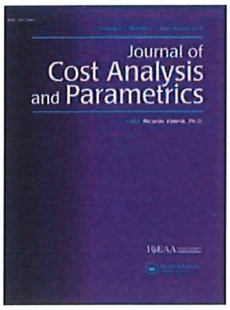

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Multiproduct Cost-Volume-Profit Model: A Resource Reallocation Approach for Decision Making

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This work addresses the problem of reallocating productive resources to maximize profit. Most contributions to the topic focus on developing or improving the Cost-Volume-Profit model to obtain solutions that provide an ideal mix of products before the data is given. In particular, some algorithms are available for the problem, such as the ones proposed by Kakumanu and Shao and Feng. However, these proposals do not consider the minimum number of units to be produced, and the reallocation of productive resources for each product is a problem found in these studies. Bearing this in mind, a new algorithm based on individual financial revenue is proposed. Computational results indicate that the proposed method can be utilized as a decision support system.

Introduction

The Industrial Revolution brought with it technological advancement in various fields of knowledge. This process provided a continuous quest to achieve more efficient operations, in other words, using productive resources to produce in the best possible way. With this in mind, various techniques and management systems became commonly used which, in turn, led to an increase in the number of organizations in various sectors (Hillier & Lieberman, 2001, p. 1).

Consequently, to deal with this situation, organizations began to provide or produce various products, the commonly known product mix. This process can be clearly seen in manufacturing organizations as it attempts to provide a set of similar products, but with specific features, observing the characteristics and needs of their customers (Kakumanu, 1998, p. 87).

Concerning this context mainly for manufacturing organizations, the definition of production is of great importance because there are not always opportunities for adjustments in production levels without financial loss. Consequently, considering market information, manufacturing organizations define their master production plan based on the Cost-Volume-Profit (CVP) model, which, in turn, defines the optimum volume for a

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product, considering the cost and profit information. Although the purpose of the model is simple, its implementation is difficult because manufacturing organizations produce various products; i.e., there are many variables to be considered and, especially in practice, there is not enough time to find a good solution or a product mix with productive resources to maximize profit.

Some algorithms have been proposed to deal with this problem, such as those by Kakumanu (1998), Shao and Feng (2007) and, more recently, Milanovic, Milanovic, Misita, Klarin, and Zunjic (2010) Although these discussions have advanced in addressing the problem of having more than one product, some gaps still remain. Considering this, focusing on complementing or corroborating these studies, the purpose of this article is to present a new algorithm that helps to define the optimal production volume, considering the CVP model in situations with various products and minimum quantities to be produced. More specifically, an algorithm attempts to determine the required volume for each product in order to achieve the highest possible profit, considering that:

- the minimum production or minimum capacity of all products, which is usually determined by the market demand or machinery capacity, is predefined by the decision makers; and
- productive resources can be reallocated from one product to another.

To fulfil this purpose, the rest of this work is organized as follows. The next section shows a brief background of the CVP. The section following that introduces the new algorithm and its proposal. After that the computational experiment and the main results are presented. Finally, the last section provides the conclusions.

Cost-Volume-Profit Model

The Cost-Volume-Profit model is understood to be a model that determines the volume required to achieve the balance between costs and revenues, as defined well by Chan (1990, p. 253). It is worth mentioning that the CVP model is usually used for short-term planning (Phillips, 1994, p. 31). Kakumanu (1998, p. 88) argues that most studies about this definition classify the CVP model by considering the number of products involved, the behavior of variables entailed, and the number of considered periods in the following classes:

- Single and multiproduct problems: For this category, problems with one product are defined as single. On the other hand, problems with more than two products are called multiproduct problems.
- Deterministic and stochastic problems: When all variables of the problem are known and do not have random elements, these problems are described as deterministic. On the other hand, problems are considered stochastic, as in a real situation values should change during the period of production.
- Single and multiperiod problems: In this category, problems for a single period are termed single and problems for the other situations are considered multiperiod problems.

By observing the classification proposed by Kakumanu (1998, p. 88), the main articles about CVP are summarized in Table 1. Consequently, it can be clearly observed that there are few works utilizing the application of the CVP model for multiproduct problems. Overall, considering the concept of CVP, Equations (1) to (5) present the idea of the original CVP model for a single product, single period, and deterministic variables.

TABLE 1 Selected studies on CVP models

Research	Research summary	Category		
		Products	Variables	Periods
Kakumanu (1998)	He develops and tests a CVP model with product limitations for multiproduct, which optimizes the rate of return on sales revenue.	Multiproduct	Deterministic	Single
Shao and Feng (2007)	They present a stochastic CVP model based on the Economic Value Added model (EVA) for uncertain situations, in which companies currently experience.	Single	Stochastic	Single
Milanovic et al. (2010)	He indicates a universal equation that shows the influential variables and their impact on the profit based on the cost-volume-profit equation.	Single	Deterministic/Stochastic	Single
Chan (1990)	He discusses the sensitive analysis for the cost-volume-profit mode by using incremental analysis using case studies and graphs.	Single	Deterministic	Single
Yuan (2009)	He presents an application of fuzzy logic in the CVP analysis to handle the imprecisions in the original model.	Single	Stochastic	Single
González (2001)	He develops an alternative model for the CVP multiproduct, by using data provided by ABC systems to keep track of some variables, to reach the required profit.	Multiproduct	Deterministic	Single
Jaedicke and Robichek (1964)	They include some concepts of probability in the CVP model creating uncertainty and making some variables no longer fixed and work with some approximations of values.	Single	Stochastic	Single
Phillips (1994)	This article examines the basic CVP model and describes how to include uncertainty during the decision-making process.	Single	Stochastic	Single
Yunker and Schofield (2005)	They analyze and apply a stochastic CVP model specifically geared towards the determination of enrollment fees for training and development.	Single	Stochastic	Single
Yunker and Yunker (2003)	They analyze and apply a CVP model under uncertainty specifically geared towards classroom instruction.	Single	Stochastic	Single

$$P = TR - TC \tag{1}$$

$$TC = TVC + FC \tag{2}$$

$$TVC = Q \cdot VC \tag{3}$$

$$TR = Q \cdot SP \tag{4}$$

$$P = (SP - VC) \cdot Q - FC \tag{5}$$

wherein:
P represents profit;
TR represents total revenue;
TC represents total costs;
TVC represents total variable costs;
FC represents fixed costs;
Q represents units sold;
VC represents variable costs; and
SP represents selling price.

According to Phillips (1994, p. 31), it is important to emphasize that the original CVP model considers the following as its premise: (a) fixed cost will remain unchanged and variable cost will change proportionately with sales volume; (b) revenue is only affected by units sold; and (c) efficiency levels remain unchanged.

Models and Algorithm

Notation

In this work, the following notation and decision variables are used.

Indices.
$i = 1,2,3,\ldots, n$ represent product i.

Variables.
n represents the number of products;
M_i represents the production volume of product i;
QR represents the remaining quantity of resources;
MPC_i represents the minimum capacity to produce product i;
IM_i represents the initial product volume of product i;
PM_i represents the profit margin of product i;
P_i represents the price of product i;
C_i represents the cost of product i;
S_i represents the production scheduling of product i;
MD_i represents the market demand of product i;
L_i represents the maximum capacity of production of product i; and
FM_i represents the final volume of product i.

Kakumanu's CVP Model

Kakumanu's CVP model was chosen because it was one of the principal articles to present a mathematician model to deal with the issue of multiproduct problems in the CVP model.

The CPV model with limited product for multiple products, as proposed by Kakumanu (1998), identifies the production volumes for each product, i.e., the optimal product mix. It considers the respective limits with the aim of optimizing the defined rate of return on sales. In summary, the pseudocode of the CVP model proposed by Kakumanu (1998) is presented below:

1. **Step 1.** Calculating the volume. Using the equations shown in his work, he computes the required volume for all of the products.
2. **Step 2.** Checking the first possibility of the solution. By checking whether all of the required volumes computed on the previous step are the same or smaller than the limit, he determinates whether the procedure stops or goes on. In case they are all the same or smaller, the best solution is found. Otherwise, he proceeds to the next step.
3. **Step 3.** Checking the second possibility of solution. By checking whether all of the required volumes computed on the previous step are the same or bigger than the limit, he determinates whether the procedure stops or goes on. In case they are all the same or bigger, the best solution is found. Otherwise, he proceeds to the next step.
4. **Step 4.** Checking the third possibility of solution. If at least one product has the required volume smaller than the limit and another one has it bigger than the limit, then a new required volume needs to be calculated.
5. **Step 5.** Calculating the new mixes. Utilizing the reminisces of the products that are over the limit, he creates new mixes of products from the ones that are under.
6. **Step 6.** Calculating the required new volume. Using the equations and the new values found earlier, he calculates an optimum mix of products.

Proposed Model

The purpose of the proposed method is to develop a mix of production based on the individual contribution of each product, considering the maximum and minimum capacity of production and, especially, the market demand. This new method differs from Kakumanu's because it relocates the excess material on the more profitable products. To meet the objective of the proposed model, the GBV model, we establish an initial solution from the minimum production. Bearing this in mind, it identifies the difference between the market demand and the initial solution, aiming at reallocating excess capacity of products that did not meet their demand. Subsequently, based on the calculation of unit profit, a new solution is defined and the total profit is calculated, comparing it with the initial income of the problem. Taking this into account, the GBV model is shown in the next section.

Step 1. Adjusting Production to the Minimum Capacity. At first, the minimum necessary should be produced so that from the remainder, the new volumes are adjusted.

$$M_i = MPC_i \quad \forall i \tag{6}$$

Step 2. Calculate the Remainder. Knowing the values of the original production and the minimum capacity of all products, the remainder is calculated from the initial production of the problem minus the minimum capacity of each product.

$$QR = \sum_{i=1}^{n} IM_i - M_i \tag{7}$$

Step 3. Calculate the Profit Margin of Each Product. In order for an optimal production sequence to be created, the profit margins of all products need to be calculated first so that the more productive ones can be defined. The profit margin can be calculated as shown in Equation (8).

$$PM_i = P_i - C_i \tag{8}$$

Step 4. Identify the Optimal Production Sequence. Based on the profit margins calculated in Step 5, the ideal sequence of production can be calculated. The following must be determined, $S_1 = \{PM_1, PM_2, \ldots, PM_i\}$, considering $i \leq n$ and that S_1 is the production scheduling of products with $PM_i \geq 0$ and $PM_1 \geq PM_2 \geq, \ldots, \geq PM_n$ and $S_2 = i$, for all products.

Step 5. Determine the Final Volume for Each Product. The final volume for each product can be calculated using Algorithm 1. It considers the production sequence defined in Step 4. Overall, the algorithm reallocates the remainder in the products until it reaches a maximum production or market demand.

Algorithm 1: Pseudocode to calculate the final mix

```
for i = 1 to n do
    FM_{S2(i)} = min(MD_{S2(i)}; L_{S2(i)})
    if QR < min(MD_{S2(i)}; L_{S2(i)}) − MPC_{S2(i)} then
        FM_{S2(i)} = MPC_{S2(i)} + QR
        QR = 0
    else
        QR = QR − (min(MD_{S2(i)}; L_{S2(i)}) − MPC_{S2(i)})
    end
end
```

Numerical Example

The proposed method will be exemplified in a problem with three products ($n = 3$), considering the information in Table 2.

Based on these data, the first two steps are taken, which define the amount of resources that will be reallocated in order to adjust the volume manufactured to the ideal volume production. This amount is obtained by the difference between the initial product volume that is determined using the characteristics of the products to be fabricated, the set-up of production, and the minimum capacity of production. In this case, it is shown in Table 3.

In Steps 3 and 4, the order of optimal production is defined with the objective of prioritizing and ranking products by using the individual profit of each product as a parameter. The result of this step is shown in Table 4.

Finally, considering the algorithm shown in Algorithm 1, the production volume required for each product is determined by optimizing resources. In other words, aiming at maximizing profit, the resources were reallocated according to the optimal production sequence, aiming at maximizing profit. The final result of the GBV model for this example is shown in Table 5.

TABLE 2 Data of the numerical example

Products (i)	Price (P_i)	Cost (C_i)	Initial product volume (IM_i)	Market demand (MD_i)	Maximum capacity of production (L_i)	Minimum capacity of production (MPC_i)
1	67.58	24.51	1,500	2,500	1,750	950
2	79.66	61.04	4,800	4,500	5,000	1,750
3	47.38	12.85	2,500	3,000	2,750	1,450

TABLE 3 Steps One and Two

Products (i)	Initial product volume (IM_i)	Minimum capacity of production (MPC_i)	Difference between (IM_i) and (MPC_i)	Remain quantity of resources (QR)
1	1,500	950	550	
2	4,800	1,750	3,050	4,650
3	2,500	1,450	1,050	

TABLE 4 Steps Three and Four

Products (i)	Price (P_i)	Cost (C_i)	Profit margin of product (PM_i)	Production scheduling (S_2)
1	67.58	24.51	43.07	1
2	79.66	61.04	18.62	3
3	47.38	12.85	34.52	2

TABLE 5 Final solution

Products (i)	Initial product volume (IM_i)	Final volume of product ($FMs_2(i)$)
1	1,500	1,750
2	4,800	4,300
3	2,500	2,750

Computational Experiment

It is important to highlight that the computational experiment was utilized to verify whether the GBV model presents good performance in response to the variability of the number of products. To perform the computational experiments of the proposed model, 50 different problems were generated in a random way. Overall, the data simulated different situations with different numbers of products and different possible scenarios regarding the maximum and minimum capacity of production. This diversification of problems was created for it to be tested in different situations. Taking this into account, Table 6 presents the data of problems. Consequently, the main results are shown in Table 7 and Figure 1.

Considering the computational experiment realized in this section, the results show that the application of the GBV algorithm can improve the financial performance in most of the cases. For example, we can see in problem number 2 that the initial production and sale of products are equal but when the algorithm is applied the profit gain has a significant increase.

In practical terms, the GBV algorithm can be used on small and medium organizations that have not integrated administration and control systems as a decision support system, especially regarding cost system.

Conclusions

In this work, we address the problem of reallocating productive resources to maximize profit, considering multiproduct CVP. In order to fulfil this purpose, we proposed the GBV

TABLE 6 Data of problems

Problem	Product	Price	Cost	Mix (initial production) (Unit)	Market demand (Unit)	Maximum capacity (Unit)	Minimum capacity (Unit)	N. products
1	1	29.00	15.00	6150	4500	145000	0	
	2	22.00	15.50	14350	14000	145000	0	
	3	8.50	4.00	4100	5500	145000	0	
	4	11.00	8.50	4100	7000	145000	0	
	5	9.50	6.25	12300	10000	145000	0	5
	1	37.44	25.42	8000	12500	13000	6000	
2	2	50.13	46.09	2000	3000	2500	1000	2
	1	62.56	40.84	1750	1650	1800	950	
	2	2.83	0.61	600	1000	750	350	
	3	48.19	22.70	1100	950	1200	500	
3	4	81.14	18.75	700	1250	1000	500	4
	1	48.66	13.31	1150	1000	1200	600	
	2	9.55	5.32	500	750	600	200	
	3	93.62	62.69	1400	1600	1800	850	
	4	81.31	38.69	550	850	800	200	
4	5	92.89	69.89	900	850	1000	500	5
	1	14.74	3.23	500	650	550	300	
	2	76.37	3.87	1000	1400	1750	600	
	3	88.55	0.51	850	1200	1000	700	
5	4	24.93	18.38	1100	1550	2000	500	4
	1	67.58	24.51	1500	2500	1750	950	
	2	79.66	61.04	4800	4500	5000	1750	
6	3	47.38	12.85	2500	3000	2750	1450	3
	1	94.88	16.28	400	500	600	300	
	2	16.90	3.77	650	1300	1000	500	
	3	30.21	5.82	500	650	700	400	
	4	57.72	41.48	650	950	800	550	
	5	94.58	77.70	1750	1500	2000	950	
	6	71.60	25.40	1200	1400	1050	850	
	7	89.01	72.03	500	950	1100	400	
7	8	29.70	22.78	1500	1150	2400	800	8
	1	16.68	2.35	800	1000	1200	500	
	2	49.85	43.11	600	800	700	450	
	3	40.75	1.37	1500	1350	1900	650	
8	4	12.71	7.02	2250	2450	2300	1600	4
	1	16.42	13.70	8000	10000	8500	6000	
9	2	76.02	57.41	6400	6000	8000	3000	2
	1	32.61	6.99	1200	1100	1500	650	
	2	4.45	0.19	1850	1400	1200	600	
	3	98.57	71.56	1650	1800	2000	1000	
	4	6.82	0.07	650	950	700	500	
	5	9.71	7.13	1000	850	1300	450	
10	6	20.41	9.69	900	1150	1200	500	6

(*Continued*)

TABLE 6 (*Continued*)

Problem	Product	Price	Cost	Mix (initial production) (Unit)	Market demand (Unit)	Maximum capacity (Unit)	Minimum capacity (Unit)	N. products
	1	21.87	14.55	2000	1100	1000	750	
11	2	32.00	20.00	6000	7000	8200	5500	2
	1	88.81	14.95	1200	1150	2000	600	
	2	83.88	8.35	600	850	1000	300	
	3	47.46	19.16	600	400	350	150	
12	4	19.52	13.89	250	350	300	150	4
	1	92.42	86.45	120	150	200	100	
	2	97.99	1.80	850	750	800	600	
	3	30.34	18.98	1000	1100	1200	800	
	4	90.40	3.73	200	150	220	80	
	5	22.81	12.88	950	1350	1000	600	
	6	21.25	2.99	400	650	420	200	
13	7	4.00	2.32	1100	950	1200	350	7
	1	71.57	48.42	2000	1150	1300	500	
	2	41.96	28.85	900	1250	1000	500	
14	3	54.94	31.00	750	950	1000	400	3
	1	69.28	16.58	1000	700	1000	400	
	2	46.89	41.97	800	1050	900	600	
	3	39.84	3.25	900	750	900	450	
	4	49.73	11.60	500	350	300	100	
15	5	95.76	64.78	900	1400	1500	650	5
	1	33.01	19.46	550	750	600	300	
	2	83.47	11.92	500	650	800	400	
	3	94.18	37.23	450	300	150	100	
	4	68.09	22.01	400	650	700	350	
	5	82.65	52.27	835	800	650	400	
16	6	31.37	27.62	800	1150	1200	500	6
	1	9.81	6.84	800	1150	900	500	
	2	32.38	26.55	850	600	750	250	
	3	83.63	27.05	700	1050	1000	500	
	4	20.31	17.11	600	750	800	300	
	5	12.33	8.63	850	600	400	250	
	6	90.26	3.02	600	1000	1200	400	
	7	62.39	53.55	1250	800	600	300	
	8	57.52	53.37	500	850	900	450	
17	9	5.21	1.88	600	900	650	500	9
	1	35.26	4.13	650	300	500	150	
	2	71.50	44.06	1150	700	650	400	
	3	52.90	7.28	650	900	1000	500	
18	4	62.49	1.72	700	1000	900	500	4
	1	67.29	47.19	800	900	1100	600	
	2	46.19	41.01	1550	850	800	600	
	3	69.13	12.84	900	1050	1200	400	

(*Continued*)

TABLE 6 *(Continued)*

Problem	Product	Price	Cost	Mix (initial production) (Unit)	Market demand (Unit)	Maximum capacity (Unit)	Minimum capacity (Unit)	N. products
	4	84.60	10.31	800	1200	950	550	
	5	52.40	36.22	950	750	800	450	
	6	46.46	1.78	550	450	400	200	
19	7	30.14	21.79	650	850	900	450	7
	1	18.40	5.48	1550	1250	1100	750	
	2	76.90	39.60	900	1100	1200	650	
20	3	58.89	33.35	1200	1500	1400	800	3
	1	67.46	42.68	1100	1350	1400	600	
	2	59.13	44.01	650	550	420	300	
	3	45.57	40.38	1000	1400	1100	700	
	4	33.28	12.41	700	650	800	250	
21	5	42.98	16.42	150	200	250	100	5
	1	67.03	43.06	700	1050	1200	500	
	2	94.91	24.01	600	1000	750	450	
	3	53.15	49.81	900	1400	1000	750	
	4	14.37	9.62	550	350	500	100	
	5	21.74	7.93	400	600	500	300	
	6	26.00	13.69	700	1050	800	400	
	7	32.07	3.71	400	150	300	50	
22	8	13.51	2.35	850	700	650	450	8
	1	42.25	28.65	600	1050	750	400	
23	2	42.45	13.50	5750	5500	6000	3000	2
	1	42.28	24.49	1500	1200	2000	800	
	2	36.79	18.43	350	500	400	200	
	3	45.33	20.57	700	1050	1200	500	
	4	65.27	8.04	500	700	600	400	
	5	34.16	25.06	1000	900	1200	450	
	6	93.67	1.56	750	1200	1700	550	
	7	100.42	76.96	1100	1350	1300	700	
	8	91.52	10.18	500	700	850	400	
24	9	96.22	62.55	600	550	700	300	9
	1	17.68	0.28	800	1500	1800	650	
	2	52.68	22.55	1500	2000	1900	1000	
	3	30.19	15.54	1300	2000	1800	750	
25	4	80.93	73.02	1700	2000	2200	900	4
	1	89.84	50.08	1500	1400	1650	500	
26	2	70.80	35.78	1200	1500	1350	800	2
	1	11.74	6.24	800	1050	900	650	
	2	26.34	1.93	750	950	1100	550	
	3	22.99	21.65	3000	2650	3800	1200	
27	4	23.38	2.75	700	1500	1750	500	6
	5	57.24	5.46	1300	1250	1400	600	
	6	69.58	45.01	800	950	1200	550	

(Continued)

TABLE 6 (*Continued*)

Problem	Product	Price	Cost	Mix (initial production) (Unit)	Market demand (Unit)	Maximum capacity (Unit)	Minimum capacity (Unit)	N. products
	1	99.38	91.60	800	1050	900	400	
	2	28.37	7.65	800	750	1000	450	
	3	40.36	25.48	450	650	600	300	
	4	43.05	40.77	900	1150	1300	600	
	5	74.98	31.13	500	900	800	350	
	6	28.34	25.95	1200	1000	1300	800	
	7	53.58	3.05	400	600	800	300	
	8	7.43	3.36	900	1150	1100	650	
	9	67.07	7.63	450	550	700	200	
28	10	100.44	65.68	700	1000	900	450	10
	1	21.93	19.10	7600	7450	8000	4000	
29	2	87.99	82.17	6000	6600	7000	3500	2
	1	11.22	2.32	650	900	800	500	
	2	33.71	21.66	500	600	800	300	
	3	94.49	35.65	750	1000	1200	550	
	4	95.72	44.06	1000	800	1200	650	
30	5	80.49	64.20	1250	1750	1600	850	5
	1	79.54	72.50	800	1050	1000	600	
	2	12.69	7.71	800	900	1100	650	
	3	60.60	40.12	950	1200	1300	750	
	4	42.25	30.33	900	1000	1200	650	
	5	100.67	1.98	1100	1400	1200	900	
31	6	66.21	35.10	750	950	850	450	6
	1	56.73	21.77	5800	6500	6000	4500	
	2	27.14	6.41	800	950	900	650	
32	3	26.70	10.28	1200	1000	1500	850	3
	1	79.44	20.25	3000	2500	3200	2000	
	2	15.08	2.65	700	850	800	400	
	3	27.13	9.18	900	1100	1000	600	
33	4	24.05	2.18	1200	900	1500	500	4
	1	67.30	5.26	800	1000	900	400	
34	2	17.92	16.96	950	1200	1400	500	7
	3	3.85	1.92	1300	1150	1500	600	
	4	8.84	0.89	1500	1350	1800	1000	
	5	77.11	59.26	5000	6300	5800	4000	
	6	3.93	3.04	800	1350	1200	500	
34	7	4.91	2.70	3200	4000	3600	2500	7
	1	13.25	6.58	6500	8000	8600	5000	
	2	7.80	7.20	5200	7000	6800	3500	
35	3	10.20	9.70	6700	5500	7000	4000	3
	1	29.60	23.87	650	800	1000	400	
	2	44.91	19.88	500	750	650	300	
	3	92.00	33.94	1100	900	1400	750	

(*Continued*)

TABLE 6 (*Continued*)

Problem	Product	Price	Cost	Mix (initial production) (Unit)	Market demand (Unit)	Maximum capacity (Unit)	Minimum capacity (Unit)	N. products
	4	79.61	17.85	500	800	750	350	
	5	79.78	70.04	1200	1050	1600	750	
	6	50.48	22.73	500	600	800	300	
	7	96.87	17.42	550	650	700	350	
36	8	38.28	15.18	600	700	900	400	8
	1	37.52	5.38	3800	3500	4000	2500	
37	2	87.21	61.43	1800	1950	2000	1200	2
	1	53.01	38.53	1200	1500	1400	750	
	2	76.09	21.02	1000	1300	1100	650	
	3	58.56	16.40	1100	1250	1500	800	
	4	81.92	55.46	600	950	800	350	
38	5	15.56	4.90	800	1250	1000	500	5
	1	22.56	9.66	1500	1850	1600	800	
	2	14.17	3.84	900	800	1200	450	
	3	74.47	46.23	1550	1400	1800	950	
39	4	98.21	49.82	500	700	600	300	4
	1	30.41	25.51	1800	1500	2000	1000	
	2	16.68	14.75	2000	2300	2200	1600	
	3	32.71	21.12	4000	5200	4800	3550	
40	4	43.19	35.80	4500	4800	5000	3850	4
	1	61.72	16.77	1300	1500	1700	950	
	2	33.60	29.19	1100	950	1350	700	
41	3	91.78	60.86	1450	1350	1600	900	6
	4	61.54	47.60	900	1200	1000	650	
	5	62.94	46.76	900	1100	950	600	
41	6	3.87	1.57	1500	1400	1750	1000	6
	1	75.52	17.01	900	700	1000	450	
	2	88.45	8.06	700	950	800	450	
	3	1.44	0.92	850	950	900	500	
	4	70.07	59.39	900	800	1100	550	
	5	56.78	26.91	400	600	500	250	
	6	17.28	10.67	1000	1150	1100	650	
	7	72.86	0.54	650	700	800	500	
	8	77.22	55.20	400	600	500	250	
42	9	57.27	27.93	300	350	500	150	9
	1	58.65	49.04	3000	4000	3600	2350	
	2	13.42	5.00	2400	2100	2800	1500	
43	3	31.12	22.84	5200	5500	5800	4350	3
	1	21.20	5.25	1000	800	1200	550	
	2	77.09	9.32	800	950	900	600	
	3	39.60	28.90	1400	1200	1500	750	
	4	23.60	20.14	800	600	1000	450	
	5	5.15	1.33	1000	1350	1200	650	

(*Continued*)

TABLE 6 (*Continued*)

Problem	Product	Price	Cost	Mix (initial production) (Unit)	Market demand (Unit)	Maximum capacity (Unit)	Minimum capacity (Unit)	N. products
	6	37.14	21.45	800	1000	900	650	
44	7	35.68	25.97	600	750	650	450	7
	1	11.16	6.69	450	550	700	200	
	2	61.18	9.48	650	900	800	550	
	3	64.55	45.93	600	650	800	400	
	4	75.63	22.08	500	600	700	300	
	5	4.05	1.17	1400	1500	1800	950	
	6	44.27	41.16	800	1100	950	650	
	7	71.08	0.01	550	800	700	350	
	8	19.54	6.65	800	850	1000	550	
	9	10.37	8.90	1000	1250	1200	650	
45	10	28.34	13.17	600	750	700	450	10
	1	66.41	18.56	1300	1450	1600	850	
46	2	55.13	30.40	800	900	1100	650	5
	3	1.74	0.14	1100	950	1300	650	
	4	59.01	26.14	1800	1650	2000	1150	
46	5	9.65	5.26	1200	1450	1300	850	5
	1	65.79	56.54	1000	1100	1300	650	
	2	11.68	9.58	2800	3000	3200	2250	
	3	28.14	8.79	2000	2500	2250	1450	
	4	68.23	57.65	2500	2300	2600	1850	
	5	64.86	55.25	1200	950	1400	700	
	6	52.18	41.30	2500	3000	2700	1625	
	7	49.31	39.92	900	1050	1000	650	
47	8	24.41	1.81	2500	3000	2650	1350	8
	1	93.84	48.52	1800	2000	2300	1250	
	2	52.97	30.35	1000	750	1200	500	
	3	72.74	27.19	600	500	750	350	
	4	16.21	4.72	700	850	750	300	
	5	87.74	49.55	2000	2500	2250	1150	
48	6	44.72	13.35	1000	800	1150	550	6
	1	5.20	4.64	800	1100	950	550	
	2	79.20	5.86	800	550	900	300	
	3	76.72	55.88	3200	2500	3500	1650	
	4	56.72	1.49	2200	3000	2400	1750	
	5	85.08	78.86	600	800	700	350	
	6	45.08	35.50	800	850	900	500	
49	7	99.66	56.23	800	650	1200	400	7
	1	84.49	14.96	2700	3000	3900	2050	
50	2	18.95	3.17	2550	1950	2200	1100	2

TABLE 7 Compared results

Problem	Initial situation (production)	Initial situation (selling)	Future situation
1	$248050.00	$215200.00	$228750.00
2	$104201.08	$104201.08	$112183.97
3	$111056.05	$105059.82	$123665.64
4	$130206.33	$123754.82	$139536.42
5	$160299.34	$160299.34	$199150.91
6	$240290.42	$234704.26	$250378.61
7	$166589.69	$159946.24	$175473.06
8	$87387.08	$81480.66	$84164.77
9	$140881.51	$133438.02	$134527.30
10	$99796.87	$94933.05	$101150.78
11	$86646.82	$80055.75	$91323.41
12	$152334.06	$142981.45	$160730.75
13	$129767.24	$115562.86	$117740.45
14	$76045.37	$56370.41	$62468.91
15	$136519.33	$109499.23	$123572.56
16	$115647.44	$106041.01	$116001.51
17	$119450.59	$113094.18	$164851.17
18	$123978.03	$100734.68	$122920.22
19	$179580.48	$168247.90	$189021.07
20	$84259.57	$80382.07	$91012.87
21	$60867.67	$58311.71	$63764.14
22	$99908.67	$90196.48	$111109.84
23	$174643.88	$167405.53	$169446.03
24	$243938.05	$236005.32	$301592.78
25	$91598.16	$91598.16	$105111.74
26	$101676.09	$97699.28	$101201.28
27	$128130.14	$125073.21	$149555.92
28	$131289.26	$129776.99	$166373.86
29	$56436.57	$56012.46	$58234.97
30	$127960.95	$117630.55	$133486.36
31	$171704.19	$171704.19	$186459.45
32	$239012.10	$235728.35	$243150.09
33	$228643.88	$192492.59	$195530.72
34	$162006.53	$160523.75	$180706.58
35	$49814.26	$49214.26	$59186.78
36	$194112.94	$181039.64	$210857.42
37	$168537.03	$158894.90	$162761.84
38	$143202.65	$143202.65	$154956.07
39	$96623.88	$91354.55	$97484.62
40	$92289.03	$90820.02	$99578.35
41	$138683.55	$134699.12	$145891.89
42	$202157.75	$189388.50	$208255.01
43	$92052.42	$89527.07	$92808.79
44	$110091.82	$104069.97	$113662.46

(*Continued*)

TABLE 7 (*Continued*)

Problem	Initial situation (production)	Initial situation (selling)	Future situation
45	$140053.96	$140053.96	$165863.80
46	$148193.07	$143021.88	$153031.32
47	$183947.11	$179428.45	$190962.55
48	$247334.82	$230851.69	$250040.24
49	$293443.95	$254007.27	$266237.59
50	$227953.80	$218490.63	$239350.11

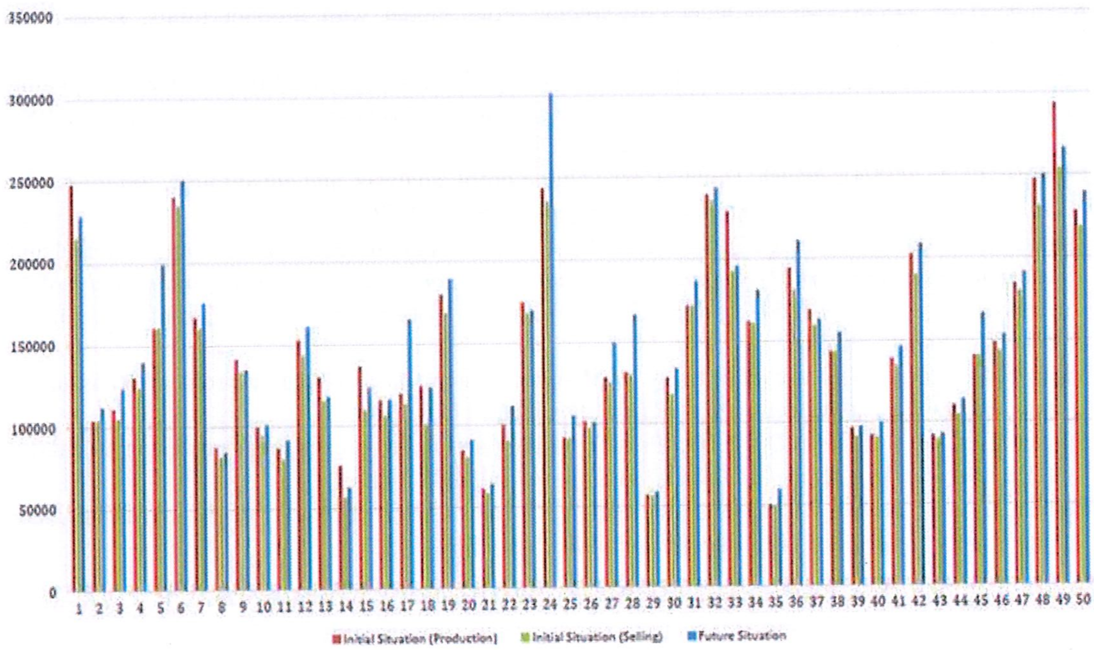

FIGURE 1 Results for each problem.

model and we carried out a computational experiment. Based on the results shown in the previous section, it can be concluded that the proposed model can be used as an important tool in decision making when it comes to balancing or adjusting production capacities, using the ideas or the assumption of the CVP model for a product and perspectives presented by Kakumanu (1998). Considering this, we believe our work contributes to the discussion in this research area because it contemplates and presents a solution to the situation in which there are multiple products. Furthermore, it is important to highlight that this situation can often be found in small manufacturers. Although our contribution to this discussion has been significant, we highlight some issues that still remain to be addressed in future studies. Among these issues, we emphasize that future studies should focus on the following problems:

- Multiple products should be considered in environments with stochastic variability, because the productive system does not always present a deterministic behaviour.

- The GBV model should be integrated with the costing systems, such as activity based costing or time-driven activity-based costing.
- The GBV model should be adjusted to consider different markets, such as perfect competition, monopolistic competition, oligopoly, and monopoly.

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