 1$, so L satisfies the group identity $w = 1$.

Since Moufang loops are *inverse property loops*, that is, $(ab)^{-1} = b^{-1}a^{-1}$ for any a, b, the map $a \mapsto a^{-1}$ is an involution (antiautomorphism of order 2) and this extends linearly to an involution of the loop ring RL which we denote $\alpha \mapsto \alpha^\sharp$.[1] Thus, for $\alpha = \sum \alpha_\ell \ell \in RL$, $\alpha^\sharp = \sum \alpha_\ell \ell^{-1}$.

Call a unit α *symmetric* if $\alpha^\sharp = \alpha$ and let $\mathcal{U}^+(RL)$ be the set of symmetric units in an alternative loop ring RL. (Conditions under which this set is actually a loop are known [GM06].) In this paper, we suppose existence of a group identity on $\mathcal{U}^+(RL)$ and begin with some basic results.

Recall that a loop is *Hamiltonian* if it is not an abelian group and every subloop is normal. Throughout, we use $\mathcal{Z}(L)$ and $\mathcal{Z}(RL)$ to denote the centres of L and RL, respectively.

Lemma 1.2. *If L is a Hamiltonian Moufang 2-loop (possibly associative) and R is any commutative, associative ring with 1, then $\mathcal{U}^+(RL)$ satisfies the group identity $(u, v) = 1$.*

Proof. Let L be a Hamiltonian Moufang 2-loop. Then $L = L_1 \times E$ is the direct product of an elementary abelian 2-group E and a loop L_1 which is the Cayley loop if L is not associative [Nor52], [GJM96, Theorem II.4.8] and the quaternion group of order 8 otherwise (see, for example, [Hal59, Theorem 12.5.4], for a proof of this classical theorem). Thus, the centre of

[1] In group rings, the map $\alpha \mapsto \alpha^\sharp$ is denoted $\alpha \mapsto \alpha^*$, but since alternative loop rings have a canonical (and different) involution $\alpha \to \alpha^*$ (see Section 4), we use $\sharp$ instead of $*$ for the involution of central interest here.

L consists precisely of the elements of L that have order 2. With s the unique nonidentity commutator of L, we note also that the inverse of a noncentral element ℓ is $s\ell$. Now the set of symmetric elements of RL is spanned by loop elements of order 2 and ring elements of the form $\ell + \ell^{-1}$, $\ell \notin \mathcal{Z}(L)$. In particular, every symmetric unit is a linear combination of elements central in L and ring elements of the form $\ell + \ell^{-1} = (1+s)\ell$, $\ell \notin \mathcal{Z}(L)$. Such elements are conjugacy class sums of RL and these span the centre of RL [GJM96, Corollary III.1.5]. So $\mathcal{U}^+(RL) \subseteq \mathcal{Z}(RL)$ and the result follows. □

2. The Finite Case

The questions we explore in this paper have been considered in the context of group rings by various authors [GSV98, SV06]. Here is a theorem of A. Giambruno, S. K. Sehgal and A. Valenti we will use later.

Theorem 2.1. [GSV98] *Let F be a field of characteristic $p \geq 0$, $p \neq 2$, and let G be a finite group. Then $\mathcal{U}^+(FG)$ satisfies a group identity if and only if the set P of p-elements of G is a normal subgroup of G and G/P is an abelian group or a Hamiltonian 2-group.*

For alternative loop rings that are not associative we have an analogous result. The proof requires the fact that for any prime p, the set L_p of p-elements in an RA loop L is a normal subloop of L and central if p is odd [CG86, proof of Theorem 6], [GJM96, Proposition V.1.1].

Theorem 2.2. *Let F be a field of characteristic $p \geq 0$ and let L be a finite RA loop. Then $\mathcal{U}^+(FL)$ satisfies a group identity if and only if*

(1) *$p = 2$, or*
(2) *L is a Hamiltonian 2-loop, or*
(3) *p is odd, $L = L_p \times L_2$, and L_2 is a Hamiltonian 2-loop.*

Proof. If $p = 2$, $\mathcal{U}(FL)$ is nilpotent [GM97] and hence satisfies a group identity. If L is a Hamiltonian 2-loop, $\mathcal{U}^+(FL)$ satisfies a group identity by Lemma 1.2. Let F be a field of odd characteristic p and suppose $L = L_p \times L_2$ with L_2 Hamiltonian. Then $FL = FL_p[L_2]$ is a loop ring of L_2 over a central coefficient ring and Lemma 1.2 says that $\mathcal{U}^+(FL)$ satisfies a group identity. This gives the theorem in one direction.

Now assume that $\mathcal{U}^+(FL)$ satisfies a group identity and that $\operatorname{char} F = p \neq 2$. Take $a, x \in L$ with $ax \neq xa$ and let z be an element of prime order $q \neq p$. Then $G = \langle a, x, z \rangle$ is a finite nonabelian group and $\mathcal{U}^+(FG)$ satisfies a group identity. If $p = 0$, the set P of p-elements is $\{1\}$ and $G = G/P$ is a Hamiltonian 2-group by Theorem 2.1. It follows that L is a Hamiltonian 2-loop. If $p > 0$, then P is central (because p is odd), so G/P is nonabelian and hence a 2-group. So $q = 2$ and we learn that the only primes dividing $|L|$ are 2 and p. Normality of L_p and L_2 shows that $L = L_p \times L_2$. Applying Theorem 2.1 to the associative subloop of L_2 generated by two noncommuting elements, we see that L_2 is Hamiltonian. □

3. Torsion Loops

In this section, L is a torsion loop that is not necessarily finite. As before, our results both use and extend a theorem about group algebras.

Theorem 3.1. [GSV98] *Let F be a field and G a torsion group with FG semiprime. Then $\mathcal{U}^+(FG)$ satisfies a group identity if and only if G is abelian or a Hamiltonian 2-group.*

In Theorems 4.2.12 and 4.2.13 of Passman's classic text [Pas77], one can find necessary and sufficient conditions for a group algebra FG over a field F to be semiprime. In characteristic 0, FG is always semiprime whereas in positive characteristic p, it is necessary and sufficient that G contain no finite normal subgroups of order divisible by p. The situation is exactly the same for RA loops [GJM96, Corollary VI.3.8] so, if an alternative loop algebra FL is semiprime and G is a group or a loop contained in L, then FG is also semiprime, in all characteristics.

This is a key idea used in the proof of our extension of Theorem 3.1 to alternative loop algebras.

Theorem 3.2. *Let F be a field and L a torsion RA loop with FL semiprime. Then $\mathcal{U}^+(FL)$ satisfies a group identity if and only if L is a Hamiltonian 2-loop.*

Proof. Lemma 1.2 gives the result in one direction. For the other, assume that $\mathcal{U}^+(FL)$ satisfies a group identity, take any $a, x \in L$ which do not commute and let z be a central element. Then $G = \langle a, x, z\rangle$ is a nonabelian torsion group and FG is semiprime. By Theorem 3.1, G is a Hamiltonian 2-group and it follows that L is a Hamiltonian 2-loop. □

4. Nontorsion Loops

In this final section, we consider RA loops with no finiteness restrictions whatsoever and present theorems concerning the symmetric units of alternative loop algebras over fields and also over the ring $\mathbb{Z}$ of rational integers. Our results make reference to the torsion elements of a loop L. If L is RA, this set forms a locally finite subloop which is finite if L is finitely generated [GM95, Lemma 2.1], [GM96a, Lemma 1.4], [GJM96, Lemma VIII.4.1].

For group rings over the integers, Giambruno, Sehgal and Valenti have shown that if $\mathcal{U}^+(\mathbb{Z}G)$ satisfies a group identity, then any torsion subgroup H of G is either abelian or a Hamiltonian 2-group and every subgroup of H is normal in G [GSV98, Theorem 4]. This helps us to characterize RA loops whose symmetric units (in $\mathbb{Z}L$) satisfy a group identity.

Theorem 4.1. *Let L be an RA loop. Then the following are equivalent:*

(1) *$\mathcal{U}^+(\mathbb{Z}L)$ satisfies a group identity.*
(2) *$\mathcal{U}(\mathbb{Z}L)$ satisfies a group identity.*
(3) *For every finitely generated group $G \subseteq L$, $\mathcal{U}(\mathbb{Z}G)$ satisfies a group identity.*

(4) *The torsion subloop T of L is either an abelian group or a Hamiltonian 2-loop, and every subloop of T is normal in L.*

When any, and hence all, of these conditions is satisfied, $\mathcal{U}(\mathbb{Z}G)$ satisfies the identity $(u^2, v^2) = 1$.

Proof. The equivalence of (2), (3) and (4) is known and each implies that $(u^2, v^2) = 1$ is a group identity [GJM96, Corollary XII.2.9]. Since (1) is an obvious consequence of (2), we have only to establish that (2) follows from (1), so assume that $\mathcal{U}^+(\mathbb{Z}L)$ satisfies a group identity. Let $t \in T$, the torsion subloop of L, and suppose $xt \neq tx$ for some $x \in L$. Let $t_0 \in T$ be any central element. The subloop $G = \langle t_0, t, x\rangle$ is a group because t_0, t and x associate and $\mathcal{U}^+(\mathbb{Z}G)$ satisfies a group identity, so the result of Giambruno et al mentioned earlier tells us that both t and t_0 are 2-elements and $x^{-1}tx \in \langle t\rangle$. In an RA loop, as with groups, the last observation shows that every subloop of T is normal in L [GJM96, Corollary IV.1.11]. In particular, every subloop of T is normal in T so, if this is a group, the result for groups shows that T is either abelian or a Hamiltonian 2-group whereas, if T is not a group, then it is a Hamiltonian Moufang 2-loop. □

Now we consider loop algebras over a field. In the associative case, Sehgal and Valenti have a helpful result.

Theorem 4.2. [SV06, Theorem 4] *Let F be an infinite field, G a nontorsion group with torsion elements the set T and suppose FG is semiprime. If $\mathcal{U}^+(FG)$ satisfies a group identity then*

(1) *if* $\operatorname{char} F = p > 2$, *then T is an abelian p'-group (that is, all elements of T have order relatively prime to p);*
(2) *if* $\operatorname{char} F = 0$, *then T is an abelian group or a Hamiltonian 2-group;*
(3) *all idempotents of FT are central in FG.*

A few preliminary remarks will help the reader with our proof of the alternative analogue. An RA loop L has a unique nonidentity *commutator/associator* s, which is necessarily central and of order 2. The map $\ell \mapsto \ell^*$ where

$$\ell^* = \begin{cases} \ell & \text{if } \ell \text{ is central} \\ s\ell & \text{otherwise} \end{cases}$$

is an involution of L that extends linearly to an involution of any loop ring RL (which we continue to denote $*$). Now L contains a group G of index 2 and so we have $L = G \cup Gu$ for any $u \in L\setminus G$. Thus elements of RL have the form $x + yu$, where x and y are in the group ring RG, and $(x+yu)^* = x^* + sy$. It is known that an element of RL is central if and only if it is invariant under $*$, so we obtain a very useful test for centrality: $x + yu$ is central if and only if $x^* = x$ and $sy = y$. This material is quite basic to the theory of RA loops and can be found, for example, in Section III.4 of [GJM96].

Theorem 4.3. *Let F be an infinite field of characteristic $p \geq 0$. If $p = 0$, assume also that F contains no solutions to $x^2 + y^2 + z^2 + w^2 = -1$. Let*

L be an RA loop with torsion subloop $T \neq L$ and suppose FL is semiprime. Then the following are equivalent:

(1) *$\mathcal{U}^+(FL)$ satisfies a group identity.*
(2) (a) *$p > 2$ and T is an abelian p'-group;*
 (b) *$p = 0$ and T is an abelian group or a (possibly associative) Hamiltonian 2-loop;*
 (c) *if $G \subseteq L$ is a group with torsion subgroup $T(G)$, then all idempotents of $FT(G)$ are central in FG.*
(3) *$p = 2$ or every idempotent of FT is central in FL.*

In the case $p \neq 2$, if $\mathcal{U}^+(FL)$ satisfies a group identity, it satisfies $(u,v) = 1$.

Proof. Assume (1), that $\mathcal{U}^+(FL)$ satisfies a group identity, and let $t_1, t_2 \in T$. Since g^2 is central for any $g \in L$ and since L contains an element of infinite order, the centre of L contains an element a of infinite order. Let $H = \langle t_1, t_2, a \rangle$ be the group generated by t_1, t_2 and a. The symmetric units of FH satisfy a group identity so, if $p > 2$, Theorem 4.2 says that the torsion of H is an abelian p'-group. Thus $t_1 t_2 = t_2 t_1$ and these elements have order prime to p. It follows that T is commutative, hence associative [GM96b], [GJM96, Corollary IV.2.4] and a p'-group. If $p = 0$ and T is not commutative, we may assume $t_1 t_2 \neq t_2 t_1$. This time Theorem 4.2 says that H is a Hamiltonian 2-group, so T is a Hamiltonian 2-loop. Now let $G \subseteq L$ be a group. Then $\mathcal{U}^+(FG)$ satisfies a group identity so, appealing again to Theorem 4.2, we learn that all idempotents of $FT(G)$ are central in FG. This establishes (2).

Now assume statement (2) and $p \neq 2$. We prove that every idempotent of FT is central in FL. This is known to be the case if $p > 0$ or T is abelian [GM] so we assume that $p = 0$ and T is a Hamiltonian 2-loop. Replacing T by the subloop generated by the support of an idempotent in FT, if necessary, we may assume that T is finitely generated and hence finite [GM95, Lemma 2.1], [GM96a, Lemma 1.4], [GJM96, Lemma VIII.4.1]. Thus $T = T_1 \times E$ is the direct product of an abelian 2-group and a subloop T_1 which is Q, the quaternion group of order 8, if T is associative, and the Cayley loop $M_{16}(Q)$ otherwise. (See the proof of Lemma 1.2 for appropriate references.) Thinking of FT as the loop algebra of T_1 with coefficients in FE, it is basic that FE is the direct sum of copies of F, so FT is the direct sum of copies of FT_1. In characteristic 0, $FQ \cong 4F \oplus (F, -1, -1)$, where $(F, -1, -1)$ is the quaternion algebra over F, and $F[M_{16}(Q)] = 8F \oplus (F, -1, -1, -1)$ where $(F, -1, -1, -1)$ is the Cayley-Dickson algebra over F [GJM96, Corollaries VII.2.3 and VII.2.4]. Since $x^2 + y^2 + z^2 + w^2 = -1$ has no solutions in F, this quaternion and this Cayley-Dickson algebra are division algebras [GJM96, Theorem I.3.4]. This already shows that every idempotent of FT is central in FT, but we want centrality in FL. Since every idempotent in FT is the sum of primitive idempotents, to obtain such centrality, it suffices to show that the primitive idempotents of FT are central in FL.

First we show that all elements of FE are central in FL by showing that E is central in L. So take $x \in L$ and $1 \neq e \in E$. Then $G = \langle x, e \rangle$ is a group and, by hypothesis, every idempotent of $FT(G)$ is central in FG. The main theorem of [CM88] says that $x^{-1}ex$ is a power of e, so $x^{-1}ex = e$. Thus e is central and we may complete the proof by showing that the primitive idempotents of FT_1 are central in FL. This task is straightforward because these idempotents appear in the literature [GM96a]. Presenting $Q = \langle a, b \mid a^4 = 1, b^2 = a^2 \rangle$, the five primitive idempotents of FQ are

$$\begin{aligned} e_1 &= \frac{\epsilon}{8}(1 + a + a^2 + a^3 + b + ab + a^2 + a^3 b) \\ e_2 &= \frac{\epsilon}{8}(1 + a + a^2 + a^3 - b - ab - a^2 - a^3 b) \\ e_3 &= \frac{\epsilon}{8}(1 - a + a^2 - a^3 + b - ab + a^2 b - a^3 b) \\ e_4 &= \frac{\epsilon}{8}(1 - a + a^2 - a^3 - b + ab - a^2 b + a^3 b) \\ e_5 &= \frac{\epsilon}{8}(1 - a^2) \end{aligned}$$

where ϵ is the identity of F which, as an idempotent of FE, is central in FL. Since $e_i^* = e_i$ for each i, each e_i is central in FL. The nine primitive idempotents of $F[M_{16}(Q)]$ are e_5 and the eight elements

$$e_{j1} = \frac{\epsilon}{2}(1 + u)e_j \quad \text{and} \quad e_{j2} = \frac{\epsilon}{2}(1 - u)e_j,$$

$j = 1, 2, 3, 4$. As before, each ϵ is central in FL. Since $e_{ij} = x + yu$, with $x, y \in FQ$, $x^* = x$ and $sy = y$, each e_{ij} is also central. This establishes (3).

Finally we assume statement (3) and prove (1). If $p = 2$, $\mathcal{U}(FL)$ is nilpotent [GM97, Theorem 3.2] and hence satisfies a group identity. Thus the subset $\mathcal{U}^+(FL)$ satisfies a group identity too. So assume $p \neq 2$ and every idempotent of FT is central in FL.

We are going to show that symmetric units commute, so we may assume that L is finitely generated and hence that T is finite. In this case, FT is the sum of simple algebras which are fields and quaternion algebras if T is associative, and fields and Cayley-Dickson algebras otherwise [GJM96, Corollary VI.4.8]. Since every idempotent of FT is central in FT, all simple components are division rings, D_i. Using [GJM96, Lemma XII.1.1], we conclude that any unit u of FL can be written in the form $\sum d_i \ell_i$, $d_i \in D_i$, $\ell_i \in L$. Thus $u^\sharp = \sum \ell_i^{-1} d_i^\sharp$ so, if u is symmetric, $\ell_i^{-1} d_i^\sharp = d_i \ell_i$ for each i. It follows that ℓ_i^2 has support in FT, so $\ell_i \in T$ and $u \in FT$. If $p > 0$, T is an abelian group [GM96a, Theorem 2.3] and if $p = 0$, T is a Hamiltonian 2-loop [GM96a, Theorem 3.3]. In either case, symmetric units of FT commute (using Lemma 1.2 in the Hamiltonian case). The proof is complete. □

Remark 4.4. It is pleasant to compare the equivalence of (1) and (3) in Theorem 4.3 with a previous result of the authors which states that the full unit loop $\mathcal{U}(FL)$ satisfies a group identity if and only if $p = 2$ or T *is an abelian group and* every idempotent of FT is central in FL [GM].

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