STUDY OF EFFECTIVE PROPERTIES IN MULTILAYERED COMPOSITES WITH DIFFERENT FRACTURE MODES AT THE INTERFACE

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Abstract. *This manuscript offers a methodology to compute the effective properties of multilayered laminated composite with crack failure at the interface. In order to achieve the objective, the equilibrium problem for a curvilinear three-dimensional composite structure with generalized periodicity is considered. The problem is subjected to different conditions for the components like elasticity and piezoelectricity with perfect and imperfect contact at the interfaces. The two-scales asymptotic homogenization method (AHM) is used to derive the expressions of the local problems and the effective coefficients. In order to study the influence of the crack in the effective properties, different cases of geometry for the crack are considered and compared. The general expressions of the homogenized problem are derived for all the aforementioned cases. To validate the model, some numerical examples are studied, and the results are compared with the values obtained by finite elements method (FEM).*

1. INTRODUCTION

Due to the great influence in the behavior and properties of the composite materials, the cracks and imperfections between the components of the solid structures have been widely studied. Many researches have focused their attention in the mechanical effective properties of composite materials with different fracture modes. In [1, 2], the authors have developed mathematical model to study the effective properties of elastic and piezoelectric composite materials using the two-scales asymptotic homogenization method (AHM) and finite elements method (FEM) considering different fracture modes at the interface. In this manuscript, the methodology is extended to the case of piezocomposite with complex geometrical shapes. In section 2, the general equations of the equilibrium problem for a piezoelectric composite with generalized periodicity are presented. The AHM is used to derive the expressions of the effective coefficients and the homogenized problem in section 3. Finally, three numerical examples of laminate composite with different fracture modes are considered. In a first application of the methodology, a wavy piezocomposite is studied, considering uniform imperfect contact at the interface. As a second example, an elastic rectangular composite structure with a crack between the layers is presented as limit case of a piezoelectric structure. Finally, a similar elastic structure is considered but with non-uniform imperfect contact at the interface.

2. EQUILIBRIUM PROBLEM FOR A PIEZOCOMPOSITE

In the present work, a periodic laminate piezocomposite is studied. The coordinates system $\mathbf{x}=(x_1,x_2,x_3)$ is used to describe the geometry of the compact connected solid $\Omega\subset\mathbb{R}^3$, bounded by the closed surface $\partial\Omega=\Sigma_u\cup\Sigma_t=\Sigma_\phi\cup\Sigma_\tau$, where $\Sigma_u\cap\Sigma_t=\Sigma_\phi\cap\Sigma_\tau=\varnothing$. The equilibrium equation for a solid is given by

$$\Delta\boldsymbol{\sigma}+\mathbf{f}=0,\quad \Delta\mathbf{D}=0, \tag{1}$$

with boundary conditions

$$\mathbf{u}=\mathbf{u}_0 \quad \text{on} \quad \Sigma_u,\quad \boldsymbol{\sigma}\cdot\mathbf{n}=\mathbf{S}_0 \quad \text{on} \quad \Sigma_t, \tag{2}$$

$$\phi=\phi_0,\quad \text{on} \quad \Sigma_\phi,\quad \mathbf{D}\cdot\mathbf{n}=-\tau_0,\quad \text{on} \quad \Sigma_\tau, \tag{3}$$

and interface conditions given by

$$\boldsymbol{\sigma}\cdot\mathbf{n}=\mathbf{K}\cdot\left[\left[\mathbf{u}\right]\right],\quad \left[\left[\boldsymbol{\sigma}\cdot\mathbf{n}\right]\right]=0,\quad \text{on} \quad \Gamma, \tag{4}$$

$$\mathbf{D}\cdot\mathbf{n}=M\left[\left[\phi\right]\right],\quad \left[\left[\mathbf{D}\cdot\mathbf{n}\right]\right]=0,\quad \text{on} \quad \Gamma, \tag{5}$$

where Δ is divergence operator, $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{f}$ are the body forces, $\mathbf{D}$ is the electric displacement, $\mathbf{u}$ represents the displacement vector, ϕ is the electrical potential; $\mathbf{u}_0$, $\mathbf{S}_0$, ϕ_0, τ_0 are the prescribed displacement, tensions, electric potential and surface charge, respectively. The surface Γ represents the interface between the components of the composite. The vector $\mathbf{n}$ denotes the corresponding normal vector and $\left[\left[\bullet\right]\right]=(\bullet)^{(1)}-(\bullet)^{(2)}$ represents the difference of the value of the function at the different components. The matrix $\mathbf{K}$ and the parameter M characterize the imperfect mechanical and electric contact at the interface Γ, respectively.

The constitutive relations between stress $\boldsymbol{\sigma}$ and electrical displacement $\mathbf{D}$ with strain $\boldsymbol{\epsilon}$ and electric field $\mathbf{E}$ have the following expression

$$\boldsymbol{\sigma}=\mathbf{C}:\boldsymbol{\epsilon}-\mathbf{e}\cdot\mathbf{E},\quad \mathbf{D}=\mathbf{e}:\boldsymbol{\epsilon}+\boldsymbol{\kappa}\cdot\mathbf{E}, \tag{6}$$

where ":" and "$\cdot$" denote the tensor and dot product respectively; $\mathbf{C}$, $\mathbf{e}$ and $\boldsymbol{\kappa}$ denote the stiffness, piezoelectric and permittivity tensors, respectively. For a periodic composite structure the coefficients $\mathbf{C}$, $\mathbf{e}$ and $\boldsymbol{\kappa}$ depend on the macro-coordinate variable $\mathbf{x}$ and they are periodic with respect the micro-coordinate $\mathbf{y}=\boldsymbol{\rho}\left(\mathbf{x}\right)/\varepsilon$, where $\boldsymbol{\rho}$ describes the geometry of the microstructure and ε characterizes the periodicity. Also, for small deformations $\boldsymbol{\epsilon}$ and $\mathbf{E}$ relate to the displacement vector $\mathbf{u}$ and the electric potential ϕ

$$\boldsymbol{\epsilon}=\frac{1}{2}\left(\nabla\mathbf{u}+\nabla\mathbf{u}^T\right),\qquad \mathbf{E}=-\nabla\phi. \tag{7}$$

Substituting the equations (6)-(7) into (1)-(5), the components of the equilibrium problem are

$$\left(\frac{\rho_{m,j}}{\varepsilon}C_{ijkl|m}+C_{ijkl,j}\right)u_{k,l}+C_{ijkl}u_{k,lj}+\left(\frac{\rho_{l,j}}{\varepsilon}e_{kij|l}+e_{kij,j}\right)\phi_{,k}+e_{kij}\phi_{,kj}+f_i=0, \tag{8}$$

$$\left(\frac{\rho_{s,i}}{\varepsilon}e_{ikl|s}+e_{ikl,i}\right)\left(u_{k,l}\right)+e_{ikl}u_{k,li}-\left(\frac{\rho_{s,i}}{\varepsilon}\kappa_{ij|s}+\kappa_{ij,i}\right)\phi_{,j}-\kappa_{ij}\phi_{,ji}=0, \tag{9}$$

with boundary conditions

$$u_i=u_i^0 \quad \text{on} \quad \Sigma_u,\quad \sigma_{ij}n_j=S_i^0 \quad \text{on} \quad \Sigma_t, \tag{10}$$

$$\phi=\phi_0,\quad \text{on}\quad \Sigma_\phi,\quad D_i n_i=-\tau_0,\quad \text{on}\quad \Sigma_\tau, \tag{11}$$

and interface conditions given by

$$\sigma_{ij}n_j=K_{ij}\left[\left[u_j\right]\right],\quad \left[\left[\sigma_{ij}n_j\right]\right]=0,\quad \text{on}\quad \Gamma, \tag{12}$$

$$D_i n_i=M\left[\left[\phi\right]\right],\quad \left[\left[D_i n_i\right]\right]=0,\quad \text{on}\quad \Gamma, \tag{13}$$

where $(\bullet)_{,j}=\partial(\bullet)/\partial x_j$ and $(\bullet)_{|j}=\partial(\bullet)/\partial y_j$.

The coefficients of the system (8)-(13) are rapidly oscillating functions. Following the methodology described in Chapter 6 of [3], the two-scales asymptotic homogenization method is used to derive the homogenized equilibrium problem.

3. ASYMPTOTIC HOMOGENIZATION METHOD

The solutions $[\mathbf{u},\phi]$ of the equilibrium problem (8)-(13) admits the following asymptotic expansions

$$\mathbf{u}=\mathbf{v}+\sum_{k=1}^{\infty}\varepsilon^k\left[\nabla^k\mathbf{v}:\mathbf{N}_{(k)}+\nabla^k\varphi:\mathbf{\Pi}_{(k)}\right],\quad \phi=\varphi+\sum_{k=1}^{\infty}\varepsilon^k\left[\nabla^k\mathbf{v}:\mathbf{\Psi}_{(k)}+\nabla^k\varphi:\mathbf{\Theta}_{(k)}\right], \tag{14}$$

where $\mathbf{v}\equiv\mathbf{v}(\mathbf{x})$ and $\varphi\equiv\varphi(\mathbf{x})$. The functions $\mathbf{N}_{(k)}\equiv\mathbf{N}_{(k)}(\mathbf{x},\mathbf{y})$, $\mathbf{\Pi}_{(k)}\equiv\mathbf{\Pi}_{(k)}(\mathbf{x},\mathbf{y})$, $\mathbf{\Psi}_{(k)}\equiv\mathbf{\Psi}_{(k)}(\mathbf{x},\mathbf{y})$ and $\mathbf{\Theta}_{(k)}\equiv\mathbf{\Theta}_{(k)}(\mathbf{x},\mathbf{y})$ are periodic with respect the micro-variable $\mathbf{y}$ functions, i.e. $\left\langle\mathbf{N}_{(k)}\right\rangle=\left\langle\mathbf{\Pi}_{(k)}\right\rangle=\left\langle\mathbf{\Psi}_{(k)}\right\rangle=\left\langle\mathbf{\Theta}_{(k)}\right\rangle=0$, where $\langle\cdot\rangle=1/|\mathbf{Y}|\int_{\mathbf{Y}}(\cdot)d\mathbf{y}$ denotes the average operator with respect the variable $\mathbf{y}$.

3.1. Local Problems

Substituting the expansions (14) into the equilibrium problem (8)-(13), a family of recurrent problems for the different power of ε and the local functions $\mathbf{N}_{(k)}$, $\mathbf{\Pi}_{(k)}$, $\mathbf{\Psi}_{(k)}$ and $\mathbf{\Theta}_{(k)}$ are derived. In order to avoid singularities when $\varepsilon\to 0$, the coefficients for ε^{-1} are equated to zero. This system of partial differential equations is known as the local problems and for a piezocomposite has the following form

$$\left(\rho_{t,j}C_{ijmn}+\rho_{r,l}C_{ijkl}\rho_{t,j}N^{mn}_{(1)k|r}+\rho_{t,j}e_{kij}\rho_{r,k}\Psi^{mn}_{(1)|r}\right)_{|t}=0, \tag{15}$$

$$\left(\rho_{t,j}e_{jmn}+\rho_{t,j}e_{jkl}\rho_{r,l}N^{mn}_{(1)k|r}-\rho_{t,j}\kappa_{jk}\rho_{r,k}\Psi^{mn}_{(1)|r}\right)_{|t}=0, \tag{16}$$

$$\left(\rho_{t,j}e_{mij}+e_{kij}\rho_{r,k}\Theta^{m}_{(1)|r}\rho_{t,j}+\rho_{t,j}C_{ijkl}\rho_{r,l}\Pi^{m}_{(1)k|r}\right)_{|t}=0, \tag{17}$$

$$\left(\rho_{t,j}e_{jkl}\rho_{r,l}\Pi^{m}_{(1)k|r}-\rho_{t,j}\kappa_{jm}-\rho_{t,j}\kappa_{jk}\rho_{r,k}\Theta^{m}_{(1)|r}\right)_{|t}=0, \tag{18}$$

with interface conditions

$$\left(C_{ijmn}+\rho_{r,l}C_{ijkl}N^{mn}_{(1)k|r}+e_{kij}\rho_{r,k}\Psi^{mn}_{(1)|r}\right)n_j=K_{ij}\left[\!\left[N^{mn}_{(1)j}\right]\!\right], \tag{19}$$

$$\left(C^{ijmn}\rho_{r,n}\Pi^{k}_{(11)m|r}+e^{kij}+e^{nij}\rho_{r,n}\Theta^{k}_{(11)|r}\right)n_j=K_{ij}\left[\!\left[\Pi^{k}_{(11)j}\right]\!\right], \tag{20}$$

$$\left(e^{ilk}+e^{imn}\rho_{r,m}N^{lk}_{(11)n|r}-k^{ij}\rho_{r,j}\Psi^{lk}_{(11)|r}\right)n_i=M\left[\!\left[\Psi^{lk}_{(11)}\right]\!\right], \tag{21}$$

$$\left(e^{ikn}\rho_{r,n}\Pi^{l}_{(11)k|r}-\kappa^{il}+\kappa^{ij}\rho_{r,j}\Theta^{l}_{(11)|r}\right)n_i=M\left[\!\left[\Theta^{l}_{(11)}\right]\!\right]. \tag{22}$$

The system of equations (15)-(22), satisfies the conditions of the Theorem 1, p. 346 of [4], that guarantees the existence of the solution of the system.

3.2. Effective coefficients and homogenized problem

The effective coefficients of the homogenized problem associated with (8)-(13) are obtained using the average operator $\langle\cdot\rangle$ to the coefficients for ε^0. The non-identically zero expressions lead to the following effective coefficients,

$$\hat{C}_{ipmn} = \left\langle C_{ipmn} + C_{ipkl}\rho_{r,l}N^{mn}_{(1)k|r} + e_{kip}\rho_{r,k}\Psi^{mn}_{(1)|r}\right\rangle, \tag{23}$$

$$\hat{e}_{min} = \left\langle e_{min} + e_{kin}\rho_{r,k}\Theta^{m}_{(1)|r} + C_{inkl}\rho_{r,l}\Pi^{m}_{(1)k|r}\right\rangle = \left\langle e_{min} + e_{mkl}\rho_{r,l}N^{in}_{(1)k|r} - \kappa_{mk}\rho_{r,k}\Psi^{in}_{(1)|r}\right\rangle, \tag{24}$$

$$\hat{\kappa}_{mn} = \left\langle \kappa_{mn} + \kappa_{nk}\rho_{r,k}\Theta^{m}_{(1)|r} - e_{nkl}\rho_{r,l}\Pi^{m}_{(1)k|r}\right\rangle. \tag{25}$$

The expressions (23)-(25) are known as the effective coefficients of the homogenized problem,

$$\left(\hat{C}_{ijkl}v_{k,l} + \hat{e}_{kij}\varphi_{,k}\right)_{,j} + f_i = 0, \qquad \left(\hat{e}_{ikl}v_{k,l} - \hat{\kappa}_{ij}\varphi_{,j}\right)_{,i} = 0, \tag{26}$$

with boundary conditions

$$v_i = u_i^0 \quad \text{on} \quad \Sigma_u, \quad \hat{\sigma}_{ij}n_j = S_i^0 \quad \text{on} \quad \Sigma_t, \tag{27}$$

$$\varphi = \phi_0, \quad \text{on} \quad \Sigma_\phi, \quad \hat{D}_i n_i = -\tau_0, \quad \text{on} \quad \Sigma_\tau, \tag{28}$$

where $\hat{\sigma}_{ij}$, $\hat{D}_i$ are the effective stress and effective electrical displacement respectively.

4. NUMERICAL CALCULATIONS

4.1. Piezoelectric with wavy geometry

In this section a piezoelectric wavy laminate composite is studied. The structure is periodically distributed along x_2. The function $\rho(x_1,x_2) = x_2 + H\sin(2\pi x_1)$ described the geometry of microstructure, where H represent the wave height, [6]. The surface Γ denotes the interface between the two components, Γ_1 and Γ_2 are the surfaces at the boundary of the unit cell Y, (see Figure 1).

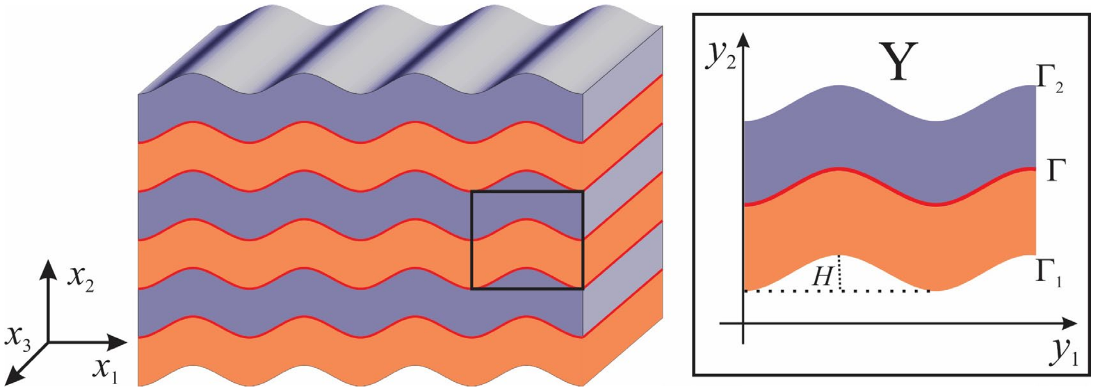

Figure 1 – Wavy piezocomposite with imperfect contact at the interface.

The piezocomposite is made of the piezoelectric material PZT-7A with symmetry 6mm and the other component is Epoxy. The mechanical properties of the elements are reported in [5], see Table 1.

Table 1 – Material properties of the composite constituents PZT-7A and Epoxy.

	C_{1111}	C_{1122}	C_{1133}	C_{3333}	C_{2323}	C_{1212}	e_{113}	e_{133}	e_{333}	κ_{11}	κ_{33}
PZT-7A	154.84	83.24	82.71	131.39	25.7	35.8	9.35	-2.12	9.52	4.07	2.08
Epoxy	8.0	4.4	4.4	8.0	1.8	1.8	-	-	-	0.037	0.037

In Figure 2, a comparison of the effective coefficients $\hat{C}_{1111}$, $\hat{C}_{1313}$, $\hat{e}_{113}$, $\hat{e}_{322}$ is shown. In order to illustrate the influence of the damage at the interface in the effective properties, two cases of imperfect contact are compared with the effective coefficients with perfect contact at the interface. The figure illustrates the effective coefficients considering imperfect contact for $K_{11} = K_{33} = M = 1$ (blue line) and $K_{11} = K_{33} = M = 100$ (red line); the perfect contact case (yellow line) is a limit case when the $\left|K_{ij}\right| \to \infty$ and $|M| \to \infty$. On the other hand, the figure shows the influence of the geometry and the function $\rho(x_1, x_2)$ in the effective properties.

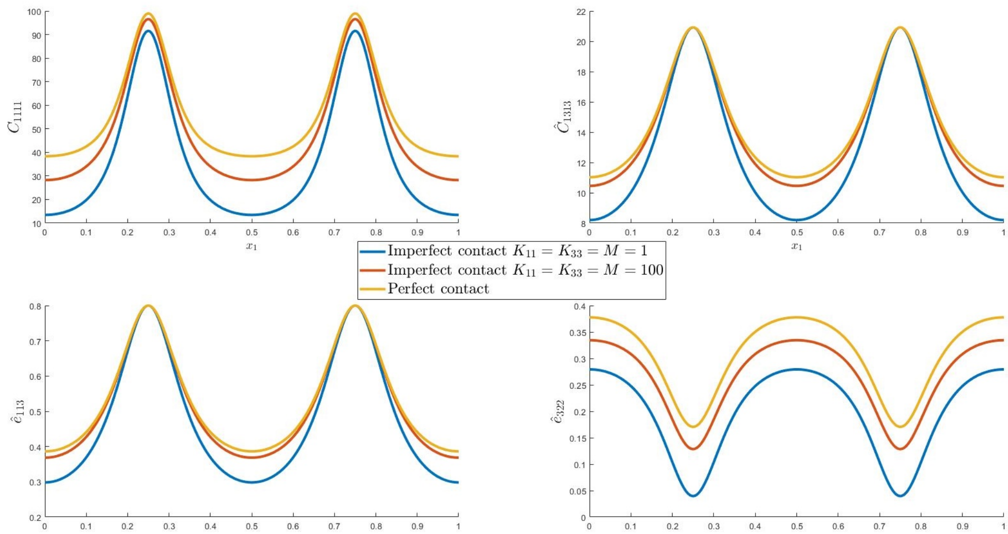

Figure 2 – Effective coefficients $\hat{C}_{1111}$, $\hat{C}_{1313}$, $\hat{e}_{113}$, $\hat{e}_{322}$ for a wavy piezocomposite considering imperfect contact at the interface.

4.2. Elastic rectangular composite with a crack at the interface

In this section, a three-dimensional elastic laminate shell composite with a crack at the interface are studied. The composite is made of aluminum and stainless steel, with Young's modulus $E_1 = 206.74$, $E_2 = 72.04$ and Poisson's ratio $\nu_1 = 0.3$, $\nu_2 = 0.35$ respectively. The volume fraction for aluminum is $V_1 = 0,8$ and for stainless steel is $V_1 = 0,2$. The layers are periodically distributing along x_3 and the properties of the materials are constant along x_2, see Figure 3.

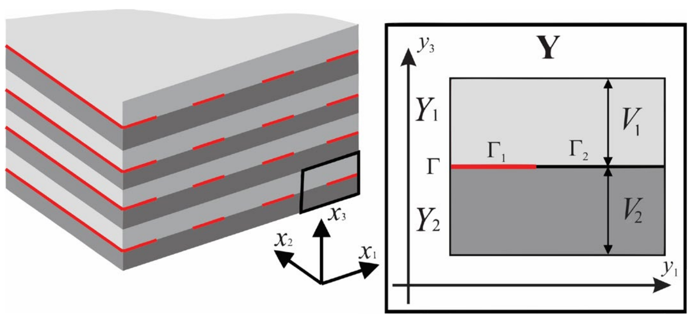

Figure 3 – Laminate elastic composite with cracks at the interface.

The periodic cell $\mathbf{Y}$ with the crack is compose by two main components Y_1 and Y_2 with thickness t_1 and t_2 respectively. The interface between the layers is denoted by $\Gamma = \Gamma_1 \cup \Gamma_2 = [0,1]$. In order to study the influence of the cracks in the effective properties, the spring type imperfect contact condition is considered at $\Gamma_1 = [0, 0.2]$ (crack) and perfect contact on the rest of the interface, $\Gamma_2 = (0.2, 1]$.

The layers Y_a are assume isotropic materials for $a = 1, 2$. For the points in Γ, the matrix $\mathbf{K}$ characterized the spring type imperfect contact at Γ_1. It has the expression $\mathbf{K} = [K_{ij}] = [40\mu\delta_{ij} + 40(\lambda + \mu)\delta_{i3}\delta_{j3}]$, where δ_{ij} is the Kronecker delta, $\mu = 0.5\mu_1 + 0.5\mu_2$, $\lambda = 0.5\lambda_1 + 0.5\lambda_2$; μ_1, λ_1 and μ_2, λ_2 are the Lame's constant of the elements Y_1 and Y_2 respectively. In Γ_2, the norm of the matrix $\left|K_{ij}\right| \to \infty$ (perfect contact).

In Table 2 and Table 3, a comparison between the effective coefficients $\hat{\mathbf{C}}$ obtained using the asymptotic homogenization method (AHM) with the results computed by finite elements method (FEM) is shown. A good coincidence between the solutions is appreciated. A detailed description of the model for an elastic composite is described in [1].

Table 2 – Effective coefficients $\hat{C}_{1111}$, $\hat{C}_{3333}$, $\hat{C}_{1122}$ for an elastic composite with crack at the interface.

$\hat{\mathbf{C}}$	$\hat{C}_{1111}$		$\hat{C}_{3333}$		$\hat{C}_{1122}$	
Interval	Γ_1	Γ_2	Γ_1	Γ_2	Γ_1	Γ_2
AHM	155.4646	192.8182	3.5191	163.4246	49.2677	86.6213
FEM	155.5800	192.8400	5.3548	163.4300	49.6000	86.6460

Table 3 – Effective coefficients $\hat{C}_{1133}$, $\hat{C}_{2323}$, $\hat{C}_{1212}$ for an elastic composite with crack at the interface.

$\hat{\mathbf{C}}$	$\hat{C}_{1133}$		$\hat{C}_{2323}$		$\hat{C}_{1212}$	
Interval	Γ_1	Γ_2	Γ_1	Γ_2	Γ_1	Γ_2
AHM	2.3111	78.9557	1.2848	39.9437	53.0984	53.0984
FEM	2.5894	79.0110	1.3174	39.9760	52.9920	53.0980

4.3. Elastic composite with non-uniform imperfect contact

In contrast to the composite presented in subsection 4.2, here an elastic composite with non-uniform imperfect contact condition is considered. The structure has the same elements, distribution of the layers and volume fraction that the composite material mentioned in subsection 4.2. However, for the new structure the imperfect contact matrix is characterized by a function $\tau(y_1)$.

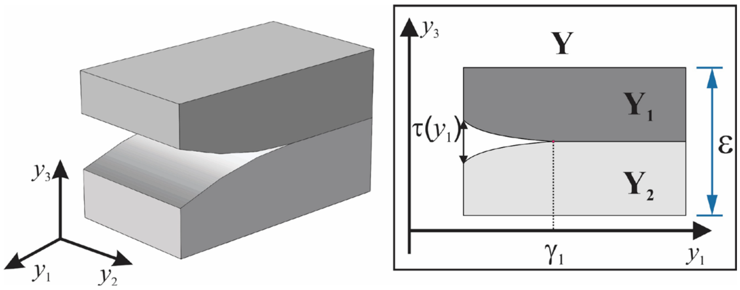

Figure 4 – Unit cell of an elastic composite with non-uniform imperfect contact at the interface.

In Figure 4, a three and a two-dimensional representations of the unit cell is shown. The parameter $\gamma_1 \in (0,1)$ is the endpoint of the imperfect contact at the interface, i.e. $\Gamma_1 = [0, \gamma_1]$. The function τ has the following expression

$$\tau(y_1) = \frac{\alpha}{\gamma_1^2}(y_1 - \gamma_1)^2 I_{[0,\gamma_1]}(y_1), \tag{29}$$

where $\alpha = \tau(0)$ is the maximum distance between Y_1 and Y_2, $I_{[0,\gamma_1]}(y_1) = 1$ if $y_1 \in \Gamma_1$ and $I_{[0,\gamma_1]}(y_1) = 0$ if $y_1 \in \Gamma_2$. The matrix $\mathbf{K}$ for the structure described in Figure 4, has the following form

$$\mathbf{K} = \frac{1}{\tau(y_1)}[\mu\delta_{ij} + (\lambda + \mu)\delta_{i3}\delta_{j3}]. \tag{30}$$

In Figure 5, a comparison between the effective coefficients $\hat{C}_{1111}, \hat{C}_{1122}, \hat{C}_{3333}, \hat{C}_{1313}$ of an elastic composite with non-uniform imperfect contact at the interface and different values of the parameter γ_1 is shown. The influence of the imperfection is appreciated when γ_1 moves between $(0,1)$.

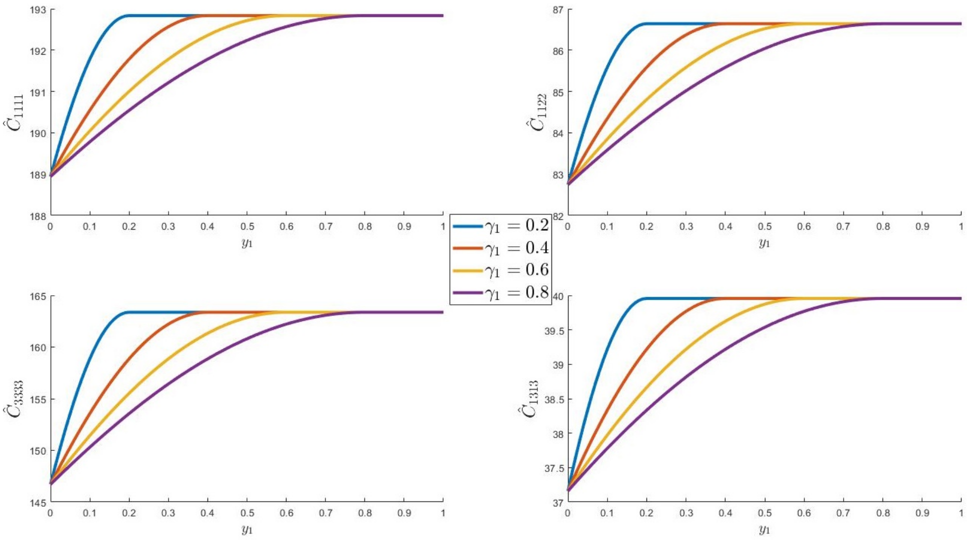

Figure 5 – Effective coefficients $\hat{C}_{1111}, \hat{C}_{1122}, \hat{C}_{3333}, \hat{C}_{1313}$ of an elastic composite with non-uniform imperfect contact at the interface and different values of the parameter γ_1.

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