Observação. As afirmações 2 e 3 podem ser provadas por indução sobre i e j respectivamente. As outras são imediatas.

Comentário final. Este resultado mostrou-se útil no estudo da estabilidade da origem (segundo Liapunov) de certas famílias de Equações Diferenciais Ordinárias "do tipo Hill", permitindo em tais casos a construção de Funções de Liapunov para a Estabilidade. — (*9 de abril de 1991*).

CENTER-MASS DISPLACEMENT AND CONTROL

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The aim of this note is to study situations in which it is possible to move optimally a mass-charged point by replacing the effect due to an explicit "external" force u according to the process

$$\ddot{z}(t) = f_1(\dot{z}, z) + u(t)$$

with

$$(z(t_0), \dot{z}(t_0), \ddot{z}(t_0)) = z_0;$$
$$(z(t_1), \dot{z}(t_1), \ddot{z}(t_1)) = z_1$$

by the force effect due to the displacement of the center-mass.

Setting the equations of the two-coupled motion

$$(1) \quad \begin{aligned} m_1\ddot{x} &= f(x, \dot{x}, y, \dot{y}, \ddot{y}) \\ m_2\ddot{y} &= g(x, \dot{x}, \ddot{x}, y, \dot{y}), \end{aligned}$$

with

$$(x(t_0), \dot{x}(t_0), \ddot{x}(t_0), \ddot{x}(t_0)) = z_0$$

and

$$(x(t_1), \dot{x}(t_1), \ddot{x}(t_1)) = z_1,$$

let $r(t)$ be the origin point of a frame, and $s(t)$ the internal coordinates:

$$s_1 = x - r = aQ + bP;$$
$$s_2 = y - r = aQ - bP$$

(where $a, b \in \mathbb{R}$), chosen in such a way that the system (1) can be written as:

$$\ddot{Q} = F(Q, \dot{Q});$$
$$\ddot{P} = G(Q, \dot{Q}).$$

The existence of the above entities is assurred by the implicit function theorem – if f and g are of class C^q, for large enough q – and by the possibility of integrating the system.

Observing that $s_1 = -s_2 + 2aQ$, then we have defined the dynamics $m_2\ddot{s}_2 = g_1(s_2, \dot{s}_2) + h(Q(t))$. We conclude, by using theorems like the Pontryaguin MP, with the class of the admissible controls h_0Q.

Examples

1). Coupling two harmonic oscillators: the entities

$$Q = \frac{1}{2}(x + y),$$
$$P = \frac{1}{2}(x - y),$$
$$h(Q(t)) = v.\sin wt$$

(v, w, variables), and $r(t)$, are defined as in "New Foundations for Classical Mechanics" by D. Hestenes (D. Riedel Pubs., 1987), p. 361.

2). Looking for the small oscillations of a double pendulum, with P_1 and P_2 the mass-charged points: let be m_i, l_i, p_i respectivelly the mass, the distance from the point of attachament, and the angle with the vertical line of $P_i (i = 1, 2)$ and g the gravitational force constant. If we choose $(r, \ddot{r})$, colinear with $(p_1, \ddot{p}_1)$ and $(p_2, \ddot{p}_2)$, then we have:

(a) the equation (1) is:

$$(m_1 + m_2)l_1\ddot{p}_1 + m_2l_1l_2\ddot{p}_2 + (m_1 + m_2)l_1gp_1 = 0$$

and

$$m_2l_2\ddot{p}_2 + m_2l_1l_2\ddot{p}_1 + m_2l_2gp_2 = 0.$$

(b) $Q = am_1s_1 + bm_2s_2 \quad (a, b \in \mathbb{R})$.

(c) $m_1\ddot{s}_1 = (k_1 - k_2\frac{a}{b})s_1 + h(Q(t))$, where

$$h(Q(t)) = \frac{Q(t)}{b} = \frac{m + ne}{b}.$$

Now, to finish the example, we need to choose the "good" control h_0Q, and to rewrite the equation in (c) above in terms of the original frame.

Finally: from the point of view of controllability it is easy to see – supposing in (1), the point moving in accordance with the first equation as having a "natural" motion – that the choice of an appropriate law for the displacing of the second mass-charged point will allow us to induce one "good" control for the whole system. — (*9 de abril de 1991*).