SEMIGROUP IDENTITIES IN THE GROUP OF UNITS OF INTEGRAL GROUP RINGS

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Let RG be the group ring of the group G over the commutative unital ring R, and let $U(RG)$ be its group of units. It is known, see [4] and [5], that when $U(RG)$ satisfies some group theoretical condition then G is somewhat restricted, and a nontrivial ring theoretical property of RG holds. In this direction there is the following conjecture, due to B. Hartley [5], problem 52:

Conjecture: Let $R = K$ be a field, and let G be a torsion group. If $U(KG)$ satisfies a group identity then KG satisfies a polynomial identity.

In [3], in a joint work with Mandel, we proved:

Theorem: If K is infinite and $U(KG)$ satisfies a semigroup identity then the Conjecture is true.

In this note we want to communicate the following result [1], obtained in colaboration with Dokuchaev:

Theorem: If $R = \mathbb{Z}$, the ring of integers, the Conjecture is true.

It just came to our attention a pre-print [2], in which Giambruno, Jespers and Valenti show that the Conjecture is true when K is infinite, char$K = p$ and G has no p-elements.

References

[1] Dokuchaev, M. A. and Gonçalves, J. Z., *Semigroup identities on units of integral group rings.* Pre-print.

[2] Giambrano, A., Jespers, E. and Valenti, A., *Group identities on units of rings.* Pre-print.

[3] Gonçalves, J. Z. and Mandel, A., *Semigroup identities on units of group algebras.* Arch. Math. 57 (1991), 539-545.

[4] Sehgal, S. K., *Topics in group rings.* Marcel Dekker, New York, 1978.

[5] Sehgal, S. K., *Units in integral group rings.* Longman, Essex, 1993.