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Hypotheses Testing based on a Corrected Score Function for Errors-in-Variables Models

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SUMMARY

In this paper, hypotheses testing based on a corrected score function are considered. Five different testing statistics are proposed and their asymptotic distributions are investigated. It is shown that the statistics are asymptotically distributed according to the chisquare distribution or can be written as a linear combination of chisquare random variables with one degree of freedom. A small scale numerical Monte Carlo study is presented in order to compare the empirical size and power of the proposed tests. A comparative calibration example is used to illustrate the results obtained.

Keywords: ASYMPTOTIC TESTS; COMPARATIVE CALIBRATION; CONSISTENT ESTIMATOR; MEASUREMENT ERROR; NAIVE TEST.

1. INTRODUCTION

Most studies in life sciences, biology, engineering, demography and economics involve covariates that can not be recorded exactly. Errors arise, most notably as measurement errors. Examples include a follow-up study of A-bomb survivors where the variable radiation received is measured with error (Okajima, Mine and Nakamura, 1985, Pierce et. al., 1992), amount of nitrogen in the soil in a study related to the yield of a certain grain (Fuller, 1987), biologic covariates, such as sistolic blood pressure, daily intake of saturated fat in the famous Framingham Heart prospective study dealing with cardiovascular disease (Gordon and Kannel, 1968). Frequently, interests are on assessing the statistical relationship between the unobserved covariates and the response.

The present paper is primary concerned with testing for association between the true covariates and the response variable. A simple approach considers the naive statistic test obtained from substituting the unobserved covariates by the observed ones. Tosteson and Tsiatis (1988) have compared the local power, assuming a general measurement error structure, of the naive score test and the optimal score test obtained by a flexible procedure in generalized linear models. Lagakos (1988) has also computed the efficiency loss for naive tests in univariate linear, Cox and logistic regression models. Stefanski and Carroll (1990) have considered Wald tests. They have compared the naive Wald test and a corrected naive Wald (Stefanski, 1985) assuming an additive measurement error structure.

Nakamura (1990) introduced an approach which allows the derivation of consistent and asymptotically normal estimators for the parameters of a linear or nonlinear measurement errors-in-variables model. Additional results on corrected score functions are established by Gimenez and Bolfarine (1997). We recall that most of the approaches considered for estimation in such models produces only approximate unbiased estimates, with no formal theorethical justification, such as the regression calibration (Carroll and Stefanski, 1990) or James-Stein (Whittemore, 1989) type estimators. This less biased estimators are used to avoid the attenuation problem typically associated with the naive or ordinary regression estimators. Resampling techniques are then required for obtaining the estimated standard errors associated with such estimates, making it difficult to obtain general valid asymptotic results to be used in conjunction with such estimators. Nakamura's approach allows its use in more general situations without making assumptions concerning the true covariates, having associated general expressions for the asymptotic covariance matrix. The main object of this paper which is associated to the derivation of asymptotically valid tests (Carroll et al., 1995, pp 207), is assured by the asymptotic distributions associated with the procedure. A review of the approach is considered in Section 2. In Section 3 the asymptotic tests are formally obtained by using the asymptotic properties of the estimators. Wald, score and likelihood type statistics are proposed. As shown, the Wald and score type statistics are asymptotically chisquare distributed and the likelihood type statistics is asymptotically distributed according to a linear combination of chisquare variables with one degree of freedom. A small scale numerical Monte Carlo study is presented in Section 4 comparing the asymptotic tests. The applicability of these results is illustrated in a comparative calibration model in Section 5.

2. CORRECTED SCORE ESTIMATOR APPROACH

Let $\mathbf{Z} = (\mathbf{z}_1^{'}, \ldots, \mathbf{z}_n^{'})'$ denote the matrix of independent variables (covariates), $\mathbf{Y} = (y_1, \ldots, y_n)'$ the vector of dependent variables and $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_p)'$ the p-dimensional vector of unknown parameters, lying in a parameter space Θ. The notation considered above is used for simplicity. However, more general situations where $\mathbf{Z}$ and $\mathbf{Y}$ are matrices, leading to multivariate models, for example, can be handled similarly. Moreover, let $L(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{Y})$ and $l(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{Y})$ be the likelihood and log-likelihood functions respectively, corresponding to the observed sample and

$$\mathbf{U}(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{Y}) = \frac{\partial l(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{Y})}{\partial \boldsymbol{\theta}} \quad \text{and} \quad \mathbf{I}(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{Y}) = -\frac{\partial \mathbf{U}(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{Y})}{\partial \boldsymbol{\theta}}, \quad \boldsymbol{\theta} \in \mathcal{F},$$

the score and observed information matrix, repectively, where $\mathcal{F}$ is a subspace of Θ. Let $\boldsymbol{\theta}_z$ be the maximum likelihood estimator of $\boldsymbol{\theta}$, that is, the value of $\boldsymbol{\theta}$ such that $\mathbf{U}(\boldsymbol{\theta}_z; \mathbf{Z}, \mathbf{Y}) = 0$ and $\boldsymbol{\theta}_0 \in \mathcal{F}$, be the true parameter value. Let $E^+(.)$ the expectation with respect to the vector $\mathbf{Y}$ given $\mathbf{Z}$. Under some regularity conditions the maximum likelihood estimator (MLE) is consistent and asymptotically normal. These important properties of the MLE is based strongly on the fact that $E^+\{\mathbf{U}(\boldsymbol{\theta}_0; \mathbf{Z}, \mathbf{Y})\} = \mathbf{0}$.

We are concerned with the situation that $\mathbf{Z}$ can not be recorded directly, but instead we observe a surrogate $\mathbf{X}$ having measurement error. Considering an additive error model

$$\mathbf{x}_i = \mathbf{z}_i + \mathbf{u}_i, \quad i = 1, \ldots, n$$

where the random error measurement $\mathbf{u}_1, \ldots, \mathbf{u}_n$, is independent of $\mathbf{Z}$ and $\mathbf{Y}$, has normal distribution with zero mean and covariance matrix Σ_u. This covariance matrix may be assumed known or estimated from replications of the $\mathbf{x}_i$s. Thus, calling $\mathbf{U}(\boldsymbol{\theta}; \mathbf{X}, \mathbf{Y})$ the naive score function, we have that, in general, $E\{\mathbf{U}(\boldsymbol{\theta}_0; \mathbf{X}, \mathbf{Y})\} \neq \mathbf{0}$, implying that $\boldsymbol{\theta}_x$ which solves $\mathbf{U}(\boldsymbol{\theta}; \mathbf{X}, \mathbf{Y}) = \mathbf{0}$ is not necessarily a consistent estimator of $\boldsymbol{\theta}$.

Nakamura (1990) considers a correction for score functions. The corrected score method depends on the existence of a corrected score function $\mathbf{U}^*(\boldsymbol{\theta}; \mathbf{X}, \mathbf{Y})$ such that

$$E\{\mathbf{U}^*(\boldsymbol{\theta}; \mathbf{X}, \mathbf{Y})/\mathbf{Y}, \mathbf{Z}\} = \mathbf{U}(\boldsymbol{\theta}; \mathbf{Z}, \mathbf{Y}), \tag{2.1}$$

for all $\mathbf{Y}, \mathbf{Z}$ and θ. Moreover, being $\mathbf{U}^*$ differentiable in $\mathcal{F}$, we have that

$$\mathbf{I}^*(\theta; \mathbf{X}, \mathbf{Y}) = -\frac{\partial \mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y})}{\partial \theta}.$$

It follows under the true model that $E\{\mathbf{U}^*(\theta_0; \mathbf{X}, \mathbf{Y})\} = 0$. Since $\mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y})$ is unbiased, regularity conditions ensures the existence of a consistent sequence of estimators $\hat{\theta}$ satisfying $\mathbf{U}^*(\hat{\theta}; \mathbf{X}, \mathbf{Y}) = 0$. The following proposition showing the asymptotic properties of $\hat{\theta}$ under various regularity conditions have been established in Gimenez and Bolfarine (1997).

Proposition 2.1. *Let $\mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y})$ be a function satisfying property (2.1). Under some regularity conditions, there exists a solution $\hat{\theta}$ of the system of equations $\mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y}) = 0$ which is consistent and asymptoticaly normal with mean θ_0 and covariance matrix $n^{-1}\Omega_n(\theta_0)$, where*

$$\Omega_n(\theta_0) = \bar{\Lambda}_n^{-1}(\theta_0)\bar{\Gamma}_n(\theta_0)\{\bar{\Lambda}_n^{-1}(\theta_0)\}', \tag{2.2}$$

with

$$\bar{\Lambda}_n(\theta) = \frac{1}{n}E\{\mathbf{I}^*(\theta; \mathbf{X}, \mathbf{Y})\} = \frac{1}{n}\sum_{i=1}^{n} E\{\mathbf{I}_i^*(\theta; \mathbf{X}, \mathbf{Y})\}$$

and

$$\bar{\Gamma}_n(\theta) = \frac{1}{n}\sum_{i=1}^{n} E\{\mathbf{U}_i^*(\theta; \mathbf{X}, \mathbf{Y})\mathbf{U}_i^*(\theta; \mathbf{X}, \mathbf{Y})'\}.$$

In cases where the corrected score function is obtained as $\mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y}) = \partial l^*(\theta; \mathbf{X}, \mathbf{Y})/\partial \theta$, where $l^*(\theta; \mathbf{X}, \mathbf{Y})$ is the corrected log-likelihood function such that $E\{l^*(\theta; \mathbf{X}, \mathbf{Y})/\mathbf{Z}, \mathbf{Y}\} = l(\theta; \mathbf{Z}, \mathbf{Y})$, for all $\mathbf{Y}, \mathbf{Z}$ and θ, the appropriate $\hat{\theta}$ would have been dictated by the maximum principle. On the other hand, it may be possible to find a corrected score function $\mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y})$ that can not be obtained as $\partial l^*/\partial \theta$ and then care must be taken when defining $\hat{\theta}$. The numerical application in Section 5 illustrates this last situation and presents a model for which the corrected score method can be employed without assuming $\mathbf{\Sigma}_u$ known or previously estimated.

The corrected score method depends critically on the assumed normality of the measurement error. Corrected score functions satisfying (2.1) may not exist and finding them may not be an easy task. These issues are studied in details in Stefanski (1989). In order to simplify notation, $\mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y})$, $\mathbf{I}^*(\theta; \mathbf{X}, \mathbf{Y})$ and $l^*(\theta; \mathbf{X}, \mathbf{Y})$ are denoted by just $\mathbf{U}^*(\theta)$, $\mathbf{I}^*(\theta)$ and $l^*(\theta)$ respectively, in the remaining of the paper.

3. TESTS BASED ON A CORRECTED SCORE FUNCTION

In this section, we consider five different test statistics for proving hypotheses of interest when the corrected score approach can be used. The asymptotic properties of the proposed statistics are based on the results established in the previous section. Lets consider the partition $\boldsymbol{\theta} = (\boldsymbol{\psi}', \boldsymbol{\lambda}')' \in \mathcal{R}^p$, where $\boldsymbol{\psi}$ is the $s \times 1$ parameter of interest, representing the object of the research and $\boldsymbol{\lambda}$ is the $(p-s) \times 1$ vector of nuisance parameters. Thus, given the n observations $(y_1, \mathbf{x}_1), \ldots, (y_n, \mathbf{x}_n)$, the main object is testing the null hypothesis $H_0 : \boldsymbol{\psi} = \boldsymbol{\psi}_0$, in the presence of the nuisance parameter vector $\boldsymbol{\lambda}$. It is considered that as n approaches infinity, the matrices $\bar{\boldsymbol{\Lambda}}_n(\boldsymbol{\theta})$ and $\bar{\boldsymbol{\Gamma}}_n(\boldsymbol{\theta})$ converge to positive definite matrices $\boldsymbol{\Lambda}(\boldsymbol{\theta})$ and $\boldsymbol{\Gamma}(\boldsymbol{\theta})$ respectively. Using notation typically associated with partitioned matrices, we write

$$\boldsymbol{\Lambda}(\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\Lambda}_{\psi\psi}(\boldsymbol{\theta}) & \boldsymbol{\Lambda}_{\psi\lambda}(\boldsymbol{\theta}) \\ \boldsymbol{\Lambda}_{\lambda\psi}(\boldsymbol{\theta}) & \boldsymbol{\Lambda}_{\lambda\lambda}(\boldsymbol{\theta}) \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Gamma}(\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\Gamma}_{\psi\psi}(\boldsymbol{\theta}) & \boldsymbol{\Gamma}_{\psi\lambda}(\boldsymbol{\theta}) \\ \boldsymbol{\Gamma}_{\lambda\psi}(\boldsymbol{\theta}) & \boldsymbol{\Gamma}_{\lambda\lambda}(\boldsymbol{\theta}) \end{pmatrix},$$

with the partitioning dimensions following the dimensions of $\boldsymbol{\psi}$ and $\boldsymbol{\lambda}$ respectively, that is, $\boldsymbol{\Lambda}_{\psi\psi}(\boldsymbol{\theta})$ is of dimension $s \times s$ and so on. In a similar fashion, we can write

$$\mathbf{U}^*(\boldsymbol{\theta}) = \begin{pmatrix} \mathbf{U}^*_{\psi}(\boldsymbol{\theta}) \\ \mathbf{U}^*_{\lambda}(\boldsymbol{\theta}) \end{pmatrix}.$$

Moreover, let $\hat{\boldsymbol{\theta}} = (\hat{\boldsymbol{\psi}}', \hat{\boldsymbol{\lambda}}')'$ and $\hat{\boldsymbol{\theta}}_0 = (\boldsymbol{\psi}_0', \hat{\boldsymbol{\lambda}}_0')'$, consistent estimators of $\boldsymbol{\theta}$ satisfying

$$\mathbf{U}^*(\hat{\boldsymbol{\theta}}) = \mathbf{0} \tag{3.1}$$

and

$$\mathbf{U}^*_{\lambda}(\hat{\boldsymbol{\theta}}_0) = \mathbf{0}. \tag{3.2}$$

The following testing statistics can be defined:

$$Q_e = n(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_0)'\hat{\boldsymbol{\Lambda}}_{\psi\psi.\lambda}(\hat{\boldsymbol{\theta}}_0)(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_0), \tag{3.3}$$

$$Q_c = n^{-1}\mathbf{U}^*_{\psi}(\hat{\boldsymbol{\theta}}_0)'\hat{\boldsymbol{\Lambda}}^{-1}_{\psi\psi.\lambda}(\hat{\boldsymbol{\theta}}_0)\mathbf{U}^*_{\psi}(\hat{\boldsymbol{\theta}}_0), \tag{3.4}$$

such that

$$\hat{\boldsymbol{\Lambda}}_{\psi\psi.\lambda}(\hat{\boldsymbol{\theta}}_0) = \hat{\boldsymbol{\Lambda}}_{\psi\psi}(\hat{\boldsymbol{\theta}}_0) - \hat{\boldsymbol{\Lambda}}_{\psi\lambda}(\hat{\boldsymbol{\theta}}_0)\hat{\boldsymbol{\Lambda}}^{-1}_{\lambda\lambda}(\hat{\boldsymbol{\theta}}_0)\hat{\boldsymbol{\Lambda}}_{\lambda\psi}(\hat{\boldsymbol{\theta}}_0), \tag{3.5}$$

$$W_e = n(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_0)'\hat{\boldsymbol{\Omega}}^{-1}_{\psi\psi}(\hat{\boldsymbol{\theta}}_0)(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_0) \tag{3.6}$$

and

$$W_c = n^{-1}\mathbf{U}^*_{\psi}(\hat{\boldsymbol{\theta}}_0)'\hat{\boldsymbol{\Lambda}}^{-1}_{\psi\psi.\lambda}(\hat{\boldsymbol{\theta}}_0)\hat{\boldsymbol{\Omega}}^{-1}_{\psi\psi}(\hat{\boldsymbol{\theta}}_0)\hat{\boldsymbol{\Lambda}}^{-1}_{\psi\psi.\lambda}(\hat{\boldsymbol{\theta}}_0)\mathbf{U}^*_{\psi}(\hat{\boldsymbol{\theta}}_0), \tag{3.7}$$

such that

$$\hat{\boldsymbol{\Omega}}(\hat{\boldsymbol{\theta}}_0) = \hat{\boldsymbol{\Lambda}}^{-1}(\hat{\boldsymbol{\theta}}_0)\hat{\boldsymbol{\Gamma}}(\hat{\boldsymbol{\theta}}_0)\hat{\boldsymbol{\Lambda}}^{-1}(\hat{\boldsymbol{\theta}}_0),$$

where $\hat{\Lambda}(\hat{\theta}_0)$ and $\hat{\Gamma}(\hat{\theta}_0)$ are consistent estimators of $\Lambda(\theta_0)$ and $\Gamma(\theta_0)$ respectively. In the special case where the corrected score is the gradient of the corrected likelihood $l^*(\theta)$, that is, $U^*(\theta) = \partial l^*(\theta)/\partial\theta$, it is also possible to define a likelihood ratio type statistic, which we write as

$$Q = 2\{l^*(\hat{\theta}) - l^*(\hat{\theta}_0)\}, \tag{3.8}$$

where $\hat{\theta}$ is a global maximum of $l^*(\theta)$ and $\hat{\theta}_0$ maximizes $l^*(\theta)$ under H_0. As shown next, Q_e, Q_c and Q are asymptotically equivalent. W_e and W_c are also shown to be asymptotically equivalent.

Theorem 3.1. *Let $\hat{\theta}$ and $\hat{\theta}_0$ consistent roots of equations in (3.1) and (3.2) respectively. Then, under some regularity conditions and H_0, it follows*
a) $Q_e \xrightarrow{D} \sum_{i=1}^{s} \mu_i V_i$, where $V_1, \ldots, V_s$ are iid chisquare random variables with one degree of freedom and $\mu_1, \ldots, \mu_s$ are the eigenvalues of the matrix

$$\Lambda_{\psi\psi.\lambda}(\theta_0)\Omega_{\psi\psi}(\theta_0). \tag{3.9}$$

Moreover, Q_e is asymptotically equivalent to Q_c and Q.
b) $W_e \xrightarrow{D} \chi^2_s$, where χ^2_s denotes a random variable distributed according to the chisquare distribution with s degrees of freedom. Moreover, W_e is asymptotically equivalent to W_c.

The proof can be found in the Appendix. The following remarks are direct consequences of the above results.

Remark 1. If the matrix in (3.9) is the identity matrix then $Q_e \xrightarrow{D} \chi^2_s$ as in the classical case. Thus, if the product $\Lambda(\theta_0)\Omega(\theta_0)$ is close to the identity matrix, then the asymptotic distribution of Q, Q_e and Q_c can be approximated by a χ^2_s distribution.

Remark 2. If $\Lambda(\theta_0)$ is a block diagonal matrix, that is, $\Lambda_{\psi\lambda}(\theta_0) = 0$ then (3.9) reduces to $\Gamma_{\psi\psi}(\theta_0)\Lambda^{-1}_{\psi\psi}(\theta_0)$. This is the case when $p - s = 0$, that is, no nuisance parameters are in the model.

Remark 3. If $\Lambda_{\psi\lambda}(\theta_0) = 0$ and $s = 1$, then (3.9) equals the ratio $\Gamma_{\psi\psi}(\theta_0)/\Lambda_{\psi\psi}(\theta_0)$ which can be seen as a correction factor of the usual χ^2_1 distribution.

Remark 4. In the estimation of matrices $\Gamma(\theta_0)$ and $\Lambda(\theta_0)$ any consistent estimator of θ_0 can be used. Notice that, if the statistic Q_c is used then the obvious choice would be $\hat{\theta}_0$, since in this case, the parametric model has only to be adjusted under H_0.

Remark 5. We need to compute quantiles of the distribution of $\sum_{i=1}^{s} \hat{\mu}_i V_i$, where $\hat{\mu}_i$ are the eigenvalues of the consistent estimator of the matrix $\Lambda_{\psi\psi.\lambda}(\theta_0)\Omega_{\psi\psi}(\theta_0)$. In the case where $p > 1$, some algorithms in Marazzi (1980) and Griffiths and Hill (1985) can be used. Another approach would be to simulate from the distribution of $\sum_{i=1}^{s} \hat{\mu}_i V_i$.

Remark 6. In the special case that $s = 1$, $\mu_1 = \Lambda_{\psi\psi.\lambda}(\theta_0)\Omega_{\psi\psi}(\theta_0)$, implies that

$$\frac{Q_e}{\hat{\mu}_1} = n\frac{(\hat{\psi} - \psi_0)^2}{\hat{\Omega}_{\psi\psi}(\hat{\theta}_0)} = W_e \quad \text{and} \quad \frac{Q_c}{\hat{\mu}_1} = \frac{n^{-1}\mathbf{U}^*_\psi(\hat{\theta}_0)^2}{\hat{\Lambda}_{\psi\psi.\lambda}(\hat{\theta}_0)\hat{\Omega}_{\psi\psi}(\hat{\theta}_0)} = W_c.$$

Thus, $Q_e/\hat{\mu}_1 = W_e$ and $Q_c/\hat{\mu}_1 = W_c$ are asymptotically distributed according to the χ^2_1 distribution.

4. SIMULATION STUDY

In this section we perform a Monte Carlo simulation study for comparing the empirical power and size of the test statistics Q, Q_e and Q_c presented in Section 3. The simulation study is based on an exponential regression model for lifetime data.

A set of independent random variables $\mathbf{T}' = (T_1, ..., T_{20})$ is generated for each repetition. $\mathbf{T}$ is a vector of realizations of an exponential distribution with parameter $exp(\alpha + \beta z)$ and the null hypothesis of interest is $\beta = 0$. The true covariate is generated as a standard normal and the error variable as a normal with mean 0 and variance σ^2_u. The parameter α in this study is set equal to zero and 1000 replications are run for each simulation. Note that according to the Remark 6 in Section 3, for this model, $W_e = Q_e/\hat{\mu}_1$ and $W_c = Q_c/\hat{\mu}_1$.

The simulations are performed for several values of the error variance ($\sigma^2_u = 0, 0.1, 0.3, 0.5$). Table 1 displays the empirical levels of the tests for a 5% nominal level. Even for a small sample of size 20 the empirical sizes of the likelihood ratio and score tests are very close to the nominal levels. The same does not happen with the Wald test.

An adjustment was performed using the empirical distribution of the statistics such that the empirical size is corrected to 5%. Figure 1 displays the empirical power of these three tests for the values $\sigma^2_u = 0, 0.1, 0.3, 0.5$, respectively. It can be observed that Wald test is losing more power than the other two as the error variance increases. It seems that likelihood ratio test has a better performance than the score test.

5. NUMERICAL APPLICATION

The results obtained in the previous sections are now illustrated by considering the comparative calibration problem which is presented in Jaech (1985). Such experiment aims at comparing different ways of measuring the same unknown quantity z in a group of n subjects. The densities of 43 cylindrical nuclear reactor fuel pellets of sintered uranium were measured by different methods. Full details of the the experiment can be found in the reference just mentioned. We use the data corresponding to three methods: Method 1, a geometric method based on weighting the pellet and finding its volume by

Table 1 - Empirical size for a 5% nominal level

σ_u^2	TRV	WALD	SCORE
0.0	5.6	12.0	6.6
0.1	5.7	12.2	6.9
0.3	5.6	12.6	6.8
0.5	4.4	10.4	6.4

measuring the pellet diameter and length and other two methods, named Method 2 and Method 3. The Method 1 measures the true quantity without bias. In particular, we want to test the hypothesis that the Methods 2 and 3 are measuring with no bias the quantity z.

A model usually considered in the literature for p different methods (Barnett, 1969, Kimura, 1992) is

$$\mathbf{y}_i = \boldsymbol{\alpha} + \boldsymbol{\beta} z_i + \boldsymbol{\epsilon}_i, \tag{5.1}$$

$$x_i = z_i + u_i, \tag{5.2}$$

for $i = 1, \ldots, n$, where $\mathbf{y}_i = (y_{i1}, \ldots, y_{ip})'$, $\boldsymbol{\epsilon}_i = (\varepsilon_{i1}, \ldots, \varepsilon_{ip})'$, are p-random vectors with $\boldsymbol{\epsilon}_i \overset{\text{iid}}{\sim} N(0, \Sigma)$, where $\Sigma = \operatorname{diag}(\sigma_1^2, \ldots, \sigma_p^2)$, $u_i \overset{\text{iid}}{\sim} (0, \sigma_u^2)$, $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_p)'$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)'$. Expression (5.1) considers that the additive and multiplicative bias corresponding to method k are α_k and β_k, respectively, $k = 1, \ldots, p$. Expression (5.2) considers that one of the methods measures the unknown quantity z_i, corresponding to the i^{th} individual without bias. In order to make the model identifiable, we consider the situation where $\lambda_k = \sigma_k^2/\sigma_u^2$, $k = 1, \ldots, p$, are known and taken without loss of generality equals to one, $i = 1, \ldots, n$, that is, $\sigma_k^2 = \sigma_u^2 = \phi$, $k = 1, \ldots, p$.

Interest centers on testing the hypothesis that the methods are measuring the quantity z without bias, which means that

$$H_0 : \begin{pmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{1}_p \end{pmatrix}.$$

where $\mathbf{1}_p = (1, \ldots, 1)'$ and we can use the Wald or the score statistics based on the corrected score function defined in (3.6) and (3.7) respectively. In order to evaluate the expressions of these statistics we have initially to obtain the corrected score estimator and its corresponding asymptotic covariance matrix. Maximum likelihood estimation is discussed in Kimura (1992) and Bolfarine and Galeas-Rojas (1995).

The unobserved log-likelihood for the model (5.1)-(5.2) can be written as

$$l(\theta; \mathbf{Z}, \mathbf{Y}) = \sum_{i=1}^{n} \ell_i(\theta; z_i, \mathbf{y}_i)$$

where $\ell_i(\theta; z_i, \mathbf{y}_i) = -\frac{p}{2}\log(2\pi) - \frac{p}{2}\log\phi - \frac{1}{2\phi}(\mathbf{y}_i - \alpha - \beta z_i)'(\mathbf{y}_i - \alpha - \beta z_i)$, with $\theta = (\alpha', \beta', \phi)'$ of dimension $2p+1$, leading to $\mathbf{U}_i(\theta) = (\mathbf{U}_{i\alpha}(\theta)', \mathbf{U}_{i\beta}(\theta)', U_{i\phi}(\theta))'$ with

$$\begin{aligned}
\mathbf{U}_{i\alpha}(\theta) &= \frac{\partial \ell_i(\theta)}{\partial \alpha} = \frac{1}{\phi}(\mathbf{y}_i - \alpha - \beta z_i),\\
\mathbf{U}_{i\beta}(\theta) &= \frac{\partial \ell_i(\theta)}{\partial \beta} = \frac{1}{\phi}(\mathbf{y}_i - \alpha - \beta z_i) z_i,\\
U_{i\phi}(\theta) &= \frac{\partial \ell_i(\theta)}{\partial \phi} = -\frac{p}{2\phi} + \frac{1}{2\phi^2}(\mathbf{y}_i - \alpha - \beta z_i)'(\mathbf{y}_i - \alpha - \beta z_i),
\end{aligned}$$

for $i = 1, \ldots, n$. Thus, considering $\mathbf{U}_i^*(\theta) = (\mathbf{U}_{i\alpha}^*(\theta)', \mathbf{U}_{i\beta}^*(\theta)', U_{i\phi}^*(\theta))'$ with

$$\mathbf{U}_{i\alpha}^*(\theta) = \frac{1}{\phi}(\mathbf{y}_i - \alpha - \beta x_i), \tag{5.3}$$

$$\mathbf{U}_{i\beta}^*(\theta) = \frac{1}{\phi}(\mathbf{y}_i - \alpha - \beta x_i) x_i + \beta, \tag{5.4}$$

$$U_{i\phi}^*(\theta) = \frac{1}{2\phi^2}(\mathbf{y}_i - \alpha - \beta x_i)'(\mathbf{y}_i - \alpha - \beta x_i) - \frac{1}{2\phi}(\beta'\beta + p), \tag{5.5}$$

for $i = 1, \ldots, n$. It follows that

$$E[\mathbf{U}_i^*(\theta; x_i, \mathbf{y}_i) | z_i, \mathbf{y}_i] = \mathbf{U}_i(\theta; z_i, \mathbf{y}_i), \; i = 1, \ldots, n,$$

which implies that $\mathbf{U}^*(\theta; \mathbf{X}, \mathbf{Y}) = \sum_{i=1}^{n} \mathbf{U}_i^*(\theta; x_i, \mathbf{y}_i)$ is a corrected score function. It can be observed that,

$$\frac{\partial \ell_i^*(\theta)}{\partial \phi} = -\frac{p}{2\phi} + \frac{1}{2\phi^2}(\mathbf{y}_i - \alpha - \beta x_i)'(\mathbf{y}_i - \alpha - \beta x_i) \neq U_{i\phi}^*(\theta).$$

implying that the corrected score function can not be obtained throught differentiating the corrected log-likelihood.

Solving $\mathbf{U}^*(\theta) = 0$, the corrected score estimator $\hat{\theta} = (\hat{\alpha}', \hat{\beta}', \hat{\phi})'$ is obtained as the solution to the following system of equations:

$$\alpha = \bar{\mathbf{y}} - \beta \bar{x}, \tag{5.6}$$

$$\mathbf{S}_{xy} - \beta S_{xx} + \beta\phi = 0, \tag{5.7}$$

$$\phi = \frac{1}{(\beta'\beta + p)} \sum_{k=1}^{p} (S_{y_k y_k} + \beta_k^2 S_{xx} - 2\beta_k S_{xy_k}), \tag{5.8}$$

where

$$\bar{\mathbf{y}} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{y}_i, \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

$$\mathbf{S}_{xy} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(\mathbf{y}_i - \bar{\mathbf{y}}), \quad S_{xx} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2,$$

$$\mathbf{S}_{yy} = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{y}_i - \bar{\mathbf{y}})(\mathbf{y}_i - \bar{\mathbf{y}})'.$$

The solution of the system of equations (5.6)-(5.8) can be obtained easily by using an iterative method. Let $\widehat{\beta}^{(m)} = (\widehat{\beta}_1^{(m)}, \ldots, \widehat{\beta}_p^{(m)})'$ be the solution of (5.7) at step m, in the step $m+1$ the algorithm proceeds as follows:

1: Find $\widehat{\phi}^{(m+1)} = \phi(\widehat{\beta}^{(m)})$, according to (5.8).

2: Find $\widehat{\beta}_k^{(m+1)}$, $k = 1, \ldots, p$, as the solution to (5.7), that is,

$$\widehat{\beta}_k^{(m+1)} = \frac{S_{xy_k}}{S_{xx} - \widehat{\phi}^{(m+1)}}, \quad k = 1, \ldots, p.$$

An estimate of α follows directly from (5.6).

The system of equations (5.6)-(5.8) has multiple roots. Therefore a careful choice must be made in order to pick a consistent root. Taking the naive estimate as an initial guess, an appropriate root is usually obtained after two or three steps in the numerical process (Stefanski, 1989).

Under the assumptions that

$$\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i \to \mu < \infty \quad \text{and} \quad \frac{1}{n}\sum_{i=1}^{n} (z_i - \bar{z})^2 \to \nu^2 < \infty, \tag{5.9}$$

it follows after some algebra that

$$\overline{\Lambda}_n(\theta) \to \Lambda(\theta) \quad \text{and} \quad \overline{\Gamma}_n(\theta) \to \Gamma(\theta),$$

with

$$\Lambda(\theta) = \frac{1}{\phi}\begin{pmatrix} \mathbf{A} \otimes \mathbf{I}_p & (0,-1)' \otimes \beta \\ 0 & \frac{\beta'\beta + p}{2\phi} \end{pmatrix}, \tag{5.10}$$

$$\Gamma(\theta) = \frac{1}{\phi}\begin{pmatrix} (\mathrm{A}+2\mathrm{B})\beta\beta' + (\mathrm{A}+\mathrm{B})\mathrm{I}_p & (\mathrm{C}\otimes\beta) \\ (\mathrm{C}'\otimes\beta') & \frac{1}{2}(\beta'\beta)^2 + \beta'\beta + \frac{p}{2} \end{pmatrix},$$

where

$$\mathrm{A} = \begin{pmatrix} 1 & \mu \\ \mu & \nu^2+\mu^2 \end{pmatrix}, \quad \mathrm{B} = \begin{pmatrix} 0 & 0 \\ 0 & \phi \end{pmatrix}, \quad \mathrm{C} = -\begin{pmatrix} 0 \\ \beta'\beta+1 \end{pmatrix},$$

and I_p denotes the p-dimensional identity matrix. It follows from Proposition 2.1 that $\widehat{\theta} = (\widehat{\alpha}', \widehat{\beta}', \widehat{\phi})'$, solution of (5.6)-(5.8), is consistent and asymptotically normal with a mean vector $\theta_0 = (\alpha_0', \beta_0', \phi_0)'$ and covariance matrix $n^{-1}\Omega_n = n^{-1}\overline{\Lambda}_n^{-1}\overline{\Gamma}_n\overline{\Lambda}_n^{-1}$.

According to the notation of Section 3, $\psi = (\alpha', \beta')'$ is the parameter of interest and $\lambda = \phi$ is the nuisance parameter. $\widehat{\theta} = (\widehat{\alpha}', \widehat{\beta}', \widehat{\phi})'$ is the unrestricted corrected score estimator, that is, is the solution of the system of equations (5.6)-(5.8) and $\widehat{\theta}_0 = (0', 1_p', \widehat{\phi}_0)'$ is the restricted estimator under H_0, such that, from (5.8)

$$\widehat{\phi}_0 = \frac{1}{2p}\sum_{k=1}^{p}(S_{y_k y_k} + S_{xx} - 2S_{xy_k}).$$

The Wald statistic W_e can be written as

$$W_e = n[\widehat{\alpha}', (\widehat{\beta} - 1_p)']\widehat{\Omega}_{\psi\psi}^{-1}(\widehat{\theta}_0)[\widehat{\alpha}', (\widehat{\beta} - 1_p)']',$$

where $\widehat{\Omega}_{\psi\psi}^{-1}$ is an estimator of the matrix $\Omega_{\psi\psi}^{-1}$. After some algebraic manipulations, $\Omega_{\psi\psi}^{-1}$ can be rewritten as

$$\Omega_{\psi\psi}^{-1}(\theta) = \frac{1}{\phi}\begin{pmatrix} (\mathrm{I}_p+\beta\beta')^{-1} & \mu(\mathrm{I}_p+\beta\beta')^{-1} \\ \mu(\mathrm{I}_p+\beta\beta')^{-1} & \mu^2(\mathrm{I}_p+\beta\beta')^{-1} + \frac{\nu^4}{\nu^2+\phi}(\mathrm{I}_p + r\beta\beta')^{-1} \end{pmatrix}, \tag{5.11}$$

where

$$r = \frac{1}{\nu^2+\phi}\left[\nu^2 + \frac{2p(p-1)\phi}{(\beta'\beta+p)^2}\right].$$

In order to obtain a consistent estimator of $\Omega_{\psi\psi}^{-1}$ it is enough to get consistent estimators of μ and ν^2. Under the assumptions (5.9) and using the weak law of large numbers,

$$\bar{x} \xrightarrow{\mathrm{P}} \mu \quad \mathrm{e} \quad S_{xx} - \phi \xrightarrow{\mathrm{P}} \nu^2, \text{ as } n \to \infty. \tag{5.12}$$

Therefore, $\Omega_{\psi\psi}$ can be consistently estimated. Thus, W_e can be rewritten as

$$\begin{aligned} W_e &= \frac{n}{\widehat{\phi}_0}\{\widehat{\alpha}'(\mathrm{I}_p + 1_p 1_p')^{-1}\widehat{\alpha} + 2\bar{x}\widehat{\alpha}'(\mathrm{I}_p + 1_p 1_p')^{-1}(\widehat{\beta} - 1_p) \\ &\quad +(\widehat{\beta} - 1_p)'[\bar{x}^2(\mathrm{I}_p + 1_p 1_p')^{-1} + \frac{(S_{xx} - \widehat{\phi}_0)^2}{S_{xx}}(\mathrm{I}_p + \widehat{r}1_p 1_p')^{-1}](\widehat{\beta} - 1_p)\} \end{aligned} \tag{5.13}$$

where $\hat{r} = 1 - \frac{(p+1)-\hat{\phi}_0}{2pS_{xx}}$.

The score statistic W_c is given by

$$W_c = n^{-1}\mathbf{U}^*_\psi(\hat{\theta}_0)'\hat{\Lambda}^{-1}_{\psi\psi.\phi}(\hat{\theta}_0)\hat{\Omega}^{-1}_{\psi\psi}(\hat{\theta}_0)\hat{\Lambda}^{-1}_{\psi\psi.\phi}\mathbf{U}^*_\psi(\hat{\theta}_0),$$

where from (5.3) and (5.4),

$$\mathbf{U}^*_\psi(\hat{\theta}_0) = \frac{n}{\hat{\phi}_0}\begin{pmatrix} \bar{\mathbf{y}} - \mathbf{1}_p\bar{x} \\ \mathbf{S}_{xy} + \bar{x}\,\bar{\mathbf{y}} - \mathbf{1}_p(S_{xx} - \hat{\phi}_0 + \bar{x}^2) \end{pmatrix}, \tag{5.14}$$

and from (5.10),

$$\Lambda^{-1}_{\psi\psi.\phi} = \phi(\mathbf{A}^{-1} \otimes \mathbf{I}_p), \tag{5.15}$$

where $\mathbf{A}^{-1} = \frac{1}{\nu^2}\begin{pmatrix} \nu^2 + \mu^2 & -\mu \\ -\mu & 1 \end{pmatrix}$.

W_c can be rewritten, according to (5.11), (5.12), (5.14) and (5.15), as

$$\begin{aligned} W_c \;=\; & \frac{n}{\hat{\phi}_0}\Big\{(\bar{\mathbf{y}} - \mathbf{1}_p\bar{x})'\left[(\mathbf{I}_p + \mathbf{1}_p\mathbf{1}_p')^{-1} + \frac{\bar{x}^2}{S_{xx}}(\mathbf{I}_p + \hat{r}\mathbf{1}_p\mathbf{1}_p')^{-1}\right](\bar{\mathbf{y}} - \mathbf{1}_p\bar{x}) \\ & -\frac{2\bar{x}}{S_{xx}}[\mathbf{S}_{xy} + \bar{x}\,\bar{\mathbf{y}} - \mathbf{1}_p(S_{xx} - \hat{\phi}_0 + \bar{x}^2)]'(\mathbf{I}_p + \hat{r}\mathbf{1}_p\mathbf{1}_p')^{-1}(\bar{\mathbf{y}} - \mathbf{1}_p\bar{x}) \\ & +\frac{1}{S_{xx}}[\mathbf{S}_{xy} + \bar{x}\,\bar{\mathbf{y}} - \mathbf{1}_p(S_{xx} - \hat{\phi}_0 + \bar{x}^2)]'(\mathbf{I}_p + \hat{r}\mathbf{1}_p\mathbf{1}_p')^{-1}[\mathbf{S}_{xy} + \bar{x}\,\bar{\mathbf{y}} \\ & -\mathbf{1}_p(S_{xx} - \hat{\phi}_0 + \bar{x}^2)]\Big\}. \end{aligned} \tag{5.16}$$

Returning to our comparative calibration problem with $n = 43$ and $p = 2$, we have the following statistics

$$\begin{aligned} &\bar{x} = 4.397, \quad \bar{\mathbf{y}} = (4.370, 4.436)' \\ &S_{xx} = 0.04391, \quad \mathbf{S}_{xy} = (0.03449, 0.03933)' \quad \text{and} \\ &\mathbf{S}_{yy} = \begin{pmatrix} 0.04013 & 0.03483 \\ & 0.06442 \end{pmatrix}. \end{aligned}$$

Solving the system of equations (5.6)-(5.8), we obtain the corrected score estimates:

$$\begin{aligned} \hat{\alpha} &= (-0.21963, -0.79769)', \\ \hat{\beta} &= (1.04381, 1.19029)' \quad \text{and} \\ \hat{\phi} &= 0.01397. \end{aligned}$$

The values of the Wald and score statistics given by (5.13) and (5.16) respectively, are $W_e = 10.46$ and $W_c = 10.62$, respectively. Comparing with a chi-squared with four degrees of freedom, we can conclude at a 5% level that Methods 2 and 3 are measuring with bias the density of cylindrical nuclear reactor fuel pellets of sintered uranium.

APPENDIX: Proof of Theorem 3.1

Replacing $\hat{\mathbf{\Lambda}}(\hat{\boldsymbol{\theta}}_0)$ and $\hat{\mathbf{\Gamma}}(\hat{\boldsymbol{\theta}}_0)$ by $\mathbf{\Lambda}(\boldsymbol{\theta}_0)$ and $\mathbf{\Gamma}(\boldsymbol{\theta}_0)$ (denoted just by $\mathbf{\Lambda}$ and $\mathbf{\Gamma}$ to simplify notation), in the expressions for Q_e, Q_c, W_e and W_c, asymptotically equivalent statistics are obtained. We denote these statistics by Q_e^*, Q_c^*, W_e^* and W_c^*, respectively, and find their asymptotic distributions.

The proof of a) is considered first. A Taylor series expasion of $\mathbf{U}^*(\boldsymbol{\theta})$ about the point $\hat{\boldsymbol{\theta}}$ yields

$$\frac{1}{\sqrt{n}}\mathbf{U}^*(\boldsymbol{\theta}_0) = \frac{1}{\sqrt{n}}\mathbf{U}^*(\hat{\boldsymbol{\theta}}) + \mathbf{I}_n^*(\boldsymbol{\theta}^*)\sqrt{n}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0), \tag{A.1}$$

where $\boldsymbol{\theta}^*$ is such that $||\boldsymbol{\theta}^* - \boldsymbol{\theta}_0|| < ||\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0||$. Under some regularity conditions

$$\bar{\mathbf{I}}_n^*(\boldsymbol{\theta}^*) \xrightarrow{P} \mathbf{\Lambda}. \tag{A.2}$$

Moreover, since

$$\sqrt{n}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) = O_p(1) \tag{A.3}$$

(see Proposition 2.1), if follows from (3.1), (A.1), (A.2) and (A.3) that

$$\frac{1}{\sqrt{n}}\mathbf{U}^*(\boldsymbol{\theta}_0) = \mathbf{\Lambda}\sqrt{n}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) + o_p(1). \tag{A.4}$$

From (A.4) and a similar expansion about $\hat{\boldsymbol{\theta}}_0$, it can be verified that

$$\hat{\boldsymbol{\lambda}}_0 = \hat{\boldsymbol{\lambda}} + \mathbf{\Lambda}_{\lambda\lambda}^{-1}\mathbf{\Lambda}_{\lambda\psi}(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_0). \tag{A.5}$$

Thus, it follows that

$$\sqrt{n}(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}_0) = \frac{1}{\sqrt{n}}\mathbf{\Lambda}_{\psi\psi.\lambda}^{-1}\mathbf{U}_\psi^*(\hat{\boldsymbol{\theta}}_0) + o_p(1), \tag{A.6}$$

with $\mathbf{\Lambda}_{\psi\psi.\lambda}$ as defined in (3.5). Moreover, replacing (A.6) in the expression for Q_e^*, it follows that

$$Q_e^* = n^{-1}\mathbf{U}_\psi^*(\hat{\boldsymbol{\theta}}_0)'\mathbf{\Lambda}_{\psi\psi.\lambda}^{-1}\mathbf{U}_\psi^*(\hat{\boldsymbol{\theta}}_0) + o_p(1),$$

that is, $Q_e^* = Q_c^* + o_p(1)$, so that Q_e^* and Q_c^* are asymptotically equivalent.

Now, if $\mathbf{U}^*(\boldsymbol{\theta}) = \partial l^*(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ then a Taylor series expansion of $l^*(\boldsymbol{\theta}_0)$ about $\hat{\boldsymbol{\theta}}$ yields

$$2\{l^*(\boldsymbol{\theta}_0) - l^*(\hat{\boldsymbol{\theta}})\} = -n(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0)'\bar{\mathbf{I}}_n^*(\boldsymbol{\theta}^*)(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0),$$

where θ^* is such that $||\theta^* - \theta_0|| < ||\hat{\theta} - \theta_0||$. Thus, (A.2) and (A.3) implies that

$$2\{l^*(\theta_0) - l^*(\hat{\theta})\} = -n(\hat{\theta} - \theta_0)'\Lambda(\hat{\theta} - \theta_0) + o_p(1). \qquad (A.7)$$

From (A.7) and the analogous expansion about $\hat{\theta}_0$, it follows that

$$Q = 2\{l^*(\hat{\theta}) - l^*(\hat{\theta}_0)\} = n(\hat{\theta} - \theta_0)'\Lambda(\hat{\theta} - \theta_0)$$
$$-n(\hat{\lambda}_0 - \lambda_0)'\Lambda_{\lambda\lambda}(\hat{\lambda}_0 - \lambda_0) + o_p(1). \qquad (A.8)$$

Replacing (A.5) in (A.8) and considering that $\Lambda_{\lambda\psi} = \Lambda'_{\psi\lambda}$, we can write

$$Q = n(\hat{\psi} - \psi_0)'\Lambda_{\psi\psi.\lambda}(\hat{\psi} - \psi_0) + o_p(1).$$

Thus, $Q = Q_e^* + o_p(1)$, that is, Q and Q_e^* are asymptotically equivalent. In order to derive the distribution of these statistics it is easier to deal with Q_e^*. From Proposition 2.1, it follows that

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{D} N_p(0, \Omega),$$

so that, in particular

$$\sqrt{n}(\hat{\psi} - \psi_0) \xrightarrow{D} N_s(0, \Omega_{\psi\psi}). \qquad (A.9)$$

Then, a) follows from (A.9) and some quadratic forms results for the multivariate normal distribution (Rao, 1973).

To prove b), it follows by using (A.9) that

$$W_e^* = n(\hat{\psi} - \psi_0)'\Omega_{\psi\psi}^{-1}(\hat{\psi} - \psi_0) \xrightarrow{D} \chi_s^2.$$

Moreover, it can be shown that the statistic

$$W_c^* = n^{-1}\mathbf{U}_\psi^*(\hat{\theta}_0)'\Lambda_{\psi\psi.\lambda}^{-1}\Omega_{\psi\psi}^{-1}\Lambda_{\psi\psi.\lambda}^{-1}\mathbf{U}_\psi^*(\hat{\theta}_0)$$

is asymptotically equivalent to W_e^*.

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