Linear Algebra and its Applications 730 (2026) 313–317

Contents lists available at ScienceDirect

Linear Algebra and its Applications

journal homepage: www.elsevier.com/locate/laa

Corrigendum

Corrigendum to "Nilpotent linear spaces and Albert's Problem" [Linear Algebra Appl. 518 (2017) 57–78]

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ARTICLE INFO

Article history:
Received 15 September 2025
Received in revised form 9 October 2025
Accepted 11 October 2025
Available online 28 October 2025
Submitted by C. de Seguins Pazzis

MSC:
17A05
17A30
15A04

Keywords:
Nilpotent linear space
Nilalgebra
Albert's Problem
Power-associative

ABSTRACT

In our article Nilpotent Linear Spaces and Albert's Problem [Linear Algebra Appl. 518 (2017) 57–78], the proof of Theorem 6 was incomplete, as a case was omitted. Here we supply the missing argument. The statement of Theorem 6, and all subsequent results depending on it, remain valid.
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DOI of original article: https://doi.org/10.1016/j.laa.2016.12.026.
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https://doi.org/10.1016/j.laa.2025.10.010
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1. Preliminaries

The proof of Theorem 6 in [3] was based on a classification by M.A. Fasoli [1] of maximal vector subspaces of $M_4(\mathbb{C})$ consisting entirely of nilpotent matrices. That classification turned out to be incomplete, and a complete version has recently been obtained in [2]. This complete classification includes an additional subspace

$$\mathcal{C}_7 = \left\{ \begin{bmatrix} 0 & x & 0 & 0 \\ 0 & 0 & x & 0 \\ y & z & 0 & x \\ 0 & -y & -z & 0 \end{bmatrix} \quad : \quad x, y, z \in \mathbb{C} \right\},$$

which was not described in Fasoli's work. This case was not treated in [3], and its consideration is necessary for the completeness of the proof of Theorem 6.

Let $\mathcal{A}$ be a commutative power-associative nilalgebra over the complex field. For $a \in \mathcal{A}$, let $L_a : \mathcal{A} \to \mathcal{A}$ denote the left multiplication operator defined by $L_a(x) = ax$ for all $x \in \mathcal{A}$. As shown in [3], for every positive integer r,

$$\begin{aligned} 3L_{a^{r+2}} = 8L_{a^{r+1}}L_a - L_{a^r}L_{a^2} - 2L_{a^r}L_a^2 + 4L_{a^2}L_{a^r} \\ - 2L_aL_{a^{r+1}} - 2L_aL_{a^r}L_a - 2L_a^2L_{a^r}. \end{aligned} \tag{1}$$

Theorem 6. *Let $\mathcal{A}$ be a commutative power-associative nilalgebra over the complex field with dimension $n \geq 9$ and nilindex $n-3$. Take $a \in \mathcal{A}$ an element of maximal nilindex, that is $a^{n-4} \neq 0$. If M is the subalgebra of $\mathcal{A}$ generated by the element a, then there exists a proper subalgebra B of $\mathcal{A}$ containing properly M.*

First, we recall some notations and results from [3], that will be used in the proof of this case. For each $b \in \mathcal{A}$, we write $\overline{b} = b + M$ for the coset of b in the quotient space $\mathcal{A}/M$. For a matrix A, we denote by $A[i,j]$ its (i,j)-entry. If $b \in \mathcal{A}$ satisfies $bM \subseteq M$, we define the induced linear operator

$$\overline{L}_b : \mathcal{A}/M \to \mathcal{A}/M, \quad \overline{L}_b(x + M) = bx + M \text{ for all } x \in \mathcal{A}.$$

Let $\Phi = (\overline{w}_1, \overline{w}_2, \overline{w}_3, \overline{w}_4)$ be a fixed basis of $\mathcal{A}/M$, and let V denote the set of matrix representations of all operators in $\mathcal{M}_M = \{\overline{L}_x : x \in \mathcal{A}, xM \subseteq M\}$ with respect to the basis Φ. For each $k \in \{1, 2, \ldots, n-4\}$, let A_k be the matrix of $\overline{L}_{a^k}$ in this basis. Since a may be replaced by an element of the form

$$b = \sum_{k=1}^{n-4} \lambda_k a^k, \qquad (\lambda_1 \neq 0),$$

and M remains the subalgebra of $\mathcal{A}$ generated by b, we may assume that the following property holds for all $i, j \in \{1, 2, 3, 4\}$:

$$\text{If } A_1[i,j]=0, \text{ then } A_k[i,j]=0 \text{ for all } k\geq 2. \tag{2}$$

We now treat the case that was omitted in [3].

Completion of the proof of theorem. Case 6. Suppose that, up to conjugacy, the space V is contained in $\mathcal{C}_7$. By replacing Φ with a suitable basis if necessary, we may assume that

$$V \subseteq \mathcal{C}_7 = \{T_7(\alpha,\beta,\gamma) \,:\, \alpha,\gamma,\beta \in \mathbb{C}\},$$

where

$$T_7(\alpha,\beta,\gamma) = \begin{pmatrix} 0 & \gamma & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ \beta & \alpha & 0 & \gamma \\ 0 & -\beta & -\alpha & 0 \end{pmatrix}.$$

There exist scalars $\alpha_i, \beta_i, \gamma_i \in \mathbb{C}$ such that

$$A_1 = T_7(\alpha_1,\beta_1,\gamma_1), \quad A_2 = T_7(\alpha_2,\beta_2,\gamma_2).$$

From equation (1) with $r=1$, we obtain

$$A_3 = 4A_2A_1 - A_1A_2 - 2(A_1)^3$$

whose $(1,4)$-entry equals $-2\gamma_1^3$. Because $A_3 \in V$, the $(1,4)$-entry must be zero; therefore, $\gamma_1 = 0$. By property (2), it follows that $\gamma_2 = 0$ as well. Applying relation (1) for $r = 1, 2$ yields

$$A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha_1\beta_2 - 4\alpha_2\beta_1 & -3\alpha_1\alpha_2 & 0 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha_2\beta_2 & -\alpha_2^2 & 0 & 0 \end{pmatrix}.$$

Since both A_3 and A_4 belong to V, we must have

$$A_3 = 0, \qquad A_4 = 0. \tag{3}$$

Using (3), together with relation (1) and an inductive argument, we obtain that $A_k = 0$ for all $k \geq 3$. This means that $M^3\mathcal{A} \subseteq M$. On the other hand, relation (3) immediately yields $\alpha_2 = 0$ and $\alpha_1\beta_2 = 0$. Consequently,

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_1 & \alpha_1 & 0 & 0 \\ 0 & -\beta_1 & -\alpha_1 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \beta_2 & 0 & 0 & 0 \\ 0 & -\beta_2 & 0 & 0 \end{pmatrix}, \quad \alpha_1\beta_2 = 0.$$

Since $\alpha_1\beta_2 = 0$, we deduce that either $\alpha_1 = 0$ or $\beta_2 = 0$. We analyze each case separately.

Case 6.1: $\alpha_1 = 0$. In this situation we have $w_3M \subseteq M$ and $w_4M \subseteq M$. Let $T_7(\alpha', \beta', \gamma')$ and $T_7(\alpha'', \beta'', \gamma'')$ denote the matrices of $\overline{L}_{w_3}$ and $\overline{L}_{w_4}$, respectively, with respect to the basis Φ. Then

$$\gamma' w_3 + M = \overline{L}_{w_3}(\overline{w}_4) = w_3w_4 + M = \overline{L}_{w_4}(\overline{w}_3) = \gamma'' w_2 - \alpha'' w_4 + M.$$

Therefore, $\gamma'' = 0$. It follows that

$$0 + M = \overline{L}_{w_4}(\overline{w}_4) = w_4^2 + M,$$

so that $w_4^2 \in M$. Hence $B = M \oplus \mathbb{C}w_4$ is a subalgebra of $\mathcal{A}$ that properly contains M, as required.

Case 6.2: $\alpha_1 \neq 0$ *and hence* $\beta_2 = 0$. In this case we have $A_2 = 0$, and consequently $M^2\mathcal{A} \subset M$. Let $w = \alpha_1^2 w_1 - \alpha_1\beta_1 w_2 + \beta_1^2 w_3$. Then $wM \subseteq M$ and $w_4M \subseteq M$. Let $T_7(\alpha', \beta', \gamma')$ and $T_7(\alpha'', \beta'', \gamma'')$ denote the matrices of $\overline{L}_w$ and $\overline{L}_{w_4}$, respectively, with respect to the basis Φ. We then obtain the relation

$$\begin{aligned}\gamma' \overline{w_3} &= \overline{L}_w(\overline{w_4}) = ww_4 + M = \overline{L}_{w_4}(\overline{w}) \\ &= -\gamma''\alpha_1\beta_1\overline{w_1} + \gamma''\beta_1^2\overline{w_2} + (\beta''\alpha_1 - \alpha''\beta_1)\alpha_1\overline{w_3} + (\beta''\alpha_1 - \alpha''\beta_1)\beta_1\overline{w_4}.\end{aligned}$$

This forces the condition

$$\gamma''\beta_1 = 0.$$

If $\gamma'' = 0$, then $B = M \oplus \mathbb{C}w_4$ is a subalgebra of $\mathcal{A}$ that properly contains M.

If $\gamma'' \neq 0$, then necessarily $\beta_1 = 0$. Hence $w = \alpha_1^2 w_1$, and since $\alpha_1 \neq 0$, we obtain $w_1M \subseteq M$. In the basis Φ, we have

$$[\overline{L}_a] = A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & -\alpha_1 & 0 \end{pmatrix}, \quad [\overline{L}_{w_4}] = \begin{pmatrix} 0 & \gamma'' & 0 & 0 \\ 0 & 0 & \gamma'' & 0 \\ \beta'' & \alpha'' & 0 & \gamma'' \\ 0 & -\beta'' & -\alpha'' & 0 \end{pmatrix}.$$

Moreover,

$$\begin{aligned} (w_4)^4 &\equiv \overline{L}_{w_4}^3(w_4) \equiv (\gamma'')^3 w_1 - (\gamma'')^2\beta'' w_4 \equiv (\gamma'')^2\big(\gamma'' w_1 - \beta'' w_4\big) \mod M, \\ (w_4)^4 M &\subseteq (\gamma'')^2\big(\gamma'' w_1 - \beta'' w_4\big)M + M^2 \subseteq w_1M + w_4M + M = M, \\ (w_4)^4 \cdot (w_4)^4 &= (w_4)^8 \equiv \overline{L}_{w_4}^7(\overline{w}_4) \equiv 0 \mod M, \text{ so } (w_4)^4 \cdot (w_4)^4 \in M. \end{aligned}$$

Therefore, $B = M \oplus \mathbb{C}(w_4)^4$ is a subalgebra of $\mathcal{A}$ that properly contains M, as required. This completes the proof of the theorem. □

Declaration of competing interest

The authors declare no competing interests.

Acknowledgements

The first author was partially supported by the project 2024/14914-9, the São Paulo Research Foundation (FAPESP).

The second author was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

Data availability

No data was used for the research described in the article.

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