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ABSTRACT. A famous result of Kostant [K] states that the universal enveloping algebra of a semisimple Lie algebra is a free module over its center. We generalize this result for the class of so-called special PBW algebras including the restricted Yangians and current algebras associated with the general linear Lie algebra. This also generalizes the result of Geoffriau for Takiff algebras [G1].

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1. INTRODUCTION

Let R be an associative algebra over a field $\mathbf{k}$ and Γ be its commutative subalgebra. In the representation theory of R one often studies Harish-Chandra modules with respect to Γ, i.e. such R-modules V that admit a decomposition $V = \oplus_\chi V_\chi$ with χ parametrizing the isomorphism classes of simple Γ-modules and with V_χ being a Γ-module with simple subquotients corresponding to χ. One of the first problems in the study of Harish-Chandra modules is to determine when a given χ can be lifted to an irreducible Harish-Chandra module V with $V_\chi \neq 0$. The most interesting case is when there exists such lifting for any χ. This is always true when R is a free module over Γ. For example, consider the universal enveloping algebra $U(\mathbf{g})$ of a semisimple Lie algebra $\mathbf{g}$ over an algebraically closed field of characteristic 0. Then a well-known result of Kostant [K] shows that $U(\mathbf{g})$ is a free module over its center. Hence irreducible $\mathbf{g}$-modules exist with an arbitrary central character. For the Yangian of the general linear Lie algebra gl_n, which is a deformation of the universal eneveloping algebra of the current algebra $\mathrm{gl}_n \otimes \mathbf{k}[x]$, the freeness over the center ([KS]) was shown in [MNO]. The case of restricted Yangian of level p is more complicated since it corresponds to the restricted current algebra of level p. Particular case ($p = 2$) was considered by Geoffriau ([G1]) who showed that the universal enveloping algebra of a Takiff algebra is free over its center. Another example of such situation was considered in [O1] where the problem of extension of a character of the Gelfand-Tsetlin subalgebra of $U(\mathrm{gl}_n)$ to an irreducible gl_n-module was studied. In particular, it was shown that such extension always exists. Besides, it was shown in [O2] that $U(\mathrm{gl}_n)$ is a free left (right) module over the Gelfand-Tsetlin subalgebra.

The knowledge of the freeness of a given algebra over its certain subalgebra often allows to obtain other important results, e.g. to find the annihilators of Verma modules [Di]. Also in [O2] the finiteness of the number of liftings to irreducible modules from a given character of the Gelfand-Tsetlin subalgebra was proved for $U(\mathrm{gl}_n)$. In fact, there was introduced a technique which generalizes Kostant methods ([K], see also [G1]) and which allows to study the universal enveloping algebras of Lie algebras as modules over its certain commutative subalgebras.

In the present paper we develop a graded version of the techniques from [O2] which can be applied to a more general class of algebras, so-called special PBW algebras. We prove that in order to establish their freeness over certain commutative subalgebra it is enough to show that the images of the generators of this subalgebra form a regular sequence in the associated polynomial algebra or equivalently that the corresponding variety is equidimensional. We apply our result to the restricted Yangians of any level p and to the universal enveloping algebra of restricted current algebras of any level m, associated with the general linear Lie algebra, and show that these algebras are free as modules over their centers.

The structure of the paper is the following. In Section 2 we collect necessary facts about regular sequences. In Section 3 we introduce our main class of special PBW algebras and study their basic properties. In Section 4 we establish the key result for special PBW algebras (Theorem 1) showing their freeness over certain commutative subalgebra generated by a sequence of elements that becomes regular in the associated graded algebra. This generalizes the Kostant theorem for semisimple Lie algebras. In Section 5 we apply our theory to the case of restricted Yangian of any level p for gl_n and show that it is free as a module over its center (Theorem 2). Finally, in Section 6 we consider the restricted current algebras of any level m for gl_n and show that their universal enveloping algebras are free modules over the center (Theorem 3). This generalizes the result of Geoffriau for Takiff algebras.

Throughout the paper we fix an algebraically closed field $\mathbf{k}$ of characteristic 0. For an associative and commutative algebra R we denote by $\mathrm{Specm}\, R$ the variety of all maximal ideals of R. If R is a polynomial algebra in n variables, then we identify $\mathrm{Specm}\, R$ and $\mathbf{k}^n$.

2. Regular sequences

In this section we collect several facts about regular sequences in polynomial rings which will be needed later. Let $\Lambda = \mathbf{k}[X_1, \ldots, X_n]$ be a polynomial algebra. For an ideal $I \subset \Lambda$ denote by $V(I) \subset \operatorname{Specm}\Lambda$ a set of all zeroes of I, $V(I) = \{\mu \in \operatorname{Specm}\Lambda | I \subset \mu\}$.

A sequence $g_1, \ldots, g_t \in \Lambda$ is called *regular* (in Λ) if the class of g_i in $\Lambda/(g_1, \ldots, g_{i-1})$ is non-invertible and is not a zero divisor for any $i = 1, \ldots, t$. For standard properties of regular sequences we refer to [BH], [Ei]. Let $g_1, \ldots, g_t \in \Lambda$ be a regular sequence and let $\mathbf{e}_1, \ldots, \mathbf{e}_t$ be a standard basis in the free module Λ^t. Then the following exact sequence is a part of the Koszul resolution:

$$\cdots \longrightarrow \wedge^2(\Lambda^t) \xrightarrow{\partial} \Lambda^t \xrightarrow{\eta} \Lambda \xrightarrow{\pi} \Lambda/(g_1, \ldots, g_t) \longrightarrow 0,$$

where $\wedge^2$ is the exterior square, $\partial(\mathbf{e}_i \wedge \mathbf{e}_j) = g_i\mathbf{e}_j - g_j\mathbf{e}_i$, $\eta(\lambda_1, \ldots, \lambda_t) = \sum_{i=1}^t \lambda_i g_i$ and π is the projection. In particular, if $\sum_{i=1}^{t} f_i g_i = 0$, then

$$(f_1, \ldots, f_t) \in \sum_{1 \leqslant i < j \leqslant t} \Lambda(g_i\mathbf{e}_j - g_j\mathbf{e}_i).$$

Proposition 1. *([BH])*

1. *Let $g_1, \ldots g_t$ be a regular sequence in Λ, $L \in GL_t(\Lambda)$ and $(h_1, \ldots, h_t) = L(g_1, \ldots g_t)$. Then the sequence $h_1, \ldots, h_t$ is regular.*
2. *A sequence $g_1, \ldots g_t$ is regular in Λ if and only if the variety $V(g_1, \ldots, g_t)$ is equidimensional of dimension $n - t$.*
3. *Any subsequence of a regular sequence is regular.*

The following result follows immediately from Proposition 1, (1).

Lemma 2.1. *The sequence $X_1, \ldots, X_r, G_1, \ldots G_t$ with $G_1, \ldots, G_t \in \Lambda$ is regular in Λ if and only if the sequence $g_1, \ldots, g_t$ is regular in $\mathbf{k}[X_{r+1}, \ldots, X_n]$, where $g_i(X_{r+1}, \ldots, X_n) = G_i(0, \ldots, 0, X_{r+1}, \ldots, X_n)$.*

3. Special PBW algebras

In this section we consider the following class of associative PBW-algebras. Let U be an associative algebra over $\mathbf{k}$, endowed with an increasing filtration $\{U_i\}_{i \in \mathbf{Z}}$, $U_{-1} = \{0\}$, $U_0 = \mathbf{k}$, $U_iU_j \subset U_{i+j}$. For $u \in U_i \setminus U_{i-1}$ set $\deg u = i$. Let $\overline{U} = \operatorname{gr} U$ be the associated graded algebra $\overline{U} = \bigoplus_{i=0}^{\infty} U_i/U_{i-1}$. For $u \in U$ denote by $\overline{u}$ its image in $\overline{U}$ and for a subset $S \subset U$ denote $S_i = S \cap U_i$, $\overline{S} = \{\overline{s} \,|\, s \in S\} \subset \overline{U}$. Set $\overline{U}_{(i)} = U_i/U_{i-1}$ and for any $T \subset \overline{U}$ denote $T_{(i)} = T \cap \overline{U}_{(i)}$.

Recall that algebra U is called a PBW algebra if any element of U can be written uniquely as a linear combination of ordered monomials in some fixed generators of U. *We will assume for the rest of the paper that U is a PBW algebra and that $\overline{U}$ is a polynomial algebra.* Such PBW algebra will be called *special.*

Note that the grading on $\overline{U}$ does not coincide in general with its standard grading as a polynomial algebra. In particular, if $d_1, \dots d_n$ are positive integers then $\Lambda = \overline{\Lambda} = \mathbf{k}[X_1, \dots, X_n]$, endowed with a grading $\deg X_i = d_i$, is a special PBW algebra with respect to the corresponding filtration. When all $d_i = 1$ we get a standard grading on Λ. As above $\Lambda_{(i)}$ will denote the i-th graded component of Λ and Λ_i will denote the i-th member of the corresponding filtration. Note that due to the Poincare-Birkhoff-Witt theorem the universal enveloping algebra of any finite-dimensional Lie algebra is a special PBW algebra.

Consider commuting elements $g_1, \dots, g_t \in U$ such that $\overline{g}_1, \dots, \overline{g}_t$ is a regular sequence in $\overline{U}$ and let $\Gamma \subset U$ be the polynomial algebra $\mathbf{k}[g_1, \dots, g_t]$.

Introduce the mapping $(\ ,\ )_U : U^t \times U^t \longrightarrow U$, s.t. for $\mathsf{u} = (u_1, \dots, u_t)$, $\mathsf{v} = (v_1, \dots, v_t)$ holds $(\mathsf{u}, \mathsf{v})_U = \sum_{i=1}^{t} u_i v_i$.

Lemma 3.1. *Let U be a special PBW algebra and let $I \subset U$ be a left ideal generated by $g_1, \dots, g_t$.*

(1) *Any $f \in I$ can be written in the form $f = \sum_{i=1}^{t} f_i g_i$ for some $f_i \in U$, $i = 1, \dots, t$ such that $\deg f = \max_{1 \le i \le t} \deg f_i g_i$.*

(2) *If $f_1 g_1 + \dots + f_t g_t = c$ for some $f_i \in U$, $c \in \mathbf{k}$, then $c = 0$ and $(f_1, \dots, f_t) \in \sum_{1 \leqslant i < j \leqslant t} U(g_i \mathsf{e}_j - g_j \mathsf{e}_i)$.*

Proof. Choose a presentation $f = f_1 g_1 + \dots + f_t g_t$ with the minimal possible $d = \max_{i=1,\dots,t} \deg f_i g_i$. We may assume that $d = \deg f_i g_i$ if and only if $i = 1, \dots, r$. Denote $\mathsf{f} = (f_1, \dots, f_t), \mathsf{g} = (g_1, \dots, g_t)$, and for a vector $\mathsf{s} = (s_1, \dots, s_t) \in U^t$ denote $\overline{\mathsf{s}}$ the vector $(\overline{s}_1, \dots, \overline{s}_r, 0, \dots, 0) \in \overline{U}^t$. If $d > \deg f$ then from the condition $(\mathsf{f}, \mathsf{g})_U = f$ follows $(\overline{\mathsf{f}}, \overline{\mathsf{g}})_{\overline{U}} = 0$. Since the sequence $\overline{g}_1, \dots, \overline{g}_r$ is regular in $\overline{U}$ by Proposition 1, (3), then there exist $g_{ij} \in U$, $1 \leqslant i < j \leqslant r$ such that $\overline{\mathsf{f}} = \sum_{1 \leqslant i < j \leqslant r} \overline{g}_{ij}(\overline{g}_i \mathsf{e}_j - \overline{g}_j \mathsf{e}_i)$.

Consider $\mathsf{k} = \sum_{1 \leqslant i < j \leqslant r} g_{ij}(g_i \mathsf{e}_j - g_j \mathsf{e}_i) \in U^t$. Then $\overline{\mathsf{f}} = \overline{\mathsf{k}}$, hence for $\mathsf{h} = (h_1, \dots, h_t) = \mathsf{f} - \mathsf{k}$ holds $\deg h_i \leqslant \deg f_i$ for all i and $\deg h_i < \deg f_i$ for $i = 1, \dots, r$. Note that $(g_i \mathsf{e}_j - g_j \mathsf{e}_i, \mathsf{g})_U = g_i g_j - g_j g_i = 0$, hence

$(\mathsf{k},\mathsf{g})_U = 0$. Then

$$\sum_{i=1}^{t} h_i g_i = (\mathsf{h},\mathsf{g})_U = (\mathsf{f}-\mathsf{k},\mathsf{g})_U = (\mathsf{f},\mathsf{g})_U = \sum_{i=1}^{t} f_i g_i = f.$$

Since $\max_{i=1,\dots,t} \deg h_i g_i < \max_{i=1,\dots,t} \deg f_i g_i$ the obtained equality contradicts the minimality of d. This proves (1).

The statement (2) follows immediately from the proof of statement (1) and the fact that $\deg f_i \geq 1$ for all i. □

Corollary 1. *Let $\Lambda = \mathbf{k}[X_1,\dots,X_n]$, $g_1,\dots,g_t \in \Lambda$ be such that the sequence $\overline{g}_1,\dots,\overline{g}_t$ is regular in $\Lambda = \overline{\Lambda}$. Then the sequence $g_1,\dots,g_t$ is regular in Λ.*

Proof. Suppose that for some $i \in \{1,\dots,n\}$, g_i is a zero divisor in $\Lambda/(g_1,\dots,g_{i-1})$. Then there exists an element $f \in \Lambda$, $f \notin (g_1,\dots,g_{i-1})$ such that $fg_i = \sum_{k=1}^{i-1} f_k g_k$ for some $f_k \in \Lambda$. Assume that f has the minimal possible degree. Then following Lemma 3.1, (1), $\deg fg_i = \max_{1\leq k\leq i-1} \deg f_k g_k$. We may assume without loss of generality that $\overline{f}\overline{g}_i = \sum_{k=1}^{r} \overline{f}_k \overline{g}_k$ for some $1 \leq r \leq i-1$. Since the sequence $\overline{g}_1,\dots,\overline{g}_t$ is regular then the sequence $\overline{g}_1,\dots,\overline{g}_r$ is regular also. Therefore $\overline{f} = \sum_{k=1}^{r} \overline{h}_k \overline{g}_k$ for some $h_k \in \Lambda$. It follows that $f' = f - \sum_{k=1}^{r} h_k g_k$ has a smaller degree than f and that $f' g_i \in (g_1,\dots,g_{i-1})$. This contradiction completes the proof. The case when the image of g_i in $\Lambda/(g_1,\dots,g_{i-1})$ is invertible is treated analogously. □

Lemma 3.2. *Let $\Lambda = \mathbf{k}[X_1,\dots,X_n]$, $\mathsf{G} = \{g_1,\dots,g_t\}$ be a regular sequence in Λ consisting of homogeneous polynomials, $\Gamma = \mathbf{k}[g_1,\dots,g_t]$, $\mu \in \operatorname{Specm}\Gamma$, $\mu_i = g_i(\mu)$, $i = 1,\dots,t$ and let $I_\mu \subset \Lambda$ be the ideal generated by $g_1-\mu_1,\dots,g_t-\mu_t$.*

(1) *$I_\mu \cap \Lambda_m = \sum_{i=1}^{t} \Lambda_{m-d_i}(g_i-\mu_i)$, where $d_i = \deg g_i$, $i = 1,\dots,t$.*
(2) *$g_1-\mu_1,\dots,g_t-\mu_t$ is a regular sequence in Λ.*
(3) *$\overline{I}_\mu$ is an ideal generated by $\overline{g}_1,\dots,\overline{g}_t$ and hence $\overline{I}_\mu$ does not depend on $\mu \in \operatorname{Specm}\Gamma$. In particular $I_\mu \neq \Lambda$.*
(4) *The regular map $p_{\mathsf{G}} : \operatorname{Specm}\Lambda \longrightarrow \operatorname{Specm}\Gamma = \mathbf{k}^t$, induced by the inclusion $i_{\mathsf{G}} : \Gamma \hookrightarrow \Lambda$ is an epimorphism and $\dim p_{\mathsf{G}}^{-1}(\mu) = n-t$ for any $\mu \in \operatorname{Specm}\Gamma$.*
(5) *There exists an open dense $\mathsf{U}_{\mathsf{G}} \subset \operatorname{Specm}\Gamma$ such that for any $\mu \in \mathsf{U}_{\mathsf{G}}$ the ideal I_μ is radical.*

Proof. Statement (1) follows from Lemma 3.1,(1) while the statement (2) follows from Corollary 1. Statement (3) follows immediately from

(1). Since the sequence $g_1-\mu_1,\dots,g_t-\mu_t$ is regular by (2) and $p_G^{-1}(\mu) = \mathsf{V}(g_1-\mu_1,\dots,g_t-\mu_t)$ then $p_G^{-1}(\mu)$ is equidimensional of dimension $n-t$. This implies (4). To prove (5) consider the *Jacobian matrix* $J = J(X_1,\dots,X_n) = \left(\dfrac{\partial(g_i-\mu_i)}{\partial X_j}\right)$, $1 \leqslant i \leqslant t$, $1 \leqslant j \leqslant n$. Denote by $\mathfrak{I}$ the ideal of Λ, generated by all $(t\times t)-$minors of J and set $J_\mu = I_\mu + \mathfrak{I}$. It is known (see for example Theorem 18.15, [Ei]) that ideal I_μ is radical if and only if $\dim \mathsf{V}(J_\mu) < n-t$. Denote by Ω the set of all μ's such that I_μ is not radical. Since J is the Jacobian matrix of the epimorphism p_G then $V(\mathfrak{I})$ is a proper closed subset in $\operatorname{Specm}\Lambda$. Let $p: V(\mathfrak{I}) \to \operatorname{Specm}\Gamma$ is the restriction of p_G on $V(\mathfrak{I})$. Obviously, $\mu\in\Omega$ if and only if $\dim p^{-1}(\mu) = n-t$. It follows immediately from the upper semicontinuity of $\dim p^{-1}(\mu)$ on μ that the set Ω is constructible, i.e. the disjoint union of locally closed subsets. If $\overline{\Omega} = \operatorname{Specm}\Gamma$ then Ω contains an open dense in $\operatorname{Specm}\Gamma$ set B. Since $\dim B = t$ and for every $\mu\in B$, $\dim p^{-1}(\mu) = n-t$, applying to p the theorem about the dimension of general layer ([Ei], 14.3), we obtain

$$\dim V(\mathfrak{I}) = \dim B + \dim p^{-1}(\mu) = t + (n-t) = n = \dim\Lambda.$$

But this contradicts the fact that $V(\mathfrak{I})$ is a proper closed subset of Λ. We conclude that the closure $\overline{\Omega} \neq \operatorname{Specm}\Gamma$, then we set $\mathsf{U}_G = \operatorname{Specm}\Gamma - \overline{\Omega}$. Thus for any $\mu\in\mathsf{U}_G$ the intersection of $\mathsf{V}(I_\mu) = p_G^{-1}(\mu)$ with $\mathsf{V}(\mathfrak{I})$ has dimension $< n-t$ implying the radicality of I_μ. □

We have the following useful properties of special PBW algebras.

Proposition 2. *Let U be a special PBW algebra and $g_1,\dots,g_t\in U$ be mutually commuting elements such that $\overline{g}_1,\dots,\overline{g}_t$ is a regular sequence in $\overline{U}$, $\Gamma = \mathbf{k}[g_1,\dots,g_t]$, $\mu\in\operatorname{Specm}\Gamma$, $\mu_i = g_i(\mu)$, $i=1,\dots,t$, $I_\mu = U(g_1-\mu_1)+\dots+U(g_t-\mu_t)$.*

(1) $I_\mu\cap U_m = \sum_{i=1}^{t} U_{m-d_i}(g_i-\mu_i)$, *where* $d_i = \deg g_i$.
(2) $\overline{I}_\mu = (\overline{g}_1,\dots,\overline{g}_t)$ *and does not depend on* $\mu\in\operatorname{Specm}\Gamma$.
(3) *For any* $\mu\in\operatorname{Specm}\Gamma$ *holds* $I_\mu\neq U$.
(4) *For any* $\mu\in\operatorname{Specm}\Gamma$ *there exists a simple left $U-$module M generated by $m\in M$ such that for every $\gamma\in\Gamma$ holds $\gamma m = \gamma(\mu)m$.*

Proof. The proof of (1) coincides with the proof of (1) in Lemma 3.1. The implications (1) ⇒ (2) ⇒ (3) are obvious. To obtain from (3) the statement (4) we consider a maximal proper left ideal $\mathfrak{m}$ in U, containing I_μ and set $M = U/\mathfrak{m}$, $m = 1+\mathfrak{m}$.

□

4. An analogue of Kostant theorem for special PBW algebras

In this chapter we follow closely the notations and techniques from [Di], 8.2.3 and [O2]. Let $\Lambda = \mathbf{k}[X_1, \dots, X_n]$. We say that $h_1, \dots, h_k \in \Lambda$ are *linearly independent over an ideal* $I \subset \Lambda$, provided $h_1 + I, \dots, h_k + I \in \Lambda/I$ are linearly independent over $\mathbf{k}$. Lemmas 4.2 and 4.3 below are analogous to [Di], 8.2.1, 8.2.2.

We use below the following fact.

Lemma 4.1. *Let V be a finite-dimensional space, $V' \subset V$ be its subspace, $Gr_k(V)$ be the Grassmanian of k-dimensional subspaces of V, $F : \operatorname{Specm}\Lambda \longrightarrow Gr_k(V)$ be a regular map and*

$$d = \min_{\mu \in \operatorname{Specm}\Lambda} \dim V' \cap F(\mu).$$

Then the set $U = \{\mu \in \operatorname{Specm}\Lambda \mid \dim V' \cap F(\mu) = d\}$ is nonempty and open in $\operatorname{Specm}\Lambda$.

Proof. Let $l = \dim V'$. Fix a basis in V which defines an open covering of $Gr_k(V)$ by affine subsets $U_{i_1,\dots,i_k} \simeq \mathsf{A}^{k(n-k)}$, $1 \le i_1 < \dots < i_k \le n$, such that any subspace $W \in Gr_k(V) \cap U_{i_1,\dots,i_k}$ is represented as the row space of a unique matrix of the form ([Ha], p.65)

$$\begin{pmatrix} X_{11} & X_{12} & \dots & \overbrace{1}^{i_1} & \dots & \overbrace{0}^{i_2} & \dots & \overbrace{0}^{i_k} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & 0 & \dots & 1 & \dots & 0 & \dots & X_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{k1} & X_{k2} & \dots & 0 & \dots & 0 & \dots & 1 & \dots & X_{kn} \end{pmatrix},$$

where the columns numbered by $i_1, \dots, i_k$ form the unit $k \times k$-matrix and all other entries are arbitrary. Denote $Z = \overline{\operatorname{Im} F}$. Consider the set Σ of all $W \in Gr_k(V)$ such that $\dim(W + V') \ge l + k - d$. Then Σ is an open set in $Gr_k(V)$ since it is defined by the condition that there exists a non-zero $k \times k$ minor of the following $(k + l) \times n$- matrix

$$\begin{pmatrix} X_{11} & X_{12} & \dots & \overbrace{1}^{i_1} & \dots & \overbrace{0}^{i_2} & \dots & \overbrace{0}^{i_k} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & 0 & \dots & 1 & \dots & 0 & \dots & X_{2n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ X_{k1} & X_{k2} & \dots & 0 & \dots & 0 & \dots & 1 & \dots & X_{kn} \\ \hline v_{11} & v_{12} & \dots & v_{1i_1} & \dots & v_{1i_2} & \dots & v_{1i_k} & \dots & v_{1n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ v_{l1} & v_{l2} & \dots & v_{li_1} & \dots & v_{li_2} & \dots & v_{li_k} & \dots & v_{ln} \end{pmatrix},$$

where the rows $(v_{11},\dots,v_{1n})$, ..., $(v_{l1},\dots,v_{ln})$ form a basis of V'. In particular, $Z\cap\Sigma$ is open in Z and hence $\mathsf{U}=f^{-1}(Z\cap\Sigma)$ is open in $\operatorname{Specm}\Lambda$, where $f:\operatorname{Specm}\Lambda\longrightarrow Z$ is induced by F. The statement of lemma follows. □

Lemma 4.2. *Let $G=\{g_1,\dots,g_t\}$ be a regular sequence of homogeneous elements in Λ, $\Gamma=\mathbf{k}[g_1,\dots,g_t]$ and let $h_1,\dots,h_k\in\Lambda$ be linearly independent over $I=(g_1,\dots,g_t)$. Then*

(1) *There exists an open dense set $\mathsf{U}_1\subset\operatorname{Specm}\Gamma$ such that $h_1,\dots,h_k$ are linearly independent over I_μ for every $\mu\in\mathsf{U}_1$.*
(2) *There exists an open dense $\mathsf{U}_2\subset\operatorname{Specm}\Gamma$ such that the restrictions of $h_1,\dots,h_k$ on $V(I_\mu)$ are linearly independent over $\mathbf{k}$ for each $\mu\in\mathsf{U}_2$.*

Proof. Let $l=\max_i\deg h_i$, $Y=\mathbf{k}h_1+\dots+\mathbf{k}h_k\subset\Lambda_l$. Let $V=\Lambda_l$, $\mu\in\operatorname{Specm}\Gamma$, $\mu_i=g_i(\mu)$, $i=1,\dots,t$, $I_\mu=(g_1-\mu_1,\dots,g_t-\mu_t)$ and $r=\dim I_\mu\cap\Lambda_l$. Note that by Lemma 3.2, (3), r does not depend on $\mu\in\operatorname{Specm}\Gamma$. Define $F:\operatorname{Specm}\Gamma\to Gr_r(\Lambda_l)$ such that $F(\mu)=(I_\mu)_l$ and set $V'=Y$. It follows from Lemma 3.2, (1) that F is a regular map. Then by linear independency of $h_1,\dots h_k$ over I it follows that $d=\min\limits_{\mu\in\operatorname{Specm}\Gamma}\dim Y\cap(I_\mu)_l=0$. Applying Lemma 4.1 we conclude that the set U_1 consisting of μ's, such that $\dim Y\cap(I_\mu)_l=0$, is open dense in $\operatorname{Specm}\Gamma$. Therefore statement (1) is proved. By Lemma 3.2,(5) there exists an open dence set $\mathsf{U}_G\subset\operatorname{Specm}\Gamma$ such that I_μ is radical for every $\mu\in\mathsf{U}_G$. For such μ's the linear independence of $h_1,\dots,h_k$ over I_μ is equivalent to the linear independence of the restrictions $h_1|_{V_\mu},\dots$, $h_k|_{V_\mu}$ as regular functions, where $V_\mu=V(I_\mu)$. Thus the statement (2) follows from (1) of this lemma and Lemma 3.2, (5), if we set $\mathsf{U}_2=\mathsf{U}_1\cap\mathsf{U}_G$. □

Lemma 4.3. *Let $g_1,\dots,g_t$ be a regular sequence of homogeneous elements in Λ, $\Gamma=\mathbf{k}[g_1,\dots,g_t]$, I be the ideal generated by $g_1,\dots,g_t$.*

Suppose $I = \sum_{i=0}^{\infty} I_{(i)}$ *is a graded decomposition of* I *and* $H = \sum_{i=0}^{\infty} H_{(i)}$ *is a graded complement of* I *in* Λ *as a* $\mathbf{k}-$*vector space. Then the mapping* $\pi : \Gamma \otimes_{\mathbf{k}} H \longrightarrow \Lambda$ *defined by* $\pi(\gamma \otimes h) = \gamma h$, $\gamma \in \Gamma$, $h \in H$ *is an isomorphism of* Γ*-modules. In particular,* Λ *is a free module over* Γ.

Proof. Note that $H_{(0)} = \mathbf{k}$ and $\operatorname{Im}\pi$ is a $\Gamma-$submodule in Λ containing Γ and H. We prove by induction on i that $\Lambda_{(i)} = I_{(i)} + H_{(i)} \subset \operatorname{Im}\pi$. If $f \in \Lambda_{(i)}$ then $f = f_I + f_H$, $f_I \in I_{(i)}$, $f_H \in H_{(i)} \subset \operatorname{Im}\pi$. By Lemma 3.2, (1) we have that $f_I = \sum_{j=1}^{t} f_j g_j$, where $\deg f_j < i$, $j = 1, \dots, t$. By induction, $f_j \in \operatorname{Im}\pi$ for all j and hence $f_I \in \operatorname{Im}\pi$, therefore $f = f_I + f_H \in \operatorname{Im}\pi$. It is left to show that π is a monomorphism. Suppose $h_1, \dots, h_k \in H$ are linearly independent over $\mathbf{k}$, hence $h_1, \dots, h_k$ are linearly independent over I. Let for some $\gamma_1, \dots, \gamma_k \in \Gamma$ holds $\gamma_1 h_1 + \dots + \gamma_k h_k = 0$. Consider the restriction of this equatlity on V_μ, $\mu \in \operatorname{Specm}\Gamma$. By Lemma 4.2, (2) there exists an open dense set $\mathsf{U}_2 \subset \operatorname{Specm}\Gamma$ such that for each $\mu \in \mathsf{U}_2$ the restrictions of functions $h_1|_{\mathsf{V}_\mu}$, ..., $h_k|_{\mathsf{V}_\mu}$ are linearly independent over $\mathbf{k}$. Since $\gamma_i|_{\mathsf{V}_\mu} = \gamma_i(\mu)$, $i = 1, \dots, k$, we get that $\gamma_1|_{\mathsf{V}_\mu} = \dots = \gamma_k|_{\mathsf{V}_\mu} = 0$ for every $\mu \in \mathsf{U}_2$. This implies that $\gamma_1|_{p_G^{-1}(\mathsf{U}_2)} = \dots = \gamma_k|_{p_G^{-1}(\mathsf{U}_2)} = 0$. Since $p_G^{-1}(\mathsf{U}_2)$ is dense in $\operatorname{Specm}\Lambda$ we conclude that $\gamma_1 = \dots = \gamma_k = 0$. □

We have the following analogue of Kostant theorem ([K]) for special PBW algebras.

Theorem 1. *Let* U *be a special PBW algebra and let* $g_1, \dots, g_t \in U$ *be mutually commuting elements such that* $\overline{g}_1, \dots, \overline{g}_t$ *is a regular sequence in* $\overline{U}$, $\Gamma = \mathbf{k}[g_1, \dots, g_t]$. *Then* U *is a free left (right)* $\Gamma-$*module.*

Proof. We will prove the statement for U as a left module. Right module structure is treated analogously. We apply Lemma 4.3 for $\Lambda = \overline{U}$ and the regular sequence $\overline{g}_1, \dots, \overline{g}_t$. Let $\overline{I}$ be a left ideal of $\overline{U}$ generated by $g_1, \dots, g_t$, $\overline{\Gamma} = \mathbf{k}[\overline{g}_1, \dots, \overline{g}_t]$ and $\overline{H} = \sum_{i=0}^{\infty} \overline{H}_i$ be a graded complement to $\overline{I}$ in Λ. Then the map $\overline{\pi} : \overline{\Gamma} \otimes_{\mathbf{k}} \overline{H} \longrightarrow \Lambda$ which sends $\overline{\gamma} \otimes \overline{h} \longmapsto \overline{\gamma}\overline{h}$ is an isomorphism of vector spaces. Let $H = \sum_{i=0}^{\infty} H_i$, where $H_i \subset U_i$ is such that $\operatorname{gr} : U \longrightarrow \overline{U}$ induces a $\mathbf{k}-$linear isomorphism H_i onto $\overline{H}_i$. We show that $\pi : \Gamma \otimes_{\mathbf{k}} H \longrightarrow U$ which sends $\gamma \otimes h \longmapsto \gamma h$ is an isomorphism of vector spaces. Since $H_0 = \mathbf{k}$ and $U_0 = \mathbf{k}$ then

$U_0 \subset \operatorname{Im}\pi$. Let now $f \in U_i$. Then $\overline{f} \in \overline{U}_i$ and $\overline{f} = \sum_j \overline{\gamma}_j \overline{h}_j$ for some $\gamma_j \in \Gamma$ and $h_j \in H$. Since $f - \sum_j \gamma_j h_j$ belongs to U_{j-1} and hence to $\operatorname{Im}\pi$ by induction on i, we conclude that $f \in \operatorname{Im}\pi$. This shows that π is epimorphism. Let now $B = \{h_j, j \in J\}$ be a basis of H such that $\overline{h}_j, j \in J$ form a basis in $\overline{H}$. Suppose $h_1, \ldots, h_k \in B$ are such that $\sum_{i=1}^{k} \gamma_i h_i = 0$ for some $\gamma_i \in \Gamma$, $\gamma_i \neq 0$, $i = 1, \ldots, k$. We can assume that $\deg h_1 g_1 = \ldots = \deg h_r g_r > \deg h_i g_i$ for some $r \leq k$ and any $i = r+1, \ldots, k$. Then $\sum_{j=1}^{r} \overline{\gamma}_j \overline{h}_j = 0$. Since $\overline{\pi}$ is an isomorphism and $\overline{h}_1, \ldots \overline{h}_r$ are linearly independent, we conclude that $\overline{\gamma}_1 = \ldots = \overline{\gamma}_r = 0$ and hence $\gamma_1 = \ldots = \gamma_r = 0$ which is a contradiction. □

5. Application to restricted Yangians

Let p be a positive integer. The *level p Yangian* $Y_p(\mathrm{gl}_n)$ for the Lie algebra gl_n ([D1], [C]) can be defined as the associative algebra with generators $t_{ij}^{(1)}, \ldots, t_{ij}^{(p)}$, $i, j = 1, \ldots, n$, subject to the relations

$$(1) \qquad [T_{ij}(u), T_{kl}(v)] = \frac{1}{u-v}(T_{kj}(u)T_{il}(v) - T_{kj}(v)T_{il}(u)),$$

where u, v are formal variables and

$$(2) \qquad T_{ij}(u) = \delta_{ij}\, u^p + \sum_{k=1}^{p} t_{ij}^{(k)}\, u^{p-k} \in Y_p(\mathrm{gl}_n)[u].$$

These relations are equivalent to the following

$$(3) \qquad [t_{ij}^{(r)}, t_{kl}^{(s)}] = \sum_{a=1}^{\min(r,s)} (t_{kj}^{(a-1)} t_{il}^{(r+s-a)} - t_{kj}^{(r+s-a)} t_{il}^{(a-1)}),$$

where $t_{ij}^{(0)} = \delta_{ij}$ and $t_{ij}^{(r)} = 0$ for $r \geq p+1$.

The importance of the restricted Yangian of level p is motivated by the fact that any irreducible finite-dimensional representation of the full Yangian ([D1]) is a representation of the restricted Yangian for some p [D2].

Note that the level 1 Yangian $Y_1(\mathrm{gl}_n)$ coincides with the universal enveloping algebra $U(\mathrm{gl}_n)$. Set $\deg t_{ij}^{(k)} = k$. This defines a filtration on $Y_p(\mathrm{gl}_n)$. The following analogue of the Poincaré–Birkhoff–Witt theorem for the algebra $Y_p(\mathrm{gl}_n)$ ([C],[M]), shows that $Y_p(\mathrm{gl}_n)$ is a special PBW algebra.

Proposition 3. *The associated graded algebra* $\overline{\mathrm{Y}}_p(\mathrm{gl}_n) = \mathrm{gr}\,\mathrm{Y}_p(\mathrm{gl}_n)$ *is a polynomial algebra in variables* $\bar{t}_{ij}^{(k)}$, $i,j = 1,\ldots,n$, $k = 1,\ldots,p$.

Note that the grading on $\overline{\mathrm{Y}}_p(\mathrm{gl}_n)$ induced from the grading on $\mathrm{Y}_p(\mathrm{gl}_n)$, $\deg \bar{t}_{ij}^{(k)} = k$, does not coincide with the standard polynomial grading for $p > 1$.

Set $\mathrm{T}(u) = (T_{ij}(u))_{i,j=1}^n$ and consider the following element in $\mathrm{Y}_p(\mathrm{gl}_n)[u]$, called *quantum determinant*

$$\mathrm{qdet}\,\mathrm{T}(u) = \sum_{\sigma\in S_n} \mathrm{sgn}(\sigma)\, T_{1\sigma(1)}(u)T_{2\sigma(2)}(u-1)\ldots T_{n\sigma(n)}(u-n+1). \tag{4}$$

The coefficients d_s by the powers u^{np-s}, $s = 1,\ldots,np$, of $\mathrm{qdet}\,\mathrm{T}(u)$ are algebraically independent generators of the center of $\mathrm{Y}_p(\mathrm{gl}_n)$ ([C], [M]).

For $F = \sum_i f_i u^i \in \mathrm{Y}_p(\mathrm{gl}_n)[u]$ denote $\overline{F} = \sum_i \overline{f}_i u^i \in \overline{\mathrm{Y}}_p(\mathrm{gl}_n)[u]$. To simplify the notations we denote $X_{ij}^{(k)} = \bar{t}_{ij}^{(k)}$, $X_{ij}(u) = \overline{T}_{ij}(u)$ and $X(u) = (X_{ij}(u))_{i,j=1}^n$.

Lemma 5.1. $\mathrm{gr}\,\mathrm{qdet}\,T(u) = \det X(u)$.

Proof. Define

$$dt\,\mathrm{T}(u) = \sum_{\sigma\in S_n} \mathrm{sgn}(\sigma)\, T_{1\sigma(1)}(u)T_{2\sigma(2)}(u)\ldots T_{n\sigma(n)}(u) \in \mathrm{Y}_p(\mathrm{gl}_n)[u].$$

Note that for every $s = 1,\ldots,np$, the coefficient of $dt\,\mathrm{T}(u)$ by the power u^{np-s} equals

$$\sum_{\sigma\in S_n} \mathrm{sgn}(\sigma) \sum_{k_1+\ldots+k_n=s} t_{1\sigma(1)}^{(k_1)} t_{2\sigma(2)}^{(k_2)} \cdots t_{n\sigma(n)}^{(k_n)}.$$

We assume that $t_{ij}^{(0)} = \delta_{ij}$. Clearly, all terms in this expression are linearly independent in $\overline{\mathrm{Y}}_p(\mathrm{gl}_n)$. Hence,

$$\mathrm{gr}\,dt\,\mathrm{T}(u) = \sum_{\sigma\in S_n} \mathrm{sgn}(\sigma) \sum_{k_1+\ldots+k_n=s} \bar{t}_{1\sigma(1)}^{(k_1)} \bar{t}_{2\sigma(2)}^{(k_2)} \cdots \bar{t}_{n\sigma(n)}^{(k_n)}$$

which is equal to $\det X(u)$. On the other hand, for all i, j and $a \in \mathbf{k}$ the coefficient of the polynomial $T_{ij}(u-a)$ by the power u^{p-m} equals

$$t_{ij}^{(m)} + \sum_{k=0}^{m-1} t_{ij}^{(k)}(-a)^{m-k}\binom{p-k}{m-k}.$$

Therefore $\mathrm{gr}\,\mathrm{qdet}\,T(u) = \mathrm{gr}\,dt\,\mathrm{T}(u) = \det X(u)$. □

Lemma 5.2. *The sequence* $\overline{d}_1,\ldots,\overline{d}_{np}$ *is regular in* $\overline{\mathrm{Y}}_p(\mathrm{gl}_n)$.

Proof. Due to Proposition 1,(3) it is enough to prove that the sequence

$$X_{ij}^{(k)}, i \neq j, i,j = 1,\dots,n, k = 1,\dots p;\ \overline{d}_1,\dots,\overline{d}_{np}$$

is regular in $\overline{Y}_p(\mathrm{gl}_n)$. Let c_s is a polynomial in variables $X_{11}^{(1)}, X_{11}^{(2)}, \dots, X_{11}^{(p)}, X_{22}^{(1)}, \dots, X_{nn}^{(p)}$ obtained from $\overline{d}_s$ by substituting $X_{ij}^{(k)} = 0$ for all $i \neq j$ and all k. It is easy to see from Lemma 5.1 that c_s is the coefficient by u^{np-s} in $\det \mathrm{diag}\{X_{11}(u),\dots,X_{nn}(u)\}$. Since the elements $X_{ij}^{(k)}, i,j = 1,\dots,n, i \neq j, k = 1,\dots p$ belong to the set of generators of $\overline{Y}_p(\mathrm{gl}_n)$ we only need to show the regularity of the sequence $c_1,\dots,c_{np}$ in the polynomial ring $\mathbf{k}[X_{ii}^{(k)}, i = 1,\dots,n, k = 1,\dots,p]$ due to Lemma 2.1. By Proposition 1, (2), the sequence $c_1,\dots,c_{np}$ is regular if and only if the variety $Z = V(c_1,\dots,c_{np})$ is equidimensional of dimension 0. Consider the map of algebraic varieties $\varphi : \mathbf{k}^{np} \to \mathbf{k}^{np}$ which sends $(a_{ii}^{(k)}) \in \mathbf{k}^{np}$ to the coefficients of the following monic polynomial $\prod_{i=1}^{n}(u^p + a_{ii}^{(1)}u^{p-1} + \dots + a_{ii}^{(p)})$. Since $\mathbf{k}[u]$ is a factorial domain and $\mathbf{k}$ is algebraically closed we conclude that the map φ is a finite (i.e. with a finite $\varphi^{-1}(\lambda)$ for each $\lambda \in \mathbf{k}^{np}$) epimorphism. Obviously, $Z = \varphi^{-1}(0)$ and thus $Z = \{0\}$. This completes the proof. □

Applying Theorem 1 and the Lemma 5.2 we immediately obtain the following analogue of the Kostant theorem for the restricted Yangians.

Theorem 2. *For all $n, p \geq 1$ the restricted Yangian $Y_p(\mathrm{gl}_n)$ is a free module over its center.*

6. Application to current algebras

In this section we consider the polynomial current Lie algebra $\mathbf{g} = \mathbf{g}(n) = \mathrm{gl}_n(\mathbb{C}) \otimes \mathbb{C}[x]$ and its restricted quotient $\mathbf{g}_m = \mathbf{g}_m(n)$, $m > 0$, by the ideal $\sum_{k\geq m} \mathrm{gl}_n \otimes x^k$ ([RT]).

In [M] the families of algebraically independent generators of the center of the universal enveloping algebra $U(\mathbf{g}_m)$ were constructed by using the quantum determinant and the quantum contraction for the restricted Yangian $Y_m(\mathrm{gl}_n)$. We will show that $U(\mathbf{g}_m)$ is a free module over its center for any $m > 0$. This generalizes in the case of gl_n the result of Geoffriau for Takiff algebra $\mathbf{g}_2$ ([G1],[G2]).

As in [M] let E_{ij}, $i,j = 1,\dots,n$, be the standard basis of gl_n, $E_{ij}^{(k)} = E_{ij} \otimes x^k$ with $1 \leq i,j \leq n$, $0 \leq k \leq m-1$ be a basis of $\mathbf{g}_m$. Set

$$F_{ij}^{(r)} = E_{ij}^{(r-1)},$$

$1 < r \leq m$ and

$$F_{ij}^{(1)} = E_{ij}^{(0)} - m(j-1)\delta_{ij}.$$

For each $k \in \{1, \ldots, mn\}$ let $r \in \{1, \ldots, m\}$ and $s \in \{1, \ldots, n\}$ be such that $k = m(s-1) + r$. Then the elements

$$\xi_k = \sum_{\substack{i_1 < \ldots < i_s \\ j_1 + \ldots + j_s = k}} \sum_{\sigma \in S_s} \operatorname{sgn}(\sigma) F^{(j_1)}_{i_{\sigma(1)} i_1} \ldots F^{(j_s)}_{i_{\sigma(s)} i_s}, \tag{5}$$

are algebraically independent generators of the center of $U(\mathfrak{g}_m)$ ([M]). Note that $U(\mathfrak{g}_m)$ is a special PBW algebra with respect to the standard grading.

We will show that the sequence $\overline{\xi}, \ldots, \overline{\xi}_{mn}$ is regular in $\overline{U}(\mathfrak{g}_m)$ where

$$\overline{\xi}_k = \sum_{\substack{i_1 < \ldots < i_s \\ j_1 + \ldots + j_s = k}} \sum_{\sigma \in S_s} \operatorname{sgn}(\sigma) \overline{F}^{(j_1)}_{i_{\sigma(1)} i_1} \ldots \overline{F}^{(j_s)}_{i_{\sigma(s)} i_s}. \tag{6}$$

As in the case of the Yangian $\mathrm{Y}_p(\mathrm{gl}_n)$ we complete this sequence by the elements $\overline{F}^{(l)}_{ij}$ for all $i, j = 1, \ldots, n$, $i \neq j$ and $l = 1, \ldots, m$ and apply Lemma 2.1. Hence it is enough to prove the regularity of the sequence $\gamma^m_1, \ldots, \gamma^m_{mn}$, where $\gamma^m_k = \sum_{\substack{i_1 < \ldots < i_s \\ j_1 + \ldots + j_s = k}} \overline{F}^{(j_1)}_{i_1 i_1} \ldots \overline{F}^{(j_s)}_{i_s i_s}$ (compare with (3.2) in [M]). We show that $Z = V(\gamma^m_1, \ldots, \gamma^m_{mn}) = \{0\}$. Suppose that $m = 1$. Since for each $k = 1, \ldots, n$, γ^1_k is the elementary symmetric polynomial of degree k in variables $\overline{F}^{(1)}_{11}, \ldots, \overline{F}^{(1)}_{nn}$ we have that $V(\gamma^1_1, \ldots, \gamma^1_n) = \{0\}$. Suppose now that the sequence $\gamma^{m-1}_1, \ldots, \gamma^{m-1}_{(m-1)n}$ is regular and hence $V(\gamma^{m-1}_1, \ldots, \gamma^{m-1}_{(m-1)n}) = \{0\}$. Note that γ^m_{ms} is the elementary symmetric polynomial of degree s in variables $\overline{F}^{(m)}_{11}, \ldots, \overline{F}^{(m)}_{nn}$, $s = 1, \ldots, n$. Thus for every $z \in Z$, $\overline{F}^{(m)}_{11}(z) = \ldots = \overline{F}^{(m)}_{nn}(z) = 0$. Substituting these values in γ^m_k for every k that does not divide m, we obtain the sequence $\gamma^{m-1}_1, \ldots, \gamma^{m-1}_{(m-1)n}$ which is regular by our assumption and thus $Z = 0$. By induction on m we conclude that the sequence $\overline{\xi}, \ldots, \overline{\xi}_{mn}$ is regular in $\overline{U}(\mathfrak{g}_m)$. Applying Theorem 1 we immediately obtain the following analogue of the Kostant theorem for restricted current algebras:

Theorem 3. *For all $m, n \geq 1$ the algebra $U(\mathfrak{g}_m(n))$ is a free module over its center.*

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