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COMPONENT IMPORTANCE IN A MODULATED MARKOV SYSTEM.

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COMPONENT IMPORTANCE IN A MODULATED MARKOV SYSTEM

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ABSTRACT. We consider an engineering Markov system which works in one of a set of N states with a semi-martingale representation related to its environment. The system depends on m components which are immediately minimally repaired after each failure. Filtering this representation through the family of sub σ- algebras generated by the multivariate point process of system's components, we consider the innovation gain corresponding to the dynamics of component i. The reliability importance of component i to system reliability at level l is calculate as the negative expected value of its innovation gain at its first failure.

1. INTRODUCTION

In this article we are concerned with an engineering Markov system Y_s which works in one of a set of N states with a semi-martingale representation related to its environment represented by a family of sub σ- algebras $(\Im_t)_{t\geq 0}$. The system depends of its components which are immediately minimally repaired after each failure leaving the state of the system unchanged. We assume that the multivariate point process of system's components and the system itself do not have simultaneous failures and the relationship between them is through its failure intensity.

In most cases, of practical interest, it is impossible to measure the total environmental impact to the system, thus, we consider to observe the system at component level. This also produces a semi-martingale representation related to component dynamics. The filtration through the family of sub σ- algebras generated by the multivariate point process of system's components produces both, the innovation process and innovation gain corresponding to the components evolution in time. As in Bueno (2000) we define a measure of reliability importance of the component i to the system reliability as the negative expected value of

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the innovation gain related to component i at its first failure. In considering component dynamics information this measure corresponds to Norros (1986) measure of importance and generalizes Natvig importance concept. Results on reliability importance can be found in Aven (1986), Boland and El-Neweihi (1995), Bueno (2000) and Xie (1988).

In Section 2 we give a mathematical formulation of the problem and recall the component reliability importance of the component i to system reliability from Bueno (2000), in Section 3 we calculate the component reliability and in Section 4 we give an explicit solution for the case $N = 2$.

2. Mathematical Formulation

In a number of situations an engineering system $(Y_t)_{t\geq 0}$ defined in a probability space $(\Omega, \Im, P)$, is given by a semi-martingale representation related to its environment $(\Im_t)_{t\geq 0}$, a family of sub σ- álgebras of $\Im$, which is increasing, right continuous and completed (shortly satisfies the Dellacherie conditions), that is:

$$Y_t = Y_0 + \int_0^t f_s ds + m_t \tag{1}$$

where f_t is an $\Im_t$-progressive process such that $\int_0^t |f_s| ds < \infty$ P a.s., and m_t is an $\Im_t$ 0 mean martingale.

Without loss of generality we consider $(\Im_t)_{t\geq 0}$ defined as

$$\Im_t = \Im^Y_\infty \vee \sigma\{N_i(s), s \leq t, 1 \leq i \leq m\} \tag{2}$$

where $\Im^Y_\infty = \sigma\{Y_t, t \geq 0,\}$ and $N_t(i) = \sum_{j=1}^{\infty} 1_{\{T_j \leq t\}} 1_{\{V_j = i\}}$ is the point process generated by the minimal repairs of the component i with lifetimes T_j on $\{V_j = i\}$. In reality, T_j are the component's lifetimes $S_i, 1 \leq i \leq m$ ordered on time.

In most cases of practical interest we can show directly the existence of the above representation, however, it is impossible to observe all the impact of the environmental σ-algebras $(\Im_t)_{t\geq 0}$ to system reliability. We observe the sub σ-algebras $(\Re_t)_{t\geq 0}$

$$\Re_t = \sigma\{N_s(i), s \leq t, i = 1, ..., m\}. \tag{3}$$

In the following $(N_t(1), ..., N_t(m))$ is the multivariate marked point process of system components with predictable $\Re_t$- intensities $(\lambda^p_t(1), ..., (\lambda^p_t(m))$ and $\Im_t$- intensities $(\lambda_t(1), ..., (\lambda_t(m))$.

Considering a filtration of Y_t over $\Re_t$, the structure function $Z_t = E[Y_t|\Re_t]$, and using the Theory of Innovation (Brémaud (1981)) we can prove that Z_t has, also, a semi-martingale representation

$$Z_t = Z_0 + \int_0^t g_s ds + \sum_{i=1}^{m} \int_0^t K_s(i) dM_s^p(i) \tag{4}$$

where g_t is an $\Re_t$-progressive process such that $\int_0^t |g_s| ds < \infty$ P a.s., $K_s(i)$ is $\Re_s$-predictable, $i = 1, ..., m$ and $\int_0^t K_s(i)\lambda_s^p(i) ds < \infty$ P a.s..

In the above, $M_s^p(i) = N_s(i) - \int_0^s \lambda_u^p(i) du$ is an $\Re_s$-martingale, $i = 1, ..., m$, $M_s^p = (M_s^p(1), ..., M_s^p(m))$ is the innovation process and $K_s = (K_s(1), ..., K_s(m))$ is the innovation gain. If $E[f_t|\Re_t]$ is $\Re_t$-progressive we can use $g_t = E[f_t|\Re_t]$.

Under the above conditions Brémaud (1981) shows that

$$K_t(i) = \psi_{1,t}(i) - \psi_{2,t}(i) + \psi_{3,t}(i), i = 1, ..., m$$

where $\psi_{1,t}(i)$, $\psi_{2,t}(i)$ and $\psi_{1,t}(i)$ are $\Re_t$-predictable processes defined by

$$E[\int_0^t C_s Z_s \lambda_s(i) ds] = E[\int_0^t C_s \psi_{1,s}(i) \lambda_s^p(i) ds] \tag{5}$$

$$E[\int_0^t C_s Z_s \lambda_s^p(i) ds] = E[\int_0^t C_s \psi_{2,s}(i) \lambda_s^p(i) ds] \tag{6}$$

$$E[\sum_{0 \le s \le t} C_s \Delta m_s \Delta N_s(i)] = E[\int_0^t C_s \psi_{3,s}(i) \lambda_s^p(i) ds] \tag{7}$$

where $(C_s)_{s \ge 0}$ is $\Re_s$-predictable

In the semi-martingale decomposition of Z_t, the integral can be seen as a device which updates, as time proceeds, the estimates $E[Y_t|\Re_t]$ of Y_t. If the integrand $K_s(i)$ is nonpositive for every i, every failure is experienced as a loss in this estimated value. This kind of behaviour leads us to the following definition:

Definition 2.1. The reliability importance of component i to system reliability is the negative expectation value of its innovation gain at its first failure time S_i:

$$-E[K_{S_i}(i)].$$

As in Bueno (2000) this results corresponds to Norros (1986) definition of reliability importance of a component.

Theorem 2.2. *Under the above assumptions and notations, the reliability importance of component i to system reliability is given by*

$$I(i) = -E[K_{S_i}(i)] = E\{E[Y_{S_i-}|\Re_{S_i}] - E[Y_{S_i}|\Re_{S_i}]\}.$$

Proof: *We consider the compensator functions* $\Lambda_t(i) = \int_0^{t\wedge S_i} \lambda_s(i)ds$ *and* $\Lambda_t^p(i) = \int_0^{t\wedge S_i} \lambda_s^p(i)ds$. *We have* $E[K_{S_i}(i)] = E[\psi_{1,S_i}(i) - \psi_{2,S_i}(i)]$.

As the system and its components does not have simultaneous failures we can write

$$E[\psi_{2,S_i}(i)] = E[Z_{S_i-}] = E[\int_0^\infty Z_{t-}dN_t(i)].$$

Also

$$\begin{aligned} E[\psi_{1,S_i}(i)] &= E[\frac{(Z_{S_i-})\lambda_{S_i}(i)}{\lambda_{S_i}^p(i)}] = E[\int_0^\infty \frac{(Z_{t-})\lambda_t(i)}{\lambda_t^p(i)}dN_t(i)] \\ &= E[\int_0^\infty \frac{(Z_{t-})\lambda_t(i)}{\lambda_t^p(i)}d\Lambda_t^p(i)] = E[\int_0^\infty Z_t d\Lambda_t(i)] \\ &= E[\int_0^\infty Z_t dN_t(i)]. \end{aligned}$$

The third equalities above are due to the fact that $N_t(i) - \Lambda_t^p(i)$ *is an* $\Re_t$*-martingale and the fourth follows from* $\Lambda_t^p(i)]$ *is a natural increasing process.*

Therefore

$$I(i) = -E[K_{S_i}(i)] = E[\int_0^\infty [Z_{t-} - Z_t]dN_t(i)].$$

□

Corollary 2.3. *Under the above assumptions and notations, if* τ *is the lifetime of a coherent system (as in Barlow and Proschan (1981)) and* $Y_t = E[\tau|\Im_t]$, *the reliability importance of component i to system reliability is given by*

$$I(i) = -E[K_{S_i}(i)] = COV(Y_{S_i}, \int_0^{S_i} \lambda_s^p(i)ds).$$

Proof: *Applying the Dellacherie integration formula in the above proof we have*

$$\begin{aligned} E[\psi_{2,S_i}(i)] &= E[\int_0^\infty Z_{t-}dN_t(i)] = E[\int_0^\infty Z_{t-}d\Lambda_t^p(i)] \\ &= E[\int_0^\infty Z_t d\Lambda_t^p(i)] = E[Z_{S_i}\Lambda_{S_i}^p(i)] = E[E[Y_{S_i}|\Re_{S_i}]\Lambda_{S_i}^p(i)] \\ &= E[E[\Lambda_{S_i}^p(i)Y_{S_i}|\Re_{S_i}]] = E[\Lambda_{S_i}^p(i)Y_{S_i}]. \end{aligned}$$

Also

$$E[\psi_{1,T_i}(i)] = E[\int_0^\infty Z_t dN_t(i)] = E[Z_{S_i}] = E[E[Y_{S_i}|\Re_{S_i}]] = E[Y_{S_i}].$$

As the compensators $\Lambda_t^p(i)$ at its final point S_i, $\Lambda_{S_i}^p(i)$, $i = 1, ..., m$ are independent and identically distributed random variables with standard exponential distribution we have

$$\begin{aligned} E[K_{S_i}(i)] &= E[\psi_{1,S_i}(i)] - E[\psi_{2,S_i}(i)] \\ &= E[Y_{S_i}]E[\Lambda_{S_i}^p(i)] - E[\Lambda_{S_i}^p(i)Y_{S_i}] \\ &= -COV(Y_{S_i}, \int_0^{S_i} \lambda_s^p(i)ds). \end{aligned}$$

□

3. Component Importance

In our context the change of states of the system are driven by a non-homogeneous right-continuous Markov process $(Y_t)_{t\geq 0}$ with values in $S = \{1, ..., N\}$ and infinitesimal parameters $q_i(t), q_{ij}(t) < \infty, t \geq 0$:

$$q_i(t) = \lim_{h \to 0+} \frac{1}{h} P(Y_{t+h} = i | Y_t = i),$$

$$q_{ij}(t) = \lim_{h \to 0+} \frac{1}{h} P(Y_{t+h} = j | Y_t = i), i, j \in S, i \neq j,$$

$$q_{ii}(t) = -q_i(t) = -\sum_{j \neq i} q_{ij}(t), t \geq 0.$$

Follows that $Z_t(j) = 1_{\{Y_t = j\}}$ have a semi-martingale representation:

$$Z_t(j) = 1_{\{Y_t=j\}} = 1_{\{Y_0=j\}} + \int_0^t \sum_{i=1}^N 1_{\{Y_t=i\}} q_{ij}(s) ds + m_t(j) \qquad (8)$$

where $m_t(j)$ is a real right-continuous $\Im_t$-martingale and with left limits and $m_0(j) = 0$.

As before we suppose that $\Im_t = \Im_\infty^Y \vee \sigma\{N_s(i), s \leq t, 1 \leq i \leq m\}$ where $\Im_\infty^Y = \sigma\{Y_t, t \geq 0,\}$, $N_t(i) = \sum_{j=1}^\infty 1_{\{T_j \leq t\}} 1_{\{V_j = i\}}$ is the point process generated by the minimal repairs of the component i with lifetimes T_j on $\{V_j = i\}$,. Also that $N_t = \sum_{k=1}^m N_t(k)$ and $Y_t, t \geq 0$ do not have simultaneous failures.

Besides we make the following assumption: The $\Im_t$-intensity of processes $N_t(i), i = 1, ..., m$ are given by

$$\lambda_t(i) = \mu_{Y_t}(i, t), i = 1, ..., m$$

and therefore, $N_t(i), i = 1, ..., m$ are conditional Poisson processes and $\lambda_t(i), i = 1, ..., m$ are $\Im_0$-measurable.

However, the only possible observations is through the σ-algebra

$$\Re_t = \sigma\{N_s(i), s \leq t, i = 1, ..., m\}$$

and we consider the projections on it.

We let

$$\widehat{Z}_t(j) = E[Z_t(j)|\Re_t]. \tag{9}$$

In order to state the result, we introduce the following notation:

$$K_t(j,i) = -\widehat{Z}_{t^-}(j) + \frac{\mu_j(i,t)\widehat{Z}_{t^-}(j)}{\sum_{l\in S}\mu_l(i,t)\widehat{Z}_{t^-}(j)},$$

where $\widehat{Z}_{t^-}(j) = lim_{s\uparrow t}\widehat{Z}_s(j)$.

Theorem 3.1. *Under the above assumptions and notations, the process* $\widehat{Z}_t = (\widehat{Z}_t(1), ..., \widehat{Z}_t(N))$ *is given by*

$$\widehat{Z}_t(j) = P(Y_0 = j) + \sum_{i=1}^{N}\int_0^t \widehat{Z}_s(i)q_{ij}(s)ds \tag{10}$$

$$+ \sum_{k=1}^{m}\int_0^t K_s(j,k)(dN_s(k) - \sum_{l\in S}\mu_l(k,s)\widehat{Z}_{s^-}(l)ds). \tag{11}$$

Proof: *For simplicity we do the case $m = 1$. By (8), the process $Z_t(j)$ admits the following decomposition*

$$Z_t(j) = Z_0(j) + \int_0^t \sum_{i=1}^{N} Z_s(i)q_{ij}(s)ds + m_t(j),$$

where $m_t(j)$ is a real right-continuous $\Im_t$-martingale with left limits and $m_0(j) = 0$. Using the integral representation of point processes martingales (see Brémaud, 1981) we get:

$$\widehat{Z}_t(j) = P(Y_0 = j) + \sum_{i=1}^{N}\int_0^t \widehat{Z}_s(i)q_{ij}(s)ds \tag{12}$$

$$+ \int_0^t K_s(j)(dN_s - \sum_{l\in S}\mu_l(s)\widehat{Z}_s(l)ds), \tag{13}$$

where $K_t(j)$ is the innovations gain to be computed and $E[\mu_{Y_s}(s)|\Re_s] = \sum_{l\in S}\mu_l(s)\widehat{Z}_s(l)$ is the $\Re_s$-intensity of N_s, for $l = 1, ..., N$ ($\mu_l(s)$'s are deterministic) .

The gain has the form $K = \psi_1 - \psi_2 + \psi_3$ *(see equation (5), (6) and (7)). As* $Z_t(j)$ *and* $N_t = \sum_{k=1}^{m} N_t(k)$ *do not have common jumps* $\psi_3 = 0$*. Clearly we have* $\psi_{2,t}(j) = \widehat{Z}_{t^-}(j)$*.* $\psi_{1,t}(j)$ *is computed from:*

$$E[\int_0^t C_s Z_s(j)\mu_{Y_s}(s)ds] = E[\int_0^t C_s \psi_{1,s}(j) \sum_{l \in S} \mu_s(l)\widehat{Z}_s(l)ds]. \quad (14)$$

As $\mu_{Y_s}(s)Z_s(j) = \mu_j(s)Z_s(j)$ *and* $\widehat{Z}_s(l)ds = \widehat{Z}_{s^-}(l)ds, l \in S$*, we have*

$$E[\int_0^t C_s Z_s(j)\mu_{Y_s}(s)ds] = E[\int_0^t C_s Z_s(j)\mu_j(s)ds]. \quad (15)$$

Using (14) and (15), we get

$$\psi_{1,t} = \frac{\mu_j(t)\widehat{Z}_{t^-}(j)}{\sum_{l \in S} \mu_l(t)\widehat{Z}_{t^-}(l)}.$$

Therefore

$$K_t(j) = -\widehat{Z}_{t^-}(j) + \frac{\mu_j(t)\widehat{Z}_{t^-}(j)}{\sum_{l \in S} \mu_l(t)\widehat{Z}_{t^-}(l)},$$

and the result is proven. □

Definition 3.2. The reliability importance of component i to system reliability at level $j, j \in S$, is the negative expectation value of its innovation gain at its first failure time S_i for state j:

$$-E[K_{S_i}(j,i)] = E[\widehat{Z}_{S_i^-}(j)] - E\left[\frac{\mu_j(i,S_i)\widehat{Z}_{S_i^-}(j)}{\sum_{l \in S} \mu_l(i,S_i)\widehat{Z}_{S_i^-}(j)}\right].$$

Remark 3.3. i) By theorem 3.1 we can write:

$$
\begin{aligned}
\widehat{Z}_t(j) &= P(Y_0=j)+\sum_{l\in S}\int_0^t \widehat{Z}_s(l)q_{lj}(s)ds \\
&+ \sum_{k=1}^m \int_0^t \left[-\widehat{Z}_{s^-}(j)+\frac{\mu_j(k,s)\widehat{Z}_{s^-}(j)}{\sum_{l\in S}\mu_l(k,s)\widehat{Z}_{s^-}(l)}\right]dN_s(k) \\
&- \sum_{k=1}^m \int_0^t \left[-\widehat{Z}_{s^-}(j)+\frac{\mu_j(k,s)\widehat{Z}_{s^-}(j)}{\sum_{l\in S}\mu_l(k,s)\widehat{Z}_{s^-}(l)}\right]\sum_{l\in S}\mu_l(k,s)\widehat{Z}_{s^-}(l)ds, \\
&= \underbrace{P(Y_0=j)+\sum_{l\in S}\int_0^t \widehat{Z}_s(l)q_{lj}(s)ds}_{I}+\underbrace{\sum_{k=1}^m\int_0^t \widehat{Z}_{s^-}(j)\sum_{l\in S}\mu_l(k,s)\widehat{Z}_{s^-}(l)ds}_{II}, \\
&- \underbrace{\sum_{k=1}^m\int_0^t \mu_j(k,s)\widehat{Z}_{s^-}(j)ds}_{III} \\
&+ \sum_{k=1}^m \left[\sum_{n\geq 0}\left(-\widehat{Z}_{T_n^-}(j)+\frac{\mu_j(k,T_n)\widehat{Z}_{T_n^-}(j)}{\sum_{l\in S}\mu_l(k,T_n)\widehat{Z}_{T_n^-}(l)}\right)1_{\{T_n\leq t. V_n=k\}}\right].
\end{aligned}
$$

Since $I+II-III=\int_0^t\sum_{l\in S}\widehat{Z}_s(l)\left[q_{lj}(s)+\widehat{Z}_s(j)\left(\sum_{k=1}^m(\mu_l(k,s)-\mu_j(k,s))\right)\right]ds,$

we have

$$
\begin{aligned}
\widehat{Z}_t(j) &= P(Y_0=j)+\int_0^t\sum_{l\in S}\widehat{Z}_s(l)\left[q_{lj}(s)+\widehat{Z}_s(j)\left(\sum_{k=1}^m(\mu_l(k,s)-\mu_j(k,s))\right)\right]ds \\
&+ \sum_{k=1}^m \left[\sum_{n\geq 0}\left(-\widehat{Z}_{T_n^-}(j)+\frac{\mu_j(k,T_n)\widehat{Z}_{T_n^-}(j)}{\sum_{l\in S}\mu_l(k,T_n)\widehat{Z}_{T_n^-}(l)}\right)1_{\{T_n\leq t. V_n=k\}}\right]. \qquad (16)
\end{aligned}
$$

Equivalently:

if t is between the jumps of $N_t(N_t=\sum_{k=1}^m N_t(k))$, that is, $T_n\leq t<T_{n+1}$, we have

$$\widehat{Z}_t(j)=\widehat{Z}_{T_n}(j)+\int_{T_n}^t\sum_{l\in S}\widehat{Z}_s(l)\left[q_{lj}(s)+\widehat{Z}_s(j)\left(\sum_{k=1}^m(\mu_l(k,s)-\mu_j(k,s))\right)\right]ds. \qquad (17)$$

Also, at the jumps

$$\widehat{Z}_{T_n}(j)=\sum_{k=1}^m\left[\frac{\mu_j(k,T_n)\widehat{Z}_{T_n^-}(j)}{\sum_{l\in S}\mu_l(k,T_n)\widehat{Z}_{T_n^-}(l)}1_{\{V_n=k\}}\right], j\in S. \qquad (18)$$

ii) Between the jumps $T_n \leq t < T_{n+1}, n \in \{0, 1, ...\}$, $(T_0 = 0)$, we get

$$\frac{d\widehat{Z}_t(j)}{dt} = \sum_{l \in S} \widehat{Z}_t(l)\left[q_{lj}(t) + \widehat{Z}_t(j)\left(\sum_{k=1}^{m}(\mu_l(k,t) - \mu_j(k,t))\right)\right] \tag{19}$$

4. Aplication-The case of $N = 2$ States

When $N = 2$ it is possible to get a explicit solution for the process $\widehat{Z}_t(j), j = 1, 2$. If we assume that $P(Y_0 = 1) = 1$, for $T_n < t < T_{n+1}$ (see 18) we have:

$$\frac{d\widehat{Z}_t(1)}{dt} = \sum_{l=1}^{2} \widehat{Z}_t(l)\left[q_{l1}(t) + \widehat{Z}_t(1)\left(\sum_{k=1}^{m}(\mu_l(k,t) - \mu_1(k,t))\right)\right],$$

but $\widehat{Z}_s(2) = 1 - \widehat{Z}_s(1)$ and

$$\begin{aligned}\frac{d\widehat{Z}_t(1)}{dt} &= -\widehat{Z}_t(1)\left[\widehat{Z}_t(1)\left(\sum_{k=1}^{m}(\mu_2(k,t) - \mu_1(k,t))\right)\right.\\ &+ \left. q_{12}(t) + q_{21}(t) - \sum_{k=1}^{m}(\mu_2(k,t) - \mu_1(k,t))\right] + q_{21}(t). \quad (20)\end{aligned}$$

Therefore, $\widehat{Z}_t(1)$ can be determined explicitly from (20) as the solution of a differential equation, where $\widehat{Z}_0(1) = 1$.
Now we are ready to prove

Theorem 4.1. *Let*

$$q_1(t) = q_{12}(t) > 0, q_{21}(t) = 0 \text{ and } \frac{q_{12}(t)}{\sum_{k=1}^{m}(\mu_2(k,t) - \mu_1(k,t))} = c, \tag{21}$$

*where c is constant. Then equation (20) have the following solution: Between jumps, if $T_n \leq t < T_{n+1}, n \in \{0, 1, ...\}$,
for $c \neq 1$ we have*

$$\widehat{Z}_t(1) = \frac{(c-1)\widehat{Z}_{T_n}(1)h(t)}{\widehat{Z}_{T_n}(1)(1 - h(t)) + c - 1}, \tag{22}$$

where,

$$h(t) = \exp((c-1)\int_{T_n}^{t}\sum_{k=1}^{m}(\mu_2(k,s) - \mu_1(k,s))ds).$$

For $c = 1$

$$\widehat{Z}_t(1) = \frac{\widehat{Z}_{T_n}(1)}{1 + \widehat{Z}_{T_n}(1)\sum_{k=1}^{m}\int_{T_n}^{t}(\mu_2(k,s) - \mu_1(k,s))ds}. \tag{23}$$

At the jumps the solution is

$$\widehat{Z}_{T_n}(1) = \sum_{k=1}^{m}\left[\frac{\mu_1(k,T_n)\widehat{Z}_{T_n}\text{-}(1)}{(\mu_1(k,T_n)-\mu_2(k,T_n))\widehat{Z}_{T_n}\text{-}(1)+\mu_2(k,T_n)}1_{\{V_n=k\}}\right], \quad (24)$$

where $\widehat{Z}_{T_0}(1) = 1$.

Proof: *Considering the conditions (21), we can rewrite the equation (20) in the following form:*

If $c \neq 1$

$$\frac{d\widehat{Z}_t(1)}{-\widehat{Z}_t(1)\left[\widehat{Z}_t(1)+c-1\right]} = \sum_{k=1}^{m}(\mu_2(k,t)-\mu_1(k,t))dt, \quad (25)$$

if $c = 1$,

$$\frac{d\widehat{Z}_t(1)}{\widehat{Z}_t^2(1)} = \sum_{k=1}^{m}(\mu_1(k,t)-\mu_2(k,t))dt. \quad (26)$$

Therefore, $d\widehat{Z}_t(1)$ *can be obtained explicitly as the solution of the above differential equations, for* $\widehat{Z}_0(1) = 1$ *and the result is proven.* □

Remark 4.2. i) The value of process $\widehat{Z}_t(2)$ is determined explicitly from $\widehat{Z}_t(2) = 1 - \widehat{Z}_t(1)$

ii)If $p_{12}(t) = p_{12}$, $\mu_1(k,t) = \mu_1(k)$, and $\mu_2(k,t) = \mu_2(k)$ are constant then the case $N = 2$ is similar to a problem well known as the detection, disruption or disorder problem:

At an unobservable random time τ there occurs a change in the characteristics of some observable randon phenomena. Detect "as well as possible" this time of disorder (disruption) by means of the random process observation (see Brémaud (1981)).

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