Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

ARTICLE TYPE

Optimal burn-in policy based on a set of cutoff points using mixture inverse Gaussian degradation process and copulas

Lia H. M. Morita*[1] | Vera L. Tomazella[2] | Paulo H. Ferreira[3] | Pedro L. Ramos[3] | Narayanaswamy Balakrishnan[4] | Francisco Louzada[3]

[1]Department of Statistics, Federal University of Mato Grosso, Cuiabá, Mato Grosso, Brazil
[2]Department of Statistics, Federal University of São Carlos, São Carlos, São Paulo, Brazil
[3]Institute of Mathematical and Computer Sciences, University of São Paulo, São Carlos, São Paulo, Brazil
[4]Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario, Canada

Correspondence
*Corresponding author name: Lia H. M. Morita. Email: liamorita@ufmt.br

Summary

Burn-in tests have been discussed extensively in the reliability literature, wherein we operate items until high degradation values are observed, which could separate the weak units from the normal ones before they get to the market. This concept is often referred to as a screening procedure, and it involves misclassification errors. Commonly, the underlying degradation process is assumed to be a Wiener or a gamma process, based on which several optimal burn-in policies have been developed in the literature. In this paper, we consider the mixture inverse Gaussian process, which possesses monotone degradation paths and some interesting properties. Under this process, we present a decision rule for classifying a unit under test as normal or weak based on burn-in time and a set of cutoff points. Then, an economic cost model is used to find the optimal burn-in time and the optimal cutoff points, when the estimation of model parameters is based on an analytical method or an approximate method involving copula theory. Finally, an example of a real data set on LASERs, well known in the reliability literature, is used to illustrate the model and the inferential approach proposed here.

KEYWORDS:
Burn-in test, copula, LASER degradation data , mixture inverse Gaussian process, optimal cutoff points, optimal termination time.

1 | INTRODUCTION

Very reliable items under test often result in few or no failures, but when failure can be associated directly with a quality characteristic (QC) over time, it is then possible to measure degradation over time and use it to make inference about the product's reliability. This approach allows us to get information about the lifetime distribution without actually observing failures. In degradation tests, a piece of equipment is considered to have failed if the degradation value crosses a certain threshold level, which is usually specified by the manufacturer complying with functional requirements [1]. While some degradation studies involve measuring physical degradation as a function of time (for instance, tire wear), in some other applications, it can not be seen directly, but some measure of the item's performance degradation (for instance, power output) may be available. Furthermore, degradation analysis can have one or more variables in the underlying degradation process.

Burn-in test is a technique used to increase the quality of components and systems by testing the units before fielding them into the market. Traditional burn-in tests are inefficient for very reliable items. The condition-based burn-in tests, on the other hand, are useful in this case, wherein a QC related to failure is chosen, and the items with deterioration levels below a specified threshold are considered to be standard units, while items with deterioration levels exceeding this threshold value are considered to be weak units.

In the last few decades, the manufacturing industry has been dedicating much effort in designing burn-in policies to eliminate early failures before fielding items into the market. These failures, which usually happen in a small proportion among the manufactured items, are generally caused by manufacturing defects and lead to high warranty and replacement costs. Diverse burn-in policies have been discussed in the degradation literature; see, e.g., Jensen[2], Kuo[3], and Leemis and Beneke[4].

The use of mixture distributions is an essential characteristic in burn-in policies because the lifetime distribution of the components is usually bimodal from a mixture of two distributions, wherein the weak units tend to fail earlier than the normal ones, and so the degradation values may exhibit bimodal behavior as well. The manufacturing process often causes such heterogeneity with a variety of material flaws, or due to the components coming from different suppliers. Burn-in policies are also subject to misclassification errors, which are of great importance in burn-in studies. Tseng *et al.*[5] developed an economic model based on misclassification errors and established an optimal burn-in policy from termination time and a set of cutoff points, whose degradation paths were modeled by a mixture of two Wiener processes.

In many experiments, the degradation is a continuous progression of wear and decay, and so it is natural to model the degradation path with a stochastic process[6]. In this sense, the degradation over time is often modeled by a stochastic process $\{D(t);\ t > 0\}$, to account for inherent randomness. Based on the supposition of additive accumulation of degradation, two classes of degradation processes have been well studied, namely, the Wiener and the gamma processes. Various papers in the literature suppose that the degradation paths follow the Wiener process; e.g., Tseng and Peng[7] studied an efficient burn-in approach based on an integrated Wiener process for the cumulative degradation, and Wu and Xie[8] suggested the use of receiver operating characteristic (ROC) curve for the removal of the weak group from the production.

Zhai *et al.*[9] used the Wiener process to model the underlying degradation and considered Gaussian measurement errors in the observations. The work then focused on the optimal burn-in strategies under two different cost structures, the misclassification cost model and the field failure cost model, to obtain the optimal cutoff levels. Also, Ye *et al.*[10] considered Wiener processes with linear drift, while the weak and the normal subpopulations possess distinct drift parameters. The objective of joint burn-in and maintenance decisions was to minimize the long-run average cost per unit time during field use by choosing suitably the burn-in settings and the preventive replacement intervals. In the multivariate context, Ye *et al.*[11] developed a burn-in planning method by considering normal and mortality failure modes and designed a burn-in test framework that assigns both degradation and failure data, and Ye *et al.*[12] built a degradation-based model that facilitates, for two different types of failures (normal and defect failures), the determination of optimal burn-in characteristics conditioned to heterogeneous customer behaviors.

The gamma process, which does exhibit a monotone increasing behavior, has been discussed by several authors, e.g., Singpurwalla[13], Lawless and Crowder[14], Park and Padgett[15], Park and Padgett[16] and, more recently, by Tsai *et al.*[17], who proposed a mixed gamma process for the degradation paths and presented an optimal burn-in policy for classifying LASER components based on a cost model.

Recently, the IG process has been suggested as an attractive and flexible stochastic process for degradation modeling. This process, with monotone paths, was first proposed by Wasan[18] and discussed more recently by Wang and Xu[19], Ye and Chen[20], and Peng[21], in the context of degradation modeling with random effects. In comparison with the gamma process, the IG process is more flexible in incorporating random effects and covariates and is also mathematically tractable (for instance, the lifetime distribution based on the IG process has an explicit form)[19]. Notably, Zhang *et al.*[22] proposed a mixed IG process for degradation data, wherein the optimal burn-in policy to screen out weak components are based on burn-in time and a single cutoff point in the decision rule.

Although the IG process is already widely used, this work presents a new proposal in the field of burn-in policies. We consider an inverse Gaussian mixing process, which has not yet been explored widely, and then focus on burn-in termination time and a set of cutoff points thus making a novel contribution in this direction.

As a motivating example, in this work, we consider the LASER data of an experiment described by Meeker *et al.*[23]. The QC of a LASER device is its operating current. To keep nearly constant light output, the LASER device contains a feedback mechanism for increasing the operating current when the light output degrades. The successful performance of a degradation test depends strongly on the appropriateness of the model that represents a product's degradation path. Degradation models based on gamma and Wiener processes have been extensively studied in the reliability literature, as previously mentioned. Nevertheless, when both these degradation-based processes do not suitably fit certain data[19], the IG process is considered an adequate alternative degradation model for describing the degradation path. The mixture IG process model is used to capture the heterogeneity in the degradation paths, where the degradation paths consist of two groups with different mean functions and the same volatility parameter. We state a decision rule for classifying an item as normal or weak based on burn-in time and a collection of cutoff points. Then, an economic cost model is used to determine the optimal burn-in time and the optimal cutoff points, whose estimation is based on analytical method or approximated method involving copula theory. Copulas are parametric functions that join or "couple" univariate distributions into multivariate distribution functions[24]. Many of them have convenient parametric forms, which allows the modeling of the dependence structure among marginal distribution functions[25]. The use of copula functions is an appealing way for dealing with multivariate distributions constructed from marginal densities, which is much more flexible and realistic. Additionally, we developed a simulation study which showed that the model misspecification affects the misclassification probabilities, and consequently, the computation of the costs and the achievement of an optimal burn-in policy. The optimal burn-in policy based on a collection of cutoff points is a more flexible method, which improves the way the weak components are separated from the normal ones in a production row.

This paper aims to determine the optimal burn-in policy, which includes the optimal burn-in time and a set of optimal cutoff points rather than a single cutoff point. The main novelty is that this approach exhibits several scenarios for the burn-in time and the cutoff points, in which we can choose the burn-in time and the number of cutoff points that lead to minimum cost. Moreover, the use of multiple cutoff points offers many other benefits as compared to a procedure based on a single cutoff point. Therefore, we propose a decision rule for classifying an item as normal or weak based on a cost model, wherein the main interest is to minimize the associated costs. This process involves the derivation of misclassification probabilities, which are obtained through an analytical method and an approximate method based on copula theory. The entire methodology developed here is based on the mixture IG degradation process model.

The rest of this paper is organized as follows. Section 2 describes the proposed method based on the mixture IG process. Section 3 describes the optimal burn-in procedure based on the proposed cost model. Section 4 contains a numerical example of LASER data and Section 4.2 presents some comments and concluding remarks.

2 | MIXTURE IG DEGRADATION PROCESS MODEL

In this work, we consider the IG process as an effective degradation model. This process has a monotone degradation path and was first proposed by Wasan[18], using properties of the IG distribution, see, e.g., Chhikara and Folks[26] for several of the univariate IG properties.

Often, electronic devices and other similar units consist of two groups, the weak group and the normal one, wherein the weak group has a shorter mean lifetime than the normal one[22]. Mixture distributions have been usually applied to capture this kind of heterogeneity, in which the degradation paths are modeled via a mixture degradation process.

Let $g_{\theta_1}(t)$ and $g_{\theta_2}(t)$ be the mean functions of the weak and normal groups of items, respectively. Also, let $\eta > 0$ be the same volatility (or shape) parameter. Thus, the degradation path $D(t)$ in the mixture IG process is given by

$$D(t) \sim \begin{cases} \text{IG}(g_{\theta_1}(t), \eta g_{\theta_1}^2(t)), & \text{for the weak group,} \\ \text{IG}(g_{\theta_2}(t), \eta g_{\theta_2}^2(t)), & \text{for the normal group,} \end{cases} \tag{1}$$

where $g_{\theta_1}(t) > g_{\theta_2}(t) > 0, \forall t \geq 0$, that is, the mean function of the degradation process is greater in the weak group than in the normal one, and the volatility parameter from both groups is supposed to be equal. We opted for this approach, which is the same used in Zhang *et al.*[27]. It is worth mentioning that the variances of the two processes are distinct due to properties of the IG distribution, even though the parameter η being equal. As a result of (1) and from the well-known properties of the IG distribution, the degradation increment $Y = D(t + \Delta t) - D(t)$ corresponding to the time interval $[t, t + \Delta t]$, with $\Delta t > 0$, has the following probability density function (pdf):

$$f_Y(y) = p\sqrt{\frac{\eta}{2\pi y^3}}\Delta g_{\theta_1}(t) e^{-\frac{\eta(y-\Delta g_{\theta_1}(t))^2}{2y}} + (1-p)\sqrt{\frac{\eta}{2\pi y^3}}\Delta g_{\theta_2}(t) e^{-\frac{\eta(y-\Delta g_{\theta_2}(t))^2}{2y}},$$

where $0 < p < 1$ is the mixing parameter, $\Delta g_{\theta_1}(t) = g_{\theta_1}(t + \Delta t) - g_{\theta_1}(t)$ is the time-function increment during the time interval Δt under weak units, and $\Delta g_{\theta_2}(t) = g_{\theta_2}(t+\Delta t) - g_{\theta_2}(t)$ is the time-function increment in the time interval Δt under normal units.

2.1 | Lifetime estimation

In degradation modeling, the first passage time (FPT) of a unit is the time at which its degradation path first hits a fixed threshold level ρ. The FPT distribution defined below, or equivalently, the unit's lifetime distribution, plays an important role in predicting the remaining useful life as well as in determining optimal maintenance strategies (Noortwijk[28]):

$$T = \inf\left\{t \geq 0 | D(t) \geq \rho\right\}. \tag{2}$$

Due to the monotone behavior of the IG process, the lifetime distribution for T is expressed from (2) as

$$F_T(t) = P(D(t) \geq \rho).$$

Since $D(t) \sim \mathrm{IG}(g_{\theta_k}(t), \eta g^2_{\theta_k}(t))$, with $k = 1$ for the weak group and $k = 2$ for the normal group, we can obtain the lifetime cumulative distribution function (cdf) for the k-th group as

$$F_k(t) = \Phi\left(-\sqrt{\frac{\eta}{\rho}}\left(\rho - g_{\theta_k}(t)\right)\right) - \exp\{2\eta g_{\theta_k}(t)\}\Phi\left(-\sqrt{\frac{\eta}{\rho}}\left(\rho + g_{\theta_k}(t)\right)\right), \tag{3}$$

where $\Phi(\cdot)$ is the standard normal cdf.

Thus, the pdf of the lifetime distribution for the k-th group is readily obtained from (3) as

$$\begin{aligned} f_k(t) = {} & \sqrt{\frac{\eta}{\rho}}\phi\left(-\sqrt{\frac{\eta}{\rho}}\left(\rho - g_{\theta_k}(t)\right)\right)\frac{\partial g_{\theta_k}(t)}{\partial t} - 2\eta\exp\{2\eta g_{\theta_k}(t)\}\Phi\left(-\sqrt{\frac{\eta}{\rho}}\left(\rho + g_{\theta_k}(t)\right)\right) \\ & \times\frac{\partial g_{\theta_k}(t)}{\partial t} + \sqrt{\frac{\eta}{\rho}}\exp\left\{2\eta g_{\theta_k}(t)\right\}\phi\left(-\sqrt{\frac{\eta}{\rho}}\left(\rho + g_{\theta_k}(t)\right)\right)\frac{\partial g_{\theta_k}(t)}{\partial t}, \end{aligned} \tag{4}$$

where $\phi(\cdot)$ is the standard normal pdf. Notice that (4) has a closed-form, unlike the gamma process model, wherein the pdf of the FPT must be obtained via numerical methods; see Pandey and Noortwijk[29].

Peng[21] obtained the meantime to failure (MTTF) from (4), for the special case when $g_{\theta_k}(t) = \theta_k t$, as

$$\mathrm{MTTF} = \left(\frac{\rho}{\theta_k} + \frac{1}{\eta\theta_k}\right)\Phi\left(\sqrt{\eta\rho}\right) + \sqrt{\frac{\rho}{\eta}}\frac{1}{\theta_k}\phi\left(\sqrt{\eta\rho}\right) - \frac{1}{2\eta\theta_k}. \tag{5}$$

The quantiles of the lifetime distribution and the MTTF can be easily obtained upon substituting the maximum likelihood estimates of the parameters from model (1) in equations (3) and (5), respectively.

The lifetime distribution information will be useful in establishing a warranty policy for the units.

2.2 | Inferential methods for the model parameters

Consider a sample of n units. For each unit i, we have the degradation collecting points $t_{i0} = 0, t_{i1}, \ldots, t_{in_i}$, with the corresponding degradation values $D_{i1}, \ldots, D_{in_i}$. Let $Y_{ij} = D_{ij} - D_{i,j-1}$ be the degradation increment in the time interval $[t_{i,j-1}, t_{ij}]$ for unit i. Its contribution to the likelihood function consists of all degradation values up to burn-in time t_b or the number n_i of degradation values (if $n_i < b$):

$$\begin{aligned} L(g_{\theta_1}(t), g_{\theta_2}(t), \eta, p) = \prod_{i=1}^{n} \Bigg\{ & p\prod_{j=1}^{b^*}\sqrt{\frac{\eta}{2\pi y_{ij}^3}}\Delta g_{\theta_1}(t_{ij})\exp\left\{-\frac{\eta(y_{ij} - \Delta g_{\theta_1}(t_{ij}))^2}{2y_{ij}}\right\} \\ & +(1-p)\prod_{j=1}^{b^*}\sqrt{\frac{\eta}{2\pi y_{ij}^3}}\Delta g_{\theta_2}(t_{ij})\exp\left\{-\frac{\eta(y_{ij} - \Delta g_{\theta_2}(t_{ij}))^2}{2y_{ij}}\right\}\Bigg\}, \end{aligned}$$

where $b^* = \min\{n_i, b\}$, $\Delta g_{\theta_1}(t_{ij}) = g_{\theta_1}(t_{ij}) - g_{\theta_1}(t_{i,j-1})$ is the time-function increment in the time interval $[t_{i,j-1}, t_{ij}]$ under weak units, and $\Delta g_{\theta_2}(t_{ij}) = g_{\theta_2}(t_{ij}) - g_{\theta_2}(t_{i,j-1})$ is the time-function increment in the time interval $[t_{i,j-1}, t_{ij}]$ under normal units.

Note that the maximum likelihood estimates (MLEs) are conditioned to the stopping time. Besides, the term b^* is useful in experiments where the units have different numbers of degradation values due to soft failures. Hence, the log-likelihood function is given by

$$\ell(g_{\theta_1}(t), g_{\theta_2}(t), \eta, p) = \sum_{i=1}^{n} \log \left\{ p \prod_{j=1}^{b^*} \sqrt{\frac{\eta}{2\pi y_{ij}^3}} \Delta g_{\theta_1}(t_{ij}) \exp\left\{ -\frac{\eta(y_{ij} - \Delta g_{\theta_1}(t_{ij}))^2}{2y_{ij}} \right\} \right.$$
$$\left. +(1-p) \prod_{j=1}^{b^*} \sqrt{\frac{\eta}{2\pi y_{ij}^3}} \Delta g_{\theta_2}(t_{ij}) \exp\left\{ -\frac{\eta(y_{ij} - \Delta g_{\theta_2}(t_{ij}))^2}{2y_{ij}} \right\} \right\}. \tag{6}$$

Here, the functions $g_{\theta_1}(\cdot)$ and $g_{\theta_2}(\cdot)$ need to be specified. The MLEs can then be obtained by direct maximization of (6) with regard to the model parameters. Besides, interval estimates and hypothesis tests can be developed by utilizing the asymptotic properties of the MLEs.

3 | OPTIMAL BURN-IN POLICY BASED ON A SET OF CUTOFF POINTS

Optimal burn-in policies are often determined based on one of the following four criteria[30]:

- Maximization of the mean residual lifetime of the product;
- Achievement of prescribed mission reliability;
- Minimization of cost;
- Optimization of an objective function subject to some constraints.

Most of the proposed procedures are efficient for estimating the life characteristics of the product, as long as sufficient information becomes available on lifetimes of units. However, as mentioned earlier, even the weakest units today are quite reliable and so may take a long time to fail even under an accelerated burn-in test. A suitable alternative is to base the data collection on degradation tests, which are especially useful in scenarios wherein there is a QC whose degradation over time is closely associated with the lifetime of the unit under test. For highly reliable units resulting in very few or no failures, then this procedure is a hypothesis testing of the following form:

$$\begin{cases} H_0 : \text{The unit belongs to the normal group,} \\ H_1 : \text{The unit belongs to the weak group,} \end{cases}$$

where H_0 denotes the null hypothesis, and H_1 stands for the alternative hypothesis.

A decision rule to separate the weak units from the normal ones declares when H_0 needs to be rejected. Here, the decision rule (DR) is based on the burn-in time t_b and a set of s cutoff points $\xi_1, \ldots, \xi_s$, with $1 \leq s \leq b$, and is as follows:

DR: For fixed t_b and s, a unit is considered normal if and only if

$$D(t_{b-s+w}) \leq \xi_w, \forall w = 1, \ldots, s, \text{ with } 1 \leq s \leq b, \tag{7}$$

which means that we are making use of several cutoff points rather than a single one.

The DR in (7) is subject to misclassification errors, and so we now present the misclassification probabilities associated with this DR.

Considering (1), the probability of Type I error (i.e., of misclassifying a normal unit as weak), for each t_b and s, is given by

$$\begin{aligned} \alpha(\xi_1, \ldots, \xi_s | t_b, s) &= P(D(t_{b-s+w}) > \xi_w, \text{ for some } w = 1, \ldots, s | H_0) \\ &= 1 - P\left(D(t_{b-s+1}) \leq \xi_1, \ldots, D(t_b) \leq \xi_s | H_0\right). \end{aligned} \tag{8}$$

Similarly, the probability of Type II error (i.e., of misclassifying a weak unit as normal), for each t_b and s, is given by

$$\begin{aligned} \beta(\xi_1, \ldots, \xi_s | t_b, s) &= P(D(t_{b-s+w}) \leq \xi_w, \forall w = 1, \ldots, s | H_1) \\ &= P\left(D(t_{b-s+1}) \leq \xi_1, \ldots, D(t_b) \leq \xi_s | H_1\right). \end{aligned} \tag{9}$$

These misclassification probabilities can be obtained either through analytical methods or through approximate methods, depending on the number of cutoff points s used.

3.1 | Analytical methods for determining the misclassification probabilities

For $s = 1$, we have a single cutoff point, in which case the probabilities in (8) and (9) are obtained analytically by the use of the IG cdf as follows:

$$\alpha(t_b) = P\left(D(t_b) > \xi(t_b) \middle| H_0\right) = 1 - F_{\text{IG}}(g_{\theta_2}(t_b), \eta g^2_{\theta_2}(t_b)), \tag{10}$$

$$\beta(t_b) = P\left(D(t_b) < \xi(t_b) \middle| H_1\right) = F_{\text{IG}}(g_{\theta_1}(t_b), \eta g^2_{\theta_1}(t_b)), \tag{11}$$

where $F_{\text{IG}}(\cdot)$ is as defined in (3).

For $s = 2$, we have two cutoff points, in which case the misclassification probabilities are calculated analytically by the use of the bivariate IG cdf. In this context, we use Theorem 1 stated below and due to Al-Hussaini *et al.*[31].

Theorem 1. Let X_1 and X_2 be two random variables with IG distribution, i.e., $X_1 \sim IG(\mu_{X_1}, \lambda_{X_1})$ and $X_2 \sim IG(\mu_{X_2}, \lambda_{X_2})$. Then, the joint cdf $F_{X_1,X_2}(x_1, x_2)$ is given by

$$\begin{aligned} F_{X_1,X_2}(x_1, x_2) =\ & [\Phi(a_1) + \exp\{2\lambda_{X_1}\mu_{X_1}\}\Phi(-b_1)][\Phi(a_2) + \exp\{2\lambda_{X_2}\mu_{X_2}\}\Phi(-b_2)] \\ & + 16\sqrt{\frac{\lambda_{X_1}\lambda_{X_2}}{\mu_{X_1}\mu_{X_2}}}\rho_{X_1,X_2} \exp\left\{4\left(\frac{\lambda_{X_1}}{\mu_{X_1}} + \frac{\lambda_{X_2}}{\mu_{X_2}}\right)\right\}\Phi(-\sqrt{2}b_1)\Phi(-\sqrt{2}b_2), \end{aligned}$$

where $a_k = \frac{\sqrt{\lambda_{X_k}}(x_k - \mu_{X_k})}{\mu_{X_k}\sqrt{x_k}}$, $b_k = \sqrt{\frac{4\lambda_{X_k}}{\mu_{X_k}} + a_k^2}$, for $k = 1, 2$, and $\rho_{X_1,X_2} = \text{Corr}(X_1, X_2)$.

Proof. For detailed proof, one may refer to Al-Hussaini *et al.*[31]. □

The expression for $\rho_{X_1,X_2} = \text{Corr}(X_1, X_2)$ can be obtained from Wasan[32], who showed that the covariance of any two variables coming from an IG process is given by

$$\text{Cov}(X(s), X(t)) = \text{Var}(X(s)),$$

where $0 < s < t$.

3.2 | Approximate methods using copulas for determining the misclassification probabilities

When $s > 2$, i.e., when we have more than two cutoff points, the misclassification probabilities can be determined approximately through the use of multivariate copulas with dimension equal to s.

A copula is a multivariate distribution whose marginals are all uniform on the interval $(0, 1)$. For a s-dimensional random vector $\boldsymbol{U} = \left(U_1, \dots, U_s\right)'$ on the unit cube, a copula C is given by

$$C(u_1, \dots, u_s) = P(U_1 \le u_1, \dots, U_s \le u_s).$$

Let F be a s-dimensional cdf with marginals $F_1, \dots, F_s$. From Sklar[33], there exists a s-dimensional copula C such that, for all $\boldsymbol{x} = \left(x_1, \dots, x_s\right)'$ in the domain of F, we have

$$F(x_1, \dots, x_s) = C\left(F_1(x_1), \dots, F_s(x_s)\right).$$

In this work, we make use of elliptical copulas, which allow a flexible unstructured correlation matrix. For a detailed discussion on elliptical distributions, see, e.g., Fang *et al.*[34]. Let F be the multivariate cdf of an elliptical distribution. Also, let F_w be the cdf of the w-th marginal and F_w^{-1} be its inverse or quantile function, for $w = 1, \dots, s$. Then, the elliptical copula determined by F is

$$C(u_1, \dots, u_s) = F\left(F_1^{-1}(u_1), \dots, F_s^{-1}(u_s)\right). \tag{12}$$

By differentiating (12), we find that the density of an elliptical copula is given by

$$c(u_1, \dots, u_s) = \frac{f\left(F_1^{-1}(u_1), \dots, F_s^{-1}(u_s)\right)}{\prod_{w=1}^{s} f_w\left(F_w^{-1}(u_w)\right)},$$

where f is the joint pdf of the elliptical distribution, and $f_1, \dots, f_s$ are the marginal density functions.

We use the R package *copula*[35]. The actual elliptical copula classes implemented in this package are the Gaussian and Student's t copulas, which are specified by the multivariate normal and Student's t distributions, respectively. Both these copulas have a dispersion matrix, inherited from the elliptical distribution, and Student's t copula (or, t copula) has one additional parameter, which is the degrees of freedom parameter. As copulas are invariant to monotone transformations of the marginals, the correlation matrix determines the dependence structure. Commonly used correlation matrix structures include the exchangeable, autoregressive of order 1, Toeplitz, and unstructured[35]. This method then calculates the multivariate cumulative probabilities, and subsequently gives approximate misclassification probabilities from (8) and (9) for fixed t_b and s.

3.3 | Optimal burn-in time and cutoff points

A concern in burn-in policy is to seek a DR that maximizes the economic benefits[2]. Based on this principle, we develop an economic cost model, consisting of the following components:

- C_α: the cost of Type I error, which is the per-unit cost of misclassifying a normal unit as weak;
- C_β: the cost of Type II error, which is the per-unit cost of misclassifying a weak unit as normal;
- C_{op}: the cost of operating the burn-in procedure (from 0 up to t_b) for each unit;
- C_{mea}: the cost of collecting data for each unit;
- n: the total number of units subject to the burn-in test;
- p: the proportion of weak units.

The cost parameters are inherent quantities to build a degradation test. In this paper, these costs are caught from the paper of Tsai *et al.*[17]. The misclassification cost is the average of C_α and C_β, weighted by their respective probabilities. Thus, for each t_b and s, the misclassification cost is a function of the cutoff points $\xi_1, \dots, \xi_s$ in the form:

$$MC\left(\xi_1, \dots, \xi_s | t_b, s\right) = C_\alpha n(1-p)\alpha(\xi_1, \dots, \xi_s | t_b, s) + C_\beta np\beta(\xi_1, \dots, \xi_s | t_b, s), \tag{13}$$

where $\alpha(\xi_1, \dots, \xi_s | t_b, s)$ and $\beta(\xi_1, \dots, \xi_s | t_b, s)$ are as given in (8) and (9), respectively.

Hence, the total misclassification cost is the sum of misclassification cost in (13) and additional costs for a whole sample and is of the form:

$$TC\left(\xi_1, \dots, \xi_s | t_b, s\right) = MC\left(\xi_1, \dots, \xi_s | t_b, s\right) + C_{\text{op}} n t_b + C_{\text{mea}} n(b+1). \tag{14}$$

The optimal cutoff points are the ones that result in minimal misclassification cost in (13) for t_b and s, i.e.,

$$\hat{\xi}_1, \dots, \hat{\xi}_s = \arg\min_{\xi_1, \dots, \xi_s} MC\left(\xi_1, \dots, \xi_s | t_b, s\right).$$

When $s = 1$, the optimal cutoff point is computed analytically using the result given below[22].

Theorem 2. For fixed t_b, the optimal cutoff point under model (1) is given by

$$\hat{\xi}(t_b) = \frac{\left[g_{\theta_1}(t_b) - g_{\theta_2}(t_b)\right]\left[g_{\theta_1}(t_b) + g_{\theta_2}(t_b)\right]\eta}{2\left[g_{\theta_1}(t_b) - g_{\theta_2}(t_b)\right]\eta - 2\log\left(\frac{C_\alpha(1-p)g_{\theta_2}(t_b)}{C_\beta p g_{\theta_1}(t_b)}\right)}. \tag{15}$$

Proof. For a single cutoff point ($s = 1$), the misclassification cost in (13) becomes

$$MC\left(\xi\left(t_b\right)\right) = C_\alpha n(1-p)\alpha\left(t_b\right) + C_\beta np\beta\left(t_b\right), \tag{16}$$

where $\alpha\left(t_b\right)$ and $\beta\left(t_b\right)$ are as given in (10) and (11), respectively.

Taking the first derivative of (16) with respect to $\xi(t_b)$, we get

$$\frac{\partial MC\left(\xi(t_b)\right)}{\partial \xi(t_b)} = \frac{n\left(C_\beta e^{-\frac{\left(\xi(t_b) - g_{\theta_1}(t_b)\right)^2\eta}{2\xi(t_b)}} g_{\theta_1}(t_b) p\sqrt{\frac{\eta}{\xi(t_b)}}\right)}{\xi(t_b)\sqrt{2\pi}} - \frac{n\left(C_\alpha e^{-\frac{\left(\xi(t_b) - g_{\theta_2}(t_b)\right)^2\eta}{2\xi(t_b)}} g_{\theta_2}(t_b)(1-p)\sqrt{\frac{\eta}{\xi(t_b)}}\right)}{\xi(t_b)\sqrt{2\pi}}. \tag{17}$$

Equating (17) to zero, i.e.,

$$\frac{\partial MC\left(\xi(t_b)\right)}{\partial \xi(t_b)} = 0,$$

we obtain

$$\frac{\left[-\left(\xi(t_b) - g_{\theta_1}(t_b)\right)^2 + \left(\xi(t_b) - g_{\theta_2}(t_b)\right)^2\right]\eta}{2\xi(t_b)} = \log\left(\frac{C_\alpha(1-p)g_{\theta_2}(t_b)}{C_\beta p g_{\theta_1}(t_b)}\right),$$

which leads to a linear equation with respect to $\xi(t_b)$, whose root, after some simple algebraic manipulations, can be expressed as in (15). □

However, when $s > 1$, the optimal cutoff points must be obtained numerically by iterative methods. Here, we have used the Quasi-Newton optimization method through Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm[36] in R software[37], for this numerical determination process. The optimal t_b and s are values that result in minimal total misclassification cost in (14), i.e.,

$$\hat{t}_b, \hat{s} = \arg\min_{t_b, s} TC\left(\xi_1, \ldots, \xi_s | t_b, s\right).$$

In practical applications, the functions $g_{\theta_1}(t)$ and $g_{\theta_2}(t)$, the volatility parameter η and the proportion p are all unknown quantities. For this reason, one may get the MLEs of the model parameters as described earlier in Section 2.2, and then make use of these values in (8), (9), (13) and (14).

4 | APPLICATION

In this section, we present a numerical example based on LASER data to illustrate the applicability of the proposed methodology.

4.1 | The LASER data revisited

Some devices for light amplification by stimulated emission of radiation (LASER) exhibit degradation of operating current over time, which leads to a decrease in the irradiated light. When the operating current reaches a pre-fixed threshold level, the device is considered to have failed. Meeker *et al.*[23] presented a sample of 15 gallium arsenide LASERs, whose degradation paths are shown in Figure 1, wherein a unit is considered to have failed when its degradation measure attains 10%. We note that the degradation paths have a linear pattern and can be divided into two groups: the weak group and the normal one. Then, the mixture IG degradation process model in (1) for these data can be represented by the mean functions $g_{\theta_1}(t) = \theta_1 t$ and $g_{\theta_2}(t) = \theta_2 t$, for the weak and normal groups, respectively, with $\theta_1 > \theta_2 > 0$. The MLEs for the parameters of model (1) based on the LASER data, with a burn-in time of 4,000 hours, are (standard error in parentheses): $\hat{\theta}_1 = 2.6902$ (0.1264), $\hat{\theta}_2 = 1.8004$ (0.0463), $\hat{\eta} = 18.2340$ (1.8417) and $\hat{p} = 0.2661$ (0.1143). From these results, we observe that the weak group presents higher estimated angular coefficient than the normal group (i.e., $\hat{\theta}_1 > \hat{\theta}_2$), and the estimated proportion of weak items in the sample turns out to be 26.61%.

4.2 | Generated data set

In practical situations, we have a large batch of items to be inspected. In order to apply the proposed methodology, a data set of size $n = 200$, with the same features of the LASER data, was generated using the MLEs presented in Section 4.1. The aim is to classify the units as normal or weak based on a burn-in test of $4,000$ hours of operation. The generated data set is available as supplementary material and Figure 2 displays the simulated degradation paths, indicating the critical value associated with failure.

In this artificial data set, we see that there are 58 (29%) weak units and 142 (71%) normal ones. Due to the randomness in the data generation process, the proportion of weak units in the synthetic data set is not the same as the estimated proportion of weak units in the original LASER data set. Therefore, it is usual that when the burn-in time approaches the total time of the simulated sample, the parameter estimates are close to the sample estimate, and do not necessarily become close to the parameter values used to generate it. Notably, this generated data set has an interesting aspect precisely to verify the classification efficiency of the proposed burn-in policy addressed in this work.

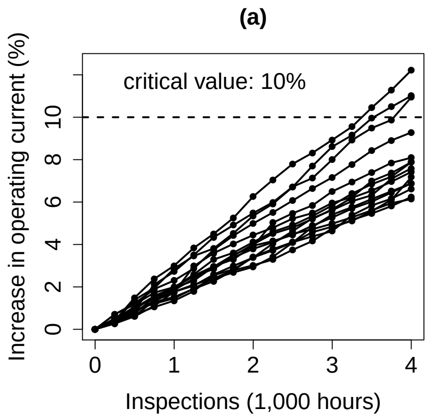

FIGURE 1 Degradation paths from the LASER data.

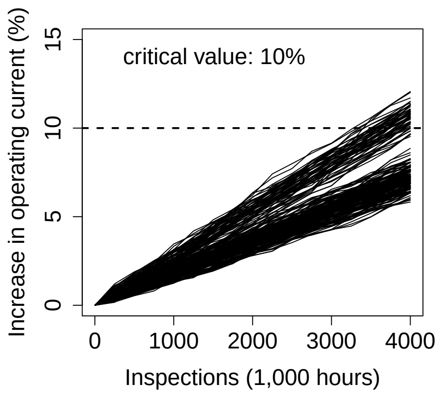

FIGURE 2 Simulated degradation paths of 200 LASER units.

From Figure 2, we see that the degradation paths show a linear pattern and are quite similar to the original ones in the LASER data.

Table 1 displays the MLEs for different t_b values (starting from 500 hours), and the true parameter values are shown in parentheses. Notice that, in general, the MLEs get closer to the corresponding true values as the burn-in time increases.

In order to develop the burn-in procedure described in Section 3.3, we set the cost parameters: $C_\alpha = 65, C_\beta = 90, C_{\text{op}} = 0.0009$ and $C_{\text{mea}} = 0.0005$. These cost parameters are caught from the paper of Tsai *et al.*[17] in an illustrative example with LASER components. The results are split into two situations: $1 \leq s \leq 2$ and $s > 2$.

- When $1 \leq s \leq 2$:

  Table 2 displays the estimated total costs and the misclassification probabilities, under different values of t_b (starting from 500 hours) and s (from 1 up to 2). The optimal burn-in time and number of cutoff points are $2,750$ hours and $s = 2$, respectively, resulting in the minimal total cost of 599.4861. The optimal cutoff points are 6.0180 and 6.5004, that is, the optimal burn-in policy consists of observing the items up to $2,750$ hours of operation, and all the units with $D(2,500) > 6.0180$ or $D(2,750) > 6.5004$ need to be rejected and not delivered to the market.

  For the sake of analysis, one can refer to the data set in the supplementary material. For instance, the weak items 5 and 61 are rejected by the optimal burn-in policy, since for unit 5, $D(2,500) = 6.62 > 6.0180$, and for unit 61, $D(2,500) = 6.96 >$

6.0180. Nevertheless, the weak items 1 and 100 are not rejected by this policy, since for unit 1, $D(2,500) = 5.69 < 6.0180$ and $D(2,750) = 6.33 < 6.5004$, and for unit 100, $D(2,500) = 5.79 < 6.0180$ and $D(2,750) = 6.39 < 6.5004$.

Under the optimal burn-in policy, three weak items are not rejected and delivered to the market, resulting in the observed probability of Type II error of 0.0517, which is higher than the estimated one (0.0081). Concerning the normal items, all of them are accepted and delivered to the market.

- When $s > 2$:

  The Gaussian and t copulas (the latter one with degrees of freedom parameter which varies from 1 to 5) were applied to compute the misclassification probabilities for more than two cutoff points (i.e., $s > 2$).

  The criterion used to select the best copula consists of finding out the copula whose estimated misclassification probabilities for $s = 2$ were the closest to the corresponding values obtained via the analytical method. Thus, we chose the t copula with 1 degree of freedom (or t_1 copula).

  Table 3 displays the estimated total costs considering the copula theory (i.e., using the t_1 copula), from which we see that the optimal burn-in time and number of cutoff points are 3,000 hours and $s = 12$, respectively, resulting in the minimal total cost of 686.6687.

  Tables 4 and 5 display the estimated probabilities of Type I and Type II errors, respectively, from which we see that the misclassification probabilities tend to decrease as the burn-in time increases. It is worth mentioning that there is an optimal combination and the probabilities of misclassification are not related to the number of cutoff points in a monotonic way.

  The optimal cutoff points associated with the minimal total cost are presented in Table 6. Hence, the optimal burn-in policy consists of observing the items up to 3,000 hours, and all the units need to satisfy $D(250) \leq \hat{\xi}_1, \ldots, D(3,000) \leq \hat{\xi}_{12}$ in order to be declared to be normal and then delivered to the market.

From the exploratory analysis of the data set available in the supplementary material, we have that the weak units 51 and 92 are rejected by the optimal burn-in policy, since for unit 51, $D(2,250) = 5.80 > \hat{\xi}_9 = 5.6336$, and for unit 92, $D(2,000) = 5.56 > \hat{\xi}_8 = 5.3350$. Nevertheless, the weak unit 1 is not rejected by the optimal burn-in policy.

Under this optimal burn-in policy, only one weak item is delivered to the market, resulting in the observed probability of Type II error of 0.0172, which is close to the estimated one (0.0127). Concerning the normal items, all of them are delivered to the market. We also observe that the probability of Type II error under this policy is lower than in the policy with two cutoff points.

Finally, we can estimate the items' lifetime distribution based on the MLEs from Table 1 and assuming a fixed threshold level $\rho = 10\%$. Table 7 shows the lifetime information for the weak and normal groups, under the burn-in times 2,750 hours (obtained by the analytical method) and 3,000 hours (obtained through the approximate method). From this table, we see that the estimated lifetime quantiles and MTTF in the weak group are distant from the ones in the normal group, and almost all the weak units are supposed to have failed by 4,175 hours. The results are analogous for different burn-in times. Figure 3 shows the plots of the estimated cdf and pdf for the burn-in time 3,000 hours, from which we see that the pdf and cdf curves are different from each other.

As mentioned earlier in Section 1, the optimization of the burn-in policy has been done in all the preceding work by considering one cutoff point in deciding between strong and weak products. Here, however, we have developed the theory for the use of multiple cutoff points and have also described the corresponding implementation part in the preceding sections. The use of multiple cutoff points offers many benefits as compared to the procedure based on a single cutoff point. For example, it can result in reducing the total cost of the experiment besides having a small Type II error (with Type I error being fixed). As an illustration, let us consider the case when $s = 1$ or 2, in which the optimal burn-in time $t_b = 2.75$, shown in Table 2. In this situation, we see that the total cost is 723.4917 for a plan with one cutoff point, but only 599.4861 for a plan with two cutoff points. Moreover, the Type I and Type II error rates for the plan with one cutoff point are 0.0131 and 0.0203, respectively, while the plan with two cutoff points has 0.0066 and 0.0081 correspondingly. Thus, the plan with the use of two cutoff points results in benefits concerning the budget as well as the error rates. Furthermore, we observe from the last two columns of Table 2 that while the cutoff point is 6.1910 for the plan with one cutoff point, the cutoff points are 6.0180 and 6.5004 for the plan with two cutoff points.

Similarly, for the case with more than two cutoff points ($s > 2$), we have the results in Table 3. In this case, $t_b = 3.00$ and the total cost varies from 686.6687 (when $s = 12$) up to 774.3641 (when $s = 11$). Moreover, the Type I and Type II error rates for the plan with less than 12 cutoff points are higher than the ones with 12 cutoff points. In practice, the burn-in policy with 12 cutoff

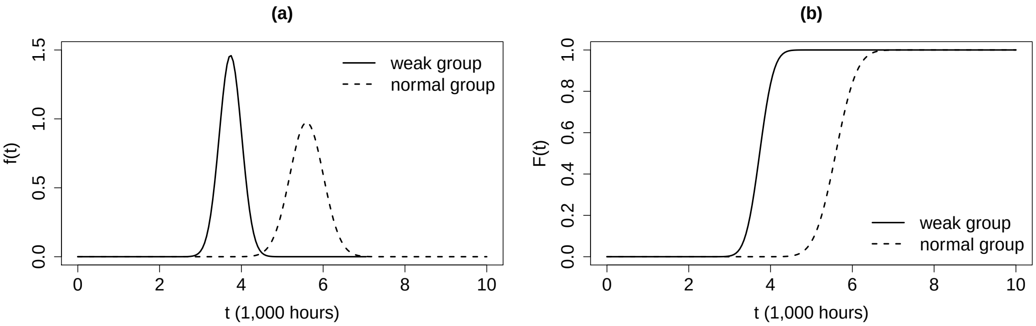

FIGURE 3 Lifetime distribution for the weak and normal groups, considering the simulated data: (a) Lifetime pdf, (b) Lifetime cdf.

points means observing the units from 0 up to 3 thousand hours and checking that their corresponding degradation values should not exceed these limits: 1.2083, 1.7824, 2.7676, 3.3106, 3.8165, 4.9439, 5.1440, 5.3350, 5.6336, 6.1722, 6.5208 and 6.8040 to be declared as normal and sent to the market.

What this highlights is that the plan with various cutoff points creates early cutoffs to wean off some weak products early and then the later cutoffs to wean off other weak products. This results in yet another benefit as it enables the removal of weak products early on due to the smaller first cutoff point values.

In Tables 2 and 3, such a benefit can be seen throughout, except when t_b (the burn-in time) itself is quite short, in which case the plan with a set of cutoff points shows no advantages as compared to the plan with one cutoff point (see, for example, the results for $t_b = 1.0$ in Table 2).

5 | CONCLUDING REMARKS AND FUTURE RESEARCH

In this paper, inspired by a real data set on LASERs, we have proposed a mixture IG process model to analyze the degradation of very reliable products. Although it is rather challenging to find the optimal burn-in time within a short period of life testing, such a problem can be adequately solved if there exists a QC whose degradation over time can be associated with the product's reliability. First, we presented a burn-in procedure whose main goal was to determine the optimal burn-in policy to screen out the weak units from the normal ones in a production row. We have considered a set of cutoff points and two methods for the calculation of misclassification probabilities; more specifically, an analytical method for the situation when we have at most 2 cutoff points and an approximate method based on copulas for the situation when we have more than 2 cutoff points. Moreover, we built up an economic cost model, in which the optimal burn-in policy was directly related to the minimum cost. Finally, we illustrated the proposed methodology by using a simulated LASER data with a size of 200 and the same features as the original one from the reliability literature. The Quasi-Newton optimization method via the BFGS algorithm available in R software was used to achieve the MLEs of the model parameters, as well as to obtain the optimal cutoff points when $s > 1$. For $s > 2$, we resorted to the R package *copula* along with the BFGS algorithm to determine the optimal cutoff points. Such methods showed convergence in the situations mentioned before. The optimal burn-in time and cutoff points were found for both analytical ($s \leq 2$) and approximate ($s > 2$) cases. The approximate method based on copulas provided better results for the simulated data set.

It is worth noting that the main objective of this work was not to point to the best model but to propose a new and more flexible methodology to obtain a decision rule for classifying a unit under test as normal or weak based on burn-in time and a set of cutoff points. However, future research may include carrying out a misspecification study to compare different degradation-based burn-in models. We also leave for further investigation, a comparison through simulations, the theoretical misclassification probabilities obtained from elliptical copulas with the empirical ones, in addition to applying goodness-of-fit criteria to compare different copula models[38]. Finally, as pointed out by an anonymous reviewer, the influence of the interval length of two adjacent cutoff points on the optimal results of the burn-in policy (with the possible existence of an optimal interval length) is another relevant issue that deserves further attention. We plan to consider all these problems for our future study.

FINANCIAL DISCLOSURE

Pedro L. Ramos acknowledges support from the São Paulo State Research Foundation (FAPESP Proc. 2017/25971-0). Francisco Louzada is supported by the Brazilian agencies CNPq (grant number 301976/2017-1) and FAPESP (grant number 2013/07375-0).

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TABLES

TABLE 1 MLEs of the parameters of model (1) according to different burn-in times t_b, based on the simulated data.

t_b	θ_1 (2.6902)	θ_2 (1.8004)	η (18.2340)	p (0.2661)
0.50	2.7317	1.7946	18.4390	0.2795
0.75	2.6346	1.7769	18.4340	0.3045
1.00	2.6991	1.7947	18.8340	0.2912
1.25	2.7041	1.7877	18.3590	0.2704
1.50	2.7106	1.7818	18.9600	0.2668
1.75	2.7057	1.7811	18.7440	0.2731
2.00	2.6893	1.7885	18.4040	0.2835
2.25	2.6787	1.7850	18.3140	0.2954
2.50	2.6826	1.7935	18.6010	0.2877
2.75	2.6909	1.7950	18.4630	0.2893
3.00	2.6858	1.7923	18.5290	0.2889
3.25	2.6777	1.7899	18.6690	0.2896
3.50	2.6770	1.7882	18.7420	0.2901
3.75	2.6753	1.7920	18.6990	0.2899
4.00	2.6779	1.7924	18.6780	0.2900

TABLE 2 Estimated total costs and probabilities of Type I and Type II errors, for different values of t_b and s.

	$TC(\hat{\xi}_1, \ldots, \hat{\xi}_s \| t_b, s)$		$\alpha(\hat{\xi}_1, \ldots, \hat{\xi}_s \| t_b, s)$		$\beta(\hat{\xi}_1, \ldots, \hat{\xi}_s \| t_b, s)$		$\hat{\xi}_1, \ldots, \hat{\xi}_s$	
t_b	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$	$s = 1$	$s = 2$
0.50	2388.8818	2389.9284	0.1208	0.1203	0.2320	0.2332	1.1586	(1.1607; 1.1608)
0.75	2085.3670	2139.5356	0.1104	0.1109	0.1736	0.1827	1.6694	(1.6731; 1.6732)
1.00	1481.0536	1552.3542	0.0731	0.0818	0.1197	0.1179	2.2677	(2.2519; 2.2520)
1.25	1183.8672	1205.4909	0.0521	0.0683	0.0954	0.0683	2.8416	(2.4899; 2.8627)
1.50	929.1636	864.4140	0.0360	0.0414	0.0657	0.0415	3.4036	(3.0558; 3.4728)
1.75	835.7277	737.7367	0.0289	0.0297	0.0501	0.0287	3.9567	(3.6281; 4.0638)
2.00	823.5396	710.2523	0.0263	0.0250	0.0427	0.0228	4.5041	(4.1950; 4.6410)
2.25	784.4533	664.5760	0.0220	0.0188	0.0332	0.0162	5.0405	(4.7578; 5.2121)
2.50	744.2518	632.0652	0.0168	0.0130	0.0265	0.0118	5.6194	(5.3566; 5.8207)
2.75	723.4917	599.4861	0.0131	0.0066	0.0203	0.0081	6.1910	(6.0180; 6.5004)
3.00	720.1626	616.3793	0.0104	0.0048	0.0160	0.0059	6.7403	(6.5888; 7.0725)
3.25	728.8519	641.7614	0.0083	0.0036	0.0127	0.0043	7.2827	(7.1517; 7.6362)
3.50	741.7310	670.8933	0.0064	0.0026	0.0098	0.0030	7.8364	(7.7268; 8.2125)
3.75	768.5201	707.4199	0.0053	0.0020	0.0082	0.0024	8.3992	(8.3094; 8.7969)
4.00	793.8918	744.0632	0.0042	0.0015	0.0064	0.0017	8.9635	(8.8952; 9.3840)

6 | SUPPLEMENTAL MATERIAL

How to cite this article: L.H.M. Morita, V.L. Tomazella, P.H. Ferreira, P.L. Ramos, N. Balakrishnan, and F. Louzada (2020), Optimal burn-in policy based on a set of cutoff points using mixture inverse Gaussian degradation process and copulas, *00:1–15*.

TABLE 3 Estimated total costs for different values of t_b and s, under t_1 copula.

t_b	$s = 3$	$s = 4$	$s = 5$	$s = 6$	$s = 7$	$s = 8$	$s = 9$
0.75	1991.6201	–	–	–	–	–	–
1.00	1416.1506	1390.1332	–	–	–	–	–
1.25	1179.9117	1126.1684	1105.0808	–	–	–	–
1.50	906.6678	882.4789	871.5606	894.8505	–	–	–
1.75	859.5467	801.9121	841.4275	794.1043	806.0926	–	–
2.00	925.6880	808.7921	832.7379	777.5438	907.7153	816.8192	–
2.25	790.1508	880.6685	861.1221	755.9416	753.5009	766.0018	755.8983
2.50	793.5968	831.0500	726.8199	726.2244	721.3627	785.5936	714.4065
2.75	774.8139	718.6646	775.0801	706.3131	719.2641	741.3324	694.3394
3.00	759.3138	717.5317	711.4677	722.9218	707.5732	704.6590	703.5339
3.25	731.6504	795.1480	792.6829	724.6223	738.7455	719.2114	726.5759
3.50	765.5756	780.0877	808.4916	739.9365	835.5413	786.9763	759.8619
3.75	774.9722	800.7395	821.6631	834.2503	882.3471	855.0226	808.4283
4.00	789.5514	811.7325	790.2724	843.6182	849.9019	870.4556	788.7533

t_b	$s = 10$	$s = 11$	$s = 12$	$s = 13$	$s = 14$	$s = 15$	$s = 16$
2.50	716.8341	–	–	–	–	–	–
2.75	712.3644	700.3879	–	–	–	–	–
3.00	705.5082	774.3641	686.6687	–	–	–	–
3.25	707.9439	740.3831	720.9134	727.9581	–	–	–
3.50	731.3081	732.3586	782.0207	732.2095	746.2420	–	–
3.75	838.4097	828.8112	796.8803	755.4353	755.4589	752.7242	–
4.00	787.3475	878.9960	803.7143	830.5440	795.4435	785.3977	777.7445

TABLE 4 Estimated probabilities of Type I error for different values of t_b and s, under t_1 copula.

t_b	$s = 3$	$s = 4$	$s = 5$	$s = 6$	$s = 7$	$s = 8$	$s = 9$
0.75	0.1152	–	–	–	–	–	–
1.00	0.0704	0.0688	–	–	–	–	–
1.25	0.0458	0.0508	0.0513	–	–	–	–
1.50	0.0316	0.0328	0.0302	0.0358	–	–	–
1.75	0.0281	0.0273	0.0296	0.0244	0.0263	–	–
2.00	0.0318	0.0231	0.0271	0.0231	0.0302	0.0370	–
2.25	0.0202	0.0268	0.0211	0.0193	0.0206	0.0242	0.0192
2.50	0.0179	0.0224	0.0153	0.0169	0.0169	0.0172	0.0169
2.75	0.0131	0.0111	0.0154	0.0126	0.0122	0.0143	0.0104
3.00	0.0123	0.0114	0.0101	0.0101	0.0099	0.0089	0.0094
3.25	0.0083	0.0118	0.0122	0.0082	0.0088	0.0082	0.0076
3.50	0.0079	0.0105	0.0080	0.0051	0.0084	0.0086	0.0068
3.75	0.0056	0.0065	0.0061	0.0095	0.0076	0.0059	0.0061
4.00	0.0041	0.0059	0.0048	0.0064	0.0072	0.0050	0.0043

t_b	$s = 10$	$s = 11$	$s = 12$	$s = 13$	$s = 14$	$s = 15$	$s = 16$
2.50	0.0156	–	–	–	–	–	–
2.75	0.0128	0.0130	–	–	–	–	–
3.00	0.0102	0.0104	0.0087	–	–	–	–
3.25	0.0070	0.0082	0.0074	0.0047	–	–	–
3.50	0.0063	0.0058	0.0096	0.0059	0.0077	–	–
3.75	0.0102	0.0061	0.0062	0.0045	0.0052	0.0048	–
4.00	0.0046	0.0051	0.0057	0.0035	0.0044	0.0039	0.0034

TABLE 5 Estimated probabilities of Type II error for different values of t_b and s, under t_1 copula.

t_b	$s = 3$	$s = 4$	$s = 5$	$s = 6$	$s = 7$	$s = 8$	$s = 9$
0.75	0.1494	–	–	–	–	–	–
1.00	0.1127	0.1108	–	–	–	–	–
1.25	0.1088	0.0868	0.0818	–	–	–	–
1.50	0.0714	0.0633	0.0660	0.0594	–	–	–
1.75	0.0584	0.0471	0.0513	0.0511	0.0497	–	–
2.00	0.0539	0.0464	0.0448	0.0395	0.0538	0.0228	–
2.25	0.0371	0.0448	0.4999	0.0327	0.0301	0.0267	0.0330
2.50	0.0347	0.0352	0.0269	0.0233	0.0222	0.0340	0.0212
2.75	0.0307	0.0243	0.0271	0.0183	0.0215	0.0222	0.0196
3.00	0.0211	0.0154	0.0154	0.0178	0.0154	0.0157	0.0146
3.25	0.0154	0.0203	0.0194	0.0132	0.0145	0.0112	0.0138
3.50	0.0135	0.0122	0.0200	0.0122	0.0249	0.0158	0.0143
3.75	0.0108	0.0126	0.0185	0.0155	0.0263	0.0250	0.0143
4.00	0.0060	0.0085	0.0616	0.0138	0.0124	0.0194	0.0055

t_b	$s = 10$	$s = 11$	$s = 12$	$s = 13$	$s = 14$	$s = 15$	$s = 16$
2.50	0.0240	–	–	–	–	–	–
2.75	0.0192	0.0161	–	–	–	–	–
3.00	0.0139	0.0275	0.0127	–	–	–	–
3.25	0.0111	0.0150	0.0129	0.0192	–	–	–
3.50	0.0080	0.0096	0.0124	0.0090	0.0090	–	–
3.75	0.0141	0.0191	0.0130	0.0073	0.0061	0.0063	–
4.00	0.0050	0.0220	0.0069	0.0150	0.0063	0.0057	0.0048

TABLE 6 Optimal cutoff points for $t_b = 3,000$ hours and $s = 12$, under t_1 copula.

$\hat{\xi}_1$	$\hat{\xi}_2$	$\hat{\xi}_3$	$\hat{\xi}_4$	$\hat{\xi}_5$	$\hat{\xi}_6$
1.2083	1.7824	2.7676	3.3106	3.8165	4.9439

$\hat{\xi}_7$	$\hat{\xi}_8$	$\hat{\xi}_9$	$\hat{\xi}_{10}$	$\hat{\xi}_{11}$	$\hat{\xi}_{12}$
5.1440	5.3350	5.6336	6.1722	6.5208	6.8040

TABLE 7 MLEs and 95% CIs of the lifetime quantiles and MTTF, under different t_b values and considering the simulated data.

		Weak group		Normal group	
t_b	Quantity	MLE	95% CI	MLE	95% CI
2.75	$t_{0.05}$	3.2774	[3.2638; 3.2910]	4.9133	[4.9092; 4.9174]
	$t_{0.5}$	3.7263	[3.6679; 3.7847]	5.5862	[5.5711; 5.6014]
	$t_{0.8}$	3.9560	[3.9127; 3.9994]	5.9306	[5.9199; 5.9414]
	$t_{0.95}$	4.1753	[4.1585; 4.1922]	6.2594	[6.2553; 6.2635]
	MTTF	3.7263	[3.5774; 3.8753]	5.5863	[5.4995; 5.6731]
3.00	$t_{0.05}$	3.2844	[3.2715; 3.2973]	4.9218	[4.9179; 4.9257]
	$t_{0.5}$	3.7333	[3.6779; 3.7888]	5.5945	[5.5801; 5.6089]
	$t_{0.8}$	3.9631	[3.9220; 4.0042]	5.9388	[5.9286; 5.9491]
	$t_{0.95}$	4.1824	[4.1665; 4.1984]	6.2675	[6.2636; 6.2714]
	MTTF	3.7334	[3.5917; 3.8750]	5.5946	[5.5119; 5.6773]

TABLE 8 LASER generated data.

	inspection times (in 1,000 hours)																	
unit	0	0.25	0.50	0.75	1	1.25	1.50	1.75	2	2.25	2.50	2.75	3	3.25	3.50	3.75	4	group
1	0	0.64	1.32	1.86	2.24	2.68	3.28	3.92	4.50	5.06	5.69	6.33	6.75	7.69	8.34	8.86	9.68	weak
2	0	0.36	1.00	1.34	1.68	2.27	2.92	3.13	3.66	4.00	4.75	5.00	5.22	5.78	6.05	6.32	6.72	normal
3	0	0.46	1.36	1.88	2.45	2.80	3.11	3.67	4.03	4.44	4.90	5.71	5.98	6.37	6.76	7.33	7.68	normal
4	0	0.58	1.11	1.49	1.81	2.31	2.82	3.16	3.72	4.07	4.52	4.75	5.06	5.39	5.80	6.31	6.72	normal
5	0	1.13	1.59	2.10	2.62	3.30	3.93	4.64	5.23	5.95	6.62	7.46	7.92	8.30	8.96	9.77	10.57	weak
6	0	0.54	0.78	1.19	1.93	2.43	2.94	3.62	4.23	4.87	5.53	6.08	6.35	6.93	7.15	7.64	8.23	normal
7	0	0.32	0.85	1.09	1.38	1.92	2.32	2.83	3.13	3.94	4.51	5.30	5.68	6.17	6.91	7.26	7.61	normal
8	0	0.35	0.74	1.16	1.51	1.86	2.08	2.54	2.90	3.43	3.71	3.96	4.27	4.47	4.98	5.59	5.94	normal
9	0	0.25	0.74	1.20	1.68	2.06	2.45	3.13	3.73	4.28	4.52	4.75	5.27	5.48	5.97	6.31	6.93	normal
10	0	0.67	1.20	1.91	2.55	2.89	3.24	3.83	4.30	4.91	5.10	5.40	5.72	6.33	6.91	7.22	7.90	normal
11	0	0.75	1.29	1.57	1.89	2.14	2.73	3.00	3.74	4.23	4.86	5.23	5.81	6.22	6.76	7.36	7.67	normal
12	0	0.51	1.22	1.46	1.90	2.29	2.57	2.93	3.43	3.98	4.41	4.98	5.40	5.89	6.29	7.07	7.63	normal
13	0	0.31	0.67	1.10	1.53	1.81	2.55	2.99	3.47	3.93	4.51	4.84	5.32	5.64	5.82	6.32	6.68	normal
14	0	0.62	1.26	1.94	2.59	3.29	3.83	4.49	4.90	5.47	6.24	7.11	7.60	8.09	8.70	9.45	10.31	weak
15	0	0.50	1.57	1.99	2.52	3.25	3.57	3.91	4.14	4.93	5.40	5.97	6.25	6.93	7.37	8.19	8.85	normal
16	0	0.93	1.65	2.28	3.09	3.69	4.46	4.93	5.46	6.01	6.52	7.00	7.53	8.51	9.05	9.72	10.33	weak
17	0	0.28	0.55	1.12	1.49	2.11	2.44	2.83	3.36	3.89	4.42	5.11	5.50	5.81	6.45	7.06	7.52	normal
18	0	0.56	0.95	1.54	2.02	2.30	2.75	3.00	3.49	3.95	4.47	4.93	5.35	5.62	6.00	6.58	7.28	normal
19	0	0.72	1.18	1.76	2.40	2.67	3.12	3.36	3.79	4.27	4.53	5.07	5.51	5.99	6.37	6.70	7.24	normal
20	0	0.18	0.54	0.81	1.44	2.01	2.36	2.85	3.11	3.55	4.40	5.17	5.49	5.89	6.51	7.11	7.52	normal
21	0	0.67	1.28	1.71	2.07	2.49	2.91	3.26	3.55	4.02	4.43	4.86	5.52	5.87	6.24	6.55	7.02	normal
22	0	0.58	0.99	1.89	2.25	3.13	3.77	4.18	5.02	5.98	6.65	7.20	8.01	8.54	8.89	9.41	10.34	weak
23	0	0.63	1.51	2.13	2.80	3.81	4.82	5.41	6.22	7.41	7.99	8.58	9.12	9.80	10.40	10.91	11.42	weak
24	0	0.86	1.64	2.51	3.00	3.85	4.44	4.92	5.66	6.42	7.25	8.10	8.67	9.16	9.86	10.42	10.97	weak
25	0	0.44	1.09	1.54	2.10	2.46	2.74	3.00	3.50	3.97	4.41	4.78	5.51	6.04	6.60	7.14	7.66	normal
26	0	0.46	1.02	1.60	2.39	2.83	3.32	4.10	4.92	5.82	6.24	7.41	8.21	8.57	9.45	10.08	10.54	weak
27	0	0.41	0.63	1.01	1.38	1.62	1.93	2.36	2.94	3.20	3.69	4.05	4.45	4.86	5.48	5.73	6.19	normal
28	0	0.29	0.68	1.02	1.35	1.71	2.20	2.60	2.87	3.31	3.73	4.15	4.83	5.45	5.78	6.13	6.43	normal
29	0	0.63	1.22	1.81	2.43	3.08	3.78	4.73	5.43	6.33	7.22	7.97	8.59	9.04	9.56	10.26	10.81	weak
30	0	0.32	0.91	1.30	1.67	2.12	2.77	3.14	3.50	3.97	4.63	4.94	5.45	6.05	6.36	6.91	7.16	normal
31	0	0.68	0.92	1.44	1.72	1.95	2.26	2.87	3.26	3.85	4.39	4.70	5.03	5.30	5.73	6.14	6.74	normal
32	0	0.31	0.78	1.24	1.79	2.15	2.49	2.79	3.22	3.84	4.19	4.66	5.05	5.32	5.65	5.98	6.37	normal
33	0	0.35	0.84	1.23	1.71	1.98	2.84	3.31	3.79	4.25	4.69	5.31	5.78	6.10	6.48	6.89	7.32	normal
34	0	0.45	1.11	1.39	1.96	2.41	2.83	3.38	3.94	4.25	4.68	5.18	5.94	6.28	6.87	7.69	8.27	normal
35	0	0.28	0.68	1.07	1.64	1.92	2.28	2.65	3.08	3.66	4.04	4.56	5.19	5.58	5.88	6.24	6.79	normal
36	0	1.00	1.75	2.37	3.05	3.74	4.35	5.06	5.66	6.12	6.89	7.68	8.23	9.18	10.52	11.27	11.70	weak
37	0	0.31	0.58	1.05	1.42	1.89	2.16	2.57	2.80	3.06	3.69	4.19	4.48	5.25	5.58	5.84	6.42	normal
38	0	0.51	0.90	1.43	1.92	2.24	2.80	3.33	3.57	3.77	4.50	4.88	5.15	5.82	6.20	6.53	6.99	normal
39	0	0.22	0.71	0.97	1.34	1.71	1.97	2.91	3.57	4.33	4.84	5.20	5.47	5.83	6.25	6.78	7.05	normal
40	0	0.27	0.57	0.93	1.35	1.55	2.21	2.52	3.15	3.90	4.42	4.95	5.70	6.26	6.55	6.86	7.30	normal

41	0	0.66	1.22	1.88	2.44	3.45	4.34	4.94	5.32	6.23	7.02	8.00	8.73	9.08	9.72	10.54	11.32	weak
42	0	0.35	0.60	1.00	1.56	2.18	2.77	3.06	3.38	3.66	3.92	4.32	4.74	5.01	5.60	6.61	7.05	normal
43	0	0.42	0.99	1.86	2.44	3.03	3.63	4.13	4.71	5.22	5.81	6.48	7.29	8.05	8.64	9.06	9.52	weak
44	0	0.25	0.75	0.99	1.43	1.61	2.08	2.36	3.03	3.57	4.03	4.48	5.24	5.77	6.16	6.76	7.19	normal
45	0	0.44	1.05	1.70	2.00	2.58	2.92	3.16	3.83	4.19	4.73	5.65	6.06	6.41	6.79	7.13	7.62	normal
46	0	0.24	0.76	1.64	2.03	2.42	2.65	3.02	3.27	3.61	4.06	4.43	4.97	5.56	5.94	6.35	6.64	normal
47	0	0.70	1.82	2.43	3.46	3.97	4.44	5.30	5.94	6.40	7.19	7.67	8.39	8.98	9.41	10.10	10.61	weak
48	0	0.72	1.13	1.35	1.81	2.16	2.61	3.02	3.44	3.94	4.63	4.99	5.48	5.86	6.07	6.60	6.96	normal
49	0	0.31	0.57	0.93	1.49	2.14	2.46	3.16	3.96	4.53	4.76	5.22	5.60	6.06	6.49	6.85	7.38	normal
50	0	0.54	1.09	1.39	1.62	2.06	2.61	3.07	3.38	3.95	4.41	4.80	5.32	5.91	6.67	7.09	7.63	normal
51	0	0.60	1.57	2.28	2.86	3.32	4.12	4.59	5.13	5.80	6.39	6.90	7.24	7.77	8.51	9.27	9.83	weak
52	0	0.34	0.84	1.20	1.81	2.03	2.47	2.93	3.27	3.74	4.08	4.56	4.99	5.62	6.01	6.31	6.86	normal
53	0	0.74	1.27	1.57	2.49	3.15	4.16	4.96	5.57	6.09	6.82	7.42	8.05	8.53	9.02	9.52	10.21	weak
54	0	0.84	1.61	2.12	2.56	3.06	3.39	4.05	4.56	5.36	6.07	6.60	7.41	7.94	8.53	9.24	9.84	weak
55	0	0.51	1.01	1.29	1.73	2.11	2.51	2.98	3.28	4.05	4.79	5.10	5.50	5.80	6.14	6.68	7.25	normal
56	0	0.40	0.70	1.95	2.28	2.64	3.02	3.88	4.58	4.89	5.49	5.79	6.18	6.53	6.90	7.54	8.04	normal
57	0	0.26	0.85	1.25	1.63	1.90	2.68	3.07	3.41	3.65	4.31	4.74	5.11	5.39	5.64	6.16	6.64	normal
58	0	0.45	1.36	2.22	3.05	3.71	4.42	5.29	6.35	7.19	7.72	8.69	9.15	9.93	10.59	11.32	12.06	weak
59	0	0.93	1.21	1.69	2.08	2.48	2.95	3.36	3.73	4.12	4.52	4.98	5.37	5.82	6.21	6.56	7.42	normal
60	0	0.26	0.70	1.12	1.38	1.70	1.97	2.49	3.12	3.45	4.00	4.36	4.69	5.06	5.57	6.13	6.33	normal
61	0	1.06	1.83	2.52	3.13	3.65	4.03	4.89	5.43	6.31	6.96	8.18	8.73	9.57	10.21	10.78	11.51	weak
62	0	0.48	0.86	1.53	2.11	2.67	3.01	3.34	3.78	4.20	4.61	4.98	5.32	5.62	6.02	6.47	7.01	normal
63	0	0.31	0.79	1.05	1.46	2.04	2.44	3.08	3.51	4.11	4.52	4.82	5.17	5.61	6.25	6.83	7.53	normal
64	0	0.49	0.80	1.11	2.05	2.91	3.33	3.71	4.21	4.86	5.42	6.01	6.45	6.82	7.07	7.48	7.81	normal
65	0	0.59	0.82	1.33	1.92	2.21	2.46	2.75	3.46	4.09	4.45	5.34	5.67	6.10	6.67	7.12	7.90	normal
66	0	0.44	0.67	1.13	1.57	1.75	2.12	2.41	2.82	3.46	4.01	4.48	4.86	5.37	5.89	6.57	6.99	normal
67	0	0.37	0.87	1.54	1.93	2.23	2.63	3.25	3.82	4.45	4.89	5.47	5.83	6.16	6.85	7.70	8.06	normal
68	0	0.40	0.82	1.46	1.93	2.34	2.69	2.95	3.52	3.78	4.08	4.35	4.67	4.87	5.39	5.72	6.34	normal
69	0	1.08	1.73	2.41	3.02	3.48	4.25	4.78	5.38	6.11	6.84	7.85	8.58	9.69	10.40	11.31	12.01	weak
70	0	0.44	1.12	1.70	2.29	2.80	3.44	4.00	4.60	5.64	6.53	7.38	8.35	8.98	9.95	10.62	11.04	weak
71	0	0.35	0.78	1.30	1.94	2.52	2.98	3.30	3.56	4.09	4.57	5.16	5.65	6.12	6.45	6.78	7.95	normal
72	0	0.17	0.98	1.26	1.49	1.95	2.55	2.95	3.39	3.77	4.16	4.60	5.29	5.83	6.39	6.73	7.18	normal
73	0	0.45	0.84	1.29	1.53	1.98	2.47	2.92	3.26	3.73	4.05	4.53	5.01	5.43	5.80	6.19	6.43	normal
74	0	0.28	0.87	1.15	1.85	2.66	3.02	3.34	3.71	4.45	5.05	5.63	6.11	6.35	6.85	7.27	7.78	normal
75	0	0.40	0.70	0.92	1.24	1.83	2.36	2.81	3.24	3.81	4.13	4.47	5.24	5.68	5.95	6.28	6.58	normal
76	0	0.42	1.24	1.76	2.23	2.67	3.04	4.10	4.62	5.18	5.89	6.57	7.21	7.71	8.42	9.26	9.97	weak
77	0	0.30	0.92	1.33	1.55	2.06	2.56	3.09	3.46	3.92	4.21	4.62	5.34	6.32	6.66	6.97	7.33	normal
78	0	1.00	1.77	2.26	2.77	3.16	3.64	4.39	5.07	5.52	6.00	6.59	7.21	7.96	8.56	8.94	9.62	weak
79	0	0.35	0.67	1.09	1.40	2.09	2.48	3.27	3.78	4.11	4.62	5.00	5.35	5.86	6.21	6.58	6.78	normal
80	0	0.35	1.17	1.89	2.29	2.66	3.40	3.76	4.32	5.00	5.36	5.78	6.12	6.60	6.88	7.37	8.06	normal
81	0	0.72	1.60	2.23	3.32	4.04	4.68	5.26	5.89	6.37	6.83	7.54	8.03	8.72	9.60	10.08	10.65	weak
82	0	0.39	0.72	1.15	1.56	2.04	2.37	2.81	3.07	3.42	3.87	4.44	4.83	5.31	5.74	5.99	6.45	normal
83	0	0.50	1.02	1.61	2.35	2.98	3.45	3.88	4.27	5.23	5.46	5.79	6.42	6.78	7.35	7.81	8.08	normal
84	0	0.65	1.25	2.02	3.07	3.68	4.45	5.42	5.97	6.44	7.11	8.00	8.78	9.39	9.91	10.47	11.20	weak

85	0	0.83	1.29	1.56	2.32	2.61	3.31	3.67	4.11	4.67	5.09	5.38	5.99	6.44	6.69	7.19	7.66	normal
86	0	0.53	0.88	1.52	2.05	2.73	3.64	4.76	6.12	6.62	7.37	7.98	8.44	9.13	9.86	10.24	11.31	weak
87	0	0.87	1.58	1.89	2.43	2.90	3.37	3.89	4.90	5.96	6.39	6.92	7.57	8.50	9.30	10.02	10.57	weak
88	0	0.27	0.56	1.14	1.79	2.17	2.45	2.71	2.93	3.15	3.53	4.00	4.29	4.97	5.55	5.84	6.34	normal
89	0	0.41	0.78	1.23	1.43	1.83	2.13	2.51	2.88	3.61	4.17	4.50	4.88	5.36	5.71	6.10	6.36	normal
90	0	0.47	0.95	1.51	2.04	2.41	2.77	3.15	3.62	4.02	4.35	4.54	4.84	5.30	5.86	6.49	7.10	normal
91	0	0.26	0.60	0.90	1.51	2.00	2.41	3.12	3.68	4.03	4.83	5.14	5.59	6.27	6.58	6.89	7.29	normal
92	0	0.39	0.98	1.78	2.39	3.22	3.96	4.68	5.56	6.18	7.14	7.54	8.24	8.68	9.59	10.24	10.84	weak
93	0	0.30	0.77	1.49	1.88	2.23	2.76	3.54	4.11	4.64	5.16	5.88	6.29	6.88	7.49	7.79	8.28	normal
94	0	0.36	0.88	1.20	1.81	2.27	2.69	3.13	3.80	4.35	4.61	5.15	5.37	5.69	6.24	6.79	7.16	normal
95	0	0.33	0.88	1.33	1.75	2.30	2.63	3.65	4.01	4.51	5.20	5.81	6.55	6.89	7.28	7.67	7.89	normal
96	0	0.71	1.05	1.58	2.01	2.25	2.55	3.28	3.69	3.91	4.43	4.97	5.62	5.94	6.37	6.82	7.23	normal
97	0	0.68	1.01	1.66	2.01	2.35	3.08	3.53	4.02	4.31	4.87	5.41	5.65	6.17	6.57	6.98	7.41	normal
98	0	0.58	0.97	1.50	2.60	3.18	4.10	4.67	5.41	5.95	6.54	7.24	7.88	8.51	9.25	9.75	10.27	weak
99	0	0.31	0.73	1.12	1.59	1.90	2.48	3.11	3.75	4.24	4.48	4.92	5.41	5.74	6.15	6.59	6.91	normal
100	0	0.46	1.11	1.50	2.06	2.81	3.20	3.87	4.47	5.04	5.79	6.39	7.53	8.17	8.68	9.17	10.06	weak
101	0	0.80	1.45	2.19	3.02	3.35	3.80	4.47	4.99	5.39	6.19	7.22	7.91	8.69	9.22	9.57	9.96	weak
102	0	0.39	1.22	1.57	2.09	2.58	2.98	3.25	3.64	4.07	4.49	5.03	5.53	6.05	6.32	6.63	7.07	normal
103	0	0.63	0.93	1.38	1.67	2.09	2.45	2.89	3.15	3.45	3.96	4.52	4.82	5.51	6.14	6.58	6.88	normal
104	0	0.41	0.99	1.50	2.23	2.47	2.84	3.72	4.38	4.73	5.19	5.61	6.11	6.44	7.26	8.04	8.49	normal
105	0	0.52	0.93	1.37	2.03	2.51	2.83	3.22	3.46	3.81	4.23	4.49	5.17	5.56	5.90	6.30	6.56	normal
106	0	0.27	0.90	1.38	1.77	2.11	2.38	3.20	3.55	3.77	4.11	4.84	5.19	5.55	5.92	6.66	7.10	normal
107	0	0.50	0.86	1.27	1.72	2.05	2.63	2.94	3.69	3.98	4.17	4.58	5.07	5.56	6.31	6.72	7.24	normal
108	0	0.41	0.73	1.41	1.72	2.09	2.71	3.29	3.69	4.28	4.83	5.13	5.71	6.17	6.47	7.15	7.62	normal
109	0	0.79	1.28	1.54	2.01	2.37	2.73	3.40	4.15	4.57	5.01	5.52	5.94	6.42	7.10	7.49	7.82	normal
110	0	0.40	1.08	1.54	2.12	2.66	3.02	3.67	4.12	4.48	4.74	5.04	5.42	5.77	6.55	7.08	7.42	normal
111	0	0.68	1.66	2.21	2.85	3.26	3.81	4.46	4.93	5.92	6.40	7.26	7.91	8.85	9.54	10.35	11.02	weak
112	0	0.40	0.81	1.20	1.71	2.31	2.91	3.31	3.74	4.33	4.71	5.09	5.57	5.97	6.25	6.66	6.88	normal
113	0	0.65	1.27	1.85	2.14	2.48	2.73	3.08	3.47	3.94	4.27	4.59	5.21	5.77	6.04	6.65	6.89	normal
114	0	0.36	0.74	1.23	1.67	2.23	2.72	3.15	3.65	4.03	4.50	4.86	5.13	5.66	6.03	6.71	7.16	normal
115	0	0.38	0.83	1.25	1.81	2.48	2.93	3.44	4.02	4.76	5.13	5.37	5.94	6.37	7.29	7.72	8.09	normal
116	0	0.33	0.99	1.31	1.75	2.11	2.68	3.19	4.08	4.45	4.78	5.02	5.50	5.75	6.15	6.47	6.91	normal
117	0	0.35	0.89	1.20	1.60	2.05	2.59	2.98	3.41	3.81	4.27	4.77	5.20	5.55	6.01	6.33	6.69	normal
118	0	0.47	0.96	1.30	1.62	2.35	2.63	3.35	3.99	4.39	4.66	5.30	6.01	6.40	6.79	7.07	7.90	normal
119	0	0.35	0.80	1.11	1.42	2.24	2.60	3.15	3.54	3.89	4.48	4.98	5.28	5.51	5.93	6.48	7.05	normal
120	0	0.54	0.92	1.26	1.89	2.68	3.46	3.95	4.48	5.07	5.40	5.73	6.08	6.50	7.00	7.55	7.76	normal
121	0	0.57	1.07	1.48	1.96	2.22	2.47	2.92	3.36	3.78	4.34	4.75	5.05	5.37	5.81	6.22	6.53	normal
122	0	0.54	1.33	1.89	2.57	3.32	4.22	4.74	5.50	6.17	6.80	7.38	8.22	8.88	9.20	9.81	10.68	weak
123	0	0.62	1.00	1.54	2.03	2.48	3.07	4.04	5.02	5.90	6.28	7.24	7.71	8.61	9.37	10.15	10.77	weak
124	0	0.33	0.86	1.93	2.19	2.50	3.03	3.23	3.56	3.73	4.12	4.39	4.65	5.01	5.38	5.94	6.67	normal
125	0	0.45	1.03	1.53	2.05	2.64	3.22	3.53	3.79	4.08	4.41	4.82	5.38	5.88	6.54	6.90	7.25	normal
126	0	0.62	1.49	2.10	3.15	3.66	4.15	4.55	5.38	6.15	6.92	8.01	8.45	9.31	10.00	10.67	11.26	weak
127	0	0.58	1.12	2.03	2.82	3.68	4.38	5.04	5.67	6.62	7.12	7.97	8.68	9.11	9.68	10.29	10.90	weak
128	0	0.44	0.73	1.28	1.60	1.97	2.27	2.54	2.81	3.05	3.74	4.15	4.70	5.19	5.89	6.55	7.11	normal

129	0	0.50	0.77	1.14	2.26	2.45	2.83	3.16	3.56	3.92	4.28	4.60	4.98	5.39	5.64	6.29	6.86	normal
130	0	0.55	1.20	1.58	1.97	2.56	2.87	3.18	3.87	4.10	4.84	5.16	5.72	6.24	6.98	7.32	7.81	normal
131	0	0.73	1.32	2.03	2.53	3.17	3.70	4.55	5.38	6.01	6.70	7.10	7.75	8.60	9.15	9.86	10.74	weak
132	0	0.69	1.07	1.80	2.75	3.45	4.05	4.82	5.38	5.92	6.46	6.99	7.60	8.64	9.74	10.21	10.76	weak
133	0	0.69	1.44	1.82	2.31	2.75	3.74	4.42	5.63	6.54	7.30	7.82	8.66	9.17	10.01	10.61	11.33	weak
134	0	0.33	0.70	1.02	1.39	1.83	2.38	2.76	3.36	3.68	4.35	4.65	4.98	5.23	5.65	6.31	6.59	normal
135	0	0.51	0.74	0.98	1.26	1.65	2.01	2.35	3.12	3.43	3.89	4.21	4.70	5.27	6.00	6.50	6.89	normal
136	0	0.40	0.88	1.18	1.48	1.88	2.30	2.54	2.91	3.50	3.92	4.36	4.60	4.97	5.36	5.57	6.08	normal
137	0	0.64	1.20	1.93	2.41	2.76	3.37	3.91	5.05	5.68	6.25	6.91	7.93	8.32	8.97	9.79	10.38	weak
138	0	1.04	1.41	1.80	2.22	2.50	2.85	3.26	3.93	4.36	4.79	5.20	5.76	6.13	6.65	6.95	7.41	normal
139	0	0.36	0.99	1.24	1.56	1.99	2.35	2.81	3.09	3.55	3.89	4.29	4.52	4.86	5.69	6.09	6.70	normal
140	0	0.81	1.39	2.16	3.06	3.85	4.61	5.12	5.43	6.02	6.59	7.38	7.88	8.55	9.61	10.24	10.92	weak
141	0	0.70	0.96	1.26	1.78	2.28	2.81	3.17	3.41	3.69	4.09	4.77	5.43	5.83	6.09	6.65	7.10	normal
142	0	0.49	0.74	1.21	1.70	1.95	2.46	2.86	3.32	3.82	4.23	4.58	4.82	5.30	5.76	6.11	6.32	normal
143	0	0.56	1.02	1.44	1.76	2.03	2.44	3.67	4.30	4.79	5.14	5.87	6.38	6.55	7.04	7.26	7.86	normal
144	0	0.42	0.87	1.43	2.13	2.38	2.64	2.92	3.19	3.48	4.10	4.39	4.85	5.30	5.72	6.40	7.15	normal
145	0	1.21	1.89	2.38	3.09	3.61	4.32	5.38	6.09	6.83	7.44	8.16	8.81	9.37	9.79	10.59	11.51	weak
146	0	0.51	1.29	1.73	2.44	2.76	3.19	3.50	3.93	4.44	4.95	5.28	5.72	6.42	6.83	7.23	7.58	normal
147	0	0.50	1.11	1.60	2.25	3.00	3.81	4.38	4.80	5.37	5.98	6.60	6.97	7.59	8.48	9.20	9.84	weak
148	0	0.63	1.19	1.86	2.31	2.62	3.10	3.43	3.86	4.17	4.77	5.45	5.82	6.17	6.36	6.76	7.29	normal
149	0	0.91	1.69	2.07	2.87	3.45	4.14	4.63	5.25	5.75	6.54	7.20	7.65	8.34	8.87	9.58	10.12	weak
150	0	0.63	1.22	1.92	2.62	3.22	3.90	4.58	5.32	6.76	7.27	7.71	8.33	9.15	9.84	10.35	10.81	weak
151	0	0.66	1.33	2.12	2.82	3.40	3.84	4.91	5.28	5.81	6.38	7.06	7.70	8.47	9.04	10.21	10.92	weak
152	0	0.92	1.30	1.88	2.97	3.41	4.24	4.88	5.53	6.31	7.07	7.85	8.65	9.68	10.20	10.72	11.26	weak
153	0	0.58	0.97	1.80	2.85	3.43	4.34	5.15	5.77	6.47	7.32	7.79	8.37	8.90	9.40	10.13	11.36	weak
154	0	0.80	1.18	1.60	1.91	2.61	3.06	3.33	3.59	4.02	4.40	4.86	5.37	5.77	6.30	6.79	7.28	normal
155	0	0.46	0.94	1.29	1.59	2.19	2.51	3.26	3.59	4.41	4.75	5.07	5.34	5.88	6.19	6.65	7.02	normal
156	0	0.24	0.90	1.10	1.51	1.98	2.32	3.04	3.93	4.60	5.38	6.02	6.42	6.78	7.29	8.20	8.61	normal
157	0	0.19	0.72	1.10	2.13	2.59	2.91	3.15	4.20	4.54	4.96	5.72	6.40	6.71	7.27	7.85	8.20	normal
158	0	0.64	1.15	1.79	2.53	3.69	4.21	4.89	5.48	5.95	6.46	7.08	7.60	8.08	8.88	9.51	10.27	weak
159	0	0.82	1.31	1.87	2.25	2.63	3.39	3.57	3.90	4.34	4.68	5.35	5.57	5.98	6.30	7.04	7.40	normal
160	0	0.76	1.37	2.09	2.65	3.44	3.91	4.29	5.21	5.68	6.10	6.56	7.03	7.60	8.16	8.82	9.64	weak
161	0	0.60	1.12	1.53	1.96	2.13	2.63	3.06	3.31	4.09	4.65	5.21	5.64	6.33	6.59	6.84	7.06	normal
162	0	0.55	1.59	2.37	2.98	3.73	4.38	5.22	6.33	6.69	7.31	7.97	8.39	8.99	9.64	10.46	11.08	weak
163	0	0.45	0.86	1.40	1.79	2.18	2.77	3.15	3.52	3.76	4.44	5.13	5.45	5.96	6.40	6.96	7.51	normal
164	0	0.62	0.99	1.46	1.91	2.43	2.90	3.34	3.67	4.00	4.52	4.98	5.30	5.93	6.23	6.68	7.28	normal
165	0	0.43	0.87	1.22	1.62	1.98	2.35	2.64	3.33	3.82	4.10	4.68	5.07	5.49	5.98	6.35	6.92	normal
166	0	0.54	0.94	1.82	2.55	3.33	3.81	4.55	5.15	5.99	6.40	6.77	7.66	8.47	9.04	10.05	10.63	weak
167	0	0.77	1.43	2.36	3.13	4.19	4.71	5.27	5.64	6.29	6.89	7.70	8.37	8.81	9.39	10.10	10.92	weak
168	0	0.45	0.77	1.06	1.45	1.78	2.33	2.81	3.27	3.69	4.30	4.85	5.82	6.37	6.61	6.97	7.22	normal
169	0	0.71	1.48	2.19	2.77	3.49	4.18	4.81	5.55	6.09	6.76	7.18	7.86	8.54	9.23	9.65	10.57	weak
170	0	0.45	0.77	0.98	1.42	2.00	2.43	2.86	3.13	3.43	3.71	4.03	4.27	4.70	5.20	5.71	6.41	normal
171	0	0.80	1.32	2.08	2.81	3.53	4.28	5.07	5.81	6.50	7.09	7.68	8.46	9.10	9.74	10.22	10.95	weak
172	0	0.45	0.92	1.61	2.02	2.47	2.87	3.31	3.74	4.13	4.94	5.17	5.79	6.25	6.84	7.33	7.63	normal

173	0	0.36	0.76	1.35	1.85	2.05	2.24	2.60	2.91	3.31	3.74	4.26	4.61	4.89	5.31	5.58	6.01	normal
174	0	0.35	0.66	1.08	1.45	1.79	2.12	2.47	3.02	3.41	3.97	4.34	4.67	5.13	5.45	6.04	6.89	normal
175	0	0.52	1.16	2.44	3.06	3.58	4.57	4.98	5.43	6.00	6.56	7.50	8.20	8.61	9.11	9.62	10.04	weak
176	0	0.58	1.10	1.55	2.33	3.07	3.73	4.69	5.23	5.82	6.98	7.49	8.12	8.56	9.08	9.61	10.32	weak
177	0	0.48	0.88	1.27	1.75	2.03	2.60	3.07	3.39	3.73	4.03	4.39	4.74	5.33	5.71	6.22	6.76	normal
178	0	0.36	0.59	0.93	1.38	1.69	2.08	2.55	2.99	3.17	3.71	3.98	4.25	4.58	4.99	5.66	5.93	normal
179	0	0.45	0.91	1.44	1.82	2.07	2.35	2.82	3.37	3.75	4.16	4.56	5.33	5.58	5.93	6.78	7.27	normal
180	0	0.22	0.99	1.31	2.07	2.61	2.87	3.34	4.08	4.30	4.53	5.35	5.88	6.13	6.71	7.00	7.60	normal
181	0	0.42	0.78	1.07	1.61	2.20	2.43	2.85	3.10	3.59	4.09	4.36	4.95	5.26	5.59	5.88	6.60	normal
182	0	0.29	0.58	1.01	1.31	1.64	2.00	2.61	3.04	3.40	3.71	4.15	4.50	5.01	5.40	5.61	5.83	normal
183	0	0.27	0.63	1.13	1.52	1.90	2.30	2.57	3.23	3.99	4.33	4.87	5.23	5.50	5.88	6.40	6.85	normal
184	0	0.42	0.66	1.19	2.12	2.43	3.00	3.38	3.95	4.48	4.96	5.18	5.35	5.69	5.95	6.10	6.38	normal
185	0	0.68	1.02	1.56	2.03	2.85	3.24	3.62	4.07	4.55	4.85	5.35	6.07	6.47	6.71	7.07	7.53	normal
186	0	0.69	1.44	1.76	2.32	2.70	3.10	3.57	4.00	4.54	5.00	5.22	5.48	5.86	6.50	6.78	7.13	normal
187	0	0.20	0.78	1.25	1.61	2.07	2.56	2.84	3.39	3.97	4.37	5.11	5.63	6.15	6.50	6.94	7.26	normal
188	0	0.45	0.64	1.23	1.51	1.71	2.19	2.64	3.11	3.71	4.13	4.38	4.85	5.29	5.62	6.05	6.62	normal
189	0	0.48	0.86	1.14	1.55	1.91	2.34	2.59	3.20	3.45	3.79	4.32	4.52	4.83	5.40	6.49	6.94	normal
190	0	0.57	1.18	1.60	2.02	2.48	2.90	3.48	3.81	4.13	4.61	4.97	5.50	5.80	6.51	7.02	7.30	normal
191	0	0.36	0.76	1.18	1.63	2.10	2.33	2.68	3.50	4.02	4.52	4.91	5.15	5.79	6.17	6.57	6.87	normal
192	0	0.42	0.87	1.22	1.55	2.26	2.94	3.25	3.69	4.08	4.42	5.10	5.53	6.04	6.51	6.83	7.34	normal
193	0	0.48	1.21	1.58	2.34	2.59	3.00	3.40	3.91	4.25	4.64	5.19	5.71	6.31	6.62	6.98	7.55	normal
194	0	0.42	1.14	1.94	2.68	3.19	3.78	4.31	4.94	5.46	5.84	6.58	7.66	8.22	9.45	10.16	10.85	weak
195	0	0.25	0.68	1.03	1.32	1.65	2.04	2.45	2.93	3.37	3.89	4.29	4.87	5.48	6.03	6.54	7.23	normal
196	0	0.42	1.15	1.60	1.89	2.22	3.09	3.53	3.78	4.16	4.59	4.99	5.44	6.02	6.40	6.65	7.02	normal
197	0	0.61	1.46	2.02	2.84	3.56	4.63	5.19	5.90	6.57	7.13	7.71	8.26	8.83	9.36	10.04	10.48	weak
198	0	0.31	0.80	1.00	1.76	2.37	2.72	3.03	3.71	4.40	4.79	5.12	5.64	6.33	6.71	7.64	7.85	normal
199	0	0.28	0.74	1.48	1.95	2.42	2.87	3.39	3.79	4.29	4.98	5.54	6.12	6.39	6.77	7.07	7.62	normal
200	0	0.33	0.71	0.98	1.42	2.09	2.44	2.81	3.33	3.69	4.18	5.46	5.90	6.49	6.88	7.29	7.59	normal