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# The dynamic of malaria at a rice irrigation system

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**Abstract**

Oliva, W.M. and Sallum, E.M. The dynamic of malaria at a rice irrigation system. Rev. Saúde Pública, ..., 1995. The authors analyse the dynamic of the population of infected hosts and mosquitoes at a rice irrigation system by the use of a deterministic mathematical model. The question arise inside the Ribeira Project developed by the Taxonomic and Systematic Research in Medical Entomology Unit of the University of São Paulo. By the computation of a parameter $\lambda$ (the spectral radius of a certain monodromy matrix) one can state that either the infection finishes naturally ($\lambda \leq 1$) or if $\lambda > 1$, the infection becomes endemic.

**Key Words:** malaria, rice irrigation, dynamics, mosquitoes.

# 1 Introduction

The present work composes the activities of the Ribeira Project developed by seachers of the Taxonomic and Systematic Research in Medical Entomology Unit of University of São Paulo (NUPTEM).

As promised in the refered project we intend to analyse the dynamic of the population of infected hosts and mosquitoes at a rice irrigation system in the Ribeira Valley Experimental Station. For this we introduce a deterministic model which considers one patch of hosts and two patches of mosquitoes containing any periodic number of individuals. The main results are stated in Theorems A and B.

In view of data published in several papers ([4], [5], [6], [7], [8], [9]) by the involved scientists, we observe those corresponding to Anopheles Cruzii and Anopheles Albitarsis.

In another section we considered a case of one patch of hosts and only one patch of mosquitoes where was possible to compute explicitely the corresponding $2 \times 2$ monodromy matrix and so, the spectrum.

In the Appendix we present the general case (system (*)) that can be used for any number of patches of hosts and mosquitoes.

All the models considered in this work are of SIS type, that is, incubation and immunity are neglected.

## 2 One patch of hosts and two patches of mosquitoes

We present here a deterministic model in order to describe the dynamic of the population of hosts and mosquitoes infected by malaria when there is a homogeneous group of hosts with a constant population $H$ and two groups of mosquitoes of different types $i = 1, 2$, with population $V_i = V_i(t)$ periodic in the time $t$ with period $T > 0$.

Let us consider the following system of ordinary differential equations:

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}t} = -\xi S + \dfrac{(H-S)}{S}(b_1' f_1 I_1 + b_2' f_2 I_2) \\ \dfrac{\mathrm{d}I_1}{\mathrm{d}t} = -\delta_1 I_1 + (V_1 - I_1)\dfrac{b_1 f_1 S}{V_1} \\ \dfrac{\mathrm{d}I_2}{\mathrm{d}t} = -\delta_2 I_2 + (V_2 - I_2)\dfrac{b_2 f_2 S}{V_2} \end{cases}$$

where:

$H$ : population of hosts;

$V_i = V_i(t) = V_i(t+T)$ : population of mosquitoes at instant $t$, periodic of period $T > 0$;

$S = S(t)$ : population of infected hosts at instant $t$;

$I_i = I_i(t)$ : population of infected mosquitoes of type $i$ at instant $t$;

$\delta_i$ : death rate of mosquitoes of type $i$;

$\xi$ : cure rate of sick hosts;

$b'_i$ : bites by one mosquito of type $i$ on hosts by unit of time;

$b_i = b_i(t)$ : bites by mosquitoes of type $i$ in one person, by unit of time, which is a periodic function with period $T > 0$;

$f'_i I_i$ : population of infected mosquitoes of type $i$ which are infective;

$f_i S$ : population of infected hosts which are infective.

Since $b_i(t)H = b'_i V_i(t)$, the system above can be written as:

$$(1)\qquad \begin{cases} \dfrac{dS}{dt} = -\xi S + \dfrac{(H-S)}{H}(b'_1 f_1 I_1 + b'_2 f_2 I_2) \\ \dfrac{dI_1}{dt} = -\delta_1 I_1 + (V_1 - I_1)\dfrac{b'_1 f_1 S}{H} \\ \dfrac{dI_2}{dt} = -\delta_2 I_2 + (V_2 - I_2)\dfrac{b'_2 f_2 S}{H} \end{cases}$$

where $H$, $\xi$, $b'_i$, $\delta_i$, $f_i$ are positive constants and $V_i = V_i(t)$ are continuous periodic functions of period $T > 0$.

That system (1) corresponds to a variation of the Ross model ([12]) and of the Dye-Hasibeder ([2], [3]) for malaria. Moreover it is also a variation of the Aronson-Mellander model ([1]) that describes the dynamics of gonorrhea.

The main results $A$ and $B$, stated below, for system (1), were obtained, with small modifications, using some techniques presented in [1] (see Appendix).

**Theorem A** *Let* $(S(t), I_1(t), I_2(t))$ *be a non zero solution of (1) such that* $0 \le S(t_0) \le H$, $0 \le I_i(t_0) \le V_i(t_0)$, $i = 1, 2$, *and some* $t_0 \ge 0$. *Then* $0 < S(t) < H$, $0 < I_i(t) < V_i(t)$, $i = 1, 2$, *and all* $t > t_0$.

We may write (1) in a matricial form

$$\dot{y} = A(t)y + N(t,y)\ , \quad y = (S, I_1, I_2)\ , \quad \text{and}$$

$$A(t) = \begin{pmatrix} -\xi & b_1' f_1 & b_2' f_2 \\ \frac{b_1' f_1 V_1}{H} & -\delta_1 & 0 \\ \frac{b_2' f_2 V_2}{H} & 0 & -\delta_2 \end{pmatrix} .$$

Let $\phi(t)$ be the matricial solution of $\dot{y} = A(t)y$ such that $\phi(0) = I_d$. The monodromy matrix $C = \phi(T)$ is positive (Lemma 2, [1]) and, by Perron's theorem ([10]), it has a simple positive eigenvalue $\lambda$ such that

$$\lambda = \max\{\mathrm{Re}\lambda_i : \det(C - \lambda_i I_d) = 0\}\ .$$

**Theorem B** *There are two possibilities for the non zero solutions* $(S(t),\ I(t)) = (S(t),\ I_1(t),\ I_2(t))$ *of (1) such that for some* $t_0 \geq 0$, $0 \leq S(t_0) \leq H$ *and* $0 \leq I_i(t_0) \leq V_i(t_0)$, $i = 1, 2$*:*

*a) If* $\lambda \leq 1$ *then* $(S(t), I(t))$ *tends to the zero solution as* $t \to \infty$*;*

*b) If* $\lambda > 1$*, then there exists a unique* $T$*-periodic solution* $(S^*, I^*)$ *such that for any* $t > t_0$ *we have* $0 < S^*(t) < H$ *and* $0 < I_i^*(t) < V_i(t)$, $i = 1, 2$*. In this case* $(S(t) - S^*(t), I(t) - I^*(t))$ *tends to zero as* $t \to \infty$*.*

In other words, Theorem B says that the infection finishes naturally when $\lambda \leq 1$; or, if $\lambda > 1$, the infection becomes endemic provided that is positive the initial number of infected of at least one group.

As usual, it is a non trivial matter to obtain the matrix $C = \phi(T)$ and an expression for $\lambda$. In the next section we will do that in another case.

## 3 One patch of hosts and one patch of mosquitoes

Let us consider the following system:

$$(2) \qquad \begin{cases} \dfrac{dS}{dt} = -\xi S + (H-S)\dfrac{b'fI}{H} \\ \dfrac{dI}{dt} = -\delta I + (V(t)-I)\dfrac{b'fS}{H} \end{cases}$$

It describes the dynamics of malaria when we have one group of mosquitoes, only, with a periodic population $V = V(t)$ interacting with a homogeneous group of individuals with a fixed population $H$. We will show that for $V = V(t)$ periodic with period $T = 12$ (months) with $V(t) = V_i$ positive and constant, $i < t < (i+1)$, $i = 1, \ldots, 12$, the eigenvalues of the fundamental solution of the associated linear system can be computed directly, showing their dependence on the data of system (2) that is, on the parameters $\xi$, $\delta$, $H$, $b'$, $f$ and on $V = V(t)$.

Let us write (2) in the following form

$$\begin{pmatrix} \dot S \\ \dot I \end{pmatrix} = A(t)\begin{pmatrix} S \\ I \end{pmatrix} + N(S,I) \quad \text{where} \quad A(t) = \begin{pmatrix} -\xi & b'f \\ \frac{b'fV(t)}{H} & -\delta \end{pmatrix} .$$

For each $i = 1, \ldots, 12$ one considers $\phi(t) = e^{A_i t}$, the fundamental solution of

$$(2)' \qquad \begin{pmatrix} \dot S \\ \dot I \end{pmatrix} = A_i\begin{pmatrix} S \\ I \end{pmatrix} \quad \text{where} \quad A_i = A(i) .$$

If we set $U = -\delta S - b'fI$, (2)' becomes

$$(3) \qquad \begin{pmatrix} \dot S \\ \dot U \end{pmatrix} = \overline{A}_i\begin{pmatrix} S \\ U \end{pmatrix} , \quad \overline{A}_i = \begin{pmatrix} \operatorname{trace} A_i & -1 \\ \det A_i & 0 \end{pmatrix} .$$

Since $A_i = M\overline{A}_iM^{-1}$, with $M = \begin{pmatrix} 1 & 0 \\ -\frac{\delta}{b'f} & -\frac{1}{b'f} \end{pmatrix}$ one obtains

$$(4) \qquad \phi_i(1) = e^{A_i} = Me^{\overline{A}_i}M^{-1} .$$

Let $\psi(t)$ be the matricial solution of $\binom{\dot S}{\dot I} = A(t)\binom{S}{I}$ such that $\psi(0) = I_d$. Then one can write

$$\begin{aligned} \psi(T) &= \phi_{12}(1)\phi_{11}(1)\phi_{10}(1)\cdots\phi_2(1)\phi_1(1) \\ (5) \qquad &= Me^{\overline{A}_{12}}e^{\overline{A}_{11}}e^{\overline{A}_{10}}\ldots e^{\overline{A}_2}e^{\overline{A}_1}M^{-1} . \end{aligned}$$

So from (2)' we have

$$\ddot{S} - (\text{trace } A_i)\dot{S} + (\det A_i)S = 0 \tag{6}$$

and so, from (6)

$$S(t) = \alpha e^{\lambda_1^i t} + \beta e^{\lambda_2^i t} \tag{7}$$

where the $\lambda_j^i$ are the eigenvalues of $A(i) = A_i$, $j = 1,2$, $i = 1,\ldots,12$. Since

$$\dot{S} = (\text{trace } A_i)S - U$$

we have $U = \alpha\lambda_2^i e^{\lambda_1^i t} + \beta\lambda_1^i e^{\lambda_2^i t}$ and then

$$e^{\overline{A}_i} = \frac{1}{\lambda_2^i - \lambda_1^i}\begin{pmatrix} \lambda_2^i e^{\lambda_2^i} - \lambda_1^i e^{\lambda_1^i} & e^{\lambda_1^i} - e^{\lambda_2^i} \\ \lambda_1^i\lambda_2^i(e^{\lambda_2^i} - e^{\lambda_1^i}) & \lambda_2^i e^{\lambda_1^i} - \lambda_1^i e^{\lambda_2^i} \end{pmatrix}. \tag{8}$$

Finally we are able to obtain the eigenvalues of $\psi(T)$ that are the ones of the following matrix:

$$\prod_{i=0}^{11} \frac{1}{\lambda_2^{12-i}\lambda_1^{12-i}} \begin{pmatrix} \lambda_2^{12-i} e^{\lambda_2^{12-i}} - \lambda_1^{12-i} e^{\lambda_1^{12-i}} & e^{\lambda_1^{12-i}} - e^{\lambda_2^{12-i}} \\ \lambda_1^{12-i}\lambda_2^{12-i}\left(e^{\lambda_2^{12-i}} - e^{\lambda_1^{12-i}}\right) & \lambda_2^{12-i} e^{\lambda_1^{12-i}} - \lambda_1^{12-i} e^{\lambda_2^{12-i}} \end{pmatrix}$$

# Appendix

Let us consider the following system

$$(*) \qquad \begin{cases} \dot{y}_1 = -\alpha_1 y_1 + (c_1(t) - y_1)(\beta_{11}(t)y_1 + \beta_{21}(t)y_2 + \cdots + \beta_{n1}(t)y_n) \\ \vdots \\ \dot{y}_n = -\alpha_n y_n + (c_n(t) - y_n)(\beta_{1n}(t)y_1 + \beta_{2n}(t)y_2 + \cdots + \beta_{nn}(t)y_n) \end{cases}$$

that can be written as

$$y' = A(t)y + N(t,y) \quad \text{where}$$

$$A(t) = \begin{pmatrix} -\alpha_1 + c_1\beta_{11} & c_1\beta_{21} & \cdots & c_1\beta_{n1} \\ c_2\beta_{12} & -\alpha_2 + c_2\beta_{22} & \cdots & c_2\beta_{n2} \\ \vdots & & & \\ c_n\beta_{1n} & c_n\beta_{2n} & \cdots & -\alpha_n + c_n\beta_{nn} \end{pmatrix}$$

Assume $\alpha_i$ to be positive constants; $c_i = c_i(t)$ to be positive continuous periodic functions of period $T > 0$; $\beta_{ji} = \beta_{ji}(t)$ to be non negative continuous periodic functions of period $T > 0$ such that $A(t)$ is an irreducible matrix ([10]), for all $t$. Moreover, assume there exists $\varepsilon > 0$ such that $\beta_{ij}(t) \geq \varepsilon$ for all $t$ provided that $\beta_{ji}(t)$ are not identically zero.

The system (*), that generalizes (1) and (2) above, is a variation of the model that appears in [1] describing the dynamics of gonorrhea.

For $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ we denote $x \leq y$ ($x < y$) if, for all $i$, $x_i \leq y_i$ ($x_i < y_i$). With the same arguments used in [1] one can state:

**I)** *If $y(t)$ and $z(t)$ are non-zero solutions of $(*)$ such that for some $t_0 \geq 0$ we have $0 \leq y(t_0) \leq z(t_0) \leq c(t_0)$ with $y(t_0) \neq z(t_0)$, then $0 < y(t) < z(t) < c(t)$ for all $t > t_0$.*

For fixed $t_0 \geq 0$ one considers the map $f_{t_0}(y_0) = y(t_0 + T, t_0, y_0)$ for $0 \leq y_0 \leq c(t_0)$. When $t_0 = 0$ we take the sequence of positive numbers $c(0) = c_0 > c_1 > c_2 > \cdots > c_n > \ldots > 0$ where $c_n = f_0(c_{n-1})$ and $Q = \lim_{n \to \infty} c_n \geq 0$. The case $Q = 0$ corresponds to $\lim_{t \to \infty} y(t, t_0, y_0) = 0$ for $0 \leq y_0 \leq c(t_0)$; if $Q \neq 0$ we have $y(t) = y(t, 0, Q)$ positive and periodic with period $T > 0$. Then by I) we have $Q > 0$.

Let $\Phi(t)$ the matrix solution of $\dot{y} = A(t)y$, $\Phi(0) = I_d$; since $A(t) = (a_{ij}(t))$ is irreducible with $a_{ij}(t) \geq 0$ for $i \neq j$, then the matrix $C = \Phi(T)$ is positive and so, by Perron's theorem, it has a simple positive eigenvalue $\lambda = \max\{\mathrm{Re}\lambda_i ; \det(C - \lambda_i I)\}$.

Like in theorem 1 of [1] one has:

**II)** *If $\lambda < 1$, there exists $K > 0$ such that $|y(t)| \leq K\lambda^{\frac{t-t_0}{T}} |y(t_0)|$ for all $t \geq t_0 \geq 0$ and any solution $y(t) = y(t, t_0, y_0)$ of $(*)$ such that $0 \leq y_0 \leq c(t_0)$.*

Consider now the case $\lambda > 1$. Let $E(t_0) = \{y \in \mathbb{R}^n : 0 \leq y \leq c(t_0)\}$, $\omega > 0$ eigenvector of $C^t$ corresponding to $\lambda$, $v(t_0)^t = \omega^t \Phi(t_0)^{-1}$ and $E_\theta(t_0) = \{y \in E(t_0) : v(t_0)^t y \geq \theta\}$. We claim that for $\theta > 0$ sufficiently small we have $f_{t_0}(E_\theta(t_0)) \subset E_\theta(t_0)$ where $0 \leq t_0 \leq T$. In fact

$$v(t_0)^t f_{t_0}(y) = \lambda v(t_0)^t y + \varphi(t_0, y) \quad \text{where}$$
$$\varphi(t_0, y) = \lambda \omega^t \int_{t_0}^{t_0+T} \Phi^{-1}(s) N(s, y(s, t_0, y)) ds \; ; \quad \text{and}$$
$$v(t_0)^t f_{t_0}(y) - v(t_0)^t y = (\lambda - 1) v(t_0)^t y + \varphi(t_0, y) \geq (\lambda - 1) v^t y + \varphi(t_0, y) \, ,$$

where $0 < v \leq v(t_0)$ for all $t_0 \in [0, T]$. Since

$$\Phi(t_0, y) = \begin{cases} \dfrac{\varphi(t_0, y)}{|y|} & \text{if } y \neq 0 \\ 0 & \text{if } y = 0 \end{cases}$$

is continuous on the compact set

$$\{(t_0, y) \mid t_0 \in [0, T], \; y \in E(t_0)\} \, ,$$

then there exist $\delta > 0$ and $N > 0$ such that $v(t_0)^t f_{t_0}(y) \geq v(t_0)^t y + N|y|$ for $t_0 \in [0, T]$ and $|y| \leq \delta$. Using the Brower fixed point theorem ([11]) one concludes that $f_{t_0}$ has a fixed point in $E_\theta(t_0)$.

Following [1], for any two solutions $y(t) = y(t, t_0, y_0)$ and $z(t) = y(t, t_0, z_0)$ of (*) such that $0 < y_0, z_0 < c(t_0)$, one has $D^+ u(t) \leq -(u - 1) \min(\psi_z, \psi_y) < 0$ for $t \geq t_0 \geq 0$ where

$$u(t) = \max_k \left\{ \max \left\{ \frac{y_k}{z_k}, \frac{z_k}{y_k} \right\} \right\}$$

and

$$\psi(y) = \min_{1 \leq m \leq n} \sum_{j=1}^{n} \beta_{jm}(t) y_j(t) \, .$$

So, one can conclude that for $\theta > 0$ sufficiently small, $f_{t_0} : E_\theta(t_0) \to E_\theta(t_0)$ has only one fixed point $Q$, and then $f_{t_0} : E(t_0) \to E(t_0)$ has one only fixed point $Q > 0$, besides the origin, that corresponds to a periodic orbit of period $T > 0$ for system (*).

For $y(t) = y(t, t_0, y_0)$ with $0 \leq y_0 \leq c(t_0)$ and $z(t) = y(t, t_0, Q)$ we have

$$u(t) - 1 \leq e^{u(t_0)}(u(t_0) - 1) e^{-p(t)} e^{-a(t - t_0)}$$

where $a = \frac{1}{T}\int_0^T \psi_x(t)dt$ and $p$ is continuous and $T$-periodic, $T > 0$.

So, for each $x_0 > 0$ sufficiently close to the origin, there exists a constant $M(x_0) > 0$ such that

$$|y(t, t_0, y_0) - y(t, t_0, Q)| \leq M(x_0)e^{-a(t-t_0)}$$

for all $y_0$, $x_0 \leq y_0 \leq c(t_0)$ and all $t \geq t_0 \geq 0$ because the $c_k(t)$ are bounded functions.

Given a compact set $K \subset E(t_0)$, one considers a point $x_0$, $0 < x_0 < c(t_0)$, such that $x_0 < y(t_0 + T, t_0, y_0) < c(t_0)$ for all $y_0 \in K$. Since there are constants $M(x_0) > 0$ and $N > 0$ such that for all $y_0 \in K$ we have

$$|y(t, t_0, y_0) - y(t, t_0, Q)| \leq M(x_0)e^{-a(t-t_0)} \ , \quad t \geq t_0 + T$$

and

$$|y(t, t_0, y_0) - y(t, t_0, Q)| \leq N \ , \quad t_0 \leq t \leq t_0 + T \ ,$$

then one has the following result:

**III)** *For $\lambda > 1$, system $(*)$ admits a unique non zero periodic solution $y(t, t_0, Q)$, which has period $T$ and a constant $a > 0$ such that for each compact set $K \subset E(t_0)$ there corresponds a constant $M_K > 0$ and we have*

$$|y(t, t_0, y_0) - y(t, t_0, Q)| \leq M_k e^{-a(t-t_0)} \quad \textit{for all } t \geq t_0 \geq 0.$$

Moreover, as in theorem 3 of [1] we state:

**IV)** *For $\lambda = 1$ there is a constant $L > 0$ such that for any solution $y(t, t_0, y_0) = y(t)$ of $(*)$ with $0 \leq y_0 \leq c(t_0)$, $t_0 \geq 0$, we have*

$$|y(t)| \leq \frac{K}{1 + (t - t_0)} \quad \textit{provided that } t \geq t_0.$$

Oliva, W.M. e Sallum, E.M. A dinâmica da malária num sistema de arroz irrigado. Rev. Saúde Pública, ..., 1995. Os autores analisam a dinâmica da população de indivíduos e mosquitos infectados num sistema de arroz irrigado por meio de um modelo matemático determinístico. A questão apareceu dentro do Projeto Ribeira desenvolvido pelo NUPTEM da Universidade de São Paulo. Pela determinação de um parâmetro $\lambda$ (o raio espectral de uma certa matriz de monodromia) pode-se estabelecer se a infecção termina naturalmente ($\lambda \leq 1$) ou se $\lambda > 1$, a infecção torna-se endêmica.

**Palavras chave:** malária, arroz irrigado, dinâmica, mosquitos.

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