

nº 09

**P-localization of some  
classes of groups**

**Augusto Reynol Filho**

**dezembro 1987**

# P-LOCALIZATION OF SOME CLASSES OF GROUPS

BY

AUGUSTO REYNOL FILHO

## Introduction

In [2] the authors constructed the theory of P-localization of nilpotent groups and some time later Ribenboim in [3] showed that it was possible to localize any group. However, as far as I know, it is not yet known whether the construction made by Ribenboim yields a theory of P-localization when restricted to the category  $\mathcal{N}$  of all nilpotent groups. Besides that, the construction presented in [3] seems to be quite abstract.

These facts led us to consider two problems.

The first one is to try to exhibit a more explicit construction of the P-localization of a group  $G$  in the category  $\mathcal{G}$  of all groups. As for this, we were successful only when  $G$  is a semi-direct product of a finite abelian group  $A$  by any group  $X$ . It turns out that  $G_P$  is thoroughly determined by  $A$ ,  $X_P$  and the action  $\ast$  of  $X$  of  $A$ .

This question is analyzed in section 2 and the main results are 2.1, 2.5 and 2.10. We should also point out that the point in the proof of the main results of this section is Proposition 2.2.

Our second concern is to build up the theory of P-localization of a group in the category  $\mathcal{C}$  of the groups which are

extensions of nilpotent groups by finite abelian groups.

Our interest in this question is due to the fact that it seems to be useful to extend the theory of  $P$ -localization of nilpotent spaces.

Likewise, we attempted to follow rather closely the technique used in [2], since those methods turned out to be particularly suitable to develop the theory of  $P$ -localization of nilpotent spaces.

We devoted section 3 to present our results in this direction. We would say that the main results are 3.12 and 3.13, notwithstanding the fact that 3.1, 3.2 and 3.11 play an important role as technical lemmas.

Finally in section 1 we introduce some basic results in order to help the development of the following sections. Nevertheless, some of the results namely 1.5, 1.11 and 1.20 are important by themselves.

The author is very much indebted to Professor Daciberg Lima Gonçalves, at whose suggestions this work was developed.

### §1. Preliminaries.

In this section we introduce some general results on  $P$ -local groups, factoring of actions and some propositions concerning group cohomology.

Proposition 1.1 - Let  $E_1, \dots, E_t, K$  be  $P$ -local groups, where  $P$  is a set of primes. Let  $\epsilon_i \in \text{Hom}(E_i, K)$ ,  $i=1, \dots, t$ . In these conditions, if  $\epsilon \in \text{Hom}(E, K)$  is the pull-back of the family  $(\epsilon_i)_{1 \leq i \leq t}$ , then  $E$  is  $P$ -local.

Proof - Straightforward.

Proposition 1.2 - Let  $\phi \in \text{Hom}(Y, F)$ , where  $Y$  is a  $P$ -local group and

$F$  is a finite group. Then,  $(\forall y \in Y)$  we have  $o(\phi(y)) = n \in P^X$ .

$(P^X = \{n \in N^* \mid p|n \rightarrow p \in P\}) =$  multiplicative set generated by  $P$ ;  
 $o(\phi(y)) =$  order of  $\phi(y)$ .

Proof - Let  $y \in Y$  and suppose that  $\exists q \in P'$  with  $q|o(\phi(y))$ . Then we may consider  $z = y^k$ , where  $o(\phi(y)) = q.k$ . It follows that  $o(\phi(z)) = q$ . The fact that  $Y$  is  $P$ -local and  $q \in P'$  enables us to state that  $\forall r > 0, \exists z_r \in Y$  such that  $z_r^{q^r} = z$ . Thus,  $\phi(z_r)^{q^r} = \phi(z) \neq 1$  and  $\phi(z_r)^{q^{r+1}} = \phi(z)^q = 1$ , so  $o(\phi(z_r)) = q^{r+1}, \forall r > 0$ . In particular,  $\{\phi(z_r) \in F : r > 0\}$  is infinite. However this is impossible, since  $F$  is finite.

Corollary 1.3 - Under the conditions of the previous proposition, we have  $|\phi(Y)| \in P^X$  (ie,  $\phi(Y)$  is a  $P$ -torsion (finite) sub-group of  $F$ ).

Remark 1.4 - According to proposition (7.1) pg 106 in [4] we have that a finite group  $F$  is  $P$ -local  $\leftrightarrow F$  is a  $P$ -torsion group.

The full subcategory of the category  $\mathcal{G}$  of all groups consisting of all nilpotent groups is denoted by  $\eta$ .

The conclusion of proposition 1.5 holds either when  $X \xrightarrow{e} X_P$  is the  $P$ -localization on  $\mathcal{G}$  or  $\eta$ .

Proposition 1.5 - Let  $X \xrightarrow{u} \text{Aut}(N)$  be an action, where  $X$  and  $N$  are groups with  $\text{Aut}(N)$  finite. Then  $\exists ! u_P$  such that the diagram

$$\begin{array}{ccc} X & \xrightarrow{u} & \text{Aut}(N) \\ \downarrow & \nearrow u_P & \\ X_P & & \end{array}$$
 is commutative  $\leftrightarrow u(X)$  is a  $P$ -torsion sub-group of  $\text{Aut}(N)$ .

Proof - ( $\rightarrow$ ) If there exists such a  $u_P$ , then  $u(X) \subset u(X_P) = P$ -torsion sub-group of  $\text{Aut}(N)$ , by corollary 1.3.

( $\leftarrow$ ) a) First of all we all suppose that  $X \xrightarrow{e} X_P$   $P$ -localizes  $X$  on  $\mathcal{G}$ .  $u(X)$  is  $P$ -local (finite) due to remark 1.4.

Then, 
$$\begin{array}{ccc} X & \xrightarrow{\bar{u}} & \mu(X) \xrightarrow{1} \text{Aut}(N) \\ e_0 \downarrow & \nearrow \bar{u}_P & \\ X_P & & \end{array}$$
 looking at  $\bar{u}(x) = u(x)$ ,  
 $\forall x \in X$ , there exists  $\bar{u}_P$

such that  $\bar{u}_P e_0 = \bar{u}$ . Therefore,  $u_P e_0 = (i\bar{u}_P) e_0 = u$ .

It's easy to see that  $u_P$  is the only one.

b)  $X \xrightarrow{e_0} X_P$  is the P-localization on  $n$ .

As  $u(X)$  is nilpotent (and P-local), the proof follows the same steps.

Now we take  $A \xrightarrow{u} G \xrightarrow{c} X$  an exact sequence of groups, where A is abelian.

Let  $X \xrightarrow{u} \text{Aut}(A)$  be the action defined by  $\mu(u(x).a) = g.u(a).g^{-1}$  (where  $c(g) = x$ ).

Fix a collection of primes P,  $n \in \mathbb{N}$  and  $x \in X$ , and define:  $\theta_n(x) = 1_A + u(x) + \dots + u(x^{n-1}) \in \text{End}(A)$ .

For this endomorphism we have:

Lemma 1.6 -  $(\mu(a).g)^n = \mu(\theta_n(c(g)).a)g^n; \forall g \in G, \forall a \in A, \forall n \in \mathbb{N}$ .

Proof - It's easy by induction on n.

Proposition 1.7 - Let P be a set of primes and let  $A \xrightarrow{u} G \xrightarrow{c} X$  be an exact sequence of groups, where A is abelian and  $u$  is the action attached to the extension. Fix the conditions: (i) G is P-local; (ii) X is P-local; (iii)  $\theta_n(x) \in \text{Aut}(A), \forall x \in X, \forall n \in P^X$ .

Then, if two of (i); (ii); (iii); hold so does the third.

Proof - (ii) + (iii)  $\rightarrow$  (i).

Fix  $n \in P^X$ . Let  $g, h \in G$  and suppose that  $g^n = h^n$ . Then,  $c(g)^n = c(h)^n \rightarrow c(g) = c(h)$  since X is P-local. So,  $g = \mu(a).h$  and  $h^n = g^n = (\mu(a).h)^n = \mu(\theta_n(c(h)).a)h^n$  (Lemma 1.6). Hence,  $\theta_n(c(h)).a = 0$ . So  $a = 0$  and  $g = h$ .

Likewise, let  $g \in G. \exists x \in X$  such that  $c(g) = x^n$ . (X P-local). Therefore,  $c(g) = x^n = c(h^n). \therefore g = \mu(a).h^n$ . Take  $b \in A$

such that  $a = \theta_n(\varepsilon(h)).b$ . Thus  $g = \mu(\theta_n(\varepsilon(h)).b).h^n = (\mu(b).h)^n$  due to 1.6. So  $g \in G \rightarrow g^n \in G$  is bijective.

(The other implications are similar).

**Proposition 1.8** - Let  $P$  be a set of primes and let  $A \xrightarrow{\mu} G \xrightarrow{\varepsilon} X$  be an exact sequence of groups, where  $A$  is finite abelian. Then,  $\theta_n(x) \in \text{Aut}(A), \forall x \in X, \forall n \in P^X$ , provided that either  $G$  is  $P$ -local or  $A$  and  $X$  are  $P$ -local.

**Proof** - (I)  $G$  is  $P$ -local.

Fix  $n \in P^X$  and  $x \in X$ . Suppose that  $\theta_n(x).a = 0$ . Let  $g \in G$  such that  $\varepsilon(g) = x$ . Then  $(\mu(a).g)^n = \mu(\theta_n(x).a)g^n = g^n$ . So  $\mu(a).g = g$  and  $a = 0$ . Thus  $\theta_n(x) \in \text{Aut}(A)$  since  $A$  is finite.

(II)  $A$  and  $X$  are  $P$ -local.

Fix  $n \in P^X$  and  $x \in X$ , and suppose  $\theta_n(x).a = 0$ . Thus,  $(\omega(x^n) - 1_A).a = (\omega(x) - 1_A) \circ \theta_n(x).a = 0 \therefore \omega(x^n).a = a$ . On the other hand,  $\omega(\omega(x)) = m \in P^X$  (prop. 1.2)  $\therefore \omega(x)^m.a = a$ . As  $\gcd(m, n) = 1$ , it follows that  $\omega(x).a = a$ , whence  $0 = \theta_n(x).a = n.a$ . Thus  $a = 0$  since  $A$  is  $P$ -local. So  $\theta_n(x) \in \text{Aut}(A)$ .

**Corollary 1.9** - In the conditions of the proposition above (1.8),  $G$  is  $P$ -local  $\Leftrightarrow A$  and  $X$  are  $P$ -local.

**Proof** - ( $\Leftarrow$ )  $A$  and  $X$   $P$ -local  $\rightarrow \theta_n(x) \in \text{Aut}(A), \forall n \in P^X, \forall x \in X$ . Thus  $G$  is  $P$ -local, by prop. 1.7.

( $\rightarrow$ )  $G$   $P$ -local  $\rightarrow \theta_n(x) \in \text{Aut}(A), \forall n \in P^X, \forall x \in X$ . In particular,  $\theta_n(1) \in \text{Aut}(A)$ . So  $A$  is  $P$ -local. Besides, we get  $X$   $P$ -local by invoking prop. 1.7.

Next we consider an extension  $N \xrightarrow{\mu} G \xrightarrow{\epsilon} X$ , where  $N$  is a finite group. Let  $X \xrightarrow{\sigma} \text{Aut}(N)$  be the action given by  $\mu(\sigma(x).a) = \sigma(x).\mu(a).\sigma(x)^{-1}$  and take  $\sigma_n(x): N \rightarrow N$  defined by  $\sigma_n(x).a = 1_A(a).(\sigma(x).a) \dots (\sigma(x)^{n-1}.a)$ ,  $x \in X$ ;  $n \in \mathbb{N}^*$ :

In this slightly different context we now describe properties which are quite similar to prop. 1.6, 1.7, 1.8 and 1.9.

Lemma 1.10 -  $(\mu(a).(x))^n = \mu(\sigma_n(x).a).\sigma(x)^n$ ;  $\forall x \in X$ ;  $\forall n \in \mathbb{N}^*$ ;  
 $\forall a \in A$ .

Proof - See prop. 1.6.

Proposition 1.11 - Let  $N \xrightarrow{\mu} G \xrightarrow{\epsilon} X$  be a split short exact sequence of groups and let  $\sigma$  be the action defined by the splitting  $\epsilon$ . Fix the statements:

(i)  $G$  is  $P$ -local; (ii)  $X$  is  $P$ -local; (iii)  $\sigma_n(x)$  is a bijection,  $\forall x \in X$ ;  $\forall n \in P \setminus X$ .

Then, if two of (i); (ii); (iii) hold, so does the third.

Proof - See prop. 1.7.

Proposition 1.12 - Let  $N \xrightarrow{\mu} G \xrightarrow{\epsilon} X$  be a split short exact sequence of groups, where  $N$  is finite. Then  $\sigma_n(x)$  is a bijection,  $\forall n \in P \setminus X$ ,  $\forall x \in X$  provided that either  $G$  is  $P$ -local or  $N$  and  $X$  are  $P$ -local.

Proof - If  $G$  is  $P$ -local, then the proof follows as (I) prop. 1.8.

So let's suppose that  $N$  and  $X$  are  $P$ -local. Let  $\sigma(X) \triangleleft \text{Aut}(N)$  and  $\bar{G} = N \rtimes_{\sigma(X)} \sigma(X)$ .  $\sigma(X)$  is a  $P$ -torsion group (cor. 1.3) and  $N$  is a

P-torsion group (remark 1.4). Thus  $\bar{G}$  is a P-torsion group. So  $\bar{G}$  is P-local (remark 1.4).

Then taking the sequence  $N \xrightarrow{\mu} \bar{G} \xrightarrow{\epsilon} \omega(X)$  and  $\xleftarrow{\alpha}$  invoking the first statement of this proposition we conclude that  $\bar{\theta}_n(\tau): N \rightarrow N$  given by  $\bar{\theta}_n(\tau) = 1_N \cdot 1(\tau) \dots 1(\tau^{n-1}) = 1_N \tau \dots \tau^{n-1}$  is a bijection,  $\forall \tau \in \omega(X), \forall n \in P^{\times}$ . So,  $\forall x \in X \forall n \in P^{\times}$  we have that  $\theta_n(x)$  is a bijection, since  $\theta_n(x) = 1_N \cdot \omega(x) \dots \omega(x^{n-1}) = \bar{\theta}_n(\tau)$ , where  $\tau = \omega(x) \in \omega(X)$ .

Corollary 1.13 - Under the condition of the previous proposition (1.12),  $G$  is P-local  $\leftrightarrow N$  and  $X$  are P-local.

Proof - See prop. 1.9.

Proposition 1.14 - Let  $N \xrightarrow{\mu} G \xrightarrow{\epsilon} X$  be an exact sequence of groups. Then,  $G$  and  $X$  P-local  $\rightarrow N$  P-local.

Proof - It follows directly from the definitions.

From now on we recall some concepts concerning group cohomology, as well as establish some results which play an important role in §3.

Let  $X \xrightarrow{\mu} \text{Aut}(A)$  and  $X \xrightarrow{\theta} \text{Aut}(B)$  be actions, where  $A$  and  $B$  are abelian. Let also  $\alpha \in \text{Hom}_{\mathbb{Z}[X]}(A, B)$ . (i.e.  $\alpha(\omega(x) \cdot a) = \omega(x) \cdot \alpha(a)$ ). Thus, if  $\zeta: A \xrightarrow{\mu} G \xrightarrow{\epsilon} X$  ( $\zeta \in H^2_{\mathbb{Z}}(X; A)$ ), then  $\alpha_* \zeta: B \xrightarrow{\nu} Q \xrightarrow{\omega} X$  where the construction of  $Q$  is described as follows:

Let  $M = B \int_{\theta} G; H = \{(-\alpha(a), \mu(a)) \in M: a \in A\}$  ( $H \triangleleft M$ ) and  $Q = M/H$ . So we have the commutative diagram

$$(1.15) \quad \begin{array}{ccccc} A & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & X \\ \downarrow \alpha & & \downarrow \beta & & \parallel \\ B & \xrightarrow{\nu} & Q & \xrightarrow{\tau} & X \end{array}, \quad \begin{array}{l} \text{where } \beta(g) = (0, g).H \\ \nu(b) = (b, 1).H \\ \tau[(b, g).H] = \epsilon(g) \end{array}$$

Proposition 1.16 - Consider the diagram

$$\begin{array}{ccccc} \zeta: A & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & X \\ \alpha \downarrow & & & & \downarrow \gamma \\ \tau: B & \xrightarrow{\nu} & Q & \xrightarrow{\tau} & Y \end{array}$$

where A and B are abelian and the rows are exact.

If there exists  $\beta \in \text{Hom}(G, Q)$  making the diagram commutative, then  $\alpha \in \text{Hom}_{\mathbb{Z}[X]}(A, B)$  and  $\alpha_* \zeta = \gamma_* \tau$ .

Conversely, if  $\alpha \in \text{Hom}_{\mathbb{Z}[X]}(A, B)$  and  $\alpha_* \zeta = \gamma_* \tau$ , then there does exist  $\beta \in \text{Hom}(G, Q)$  making the diagram commutative.

Proof - Straightforward.

Proposition 1.17 - In the diagram  $A \xrightarrow{\mu} G \xrightarrow{\epsilon} X$ , the rows

$$\begin{array}{ccccc} A & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & X \\ \alpha \downarrow & & \tau \downarrow \beta & & \downarrow \gamma \\ B & \xrightarrow{\nu} & Q & \xrightarrow{\tau} & Y \end{array}$$

are exact, A and B are abelian and  $\tau$  and  $\beta$  yield commutativity.

Then, there exists a cross homomorphism  $\kappa: X \rightarrow B$  such that  $\forall g \in G$ ,  $\beta(g) = \nu \kappa \epsilon(g) \cdot \tau(g)$ .

Proof -  $g \in G$ ,  $\nu \beta(g) = \gamma \tau(g) = \tau \tau(g)$ .  $\therefore \exists! b_g \in B$  such that  $\beta(g) = \nu(b_g) \tau(g)$ . So we define  $G \xrightarrow{\rho} B$  by  $\rho(g) = b_g$ .

Furthermore,  $g^{i-1} g \in \mu(A) \rightarrow b_g = b_{g'}$ . This fact enable us to define  $G \xrightarrow{\rho} B$   $k$  such that  $k \epsilon = \rho$ . Not it's easy to see that  $\beta(g) = \nu k \epsilon(g) \cdot \tau(g)$  and  $K$  is a cross homomorphism.

$$\begin{array}{ccc} & \rho & \\ \alpha \downarrow & \dashrightarrow & \\ X & \xrightarrow{\kappa} & B \end{array}$$

Proposition 1.18 - 
$$\begin{array}{ccccc} A & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & X \\ \alpha \downarrow & & \downarrow \tau & & \downarrow \gamma \\ B & \xrightarrow{\nu} & Q & \xrightarrow{\eta} & Y \end{array}$$
 In the commutative diagram the rows are exact and  $A$  and  $B$  abelian.

Let  $X \xrightarrow{\kappa} B$  be a cross homomorphism. In these conditions the function  $G \xrightarrow{\beta} Q$  given by  $\beta(g) = \nu\kappa(g) \cdot \tau(g)$ ,  $\forall g \in G$  is a group homomorphism.

Proof - It follows easily from the definitions.

Lemma 1.19 - Let  $Q$  be a  $P$ -torsion abelian group. Then  $H_q(Q)$  is a  $P$ -torsion abelian group,  $\forall q > 0$ . (Here  $H_q(Q)$  means the homology of the group  $Q$  with integer coefficients).

Proof - The assertion is readily checked if  $Q$  is a cyclic group. Use of Künneth Formula shows the assertion to be true for finite direct sums of cyclic groups, and therefore for arbitrary finitely generated abelian group. Finally, since  $H_q(Q)$  ( $q > 0$ ) commutes with direct limits and  $\varinjlim$  ( $P$ -torsion group) is a  $P$ -torsion group, the assertion is true for arbitrary abelian groups.

The next theorem is stated in the category  $\mathcal{A}$ .

Theorem 1.20 - Consider the commutative diagram 
$$\begin{array}{ccc} X & \xrightarrow{\alpha} & \text{Aut}(A) \\ \circ \downarrow & & \circ \\ X_P & \xrightarrow{\alpha_P} & \end{array}$$
 where  $X$  is a nilpotent group,  $A$  is a  $P$ -local abelian group and  $\alpha, \alpha_P$  are actions.

Then we have 
$$H_{\alpha_P}^n(X_P; A) \xrightarrow{\cong} H_{\alpha}^n(X; A).$$

Proof - (Induction on  $c = \text{nil } X$ .)

If  $X$  is abelian, we take the following short exact sequences:

$$0 \rightarrow \text{Ker}(e_0) \rightarrow X \xrightarrow{e_0^*} e_0(X) \rightarrow 0 \dots \text{(I)}$$

$$0 \rightarrow e_0(X) \xrightarrow{e_0''} X_P \rightarrow \text{coker}(e_0) \rightarrow 0 \dots \text{(II)}$$

(I) yields a spectral sequence (Lyndon-Hochschild-Serre) where  $E_2^{r,s} = H^r(e_0(X); H^s(\text{ker}(e_0); A))$ .

We notice that  $\text{ker}(e_0)$  acts trivially on  $A$ . So we are allowed to say that

$$0 \rightarrow \text{Ext}(H_{s-1}(\text{ker}(e_0)); A) \rightarrow H^s(\text{ker}(e_0); A) \rightarrow \text{Hom}(H_s(\text{ker}(e_0); A) \rightarrow 0$$

Since  $\text{Hom}(P'\text{-torsion}, P\text{-local}) = 0 = \text{Ext}(P'\text{-torsion}, P\text{-local})$  and  $H_s(\text{ker}(e_0))$  ( $s > 0$ ) is  $P'$ -torsion (Lemma 1.19) we conclude that  $H^s(\text{ker}(e_0); A) = (0)$ .

Moreover,  $e_0(X)$  acts trivially on  $H^s(\text{ker}(e_0); A)$ , since  $e_0(X)$  acts trivially on  $\text{ker}(e_0)$ . Thus  $E_2^{r,s} = \begin{cases} H^r(e_0(X); A); & s = 0 \\ (0) & ; s > 0 \end{cases}$

whence  $H^r(e_0(X); A) \xrightarrow{e_0^*} H^r(X; A)$ .

The same argument shows us (using (II)) that

$$H_u^s(X_P; A) \xrightarrow{e_0''} H_u^s(e_0(X); A)$$

Taking into account that the diagram commutes

$$\begin{array}{ccc} H_u^n(X; A) & \xleftarrow{e_0^*} & H_u^n(X_P; A) \\ & \searrow e_0'' & \swarrow e_0'' \\ & & H^n(e_0(X); A) \end{array}$$

we have  $e_0^*$  is an isomorphism, so that the proof is ended if  $c = \text{nil } X = 1$ .

Now let  $X = |\eta|$  with  $\text{nil } X = c > 1$  and set  $r = r^c X \neq \{1\}$ . Then, the commutative diagram with exact rows

$$\begin{array}{ccccc}
 \Gamma & \twoheadrightarrow & X & \twoheadrightarrow & X/\Gamma & \text{induces a map of the spectral} \\
 e''_0 \downarrow & & \downarrow e_0 & & \downarrow e'_0 & \text{sequences.} \\
 \Gamma_P & \twoheadrightarrow & X_P & \twoheadrightarrow & (X/\Gamma)_P
 \end{array}$$

In particular we obtain the commutative diagram

$$\begin{array}{ccc}
 E_2^{r,s} = H^r(X/\Gamma; H^s(\Gamma; A)) & \xleftarrow{e''_0} & H^r((X/\Gamma)_P; H^s(\Gamma; A)) \\
 \uparrow (e'_0, e''_0)^* & & \uparrow (e''_0)^* \\
 E_2^{r,s} = H^r((X/\Gamma)_P; H^s(\Gamma_P; A)) & \xrightarrow{(e''_0)^*} & 
 \end{array}$$

where the coefficients are trivial in all cases.

It now follows from the first step that  $H^s(\Gamma_P; A) \xrightarrow{(e''_0)^*} H^s(\Gamma; A)$ , whence  $(e''_0)^*$  is an isomorphism. Also, it follows from the inductive hypothesis that  $e'_0$  is an isomorphism, since  $H^s(\Gamma; A)$  is  $P$ -local.

So  $E_2^{r,s} \xrightarrow{(e'_0, e''_0)^*} E_2^{r,s}$  and we may infer that  $H^n(X_P; A) \xrightarrow{(e''_0)^*} H^n(X; A)$  finishing the proof.

Finally, in order to finish this section, we are going to consider the following situation (which will appear again in the next two sections):

Let  $X \xrightarrow{\mu} \text{Aut}(A)$  be an action, where  $A$  is a finite abelian  $p$ -group ( $p \neq P$ ).

Let  $P_1 = \{q \in P' : q \mid |\mu(X)|\}$  and set  $H = \langle x \in X : o(\mu(x)) \in P_1^X \rangle =$  (sub-group generated by  $x \in X$  such that  $o(\mu(x)) \in P_1^X$ ). Noticing that  $H \triangleleft X$  and setting  $\mu_H = \mu|_H : H \rightarrow X \xrightarrow{\mu} \text{Aut}(A)$  we get:

Proposition 1.21 -  $\Gamma_{\omega_H}^j$  is a  $\omega$ -sub-module of  $A$ ,  $\forall j \geq 1$ .

Proof - It follows, by induction on  $j$ , from the fact that  $\omega(x) \cdot (\omega(h) \cdot a - a) = \omega(xhX^{-1}) \cdot (\omega(x) \cdot a) - \omega(x) \cdot a$ , together with the fact that  $H \triangleleft X$  ( $a \in \Gamma_{\omega_H}^{j-1}$ ;  $h \in H$ ;  $x \in X$ ).

Next set  $\Gamma = \Gamma_{\omega_H}^r$ , where  $r$  is the smallest positive integer such that  $\Gamma_{\omega_H}^r = \Gamma_{\omega_H}^{r+1}$ .

Proposition 1.21 enables us to take  $X \xrightarrow{\bar{\omega}} \text{Aut}(A/\Gamma)$ , where  $\bar{\omega}(x)(a+\Gamma) = \omega(x) \cdot a + \Gamma$ . Then it's plain that  $\bar{\omega}|_H = \bar{\omega}|_H$  and we have:

Lemma 1.22 -  $\bar{\omega}|_H$  is trivial.

Proof - Recall that  $\Gamma_{\bar{\omega}|_H}^j = \Gamma_{\omega_H}^j / \Gamma_{\omega_H}^r$ . Thus  $\bar{\omega}|_H$  is nilpotent.

Remembering that  $A/\Gamma$  is a finite abelian  $p$ -group and invoking the prop. 7, pg. 7, [1] we conclude that  $\bar{\omega}(H) = \bar{\omega}_H(H)$  is a (finite)  $p$ -group.

Taking, on the other hand,  $h$  a generator of  $H$ , we have  $h \in X$  and  $o(\omega(h)) \in P_1^X$ . It follows that  $o(\bar{\omega}(h)) \in P_1^X$ , since  $o(\bar{\omega}(h)) | o(\omega(h))$ . Taking into account that  $P_1 \subset P'$ , it follows, at last, that  $o(\bar{\omega}(h)) = 1$  and therefore  $\bar{\omega}(h) = 1_{A/\Gamma}$ ,  $\forall h \in H$ .

Proposition 1.23 -  $\bar{\omega}(X)$  is a  $P$ -torsion sub-group of  $\text{Aut}(A/\Gamma)$ .

Proof - Suppose that  $\exists q \in P'$  with  $q | |\bar{\omega}(X)|$ . Thus  $\exists y \in X$  such that  $o(\bar{\omega}(y)) = q$ . Then we have  $o(\omega(y)) = q^s \cdot m$ ,  $\text{gcd}(q, m) = 1$  (for  $o(\bar{\omega}(y)) | o(\omega(y))$ ). Hence,  $o(\omega(y^m)) = q^s$  and therefore  $o(\bar{\omega}(y^m)) = q$ . It follows that  $\bar{\omega}(y^m) \neq 1_{A/\Gamma}$  which contradicts the statement of the preceding lemma (1.22).

Corollary 1.24 -  $\exists!$  action  $X_P \xrightarrow{\omega'} \text{Aut}(A/r)$ , with  $\omega'(X_P) = \bar{\omega}(X)$ , which yields commutativity in the following diagram

$$\begin{array}{ccc} X & \xrightarrow{\bar{\omega}} & \text{Aut}(A/r) \\ \omega_0 \downarrow & \nearrow \omega' & \\ X_P & & \end{array}$$

Proof - It follows directly from the prop. 1.5

(Remark: We should point out that this corollary (1.24) holds either in the category  $\mathcal{G}$  or  $\mathcal{n}$ , just because prop. 1.5 is true in both categories.)

Now consider the commutative diagram

$$\begin{array}{ccccc} A & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & X, \\ \alpha \downarrow & & \downarrow \beta & & \downarrow \gamma \\ B & \xrightarrow{\nu} & Q & \xrightarrow{\tau} & Y \end{array}$$

where the rows are exact and  $A$  and  $B$  are finite abelian  $p$ -groups. Let  $P_1(A) = \{q \in P' : q \mid |\omega_1(X)|\}$ ;  $P_1(B) = \{q \in P' : q \mid |\omega_2(Y)|\}$ .

Proposition 1.25 - In the conditions above,  $\alpha(r_{\omega_1|_{H_A}}^j) \in r_{\omega_2|_{H_B}}^j, \forall j \geq 1$  (In particular  $\alpha(r(H_A)) \in r(H_B)$ ).

Proof - We argue by induction on  $j$ . It's clear for  $j = 1$ . Next, take  $a \in r_{\omega_1|_{H_A}}^{j-1}$  and  $x$  a generator of  $H_A$ . Thus,  $\omega_1(x) = n \in P_1(A)^X$ . Let's suppose that  $\omega_2(\gamma(x)) = r \cdot s$  where  $r \in P^X$  and  $s \in P_1(B)^X$ . Let  $n_1 = \omega_1(x^S) \mid \omega_1(x) = n$ . Then,  $n_1 \in P_1(A)^X$ . Now,  $\omega_2(\gamma(x^S))^r \cdot \alpha(a) = \alpha(a)$  and  $\omega_2(\gamma(x^S))^{n_1} \cdot \alpha(a) = \alpha(\omega_1(x^S)^{n_1} \cdot a) = \alpha(a)$ . It follows that  $\omega_2(\gamma(x^S)) \cdot \alpha(a) = \alpha(a)$ , due to the fact that  $\gcd(r, n_1) = 1$ . Also,  $\gcd(r, s) = 1 \iff \exists k, t \in \mathbb{Z}$  such that  $kr + ts = 1$ .  $\therefore \omega_2(\gamma(x)) \cdot \alpha(a) = \omega_2(\gamma(x)^r)^k \cdot \omega_2(\gamma(x^S))^t \cdot \alpha(a) = \omega_2(\gamma(x)^r)^k \cdot \alpha(a)$ . But  $\omega_2(\gamma(x^r)) = s \in P_1(B)^X$ .  $\therefore \gamma(x^r) \in H_B$ , whence  $\gamma(x^r)^k \in H_B$ .

It follows that  $\alpha(\omega_1(x).a - a) = \omega_2(\gamma(x)).\alpha(a) - \alpha(a) = \omega_2(\gamma(x^r))^k.\alpha(a) - \alpha(a) \in \Gamma_{2|H_B}^J$ , since  $\alpha(a) \in \Gamma_{2|H_B}^{J-1}$  by the inductive hypothesis. Now it's easy to finish the proof.

## §2. P-Localization of Semi-Direct Products.

Throughout this section we just work with the theory of localization in the category  $\mathcal{G}$ , as developed in [3].

Let  $P$  be a set of primes.

**Proposition 2.1** - Let  $N \xrightarrow{h} G \xrightarrow{e} X$  be an exact sequence of groups, where  $N$  is a  $p$ -group ( $p \in P'$ ). Then,  $e = e_0$   $P$ -localizes  $G$ , where  $X \xrightarrow{e_0} X_P$   $P$ -localizes  $X$ .

**Proof** - Let  $\phi \in \text{Hom}(G, K)$ ;  $K$   $P$ -local.

$$\begin{array}{ccc}
 N \xrightarrow{h} G & \xrightarrow{e} & X \\
 \downarrow \phi & \swarrow \phi' & \downarrow e_0 \\
 K & \xrightarrow{\quad} & X_P
 \end{array}$$

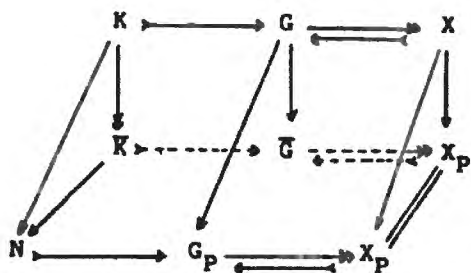
$\forall a \in N, r \geq 0$  such that  $a^{P^r} = 1$ .  
 Thus,  $\phi \mu(a)^{P^r} = 1 \in K$ . So  $\phi \mu(a) = 1$ , since  $K$  is  $P$ -local and  $p \in P'$ .

So  $\exists! \phi' \in \text{Hom}(X, K)$  such that  $\phi' e = \phi$ .

Finally, taking into account that  $K$  is  $P$ -local, we conclude that  $\exists! \bar{\phi} \in \text{Hom}(X_P, K)$  such that  $\bar{\phi} e_0 = \phi'$ . The uniqueness is obvious.

Next we prove a proposition which plays a fundamental role in the proof of the main theorems in this section.

**Proposition 2.2** - Let  $X \xrightarrow{\mu} \text{Aut}(K)$  and  $X_P \xrightarrow{\bar{\mu}} \text{Aut}(K)$  be actions, where  $K$  and  $\bar{K}$  are finite groups,  $\bar{K}$   $P$ -local. Let  $G = K \rtimes_{\mu} X$  and  $G = \bar{K} \rtimes_{\bar{\mu}} X_P$ . Consider also the commutative diagram



In the diagram  $\mu, \epsilon, \sigma, \bar{\mu}; \bar{\epsilon}, \bar{\sigma}, \epsilon_p, \sigma_p$  are defined as usual.  $e_p$  localizes  $G$ ,  $N = \ker \epsilon_p$ ,  $\mu': N \rightarrow G_p$ ,  $e'$  is defined by the restriction of  $e$ ,  $\nu$  is surjective and  $\bar{e}\nu = e'$ . Let's also suppose that  $\forall x \in X$ ,

$\forall a \in K$  we have:

$$(i) \nu(\mu(x).a) = \bar{\mu}(e_0(x)).\nu(a) \quad \text{and} \quad (ii) e(\mu(x).a) = \mu_p(e_0(x)).e'(a).$$

Under these conditions we claim that  $\bar{e}(\bar{\mu}(z).a) = \mu_p(z).\bar{e}(a)$ ,  $\forall z \in X_p$ ,  $\forall a \in K$ .

Proof -  $\bar{e} \in \text{Hom}(G, \bar{G})$  due to the hypothesis about  $\nu$ , as well as  $\bar{G}$  is  $P$ -local by corollary 1.13. So  $\exists ! \phi \in \text{Hom}(G_p, \bar{G})$  such that  $\phi e_p = \bar{e}$  and it's plain that  $\bar{\nu}\phi = \phi_p$  and  $\phi\sigma_p = \bar{\sigma}$ .

$\phi$  defines  $\phi'$  by restriction, whence we have  $N \xrightarrow{\phi'} K$  with  $\bar{\mu}'\phi' = \phi_p$  and  $\phi'e' = \nu$ . So  $\phi'\bar{e} = 1_K$ . Then we get the split short exact sequence  $B \rightarrow N \xrightarrow[\bar{e}]{\phi'} K$ ,  $B = \ker \phi'$ .

Use the proposition 6, pg. 8, [3] shows that  $X_p = \bigcup_{i=0}^{\infty} \langle I_{P^i, 1}(X_p, e_0(X)) \rangle$ .

Therefore, it's enough to show that our formula holds  $\forall z \in \langle I_{P^i, 1}(X_p, e_0(X)) \rangle \forall i \geq 0$ . We argue by induction on  $i$ .

$\langle I_{P^0, 0}(X_p, e_0(X)) \rangle = e_0(X)$  and  $\bar{e}(\bar{\mu}(e_0(x)).\bar{a}) = \mu_p(e_0(x)).\bar{e}(\bar{a})$  easily.

Now suppose  $z \in I_{P^i, 1}(X_p, e_0(X)) = I_{P^i, 1}(X_p, \langle I_{P^{i-1}, 1}(X_p, e_0(X)) \rangle)$ .

Thus,  $\exists n \in P^{\times}$  such that  $z^n \in \langle I_{P^{i-1}, 1}(X_p, e_0(X)) \rangle$ , whence  $\bar{e}(\bar{\mu}(z^n).\bar{a}) = \mu_p(z^n).\bar{e}(\bar{a})$  by inductive hypothesis.

Next we take  $m = o(\bar{\mu}(z^n)) \in P^{\times}$  by prop. 1.2.

So,  $\omega_P(z^{nm}).\tilde{e}(\bar{a}) = \tilde{e}(\bar{w}(z^n)^m.\bar{a}) = \tilde{e}(\bar{a})$ , whence  
 $\omega_P(z^{nm})|_{\tilde{e}(K)} = 1_{\tilde{e}(K)}$ .

On the other hand, taking  $\tau = \phi' \circ \omega_P(z^m) \circ \tilde{e}$  we have  
 $\tau = \bar{w}(z^m)$  and  $o(\bar{w}(z^m)) \in P^X$  by prop. 1.2 again.

But,  $\bar{w}(z^m)^n = 1_K \rightarrow o(\bar{w}(z^m))|n \therefore \bar{w}(z^m) = 1_K$ .

So,  $\phi' \circ \omega_P(z^m) \circ \tilde{e} = 1_K$ . Now recalling that  $B \xrightarrow{\phi} N \xleftarrow{\psi} K$   
 splits we have  $\omega_P(z^m).\tilde{e}(\bar{a}) = b.\tilde{e}(\bar{a}_1)$ ;  $b \in B$ ;  $\bar{a}_1 \in K$ .

Use of the  $\phi'$ .  $\omega_P(z^m).\tilde{e} = 1_K$  shows that  $\bar{a}_1 = \bar{a}$ .

$\therefore \omega_P(z^m).\tilde{e}(\bar{a}) = b.\tilde{e}(\bar{a})$ .

Applying successively  $\omega_P(z^m)$  to the expression above we  
 obtain:  $\tilde{e}(\bar{a}) = \omega_P(z^m)^n.\tilde{e}(\bar{a}) = [(\omega_P(z^m)^{n-1}.b) \dots (\omega_P(z^m).b).b]\tilde{e}(\bar{a}) =$   
 $= (\bar{\theta}_n(z^m).b^{-1})^{-1}.\tilde{e}(\bar{a})$ , (where  $\bar{\theta}_n(z^m).u = u(\omega_P(z^m).u) \dots (\omega_P(z^m)^{n-1}.u)$ ).

Use of the proposition 1.11 together with the fact that  
 $X_P$  and  $G_P$  are  $P$ -local shows that  $\bar{\theta}_n(z^m)$  is bijective, whence  $b=1$ .

So,  $\omega_P(z^m)|_{\tilde{e}(K)} = 1_{\tilde{e}(K)}$ . Also,  $\gcd(m,n) = 1 \rightarrow \exists r,s \in \mathbb{Z}$   
 such that  $rm + sn = 1$ . Thus,  $\omega_P(z).\tilde{e}(\bar{a}) = \omega_P(z^n)^s \omega_P(z^m)^r.\tilde{e}(\bar{a}) =$   
 $= \omega_P(z^n)^s.\tilde{e}(\bar{a}) = \tilde{e}(\bar{w}(z^n)^s.\bar{a}) = \tilde{e}(\bar{w}(z^n)^s \bar{w}(z^m)^r.\bar{a}) = \tilde{e}(\bar{w}(z).\bar{a}), \forall \bar{a} \in K,$   
 $\forall z \in I_{P,1}(X_P, e_o(X)).$

Now it's easy to complete the argument.

Next let  $X \xrightarrow{\bar{w}} \text{Aut}(A)$  be an action, where  $A$  is a finite  
 abelian  $p$ -group ( $p \in P$ ). Let  $P_1 = \{q \in P' : q| |e(X)|\}$ .

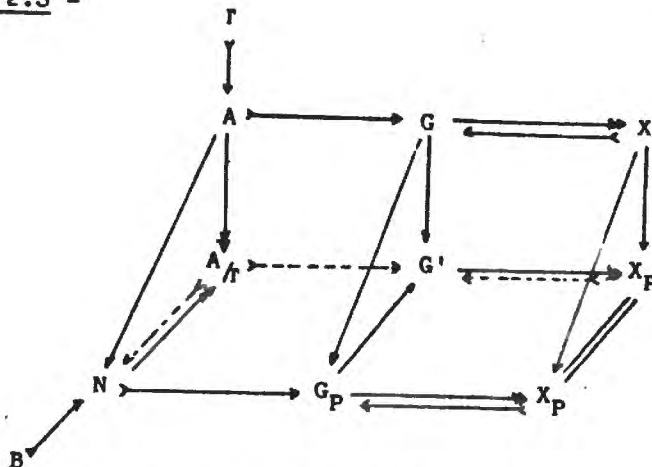
Take  $\Gamma = \Gamma(H)$  as it was defined just after prop. 1.21.

Thus, corollary 1.24 yields  $X \xrightarrow{\bar{w}} \text{Aut}(A/\Gamma)$ .

$$\begin{array}{ccc} & & \text{Aut}(A/\Gamma) \\ & \nearrow \omega' & \\ e_o \downarrow & & \\ & X_P & \end{array}$$

Let  $G = A \uparrow_{\omega'} X$  and  $G' = A/\Gamma \uparrow_{\omega'} X_P$ .

Diagram 2.3 -



In the diagram  $v$  is the natural projection  
 $(\therefore v \in \text{Hom}_{\mathbb{Z}[X]}(A, A/r))$ ,  $e'(a, x) = (v(a), e_0(x)) \therefore e' \in \text{Hom}(G, G')$ .

Thus  $\exists! f \in \text{Hom}(G_p, G')$  such that  $fe = e'$ .

The other functions are defined as usual except for  $\bar{e}$ ,  
 which is going to be defined just after the next lemma.

Lemma 2.4 -  $\bar{e}|_r = 0$ .

Proof - Let  $x$  be a generator of  $H$  and  $m = o(\nu(x)) \in P_1^X$ .

Let  $b = \nu(x) \cdot a - a$ ;  $a \in r$ .

Then,  $(b, x)^m = (e_m(x) \cdot b, x^m) = ((\nu(x)^m - 1_A) \cdot a, x^m) = (0, x^m) =$   
 $= (0, x)^m \therefore e(b, x)^m = e(0, x)^m \rightarrow e(b, x) = e(0, x)$  for  $\bar{e}$  is  $P$ -local.

Therefore,  $\bar{e}(b) = 1$ . Now it's plain that  $\bar{e}(r) = \{1\}$

Define  $\bar{e} \in \text{Hom}(A/r, N)$  such that  $\bar{e}v = \bar{e}$ . Then,  $\bar{e}v = 1_{A/r}$   
 whence  $B \rightarrow N \xrightleftharpoons[\bar{e}]{v} A/r$  splits.

Theorem 2.5 - In the conditions above,  $e': G \rightarrow G'$   $P$ -localizes  $G$ .

Proof - Set  $\phi': G' \rightarrow G_p$ , where  $\phi'(a, z) = \bar{u}\bar{e}(a) \cdot \sigma_p(z)$ .

Owing to prop. 2.2 we have  $\phi' \in \text{Hom}(G', G_r)$ , and then it's readily checked that  $\phi' = f^{-1}$ .

Finally, we analyze the situation in which  $A$  is (only) a finite abelian group.

Let  $X \xrightarrow{u} \text{Aut}(A)$  be an action, where  $A$  is a finite abelian group.

If  $|A| = p_1^{s_1} \dots p_t^{s_t}$ , then  $A = \bigoplus_{i=1}^t A_i$ ,  $A_i = p_i$ -primary component and  $\text{Aut}(A) \cong \prod_{i=1}^t \text{Aut}(A_i)$ .

Thus, there is (uniquely determined)  $\omega_i: X \rightarrow \text{Aut}(A_i)$ ;  $i = 1, \dots, t$ .

Let  $G = A \wr_u X$ ;  $G_i = A_i \wr_{\omega_i} X$ ;  $G \xrightarrow{\epsilon} X$ ;  $G_i \xrightleftharpoons[\sigma_i]{\epsilon_i} X$  as usual.

It's well-known that  $\epsilon$  is the pull-back of  $(\epsilon_i)_{1 \leq i \leq t}$ .

Let  $G \xrightarrow{\pi_i} G_i$  be the projection and  $\bar{G} \xrightarrow{\bar{\epsilon}} X_p$  be the pull-back of the arrows  $(G_i)_p \xrightarrow{(\epsilon_i)_p} X_p$ ;  $i = 1, \dots, t$ .

Since  $(\epsilon_i)_p \circ (\sigma_i)_p = 1_{X_p}$ , there does exist (only one)  $\bar{\sigma} \in \text{Hom}(X_p, \bar{G})$  such that  $\bar{\pi}_i \circ \bar{\sigma} = (\sigma_i)_p$ ,  $\forall i$  (here  $\bar{\pi}_i$  is the usual projection).

Likewise, it's plain that  $\exists! f \in \text{Hom}(G, \bar{G})$  such that  $\bar{\pi}_i \circ f = \epsilon_i \bar{\pi}_i$  ( $\epsilon_i: G_i \rightarrow (G_i)_p$  P-localizes  $G_i$ ).

It follows that  $\bar{\epsilon} f = e_0 \epsilon$  and  $f \sigma = \bar{\sigma} e_0$ . We recall that  $\bar{G}$  is P-local by prop. 1.1.

Moreover,  $\exists! \phi \in \text{Hom}(G_p, \bar{G})$  such that  $\bar{\pi}_i \circ \phi = (\pi_i)_p$ , since  $(\epsilon_i)_p \circ (\pi_i)_p = \epsilon_p$ ,  $\forall i$  (In particular  $\bar{\pi}_i$  is an isomorphism). By uniqueness we've got  $f = \phi \epsilon$ ;  $\bar{\epsilon} \phi = \epsilon_p$ ;  $\phi \sigma_p = \bar{\sigma}$  and  $\bar{\sigma} \bar{\sigma} = 1_{X_p}$  as well. ( $\therefore \bar{G} = C \wr_{\bar{\sigma}} X_p$ ).



Proof - Since  $K = \bigoplus_{i=1}^t K_i$  ( $K_i = \ker \bar{e}_i$ ), we just have to show that  $\bar{e}|_{K_i} = 0, \forall i$ .

This follows from the diagram ( $v_1$  is a splitting attached to  $v_1$ )

$$\begin{array}{ccccccc}
 & & & & A & & \\
 & & & & \swarrow & \searrow & \\
 & & & & \mu_1 & & \mu \\
 K_1 & \longrightarrow & A_1 & \longrightarrow & G_1 & \xrightarrow{v_1} & G \\
 & & \downarrow \bar{e}_1 & & \downarrow e_1 & & \downarrow e \\
 & & \ker(\varepsilon_1)_P & \xrightarrow{\mu'_1} & (G_1)_P & \xrightarrow{(v_1)_P} & G_P
 \end{array}$$

So we have  $e' \in \text{Hom}(C, N)$  with  $e'f = \bar{e}$ .

Lemma 2.9 - (i)  $\bar{f}(u(x).a) = \bar{e}(e_0(x)).\bar{f}(a)$   
 (ii)  $\bar{e}(u(x).a) = \sigma_P(e_0(x)).\bar{e}(a) \quad ; \quad \forall x \in X; \forall a \in A.$

Proof - Both statements are readily checked from the definitions.

Theorem 2.10 - In the conditions above,  $G \xrightarrow{f} \bar{G}$  P-localizes G.

Proof - Let  $\psi: \bar{G} \rightarrow G_P$  defined by  $\psi(\bar{\mu}(c).\bar{\sigma}(z)) = \mu'e'(c).\sigma_P(z)$ ;  
 $c \in C; z \in X_P.$

$\psi \in \text{Hom}(\bar{G}, G_P)$  by prop. 2.2, so that it's plain that  $\psi = \psi^{-1}$

§3 - Throughout this section we only work with the theory of localization on the category  $\mathfrak{n}$ , as developed in [2].

Proposition 3.1 - Let  $A \xrightarrow{\mu} G \xrightarrow{c} X$  be an exact sequence of groups, where A is abelian finite and X is nilpotent. Let  $X \xrightarrow{u} \text{Aut}(A)$  be the action attached to the extension and suppose  $\Gamma_u^2 = A$ .

Let also  $\beta \in \text{Hom}(G, K)$  and  $B \xrightarrow{\nu} K \xrightarrow{\kappa} Y$  be an exact

sequence, where  $B$  is finite abelian and  $Y$  nilpotent. Then, there exist  $\alpha \in \text{Hom}(A, B)$  and  $\gamma \in \text{Hom}(X, Y)$  which yield commutativity in

$$\begin{array}{ccccc} A & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & X \\ \alpha \downarrow & & \beta \downarrow & & \downarrow \gamma \\ B & \xrightarrow{\nu} & K & \xrightarrow{\kappa} & Y \end{array}$$

**Proof** - Let  $H = \langle \epsilon\mu(A) \rangle < Y$ . It follows from  $r_{\mu}^2 = A$  that  $H \in [Y, H]$ . So  $H \in [Y, H] \in r^2 Y$  and then, by induction,  $H \in r^k Y, \forall k \geq 2$ ; whence  $H = \{1\}$  since  $Y$  is nilpotent. This completes the proof.

**Proposition 3.2** -  $\forall G \in |C|, \exists ! U = U(G) \triangleleft G, U$  finite abelian with  $G/U$  nilpotent such that  $r_{\mu}^2 = U$ , provided that  $\mu$  is the action attached to the extension  $U \xrightarrow{\nu} G \xrightarrow{\epsilon} G/U$ .

**Proof** -  $A \xrightarrow{\mu} G \xrightarrow{\epsilon} X$  be an extension where  $A$  is finite abelian. An  $X$  is nilpotent. Let  $\alpha: X \rightarrow \text{Aut}(A)$  be the action attached to this extension, and set  $r = r_{\alpha}^r$  where  $r$  is the smallest positive integer such that  $r_{\alpha}^r = r_{\alpha}^{r+1}$ . Let  $U = \mu(r) \triangleleft G$ . So  $U$  is finite abelian. Furthermore,  $A/r \xrightarrow{\nu} G/U \xrightarrow{\epsilon} X$  is exact and  $X$  acts nilpotently on  $A/r$ , so that  $G/U$  is nilpotent. It's also plain that  $r_{\mu}^2 = U$ , if  $\mu(gU)u = \mu(g)u^{-1}$ . Finally we point out that the uniqueness follows in a straightforward way from proposition 3.1.

Now, let  $p$  be a prime and  $C_p$  be the full sub-category of  $C$  of all groups, which are extensions of  $X$  by  $A$ , where  $A$  is a finite abelian  $p$ -group.

**Corollary 3.3** -  $G \in |C_p| \rightarrow U = U(G)$  is a finite abelian  $p$ -group.

**Proof** - In fact,  $U = \mu(r)$  and  $r$  is a sub-group of  $A$ .

Corollary 3.4 -  $G \in |C|$ ;  $G$  is nilpotent  $\leftrightarrow U = U(G) = (1)$ .

Proof -  $(\rightarrow)$   $G \in |n| \rightarrow \omega: G/U \rightarrow \text{Aut}(U)$  is nilpotent  $\rightarrow U = \Gamma_{\omega}^2 = \dots = \Gamma_{\omega}^{c+1} = (1)$ . ( $c = \text{nil } \omega$ ).

$(\leftarrow)$   $U = (1)$   $G = G/U$  nilpotent.

Corollary 3.5 -  $G \in |n| \leftrightarrow \exists p, q$  primes,  $p \neq q$ , such that  $G \in |C_p| \wedge |C_q|$ .

Proof - It follows from cor. 3.3 and cor. 3.4.

We now define  $G_p \in |C|$  provided  $G \in |C|$ .

Fix  $\zeta: U \xrightarrow{\mu} G \xrightarrow{\epsilon} G/U$  where  $U = U(G)$  defined by prop. 3.2;  $\omega(gU)u = gug^{-1}$  and  $G/U \xrightarrow{e} (G/U)_p$   $p$ -localizes  $G/U$  in  $n$ .

We consider 3 cases:

Let  $p$  be a prime and suppose firstly  $G \in |C_p|$ .

I)  $p \in P'$ .

Set  $e = e_0 \circ \epsilon$ ,  $G \xrightarrow{\epsilon} G/U \xrightarrow{e_0} (G/U)_p$ .

Then we have:

$$\begin{array}{ccccc} \zeta: & U & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & G/U \\ & \downarrow \nu & & \downarrow \epsilon & & \downarrow e_0 \\ \zeta_p: & (0) & \longrightarrow & (G/U)_p & \cong & (G/U)_p \end{array}$$

We should point out that  $\nu_* \zeta = e_0^* \zeta_p = 0$ .

II)  $p \in P$ .

Let  $P_1 = \{q \in P' : q \mid |\omega(G/U)|\}$ ,  $H = \langle x \in G/U : \omega(x) \in P_1^x \rangle$ ,  $r = r(H)$  and  $\bar{\omega}: G/U \rightarrow \text{Aut}(U/r)$  as defined just after theorem 1.20.

Corollary 1.24 allows us to claim that  $\exists!$  action  $\omega_p$  making commutative the diagram

$$\begin{array}{ccc} G/U & \longrightarrow & \text{Aut}(U/r) \\ \downarrow & \nearrow \omega_p & \\ (G/U)_p & & \end{array}$$

Taking the natural projection  $U \rightarrow U/\Gamma$ , we have that  $\exists! \tau_p$  such that  $e_0^* \tau_p = \tau_* \tau$  where

$$H_{\omega}^2(G/U; U) \xrightarrow{\tau_*} H_{\omega}^2(G/U; U/\Gamma) \xleftarrow{e_0^*} H_{\omega_p}^2((G/U)_p; U/\Gamma)$$

Once more it is shown by prop. 1.16 that there is a commutative diagram

$$\begin{array}{ccccc} \tau: U & \xrightarrow{\mu} & G & \xrightarrow{\epsilon} & G/U \\ \tau \downarrow & & \downarrow e & & \downarrow e_0 \\ \tau_p: U/\Gamma & \xrightarrow{\bar{\mu}} & G_p & \xrightarrow{\epsilon} & (G/U)_p \end{array}$$

At this point it's important to point out that we have defined  $G \in \bigcup_p |C_p| \rightarrow G_p \in |C|$  and this definition is "good" since  $G \in |C_p| \wedge |C_q| \rightarrow G \in \eta$  (cor. 3.5) and then  $U = (1)$  (cor. 3.4).

In particular, this construction extends the one made in [2].

Example 3.6 - Let  $\omega: \mathbb{Z} \rightarrow \text{Aut}(\mathbb{Z}/3 \oplus \mathbb{Z}/5)$  given by

$\omega(1).a = 2a$  and  $\omega(1).b = 2b$ . Let  $G = (\mathbb{Z}/3 \oplus \mathbb{Z}/5) \rtimes_{\omega} \mathbb{Z}$ .

Then  $\Gamma_{\omega}^2 = \mathbb{Z}/3 \oplus \mathbb{Z}/5 = A$ , whence  $G \notin |\eta|$ . However  $G \in |C|$  and since  $U = \mu(A)$ , it follows that  $G \notin \bigcup_p |C_p|$ .

This example shows that  $|C| \setminus \bigcup_p |C_p| \neq \emptyset$ .

So we must consider

III)  $G \in |C| \setminus \bigcup_p |C_p|$ .

Now  $U$  is no longer a  $p$ -group. Nevertheless,  $U = \bigoplus_{i=1}^t U_i$  where

$U_i$  is the  $p_i$ -primary component of  $U$ .

Also,  $G/U \cong \text{Aut}(U) \cong \prod_{i=1}^t \text{Aut}(U_i)$  and  $\omega = (\omega_1, \dots, \omega_t)$ .

We have  $H_{\omega}^2(G/U; U) \xrightarrow{(\tau_1, \dots, \tau_t)^*} \prod_{i=1}^t H_{\omega_i}^2(G/U; U_i)$ ; where

$U \xrightarrow{\tau_i} U_i$  is the usual projection. Notice that  $\Gamma_{\omega}^2 = U \rightarrow \Gamma_{\omega_i}^2 = U_i$ ;

$\forall i = 1, \dots, t$ . Let  $\tau_i = \tau_{i*} \tau$  and consider the commutative diagram

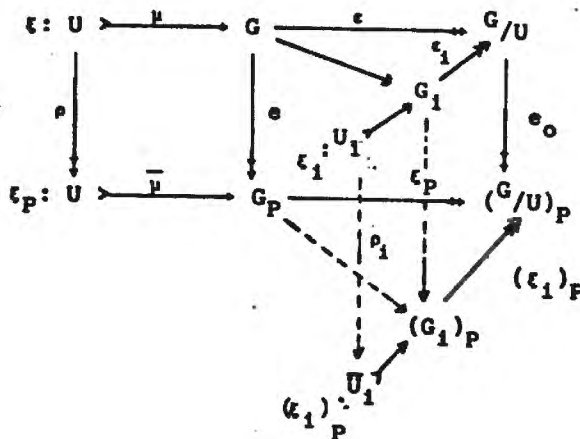
$$\begin{array}{ccc} \zeta \in H^2(G/U; U) & \xrightarrow[\cong]{(\bar{\nu}_1, \dots, \bar{\nu}_{t^*})} & (\zeta_1)_1 \in \bigoplus_{i=1}^t H^2_{U_1}(G/U; U_1) \\ \downarrow e_0^{-1} \circ \rho_0 & & \downarrow \left( \bigoplus_{i=1}^t e_0^{-1} \right) \left( \bigoplus_{i=1}^t \rho_{1*} \right) \\ \zeta_P \in H^2_{(G/U)_P}((G/U)_P; \bar{U}) & \xrightarrow[\cong]{(\bar{\nu}_P, \dots, \bar{\nu}_{t^*})} & (\zeta_1)_P \in \bigoplus_{i=1}^t H^2_{(U_1)_P}(G/U)_P((G/U)_P; \bar{U}_1) \end{array}$$

where  $\epsilon_1: U_1 \xrightarrow{\nu_1} G_1 \xrightarrow{\epsilon_1} G/U$  is defined by (I) or (II). Also,  $U = \bigoplus_{i=1}^t U_1$  and  $\exists! \zeta_P$  such that

$$\begin{array}{ccc} \epsilon_1: U_1 & \xrightarrow{\nu_1} & G_1 \xrightarrow{\epsilon_1} G/U \\ \downarrow \rho_1 & & \downarrow \rho_1 \quad \downarrow \rho_0 \\ (\zeta_1)_P: \bar{U}_1 & \xrightarrow{\bar{\nu}_1} & (G_1)_P \xrightarrow{(\zeta_1)_P} (G/U)_P \end{array}$$

$(\bar{\nu}_1, \dots, \bar{\nu}_{t^*})\zeta_P = ((\zeta_1)_P)_1$  provided that  $\bar{\nu}_1$  is the usual projection and  $\rho = \bigoplus_i \rho_i$ .

Diagram 3.7 -



By definition, we have that  $\zeta_P$  is the pull-back of the arrows  $((\zeta_1)_P)_1$  since  $\zeta_P = \rho_* \zeta$ .

As for  $G \rightarrow G_P$  defined by (I);(II);(III) we have the next two propositions.

Proposition 3.8 -  $e$  is  $P$ -surjective.

Proof - It follows directly from the definitions.

Corollary 3.9 -  $G \xrightarrow{e} G_p \xrightarrow[\underline{g}]{f} K$  with  $K$   $P$ -local.

Then  $fe = ge \rightarrow f = g$ .

Proof - Obvious.

Proposition 3.10 -  $G$   $P$ -local  $\rightarrow e$  is an isomorphism.

Proof - We have 3 cases to analyze. The only one which is not obvious is (II).

Suppose we have

$$\begin{array}{ccccc} U & \xrightarrow{\quad} & G & \xrightarrow{\quad} & G/U \\ \downarrow v & & \downarrow e & & \downarrow e_0 \\ U/r & \xrightarrow{\quad} & G_p & \xrightarrow{\quad} & (G/U)_p \end{array}$$

$G$   $P$ -local  $\rightarrow G/U$   $P$ -local (cor. 1.9). So  $\alpha(G/U)$  is a  $P$ -torsion sub-group of  $\text{Aut}(U)$ , since  $\text{Aut}(U)$  is finite and  $G/U$  is  $P$ -local (cor. 1.3). Therefore  $H = \{1\}$  and  $r = \{1\}$ .  $\therefore v = 1_U$ .  $\therefore e$  is an isomorphism.

Next we consider a commutative diagram

$$\begin{array}{ccccc} U(G) & = & U & \xrightarrow{\mu} & G & \xrightarrow{e} & G/U \\ & & \downarrow \alpha & & \downarrow \beta & & \downarrow \gamma \\ U(K) & = & V & \xrightarrow{\nu} & K & \xrightarrow{k} & K/V \end{array}$$

Let's define  $\bar{\alpha} \in \text{Hom}(\bar{U}, \bar{V})$  induced by  $(\bar{\alpha}_0 = \alpha \nu; U \xrightarrow{\mu} U)$ .

We take  $U = \bigoplus U(p)$  and  $V = \bigoplus V(p)$ ;  $p$ -primary decompositions.

Proposition 1.25 assures that  $\alpha(\Gamma_{U(p)}) \subset \Gamma_{V(p)}$ , since  $\alpha(U(p)) \subset V(p)$ .

$$\begin{array}{ccccc} \text{Actually, we consider } \zeta(p): U(p) & \xrightarrow{\quad} & G(p) & \twoheadrightarrow & G/U \\ & & \downarrow \alpha|_{U(p)} & & \downarrow \\ \zeta(p): V(p) & \xrightarrow{\quad} & K(p) & \twoheadrightarrow & K/V \end{array}$$

and then use prop. 1.25 to  $\zeta(p) = \alpha(p)_* \zeta$  and  $\zeta(p) = \alpha(p)_* \zeta$ :

We define, by restriction,  $\zeta(p): U(p) \rightarrow V(p)$ , whence we have

$$\begin{array}{ccc} U(p) & \xrightarrow{\alpha(p)} & V(p) \\ \downarrow \rho_{U(p)} & \searrow \bar{\alpha}(p) & \downarrow \rho_{V(p)} \\ U(p) = U(p)/\Gamma_{U(p)} & \xrightarrow{\quad} & V(p)/\Gamma_{V(p)} = \bar{V}(p) \end{array} \quad \text{and finally} \quad \bar{\alpha} = \bar{\alpha} \circ \bar{\alpha}(p).$$

At this point we state a fundamental proposition.

**Proposition 3.11** -  $\bar{\alpha}$  is a homomorphism of modules.

**Proof** - Let's consider the commutative diagrams.

$$\begin{array}{ccc} G/U & \xrightarrow{\bar{\alpha}} & \text{Aut}(U) \\ \downarrow \epsilon_0 & \nearrow \alpha_p & \\ (G/U)_p & & \end{array} \quad \begin{array}{ccc} K/V & \xrightarrow{\bar{\alpha}} & \text{Aut}(V) \\ \downarrow \epsilon_0 & \nearrow \alpha_p & \\ (K/V)_p & & \end{array}$$

where  $\alpha_p$  and  $\alpha_p$  are the actions given by the extensions

$$\begin{array}{ccccc} \zeta_p: U & \xrightarrow{\bar{\alpha}} & G_p & \xrightarrow{\alpha_p} & (G/U)_p \\ & & \downarrow \bar{\alpha} & & \downarrow \Gamma_p \\ \zeta_p: V & \xrightarrow{\bar{\alpha}} & K_p & \xrightarrow{\alpha_p} & (K/V)_p \end{array}$$

We ought to get that  $\bar{\alpha}(u_p(z) \cdot \bar{a}) = \alpha_p(\gamma_p(z)) \cdot \bar{\alpha}(\bar{a})$ ,  $\forall z \in (G/U)_p$  and  $\forall \bar{a} \in \bar{U}$ .

Fix  $z \in (G/U)_p$  and  $\bar{a} \in \bar{U}$ .

$G/U$  nilpotent  $\rightarrow \exists n \in P^{\times}$  such that  $z^n = e_o(x)$ . Then,

$$\begin{aligned} \bar{\alpha}(u_p(z^n) \cdot \bar{a}) &= \bar{\alpha}(u_p(e_o(x)) \cdot \bar{a}) = \bar{\alpha}(u(x) \cdot \bar{a}) = \bar{\alpha}(u(x) \cdot \bar{a}) \text{ (by definition)} \\ &= \alpha_V(u(x) \cdot \bar{a}) = \alpha_V(\alpha(\gamma(x)) \cdot \alpha(\bar{a})) \quad (\alpha \text{ is a homomorphism of modules}) = \\ &= \bar{\alpha}(\gamma(x)) \cdot \alpha(\bar{a}) = \alpha_p(e_o(\gamma(x))) \cdot \bar{\alpha}(\bar{a}) = \alpha_p \gamma_p(z^n) \cdot \bar{\alpha}(\bar{a}) \dots (*) \end{aligned}$$

On the other hand  $o(u_p(z)) = m \in P^{\times}$ , (prop. 1.2), since  $(G/U)_p$  is  $P$ -local and  $\text{Aut}(\bar{U})$  is finite.

So,  $\alpha_p(\gamma_p(z^n)^m) \bar{\alpha}(\bar{a}) = \bar{\alpha}(u_p(z^n)^m \cdot \bar{a}) = \bar{\alpha}(\bar{a})$ ,  $\forall \bar{a} \in \bar{U}$ .

$$\therefore \alpha_p(\gamma_p(z^m)^n) \Big|_{\bar{\alpha}(\bar{U})} = 1_{\bar{\alpha}(\bar{U})} .$$

Still, taking into account that in the exact sequence  $\epsilon_p: \mathbb{V} \rightarrow K_p \rightarrow (K/V)_p$ ;  $K_p$  and  $(K/V)_p$  are  $P$ -local we can state that  $\tilde{\theta}_n(\gamma_p(z^m)) = 1_{\mathbb{V}} + \alpha_p(\gamma_p(z^m)) + \dots + \alpha_p(\gamma_p(z^m)^{n-1}) \in \text{Aut}(\mathbb{V})$ ,  $\forall n \in P^{\times}$  and  $\gamma_p(z^m) \in (K/V)_p$ .

Thus  $\forall \bar{a} \in \bar{U}$  we have:

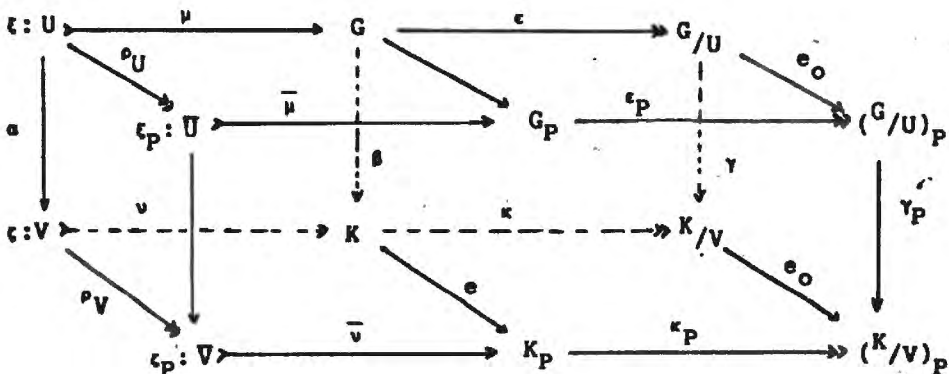
$$\begin{aligned} 0 &= \bar{\alpha}(\bar{a}) - \alpha_p(\gamma_p(z^m)^n) \bar{\alpha}(\bar{a}) = [1_{\mathbb{V}} - \alpha_p(\gamma_p(z^m)^n)] \bar{\alpha}(\bar{a}) = \\ &= \tilde{\theta}_n(\gamma_p(z^m)) \circ [1_{\mathbb{V}} - \alpha_p(\gamma_p(z^m))] \cdot \bar{\alpha}(\bar{a}) . \end{aligned}$$

$$\therefore \bar{\alpha}(\bar{a}) - \alpha_p(\gamma_p(z^m)) \cdot \bar{\alpha}(\bar{a}) = 0, \text{ whence } \alpha_p(\gamma_p(z^m)) \Big|_{\bar{\alpha}(\bar{U})} = 1_{\bar{\alpha}(\bar{U})} \dots \textcircled{A}$$

Finally  $\exists r, s \in \mathbb{Z}$  such that  $rm + sn = 1$  ( $\text{gcd}(m, n) = 1$ ).  $\therefore$

$$\begin{aligned} \bar{\alpha}(u_p(z) \cdot \bar{a}) &= \bar{\alpha}(u_p(z^n)^s \circ u_p(z^m)^r \cdot \bar{a}) = \bar{\alpha}(u_p(z^n)^s \cdot \bar{a}) = \\ &= \alpha_p(\gamma_p(z^n)^s) \bar{\alpha}(\bar{a}) = \alpha_p(\gamma_p(z^n)^s) \cdot \alpha_p(\gamma_p(\gamma_p(z^m)^r)) \cdot \bar{\alpha}(\bar{a}) = \\ &= \alpha_p(\gamma_p(z)) \cdot \bar{\alpha}(\bar{a}) . \end{aligned}$$

**Theorem 3.12** -  $G, K \in \mathcal{C}$ ;  $\exists \tau_P \in \text{Hom}(G_P, K_P)$  yielding commutativity in the diagram:



**Proof** - The uniqueness follows from corollary 3.9:

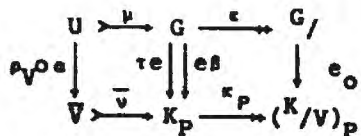
For the existence we observe that  $e_0^* \tau_P^* \tau_P = \tau^* e_0^* \tau_P = \tau^* \rho_{V^*} \tau$  (definition of  $\tau_P$ )  $= \rho_{V^*} \tau^* \tau = \rho_{V^*} \alpha_*(\tau)$  (prop. 1.16)  $= \bar{\alpha}_* \rho_{U^*} \tau = \bar{\alpha}_* e_0^* \tau_P$  (def. of  $\tau_P$ )  $= e_0^* \bar{\alpha}_* \tau_P$ .

It follows that  $\tau_P^* \tau_P = \bar{\alpha}_* \tau_P$  due to the fact that

$$H^2((G/U)_P; \mathbb{V}) \xrightarrow[\cong]{e_0^*} H^2(G/U; \mathbb{V}) \quad (\text{Th. 1.20}).$$

So by proposition 1.16,  $\exists \tau \in \text{Hom}(G_P, K_P)$  yielding commutativity in the "front face" of the diagram.

Thus  $\tau e$  and  $e \tau$  make commutative the diagram



Use of the proposition 1.17 shows that  $e: G/U \rightarrow \mathbb{V}$  a cross homomorphism such that

$$e \tau(g) = \bar{\nu} e(g) \cdot \tau e(g), \quad \forall g \in G. \text{ However, } H^1((G/U)_P; \mathbb{V}) \xrightarrow[\cong]{e_0^*} H^1(G/U; \mathbb{V}) \quad (\text{Th. 1.20}).$$

So  $\theta = \theta'_P e_O + \delta'_V$ , where  $\delta'_V(x) = v \cdot x \cdot v$ ,  $v \in V$ .

Setting  $\delta_V: (G/U)_P \rightarrow \bar{V}$ ,  $\delta_V(z) = v \cdot z \cdot v$ , it follows that

$\delta_V \circ e_O = \delta'_V$  and therefore  $\theta = \theta_P \circ e_O$ , where  $\theta_P = \theta'_P + \delta_V$ .

Now  $e\theta(g) = \bar{v} \theta_P e_O(g) \cdot \tau(g) = \bar{v} \theta_P \epsilon_P(g) \cdot \tau(g)$ ,  $\forall g \in G$ .

Thus defining  $\theta_P: G_P \rightarrow K_P$  by  $\theta_P(z) = \bar{v} \theta_P \epsilon_P(z) \cdot \tau(z)$ ,  
 $\forall z \in G_P$ , it follows from prop. 1.18 that  $\theta_P \in \text{Hom}(G_P, K_P)$  and  
 $\theta_P e = e \theta$ .

Besides,  $K_P \theta_P = \gamma_P \epsilon_P$  and  $\theta_P \bar{u} = \bar{v} \bar{a}$

**Remark:** The theorem above shows us that  $G \rightarrow G_P$  is a functor and  $e$  is a natural transformation of functors.

**Theorem 3.13** -  $G \xrightarrow{e} G_P$   $P$ -localizes  $G$  in  $C$ .

**Proof** - Let  $G, K \in |C|$ , with  $K$   $P$ -local, and  $\theta \in \text{Hom}(G, K)$ .

Owing to proposition 3.1, there exists a commutative diagram

$$\begin{array}{ccccc} U(G) = U & \xrightarrow{\mu} & G & \xrightarrow{e} & G/U \\ & \downarrow \alpha & \downarrow \beta & & \downarrow \gamma \\ U(K) = V & \xrightarrow{\nu} & K & \xrightarrow{\kappa} & K/V \end{array}$$

Now using th. 3.12 we conclude that  $\exists! \theta_P \in \text{Hom}(G_P, K_P)$  such that  $\theta_P e = e \theta$ .

So it's enough to take  $\bar{\theta} = e^{-1} \circ \theta_P$

$$\begin{array}{ccc} G & \xrightarrow{\theta} & K \\ e \downarrow & \nearrow \bar{\theta} & \downarrow \epsilon \\ G_P & \xrightarrow{\theta_P} & K_P \end{array} \quad (\text{prop. 3.10})$$

The uniqueness follows from cor. 3.9.

- 82-46 GONCALVES, J.Z. Free subgroups of units in group rings. 8 p.
- 82-47 VELOSO, J.M.M. New classes of intransitive simple Lie pseudogroups. 8 p.
- 82-48 CHEN, C.C. The generalized curvature ellipses and minimal surfaces. 10 p.
- 82-49 COSTA, R.C.F. On the derivation algebra of zygotic algebras for polyploidy with multiple alleles. 24 p.
- 83-50 GONCALVES, J.Z. Free subgroups in the group of units of group rings over algebraic integers. 3 p.
- 83-51 MANDEL, A. & GONCALVES, J.Z. Free k-triples in linear groups. 7 p.
- 83-52 BRITO, F.G.B. A remark on closed minimal hypersurfaces of  $S^4$  with second fundamental form of constant length. 12 p.
- 83-53 KIIM, J.C.S. U-structures and sphere bundles. 8 p.
- 83-54 COSTA, R.C.F. On genetic algebras with prescribed derivations. 23 p.
- 83-55 SALVITTI, R. Integrabilidade das distribuições dadas por subálgebras de Lie de codimensão finita no  $gh(n,C)$ . 4 p.
- 83-56 MANDEL, A. & GONCALVES, J.Z. Construction of open sets of free k-Tuples of matrices. 18 p.
- 83-57 BRITO, F.G.B. A remark on minimal foliations of codimension two. 24 p.
- 83-58 GONCALVES, J.Z. Free groups in subnormal subgroups and the residual nilpotence of the group of units of group rings. 9 p.
- 83-59 BELOQUI, J.A. Modulus of stability for vector fields on 3-manifolds. 40 p.
- 83-60 GONCALVES, J.Z. Some groups not subnormal in the group of units of its integral group ring. 8 p.
- 84-61 GOES, C.C. & SIMÕES, P.A.Q. Imersões mínimas nos espaços hiperbólicos. 15 p.
- 84-62 GIAN BRUNO, A.; MISSO, P. & POLCINO MILIES, F.C. Derivations with invertible values in rings with involution. 12 p.
- 84-63 FERRER SANTOS, W.R. A note on affine quotients. 6 p.
- 84-64 GONCALVES, J.Z. Free-subgroups and the residual nilpotence of the group of units of modular and p-adic group rings. 12 p.
- 84-65 GONCALVES, D.L. Fixed points of  $S^1$ -fibrations. 18 p.
- 84-66 RODRIGUES, A.A.M. Contact and equivalence of submanifolds of homogenous spaces. 15 p.
- 84-67 LOURENCO, M.L. A projective limit representation of (DFC)-spaces with the approximation property. 20 p.
- 84-68 FORNARI, S. Total absolute curvature of surfaces with boundary. 25 p.
- 84-69 BRITO, F.G.B. & WALCZAK, P.G. Totally geodesic foliations with integral normal bundles. 6 p.
- 84-70 LANGEVIN, R. & POSSANI, C. Quase-folheações e integrais de curvatura no plano. 26 p.
- 84-71 OLIVEIRA, M.E.G.G. de Non-orientable minimal surfaces in  $R^n$ . 41 p.
- 84-72 PERESI, L.A. On baric algebras with prescribed automorphisms. 42 p.
- 84-73 MIRAGLIA NETO, F. & ROCHA FILHO, G.C. The measurability of Riemann integrable-function with values in Banach spaces and applications. 27 p.
- 84-74 MERKLEN GOLDSCHMIDT, H.A. Artin algebras which are equivalent to a hereditary algebra modulo preprojectives. 38 p.
- 84-75 GOES, C.C. & SIMÕES, P.A.Q. The generalized Gauss map of minimal surfaces in  $H^3$  and  $H^4$ . 16 p.
- 84-76 GONCALVES, J.Z. Normal and subnormal subgroups in the group of units of a group rings. 13 p.
- 85-77 ARAGONA VALLEJO, A.J. On existence theorems for the  $\bar{\partial}$  operator on generalized differential forms. 13 p.
- 85-78 POLCINO MILIES, C.; RITTER, J. & SEHGAL, S.K. On a conjecture of Zassenhaus on torsion units in integral group rings II. 14 p.
- 85-79 JONES RODRIGUES, A.R. & MICHLER, G.O. On the structure of the integral Green ring of a cyclic group of order  $p^2$ . The Jacobson radical of the integral Green ring of a cyclic group of order  $p^2$ . 26 p.
- 85-80 VELOSO, J.M.M. & VERDERESI, J.A. Three dimensional Cauchy-Riemann manifolds. 19 p.
- 85-81 PERESI, L.A. On baric algebras with prescribed automorphisms II. 18 p.
- 85-82 KRUDSEN, C.A. O impasse aritmo-geométrico e a evolução do conceito de número na Grécia antiga. 43 p.
- 85-83 VELOSO, J.M.M. & VERDERESI, J.A. La géométrie, le problème d'équivalence et le classification des CR-varietés homogènes em dimension 3. 30 p.
- 85-84 GONCALVES, J.Z. Integral group rings whose group is solvable, an elementary proof. 13 p.
- 85-85 LUCIANO, O.O. Nebuleuses infinitesimement fibrées. 5p.
- 85-86 ASPERTI, A.C. & Dajczer, P. Conformally flat Riemannian manifolds as hypersurfaces of the fifth cone. 8p.

- 85-87 BELOQUI, J.A. A quasi-transversal Hopf bifurcations. 11p
- 85-88 POLCINO MILIES, C. & RAPHAËL D.M. A note on derivations with power central values in prime rings. 7p.
- 85-89 POLCINO MILIES. Torsion units in group rings and a conjecture of H.J.Zassenhaus. 14p
- 86-90 LOURENCO, M.L. Riemann domains over (DFC) spaces. 32p.
- 86-91 ARAGONA VALLEJO, A.J. & FERRANDES, J.C.D. The Hartogs extension theorem for holomorphic generalized functions. 9p.
- 86-92 CARRARA ZANETIC, V.L. Extensions of immersions in dimension two. 27p.
- 86-93 PERESI, L.A. The derivation algebra of generic and zygotic algebras for linked loci. 25p.
- 86-94 COELHO, S.P. A note on central idempotents in group ring. 5p.
- 86-95 PERESI, L.A. On derivations of basic algebras with prescribed automorphisms. 21p.
- 86-96 COELHO, F.U. A generalization of a theorem of Todorov on preprojective partitions 18p.
- 86-97 ASPERTI, A.C. A note on the minimal immersions of the two-sphere. 11p.
- 86-98 COELHO, S.P. & POLCINO MILIES, C. A note on central idempotents in group rings II. 8p.
- 86-99 EXEL, R. Hankel matrices over right ordered amenable groups. 18p.

#### NOVA SÉRIE

- 86-01 GOODAIRE, E. G. & POLCINO MILIES, C. Isomorphisms of integral alternative loop rings. 11p.
- 86-02 MALCZAK, P. G. Foliations which admit the most mean curvature functions. 11p.
- 86-03 OLIVEIRA, M.E.G.G. Minimal Klein bottles with one end in  $\mathbb{R}^3$  and  $\mathbb{R}^n$ . 12p.
- 86-04 MICALI, A. & VILLAMAYOR, O.E. Homologie de Hochschild de certaines algèbres de groupes. 17p.
- 86-05 OLIVEIRA, M.E.G.G. Minimal Klein bottles in  $\mathbb{R}^3$  with finite total curvature. 9p.
- 86-06 CARRARA ZANETIC, V. L. Classification of stable maps between 2-manifolds with given singular set image. 22p.
- 87-01 BRITO F.G.B. & MALCZAK P.G. Total curvature of orthogonal vector fields on three-manifolds. 4p.
- 87-02 BRITO F.G.B. & LEITE M.L. A remark on rotational hypersurfaces of  $S^n$ . 13p.
- 87-03 GONÇALVES J., RITTER J. & SEHGAL S. Subnormal subgroups in  $U(\mathbb{Z}G)$ . 13p.
- 87-04 ARAGONA VALLEJO, A.J. & COLOMBEAU, J.F. The interpolation theorem for holomorphic generalized functions. 12p.
- 87-05 ALMEIDA, S.C. de & BRITO, F.G.B. Immersed hypersurfaces of a space form with distinct principal curvatures. 9p.
- 87-06 ASPERTI, A.C. Generic minimal surfaces. 21p.
- 87-07 GOODAIRE, E.G. & POLCINO MILIES, F.C. Torsion units in alternative group rings. 17p.
- 87-08 MERKLEN, h.A. Algebras which are equivalent to a hereditary algebra modulo preprojectives II. 27p.
- 87-09 REYNOL FILHO, A. P-localization of some classes of groups. 30p.