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Derivations with invertible values in rings with involution

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Recently Bergen, Herstein and Lanski studied the structure of a ring R with a derivation $d \neq 0$ such that, for each $x \in R$, $d(x)=0$ or $d(x)$ is invertible. They proved that, except for a special case which occurs when $2R=0$, such a ring must be either a division ring D or the ring D_2 of 2x2 matrices over a division ring.

In this paper we address ourselves to a similar problem in the setting of rings with involution, namely: let R be a 2-torsion free semiprime ring with involution and let S be the set of symmetric elements. If $d \neq 0$ is a derivation of R such that the non-zero elements of $d(S)$ are invertible, what can we conclude about R?

We shall prove that R must be rather special. In fact we shall show the following:

THEOREM - Let R be a 2-torsion free semiprime ring with involution. Let d be a derivation of R such that $d(S) \neq 0$ and the non-zero elements of $d(S)$ are invertible in R. Then R is either:

1. a division ring D, or
2. D_2, the ring of 2x2 matrices over D, or
3. $D \oplus D^{op}$, the direct sum of a division ring and its opposite relative to the exchange involution, or
4. $D_2 \oplus D_2{}^{op}$ with the exchange involution, or
5. F_4, the ring of 4x4 matrices over a field F with symplectic involution

In case $R=F_4$ with * symplectic we shall prove that d is inner. As Herstein has pointed out, an easy example of such a ring is given by taking F to be a field in which -1 is not a square and d the inner derivation in F_4 induced by $\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ where I is the identity matrix in F_2.

Now, if $R=D\oplus D^{op}$ or $R=D_2\oplus D_2^{op}$ then $S\cong D$ or $S\cong D_2$ respectively. Thus both cases come naturally from [1].

We remark that if $d(S)=0$ then $d(\overline{S})=0$, where $\overline{S}$ is the subring generated by S; hence, if R is semiprime, by [3, theorem 2.1.5.] either S lies in the center of R (and R satisfies the standard identity of degree 4) or $d(J)=0$ for some non-zero ideal J of R.

Let R be a ring with involution; we denote by Z the center of R and by S and K the sets of symmetric and skew elements of R respectively. Throughout this paper, unless otherwise stated, R *will be a 2-torsion free semiprime ring with an involution * and d will be a derivation of R such that $d(S)\neq 0$ and the non zero elements of $d(S)$ are invertible.*

We begin with the following

LEMMA 1 - If $I=I^*$ is a non-zero ideal of R then $d(I \cap S) \neq 0$.

PROOF - Suppose, by contradiction, that $d(I \cap S)=0$ and let $t \in S$ be such that $d(t) \neq 0$. For all $s \in I \cap S$ the elements sts and st+ts lie in $I \cap S$, hence

$$0 = d(sts) = sd(t)s$$

$$0 = d(st+ts) = sd(t)+d(t)s$$

Multiplying the second equality from the left by s, we obtain $s^2d(t)=0$. Now, from our basic hypothesis on R, d(t) is invertible; hence $s^2=0$, for all $s \in I \cap S$.

Let now $x \in R$, $s \in I \cap S$. Then the element $sx+x^*s$ lies in $I \cap S$ and, so, it must be square-zero. Therefore, since $s^2=0$,

$$0 = (sx+x^*s)sx = (sx)^3 ,$$

that is, every element in the right ideal sR is nilpotent of index ≤ 3. By Levitski's Theorem [2, Lemma 1] we must have $sR=0$ and, so, $s=0$. This proves that $I \cap S=0$.

For $x \in I$, $x+x^* \in I \cap S$; hence $x=-x^*$ and $x^2 \in I \cap S=0$. This I is a nil ieal of index ≤ 2. This forces $I=0$, a contradiction.

□

At this stage we are able to prove our result in case R is not simple; in fact we have

PROPOSITION 1 - If R is not a simple ring then either $R \simeq D \oplus D^{op}$, D a division ring, or $R \cong D_2 \oplus D_2{}^{op}$ and * is the exchange involution.

PROOF - Let $I \neq R$ be an ideal of R such that $I=I^*$. If $x,y \in I$ then $xy+y^*x^* \in I \cap S$ and

$$d(xy+y^*x^*) = d(x)y+xd(y)+y^*d(x^*)+d(y^*)x^* \in I .$$

Since I doesn't contain invertible elements we must have $d(xy+y^*x^*) = 0$. This fact implies that for all $z \in I^2$, $d(z+z^*) = 0$ and so, since R is 2-torsion free, $d(I^2 \cap S) = 0$. But then, since $I^2=I^{*2}$, by lemma 1, $I^2=0$ and the semiprimeness of R forces I=0. We have proved that R doesn't contain proper *-ideals.

If R is not simple, then there exists a proper ideal $I \neq I^*$. Since $I+I^*$ is a non-zero *-ideal of R, $I+I^* = R$. Also $I \cap I^* \neq R$ is a *-ideal of R, hence $I \cap I^* = 0$. Thus we have that $R=I \oplus I^*$. Moreover since $I^2 \neq I^{*2}$ we also get $R=I^2 \oplus I^{*2}$ and, so, $I=I^2$. Now, $d(I)=d(I^2) \subset I$ says that I, and so I^*, is invariant under d. If we write $1=e+f$ with $e,f^* \in I$ then e is the unit element of I. Also, if x is in I and $d(x) \neq 0$, then $0 \neq d(x)+d(x^*) = d(x+x^*)$ is invertible in R. If $y+z$ is its inverse, where $y,z^* \in I$, we get $d(x)y=e$. Thus z^*, for every $x \in I$, $d(x)$ is either 0 or invertible. By [1, Theorem 1] I, and so I^*, is either a division ring D or D_2. If $d(I)=0$, then $d(I^*) \neq 0$ and the above argument leads to the same conclusion. Clearly the involution in R is the exchange involution.□

If R is a prime ring we denote by C the extended centroid of R and by $Q=RC$ the central closure of R (see [3, pg. 22]. Next lemma holds for arbitrary rings with involution, with a derivation $d \neq 0$.

LEMMA 2 - Let R be a prime ring with involution, with a derivation $d \neq 0$. Let $x \in R$ be such that for all $s \in S$

$$xsx^*d(R)xsx^* = 0.$$

Then either $x^*d(R)x = 0$ or $Q=RC$ has a minimal right ideal.

PROOF - For $y \in R$ let $u=x^*d(y)x$. Then if $s \in S$, $ususu=ususu^*=0$; now, if $r \in R$, $su^*r^*+rus \in S$ and, so,

$$0 = usu(su^*r^*+rus)u(su^*r^* \ldots)$$

This says that every element in the right ideal $usuR$ is nilpotent of index ≤ 3. By Levitski's theorem [2, Lemma 1.1], $usu\,R$ and so $usu=0$ for all $s \in S$. By [5, Lemma 3], if $u \neq 0$, $Q=RC$ has a minimal right ideal.

□

At the light of Proposition 1 we now make a first reduction: from now on, unless otherwise stated, *we will always assume that R is a simple ring with 1.* In this case clearly R coincides with its own central closure.

Next lemmas give us some information about the nature of the symmetric elements in the kernel of d.

LEMMA 3 - Let $a \in S$. If for all $s \in S$ we have that $asa=\lambda a$, for some $\lambda=\lambda(s) \in z$, then R has a minimal right ideal.

PROOF - Let $x \in R$. Then $a(x+x^*)a=\lambda a$, for some $\lambda \in Z$, that is $ax^*a=\lambda a-axa$. Let $\mu \in Z$ be such that $a(xax+x^*ax^*)a=\mu a$. Playing these off against each other we get

$$0 = axaxa+ax^*ax^*a-\mu a = 2axaxa-2\lambda axa+(\lambda^2-\mu)a.$$

Therefore $2(ax)^3-2\lambda(ax)^2+(\lambda^2-\mu)ax = 0$ and, since char $R \neq 2$, ax is algebraic over Z of degree at most 3. This proves that aR is an algebraic algebra of bounded degree. Thus aR satisfies a polynomial identity; hence R satisfies a generalized polynomial identity. Since R coincides with its own central closure, by a theorem of Martindale [3, Theorem 1.3.2.] R has a minimal right ideal.

□

LEMMA 4 - Suppose R does not contain minimal right ideals. If $a \in S$ is such that $d(a)=0$ then either a is invertible or $ad(R)a=0$.

PROOF - Suppose $a \neq 0$ and a is not invertible. Since $d(a)=0$ then, for all $s \in S$, $d(asa)=ad(s)a$ and it is not invertible. Hence $ad(s)a=0$.

Let now $x \in R$. Then $ad(x+x^*)a=0$ implies $ad(x)a=-ad(x^*)a$. Therefore for all $s \in S$, recalling that $d(a)=ad(s)a=0$ we get

$$asad(x)a = ad(sax)a = -ad(x^*as)a = -ad(x^*)asa = ad(x)asa.$$

We have proved that for all $x \in R$, $s \in S$

$$asa\, d(x)a = ad(x)asa \tag{1}$$

Since $d(a)=0$, $d(aR) \subset aR$; moreover if $\rho_R(a)$ is the left annihilator of a in R, $d(\rho_R(a)) \subset \rho_R(a)$; this says that d induces a derivation (which we will still denote by d) in the prime ring $R_1 = aR/\rho_R(a) \cap aR$. Moreover, for $s \in S$, if $\overline{as}$ is the image of as in R_1, from (1) we get

$$\overline{as}\, d(\overline{ax}) = d(\overline{ax})\overline{as} \quad , \quad \text{for all } \overline{ax} \in R_1.$$

By [4] since char $R \neq 2$ either $d = 0$ in R_1 or $\overline{as} \in Z(R_1)$, the center of R_1. That is, either $ad(R)a = 0$ or $asaxa = axasa$ for all $x \in R$.

If $ad(R)a=0$ we are done; therefore we may assume that $asaxa=axasa$, for all $x \in R$, $s \in S$. But then, by [3, Lemma 1.3.2.], $asa=\lambda a$, for some $\lambda \in Z$ and, by Lemma 3, R has a minimal right ideal, a contradiction.

□

We remark that since R is simple with 1 then it must be a primitive ring. Now, through a repeated application of the density theorem we will be able to prove that R is artinian.

PROPOSITION 2 - R is a simple artinian ring

PROOF - Since R is primitive it is a dense ring of linear transf transformations on a vector space V over a division ring D. By [3, Lemma 1.1.2.] to prove that R is artinian it is enough to prove that R has a minimal right ideal or equivalently that R contains a non-zero transformation of finite rank. Suppose, by contradiction, that this is not the case.

Let $s \in S$ be such that $d(s) \neq 0$ and suppose that there exist linearly independent vectors $v, w \in V$ such that

$$vs = ws = 0 \quad .$$

Since $d(s)$ is invertible, the vectors $vd(s)$ and $wd(s)$ are linearly independent over D. Moreover, since R doesn't contain non-zero transformations of finite rank, there exists a vector $u \in V$ such that $us \notin vd(s)D+wd(s)D$, i.e., $us, vd(s), wd(s)$ are linearly independent over D.

By the density of the action of R on V, there exists $x \in R$ such that

$$usx \neq 0$$
$$vd(s)x = 0$$
$$wd(s)x \neq 0 \ .$$

Let $t \in S$. Since $vd(s)x=vs=0$ then $vd(sxtx^*s)=0$; hence, since $sxtx^*s \in S$ and $d(sxtx^*s)$ is not invertible, we must have $d(sxtx^*s) = 0$. Moreover s, and so $sxtx^*s$, is not invertible. Since R has no minimal right ideals, by applying Lemma 4 to the element $sxtx^*s$, we get $sxtx^*sd(R)sxtx^*s = 0$, for all $t \in S$. Hence Lemma 2 implies $x^*sd(R)sx = 0$.

Let now $y,z \in R$. Since $x^*sd(y)sx = 0$ we have

$$0 = x^*sd(ysxz)sx = x^*syd(sxz)sx .$$

Hence $x^*sRd(sxR)sx = 0$ and, since $x^*s \neq 0$, the primeness of R forces $d(sxR)sx = 0$. If $y \in R$ we get

$$0 = d(sxy)sx = d(s)xysx+sd(xy)sx;$$

hence, since $ws = 0$, $0 = wd(sxy)sx = wd(s)xysx$. But $wd(s)x \neq 0$ and, by the density of the action of R on V, $wd(s)xR = V$; thus $0=wd(s)xRsx=Vsx$ implying $sx=0$, a contradiction.

We have proved that for every $s \in S$ with $d(s) \neq 0$, $\dim_D \ker s \leq 1$.

Let now W be a finite dimensional subspace of V such that $\dim_D W > 1$ and let $\rho=\rho_W =\{x \in R \mid Wx=0\}$; ρ is a right ideal of R.

We claim that there exists $s \in \rho \cap S$ such that $s^2 \neq 0$. In fact, suppose not and let $x \in \rho$, $s \in \rho \cap S$. Then, since $(xs+sx^*) \in \rho \cap S$ and $(xs+sx^*)^2=S^2=0$; we get $0=s(xs+sx^*)^2=s(xs)^2$, i.e., $s\rho$ is a right ideal nil of bounded index. By Levitski's theorem $s\rho=0$; hence $(\rho\cap S)\rho=0$. Now, since R has no minimal right ideals, by [3, Lemma 5.1.2.], for $v \notin W$, there exists $x \in \rho$ such that $x^* \in \rho$, $vx^*=0$ and $v(x+x^*) = vx \notin W+Dv$. But then, by density, there exists $y \in \rho$ such that $v(x+x^*)y \neq 0$, contradicting the fact that $(x+x^*)y \in (\rho \cap S)\rho = 0$. This establishes the claim

Set then $s \in \rho \cap S$ such that $s^2 \neq 0$. Since ρ is a proper right ideal of R, s is not invertible; moreover, since $\dim_D \ker s \geq \dim W > 1$, $d(s)=0$. Hence, by Lemma 4, $sd(R)s=0$.

Now, if $x \in \rho$ then $sx^*+xs \in \rho \cap S$ and $d(s)=0$ implies $0=d(sx^*+xs)=sd(x^*)+d(x)s$. Since $sd(x^*)s=0$, multiplying by s from the right we get $d(x)s^2=0$. Thus $d(\rho)s^2=0$. Now, for $x,y \in \rho$, $0=d(xy)s^2=d(x)ys^2$ forces $d(\rho)\rho s^2=0$ and, since R is prime and $s^2 \neq 0$, $d(\rho)\rho=0$. Clearly $d(\rho) \neq 0$; so, let $x \in \rho$ be such that $d(x) \neq 0$. If $vd(x) \notin W$ for some $v \in V$, then by density there exists $r \in \rho$ such that $vd(x)r \neq 0$, contradicting the fact that $d(x)r \in d(\rho)\rho=0$. Thus $Vd(x) \subset W$ and $d(x)$ is a transformation of finite rank, a contradiction. □

We are now in a position to prove the Theorem:

PROOF OF THE THEOREM - By Proposition 1 and Proposition 2 we may assume that R is a simple artinian ring. Hence, $R=D_n$, the ring of nxn matrices over a division ring D.

Suppose first that $*$ on D_n is of transpose type and assume $n > 2$. Let e_{ij} be the usual matrix units. For $i=1,\dots,n$ $e_{ii}=e_{ii}^* \in S$ implies $d(e_{ii})=e_{ii}d(e_{ii})+d(e_{ii})e_{ii}$. Thus, since rank $e_{ii}=1$, rank $d(e_{ii}) \leq 2$ and, being $n > 2$, $d(e_{ii})$ canot be invertible. Hence $d(e_{ii})=0$, $i=1,\dots,n$.

Now, if $i \neq j$, for a suitable $0 \neq c \in D$, $e_{ij}+ce_{ji} = e_{ij}+e_{ij}^* \in S$. Thus

$d(e_{ij}+ce_{ji})=d(e_{ii}(e_{ij}+ce_{ji})+(e_{ij}+ce_{ji})e_{ii})=e_{ii}d(e_{ij}+ce_{ji})+d(e_{ij}+ce_{ji})e_{ii}$; and so, rank $d(e_{ij}+ce_{ji}) \leq 2$. It follows $d(e_{ij}+ee_{ji}) = 0$ which implies $0 = d(e_{ii}(e_{ij}+ce_{ji})) = d(e_{ij})$.

We have proved that $d(e_{ij})=0$ for $i,j=1,\ldots,n$. Let now $x \in D$. If $i \neq j$, $S \ni xe_{ij}+(xe_{ij})^*=xe_{ij}+c_1x^*c_2e_{ji}$ for suitable $c_1,c_2 \in D \cap S$. We have:

$$\text{rank}(d(xe_{ij}+c_1x^*c_2e_{ji})=\text{rank}(d(x)e_{ij}+d(e_1x^*c_2)e_{ji}) \leq 2 ,$$

hence $d(xe_{ij}+e_1x^*c_2e_{ji}) = 0$ and, multiplying by e_{ji} from the right we get $d(x)e_{ii}=0$, for all $i=1,\ldots,n$. Thus $d(x)=d(xI)=\sum_i d(x)e_{ii} = 0$, i.e., $d(D)=0$. In short $d=0$ in D_n.

Suppose now that $*$ is symplectic. In this case $D=F$ is a field and suppose $n > 4$. Let $I_1=e_{11}+e_{22}$; $I_1^2=I_1 \in S$, so rank $d(I_1) = \text{rank}(I_1d(I_1)+d(I_1)I_1) \leq 4$ implies $d(I_1)=0$. Now, for i odd , $a=e_{1i}+e_{i+1,2} \in S$; hence $d(a)=d(I_1a+aI_1)=I_1d(a)+d(a)I_1$ has rank $\leq$ 4. It folows $d(a)=0$ and, so, for $i \neq 1$, $0=d(I_1a)=d(e_{1i})$. On the other hand, if i is even, $e_{1i}-e_{i-1,2} \in S$ and by the same argument we get $d(e_{1i})=0$ for $i \neq 2$. Moreover by looking at $e_{1i}+e_{i1}^*$ as above, we obtain $d(e_{i1})=0$ for $i \neq 1,2$. At this stage it easily follows $d(e_{ij})=0$ for all $i,j=1,\ldots,n$. Since clearly $d(F)=0$, then $d=0$ in F_n and we are done.

We are left with the case $R=F_4$ and $*$ symplectic. We will prove that in this case d must be inner. By a well known result on finite dimensional central simple algebres it is enough to prove that $d(F)=0$. So, suppose by contradiction that there exists $\alpha \in F$ such that $d(\alpha) \neq 0$ and let $s \in S$, $s \neq 0$, be such that $d(s)=0$. Then, since $d(\alpha) \in F$, $d(\alpha s)=d(\alpha)s \neq 0$ implying s invertible.

Therefore, for every $s \in S$, $s \neq 0$, $d(s)=0$ implies s invertible.

Now, if I is the identity matrix in F_2, $t=\begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \in S$ and, since t is not invertible, $d(t) \neq 0$. Moreover it is easy to prove that $d(t) = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}$ where $A,B \in F_2$. Let now V be a 4-dimensional vector space over F and let $\{e_1,e_2,e_3,e_4\}$ be the standard basis for V. Then since $d(t)$ is invertible, $e_1d(t),e_2d(t)$ are linearly independent over F; moreover $e_1d(t),e_2d(t) \in \mathrm{Span}_F\{e_3,e_4\}$. By density there exists $x \in F_4$ such that $e_1d(t)x=e_1x=e_2x=0$, $e_2d(t)x \in \mathrm{Span}_F\{e_3,e_4\}$ and $e_2d(t)x \neq 0$. Clearly $tx=x$ and, so, $x^*t=x^*$. Now, since $e_1d(txx^*t)=e_1d(t)xx^*t+e_1td(xx^*t)=0$, $d(txx^*t)$ cannot be invertible; hence $d(txx^*t)=0$. By the remark above, since t is not invertible, we must have $0=txx^*t=xx^*$. Thus, since $*$ is symplectic, $x=0$, a contradiction. □

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