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Correction to: Some results and examples concerning Whyburn spaces

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Abstract

We correct the proof of Theorem 2.9 of the paper mentioned in the title (published in Applied General Topology, **13** *No.1 (2012), 11-19).*

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There is an error in the proof of Theorem 2.9. A correct proof is as follows.

Theorem 2.9. If X is weakly Whyburn, then $|X| \leq d(X)^{t(X)}$.

Proof. If X is finite, the result is trivial; thus we assume that X is infinite. Suppose that $d(X) = \delta$, $t(X) = \kappa$ and $D \subseteq X$ is a dense (proper) subset of cardinality δ. Let $D = D_0$ and define recursively an ascending chain of subspaces $\{D_\alpha : \alpha < \kappa^+\}$ as follows:

Suppose that for some $\alpha \in \kappa^+$ and for each $\beta < \alpha$ we have defined dense sets D_β such that $|D_\beta| \leq \delta^\kappa$ and $D_\gamma \subseteq D_\lambda$ whenever $\gamma < \lambda < \alpha$. If α is a limit ordinal, then define $D_\alpha = \bigcup\{D_\beta : \beta < \alpha\}$ and then $|D_\alpha| \leq |\alpha|.\delta^\kappa \leq \kappa^+.\delta^\kappa = \delta^\kappa$. If on the other hand $\alpha = \beta + 1$, and $D_\beta \subsetneq X$, then since X is weakly Whyburn there is some $x \in X \setminus D_\beta$ and $B_x \subseteq D_\beta$ such that $|B_x| \leq \kappa$, $\mathrm{cl}(B_x) \setminus D_\beta = \{x\}$; thus necessarily, we have that $|\mathrm{cl}(B_x)| \leq \delta^\kappa$ and we define

$$D_\alpha = \bigcup\{\mathrm{cl}(B) : B \subseteq D_\beta,\ |B| \leq \kappa,\ |\mathrm{cl}(B)| \leq \delta^\kappa\}.$$

Clearly $D_\alpha \supsetneq D_\beta$ and since there are at most $(\delta^\kappa)^\kappa$ such sets B it follows that $|D_\alpha| \leq \delta^\kappa$. If $D_\alpha = X$ for some $\alpha < \kappa^+$, then we are done. If not, then we define $\Delta = \bigcup\{D_\alpha : \alpha < \kappa^+\}$, and clearly $|\Delta| \leq \kappa^+.\delta^\kappa = \delta^\kappa$.

Thus to complete the proof it suffices to show that $\Delta = X$. Suppose to the contrary; then, since Δ is not closed and X is weakly Whyburn and has tightness κ, there is some $z \in X \setminus \Delta$ and some set $B \subseteq \Delta$ of cardinality at most κ, such that $\mathrm{cl}(B) \setminus \Delta = \{z\}$ and hence $|\mathrm{cl}(B)| \leq \delta^\kappa$. Since the sets $\{D_\alpha : \alpha < \kappa^+\}$ form an ascending chain and $\mathrm{cf}(\kappa^+) > \kappa$, it follows that for some $\gamma < \kappa^+$, $B \subseteq \bigcup\{D_\alpha : \alpha < \gamma\}$ and hence $z \in D_{\gamma+1} \subseteq \Delta$, a contradiction. □

It should also be noted that Theorem 2.6 is not as claimed, an improvement on the cited result of Bella, Costantini and Spadaro, since a Lindelöf P-space is regular.

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