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ESTIMATING SYSTEM'S COMPONENT RELIABILITY IMPORTANCE UNDER DEPENDENCE CONDITIONS

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Summary

Using martingale notions and methods Bueno (2000) gives a new measure of reliability importance of system's component under dependence conditions. The measure is given in terms of compensator transform and, for example, corresponds to either a minimal repair or a parallel improvement of the component. In this paper we give a martingale estimator for such quantities.

1.Introduction

Birnbaum (1969) defined the importance of a component in a system (essentially) as follows. Let S and T denote the random life lengths of the component and the system respectively. Then the importance of S for T at time t is

$$I^B(S,t) = P(T > t|S > t) - P(T > t|S \leq t).$$

In the context of coherent systems we denote by $S_i, \quad i = 1, ..., n$ the life lengths of the components . As in Barlow and Proschan (1981) we assume that the system life length is given as

$$T = \Phi(\mathbf{S}) = \min_{1 \leq j \leq k} \max_{i \in K_j} S_i,$$

where $K_j, 1 \leq j \leq k$ are minimal cut sets, that is, a minimal set of components whose joint failure causes the system to fail.

This measure depends on a given point t in time and it is not quite relevant for most design or redesign decisions. Several time independent importance measures have been suggested, and most of them are weighted integrals of $I^B(S,t)$ over t (e.g., Barlow and Proschan (1975), Natvig (1985); for a survey see Boland and El-Neweihi (1990)).

Bergman (1985) pointed out that many importance measures in reliability theory can be obtained through the study of the change of the system expected life length due to different variations on component life length distributions. Assume that the components are independent, and let F_i and G_i, denote the original and modified distribution of component i, respectively. Then the importance of component i with respect to this "component improvement" is given as

$$\int_0^\infty (\overline{G}_i(t) - \overline{F}_i(t)) I^B(S_i, t) dt.$$

(We apply the usual notation $\overline{F}(t) = 1 - F(t)$). For example, in the case of Natvig's importance measure, the improvement is obtained through a minimal repair of the component.

As in Jacod (1975), Bueno and Norros (1992) and Bueno (2000) considered a bijective correspondence between the joint component life lengths $(S_1, ..., S_n)$ and its $\Im_t$-compensators at its final points, $A(\mathbf{S}) = (\mathbf{A_1(S_1)}, ..., \mathbf{A_n(S_n)})$, which are independent and identically distributed random variables with standard exponential distribution, no matter how dependent the component life lengths are and what the history, as long as simultaneous failures are ruled out, outside a null set. Bueno (2000) gives an component importance concept through compensator transform. Such importance measure is defined under a new probability measure and in what follows, using martingale methods, we propose a martingale estimator to these quantities. In Section 2 we give the necessary preliminary and in Section 3 we propose the martingale estimator.

2. The mathematical formulation

We consider the vector $\mathbf{S} = (S_1, S_2, ..., S_n)$ representing the component life lengths of a technical system which are positive random variables in a complete probability space $(\Omega, \Im, P)$. As introduced in Norros (1986), we associate with $\mathbf{S}$ the corresponding failure process, which is defined as follows. For $j = 1, ..., n$ let

$$T_0 = 0, \quad J_0 = \emptyset;$$

$$T_j = \inf\{S_i : 1 \leq i \leq n, S_i > T_{j-1}\};$$

$$J_j = \{i : S_i = T_j\}, \quad j \geq 1;$$

$$S_I = \inf\{T_j : J_j = I\};$$

$$N_I(t) = 1_{\{S_I \leq t\}}, \quad t \geq 0, \quad \emptyset \neq I \subseteq \{1, ..., n\};$$

$$\Im_t = \sigma\{N_I(s) : s \leq t, \emptyset \neq I \subseteq \{1, ..., n\}.$$

$(\Im_t)_{t\geq 0}$ represents our observations in time and is a family of sub-σ-algebras of $\Im$ which is increasing, right continuous and completed (shortly: satisfies Dellacherie's usual conditions). The marked point process (T_j, J_j) is called the failure process associated to $\mathbf{S}$.

For each I, $(N_I(t))_{t\geq 0}$ is a uniformly integrable $(P, \Im_t)$-submartingale and from Doob Meyer decomposition, there exists a unique right continuous predictable nondecreasing and integrable process $(A_I(t))_{t\geq 0}$, with $A_I(0) = 0$ and such that $(N_I(t) - A_I(t))_{t\geq 0}$ is a uniformly square integrable $(P, \Im_t)$-martingale.

$A_I(t)$ is called the $(P, \Im_t)$-compensator of the counting process $N_I(t)$. The compensator process is expressed in terms of the conditional probability, given the available information and generalize the classical notion of hazards. Intuitively, this corresponds to producing whether the failure is going to occur now, on the basis of all observations available up to, but not including, the present.

Here we shall restrict ourselves to a relatively simple special case. Throughout the rest of this paper we assume that the following conditions hold:

A) $P(0 < S_i < \infty) = 1, \quad i = 1, ..., n;$

B) For all i, j with $i \neq j, P(S_i = S_j) = 0;$

C) All compensators A_I are continuous.

Assumption B implies that if I contains more than one element, then $A_I(t) = 0$. Thus we have only to consider n compensators

$$A_i = A_{\{i\}}, \quad i = 1, ..., n.$$

Also, implies that, for $i \neq j$, the processes $(N_i(t) - A_i(t))_{t\geq 0}$ and $(N_j(t) - A_j(t))_{t\geq 0}$ are orthogonal martingales.

To consider the case of dependent components we use Girsanov Theorem:
Theorem 2.1(Brémaud (1981)) For each i we let $(\alpha_i(t))_{t\geq 0}$ be a predictable and non negative process. Define

$$B_i(t) = \int_0^t \alpha_i(s) dA_i(s)$$

and assume that $B_i(t) < \infty, \quad P - a.s.$ Then

$$L(t) = \prod_{i=1}^{n} L_i(t) = \prod_{i=1}^{n} \alpha_i^{N_i(t)}(S_i) \exp[A_i(t) - B_i(t)]$$

is a $(P, \Im_t)$ nonnegative local martingale and a $(P, \Im_t)$- super martingale. Furthermore, if $(L(t))_{t\geq 0}$ is a martingale with expectation 1, under the measure $Q_{\alpha_1,\alpha_2,...,\alpha_n}$ defined by the Radon Nikodym derivative

$$\frac{\delta Q_{\alpha_1,\alpha_2,...,\alpha_n}}{\delta P} = L(\infty)$$

$B_i(t))$ is the $\Im_t$-compensator of $N_i(t)$.

Usually, $B_i(t) = \int_0^t \alpha_i(s) dA_i(s)$ is interpreted as a hazard transformation process for S_i and $\alpha_i(t)$ can be associated with an improvement of S_i.

Bueno and Norros (1992) and Bueno (2000) considered the following general definition of component's reliability importance through compensator transform:

Under the notation $Q_{\alpha_i} = Q_{1,...,\alpha_i,...,1}$ we consider

$$\frac{\delta Q_{\alpha_i}}{\delta P} = L(S_i),$$

and

Definition 2.2 Let Q_{α_i} be defined as above, assuming that $L(t)$ is uniformly integrable, T is integrable and S_i is finite. The (α_i) importance of S_i for T is

$$I_{\alpha_i}(i) = E_{Q_{\alpha_i}}[T] - E_P[T].$$

Remarks 2.3:

(1) $I_{\alpha_i}(i)$ has the dimension of time. Its interpretation in words could be:"the amount of time by which the expectation of life length T increases when an operation corresponding to $\alpha_i(t)$, is performed on the object whose life length is S_i".

(2) The above definition is meaningful for any life lengths. In particular, no independence assumptions are needed in the standard setting where S_i and T denote the life lengths of a component and the system, respectively.

(3) Since, under the assumptions of the above definition, $EL_\infty = 1$, the importance can be written as

$$I_{\alpha_i}(i) = -COV(L(\infty), T).$$

3. The martingale estimator

Firstly we propose a martingale estimator for $E_{Q_{\alpha_i}}[T]$ (a martingale estimator for $E_P[T]$ corresponds to take $\alpha_i(s) = 1$). For that we are going to use the following Theorem:

Theorem 3.1 (Karr (1986) Let, for each i, $1 \leq i \leq n$ the sequence $(M_i^m(t))_{m\geq 1}$ of orthogonal mean zero square integral martingale and let, for each i, $1 \leq i \leq n$, $(V_i(t))_{t\geq 0}$ be continuous and nondecreasing with $V_i(0) = 0$.

Suppose that:

A) For each t and i,

$$< M_i^m >_t \rightarrow V_i(t)$$

, in distribution, as $m \rightarrow \infty$

B) There are a sequence $(c_m)_{m\geq 1}$, $c_m \rightarrow 0$ as $m \rightarrow \infty$ and

$$P(\sup |\triangle M^m(t)| \leq c_m) \rightarrow 1$$

as $m \rightarrow \infty$.

Then there exists a n-variate continuous Gaussian process M, with each component a martingale and with

$$< M_i, M_j >_t = 1_{\{i=j\}} V_i(t)$$

, such that $M^m \rightarrow M$ in distribution, as $m \rightarrow \infty$.

In what follows we assume that we observe the $\Im_t$ intensity process $(\lambda_i(t))_{t\geq 0}$ for component i, a bounded nonnegative $\Im_t$-predictable process such that $A_i(t) = \int_0^t \lambda_i(s)ds$ $1 \leq i \leq n$. That is, without loss of generality, we actualize $\Im_t$ as

$$\Im_t = \sigma\{N_i(s), \lambda_i(s) : s \leq t, i \in \{1, ..., n\}.$$

We must account for the possibility of $\alpha_i(s)\lambda_i(s)$ to be equal to zero and we assume that there is $\delta > 0$ such that for each i and s, $\alpha_i(s)\lambda_i(s) > \delta$ if $\alpha_i(s)\lambda_i(s) > 0$. In other words, besides that in Girsanov Theorem we have $\int_0^t \alpha_i(s)\lambda_i(s)ds < \infty$, *a.s.*, we are also assuming that $\int_0^t (\alpha_i(s)\lambda_i(s))^{-1}ds < \infty$, Q_{α_i} *a.s.*.

Lemma 3.2 Under the hypothesis of Theorem 2.2,

$$\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}dN_i(s) - \int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}ds$$

is an uniformly square integrable mean zero $(Q_{\alpha_i}, \Im_t)$ - martingale.

Proof We can write

$$\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}dN_i(s) - \int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}ds =$$

$$\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}dN_i(s) - \int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}(\alpha_i(s)\lambda_i(s))ds =$$

$$\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}d(N_i(s) - \int_0^s (\alpha_i(u)\lambda_i(u))du).$$

From Girsanov Theorem, $N_i(t) - \int_0^t \alpha_i(s)\lambda_i(s)ds$ is an $(Q_{\alpha_i}, \Im_t)$- martingale. As $1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}$ is an $\Im_t$-predictable process we have proved that

$$\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}dN_i(s) - \int_0^t 1_{\{\alpha_i(s)\lambda_i(s)\}}ds$$

is an $(Q_{\alpha_i}, \Im_t)$ - martingale. From Doob Meyer decomposition, it is uniformly square integrable and the Lemma is proved.

Using the Optimal Sampling Theorem, if we assume that $\alpha_i(s)\lambda_i(s) > 0$ Q_{α_i} *a.s.* we propose

$$\widehat{E}_{Q_{\alpha_i}}(T) = \int_0^T (\alpha_i(s)\lambda_i(s))^{-1}dN_i(s)$$

as an unbiased martingale estimator for $E_{Q_{\alpha_i}}[\int_0^T ds] = E_{Q_{\alpha_i}}[T]$.

The variation process for the martingale $\widehat{E}_i(t) - E_i(t)$ is

$$< \widehat{E}_i - E_i >_t = \int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}ds.$$

The mean square error is the mean of the variation process, that is

$$E_{Q_{\alpha_i}}[(\widehat{E}_i(t) - E_i(t))^2] = E_{Q_{\alpha_i}}[< \widehat{E}_i - E_i >_t] =$$

$$E_{Q_{\alpha_i}}[\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-1}ds]$$

with estimator $\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}(\alpha_i(s)\lambda_i(s))^{-2}dN_i(s)$.

We are going to introduce our statistical model applying Theorem 3.2 in a set of data constituting of independent and identically distributed copies of (N_i, λ_i), $1 \leq i \leq n$.

We assume, for each $i, 1 \leq i \leq n$ a sequence of pairs $(N_i^j(t), \lambda_i^j(t)), 1 \leq j \leq m$, of m copies independent and identically distributed as $(N_i(t), \lambda_i(t))$, and let, for each $i, 1 \leq i \leq n$,

$$N_i^m(t) = \sum_{j=1}^{m} N_i^j(t), \quad and \quad \lambda_i^m(t) = \sum_{j=1}^{m} \lambda_i^j(t).$$

To estimate $E_i(t) = E_{Q_{\alpha_i}}[\int_0^t 1_{\{\alpha_i(s)\lambda_i(s)>0\}}ds]$, we propose the martingale estimator

$$\widehat{E}_i^m(t) = \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}}(\alpha_i(s)\lambda_i^m(s))^{-1}dN_i^m(s).$$

Clearly, as proved in Corollary 3.2, it is an unbiased estimator.

A natural first question to be done is :

Under Q_{α_i}, are the martingale estimator $\widehat{E}_i^m(t)$ consistent?

Theorem 3.3 If $E_{Q_{\alpha_i}}[\int_0^t(\alpha_i(s)\lambda_i(s))^{-1}ds] < \infty$, $\widehat{E}_i^m(t)$ is a consistent estimator for $E_i(t)$, under Q_{α_i}, that is

$$E_{Q_{\alpha_i}}[\sup_{t\leq T}(\widehat{E}_i^m(t) - E_i(t))^2] \to 0$$

as $m \to \infty$.

Proof. As

$$\widehat{E}_i^m(t) - E_i(t) = \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}}(\alpha_i(s)\lambda_i^m(s))^{-1}d(N_i^m(s) - \int_0^s(\alpha_i(u)\lambda_i^m(u))du)$$

is an $(Q_{\alpha_i}, \Im_t)$-martingale, we have

$$< \widehat{E}_i^m - E_i >_t = \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}}(\alpha_i(s)\lambda_i^m(s))^{-1}ds.$$

Therefore,

$$E_{Q_{\alpha_i}}[\sup_{s\leq t}(\widehat{E}_i^m(s) - E_i(s))^2] \leq$$

$$4E_{Q_{\alpha_i}}[(\widehat{E}_i^m(t) - E_i(t))^2] = 4E_{Q_{\alpha_i}}[< \widehat{E}_i^m - E_i >_t] \leq$$

$$4E_{Q_{\alpha_i}}[\frac{1}{m}\int_0^t \frac{m}{\lambda_i^m(s)}\alpha_i^{-1}(s)ds \longrightarrow 0$$

as $m \longrightarrow \infty$ from the Stronger Law of Larger Numbers and Dominated Convergence Theorem.

As $\widehat{E}_i^m(t) - E_i(t)$ is an uniformly integrable martingale and T is a finite $\Im_t$-stopping time, we can apply the Optimal Sampling Theorem to get

$$E_{Q_{\alpha_i}}[\sup_{t\leq T}(\widehat{E}_i^m(t) - E_i(t))^2] \to 0$$

as $m \to \infty$.

A second important question is: Are the estimator error process $\widehat{E}_i^m(t) - E_i(t)$, suitable scaled, asymptotically normal , in the sense that , under Q_{α_i}, they satisfy a martingale central limit theorem?

Theorem 3.4 Assume that, for each $i, 1 \leq i \leq n$

$$\int_0^t E_{Q_{\alpha_i}}[(\alpha_i(s)\lambda_i(s))^{-1}]ds < \infty.$$

Then the processes $m^{\frac{1}{2}}[\widehat{E}^m - E]$ converge in distribution, under Q_{α_i}, to a continuous n variate Gaussian martingale M satisfying

$$< M_i, M_j >_t = 1_{\{i=j\}} \int_0^t E_{Q_{\alpha_i}}[(\alpha_i(s)\lambda_i(s))^{-1}]ds.$$

Proof. We verify the hypothesis of Theorem 3.2.

Jumps in the martingale $m^{\frac{1}{2}}[\widehat{E}^m - E]$ arise only from $\widehat{E}_i^m$, whose jumps are of size $[\lambda_i^m(s)]^{-1} \sim m^{-1}$. Therefore, part B) of Theorem is true if we take $c_m = m^{-\frac{1}{4}}$.

To check part A) we note that

$$m \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}}[\alpha_i(s)\lambda_i^m(s)]^{-2}\alpha_i(s)\lambda_i^m(s)ds =$$

$$m \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}}[\alpha_i(s)\lambda_i^m(s)]^{-1}ds \to \int_0^t E_{Q_{\alpha_i}}[\frac{1}{\alpha_i(s)\lambda_i(s)}]ds.$$

Remark 3.5 To apply Theorem 3.4 it is necessary to estimate the variation process

$$\int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}} \frac{m}{\alpha_i(s)\lambda_i^m(s)}ds$$

which converges to the process $\int_0^t E_{Q_{\alpha_i}}[\frac{1}{\alpha_i(s)\lambda_i(s)}]ds$. Its martingale estimator is

$$\int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}}(\alpha_i(s)\lambda_i^m(t))^{-2}dN_i^m(s).$$

Example 3.6 Consider the compensator change which arises from exactly one minimal repair of component i. Intuitively this means that when S_i occurs, the system is returned to the state in which it was immediately before S_i occurred. The second occurrence of S_i is considered as final, and we take it as the improved value of S_i. This improvement operation corresponds to the compensator transform

$$B_i(t) = \int_0^t \frac{A_i(s)}{1+A_i(s)} dA_i(s) = A_i(t) - \ln(1 + A_i(t)).$$

In this case $\alpha_i(s) = \frac{A_i(s)}{1+A_i(s)}$, and therefore $L_i(S_i) = \alpha_i^{N_i(S_i)}(S_i)\exp[A_i(S_i) - B_i(S_i)] = A_i(S_i)$ with $E[A_i(S_i)] = 1$ because $A_i(S_i)$ is a standard exponential random variable and, as defined,

$$I_{\alpha_i}(i) = -COV(T, A_i(S_i)).$$

If $\alpha_i(s)\lambda_i(s) > 0 \ \ Q_{\alpha_i} - a.s.$, the martingale estimator for $E_{Q_{\alpha_i}}[T]$ is

$$\widehat{E}_i^m = \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}} (\frac{1+A_i(s)}{A_i(s)}) \frac{1}{\sum_{j=1}^m \lambda_i^j(t)} dN_i^m(s).$$

Example 3.7 Consider the compensator change which arises from a parallel improvement of component i. This means that we replace the lifetime S_i by the lifetime $S_i \vee S$ resulting from a parallel improvement where S and S_i are independent and identically distributed random variables. This improvement operation corresponds to the compensator transform

$$B_i(t) = \int_0^t \alpha_i(s) dA_i(s) = \int_0^t (\frac{2 - 2\exp[-A_i(s)]}{2 - \exp[-A_i(s)]}) dA_i(s) = A_i(t) - \ln(2 - \exp[-A_i(t)]).$$

In this case $\alpha_i(s) = \frac{2-2\exp[-A_i(s)]}{2-\exp[-A_i(s)]}$ and $L_i(S_i) = \alpha_i^{N_i(S_i)}(S_i)\exp[A_i(S_i) - B_i(S_i)] = 2 - 2\exp(-A_i(S_i))$ with $E[2 - 2\exp(-A_i(S_i))] = 1$ because $A_i(S_i)$ is a standard exponential random variable and, as defined,

$$I_{\alpha_i}(i) = -COV(T, 2 - 2\exp(-A_i(S_i))).$$

If $\alpha_i(s)\lambda_i(s) > 0 \ \ Q_{\alpha_i} - a.s.$, the martingale estimator for $E_{Q_{\alpha_i}}[T]$ is

$$\widehat{E}_i^m = \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}} (\frac{2 - \exp[-A_i(s)]}{2 - 2\exp[-A_i(s)]}) \frac{1}{\sum_{j=1}^m \lambda_i^j(t)} dN_i^m(s).$$

Example 3.8 For another example, consider that the life length is minimally repaired with constant probability $1-\theta$, $0 \leq \theta \leq 1$ up to the first unsuccessful repair attempt. The corresponding compensator transform was interpreted as a proportional improvement). In this case $\alpha_i(s) = \theta$ for all s ; therefore $L(S_i) = \theta \exp[(1-\theta)A_i(S_i)]$ with $E[\theta \exp[(1-\theta)A_i(S_i)] = 1$ because $A_i(S_i)$ is a standard exponential random variable and, as defined,

$$I_{\alpha_i}(i) = -COV(T, \theta \exp[(1-\theta)A_i(S_i)]).$$

If $\alpha_i(s)\lambda_i(s) > 0 \ \ Q_{\alpha_i} - a.s.$, the martingale estimator for $E_{Q_{\alpha_i}}[T]$ is

$$\widehat{E}_i^m = \int_0^t 1_{\{\alpha_i(s)\lambda_i^m(s)>0\}} \frac{1}{\theta} \frac{1}{\sum_{j=1}^m \lambda_i^j(t)} dN_i^m(s).$$

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