

Symmetry in Many-Body Physics

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The harmony of nature is expressed through the implementation of symmetry providing optimal structures for complex systems from snowflakes to graphene lattices. However, sometimes nature breaks symmetry, for example: in sugar molecules, the predominance of people with the heart on the left side, or in various phase transitions. Symmetry plays a crucial role in many-body physics. For instance, chiral symmetry is important in the unusual properties of graphene and the theory of strong interactions. Symmetry breaking and restoration constantly occurs in the world around us. Usually, finding exact solution to the problem of interacting particles presents a fundamental challenge. Therefore, we have to restrict ourselves to approximate solutions that reflect the essential features of the entire problem as a whole and contain an indication of the range of applicability of these solutions. An important role in finding the approximate solutions is played by the knowledge of basic symmetries that determine the accuracy of the used approximations. The purpose of this issue is to demonstrate the principal role of exact and approximate symmetries in solving various problems in many-particle physics, as well as finding approximate solutions for the systems typical of condensed matter, trapped Fermi and Bose gases, nuclear matter, and field theory [1–14].

The review article “Non-Thermal Fixed Points in Bose Gas Experiments”, by L. Madeira and V.S. Bagnato, considers one of the most challenging tasks in physics for understanding the route of an out-of-equilibrium system to its thermalized state. This problem can be particularly overwhelming when one considers a many-body quantum system. Several recent theoretical and experimental studies have indicated that some far-from-equilibrium systems display universal dynamics close to the so-called non-thermal fixed point (NTFP), following a rescaling of both space and time. This opens up the possibility of a general framework for studying and categorizing out-of-equilibrium phenomena into well-defined universality classes. This paper reviews the recent advances in observing NTFPs in experiments involving Bose gases. A brief introduction is provided to the theory behind this universal scaling, focusing on the experimental observations of NTFPs. The benefits of the NTFP universality classes are presented, using the analogy with renormalization group theory in equilibrium critical phenomena.

The review article “A Review of Many-Body Interactions in Linear and Nonlinear Plasmonic Nanohybrids”, by M.R. Singh, discusses the many-body interactions in plasmonic nanohybrids made from an ensemble of quantum emitters and metallic nanoparticles. A theory of the linear and nonlinear optical emission intensity is developed by using the many-body quantum-mechanical density matrix method. The ensemble of quantum emitters and metallic nanoparticles interact with each other via the dipole–dipole interaction. Surface plasmon polaritons are located near to the surface of the metallic nanoparticles. It is shown that the nonlinear Kerr intensity enhances due to the weak dipole–dipole coupling limits. On the other hand, in the strong dipole–dipole coupling limit, the single peak in the Kerr intensity splits into two peaks. The splitting of the Kerr spectrum is due to the creation of dressed states in the plasmonic nanohybrids within the strong dipole–dipole interaction. Further, it is found that the Kerr nonlinearity is enhanced due to the interaction between



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the surface plasmon polaritons and excitons of the quantum emitters. Next, the effect of the spontaneous decay-rate enhancement is predicted due to dipole–dipole coupling. The enhancement of the Kerr intensity due to the surface plasmon polaritons can be used to fabricate nanosensors. The splitting of one peak into two peaks can be used for the fabrication of nanoswitches for nanotechnology and nanomedical applications.

In the paper “Spin Interference Effects in a Ring with Rashba Spin–Orbit Interaction Subject to Strong Light–Matter Coupling in Magnetic Field”, by M. Pudlak and R. Nazmitdinov, the authors studied electron transport through a one-dimensional quantum ring, subject to Rashba spin–orbit interaction and connected with two external leads in the presence of external fields. They include optical radiation, produced by an off-resonant high-frequency electric field, and a perpendicular magnetic field. By means of the Floquet theory of periodically driven quantum systems the interference effects under these fields are described in detail. Specific analytic conditions are found for reaching the spin-filtering effect, caused by the interplay of the external fields and Rashba spin–orbit interactions.

The paper “Long-Time Bit Storage and Retrieval without Cold Atom Technology”, by R. Friedberg and J.T. Manassah, reports computer investigations showing how the duration of memory for storage and retrieval of a classical bit can be increased to 100 times the decay time of an isolated atom, with no use of high-tech cold-atom preparations recently developed in the light-matter field. This low-tech procedure can greatly enlarge the number of experimenters able to enter this field. The role of symmetry in this procedure arises in a careful interplay of incoherent and coherent excitations of a large collection of two-level atoms, the level separation being matched by the dominant frequency of the electromagnetic fields (short pulses and continuing field) applied to the system.

In the paper “Formation of Matter-Wave Droplet Lattices in Multi-Color Periodic Confinements”, by M.R. Pathak and A. Nath, a new model is introduced that addresses the generation of quantum droplets (QDs) in binary Bose–Einstein condensate (BEC) mixtures with mutually symmetric spinor components, loaded in multi-color optical lattices (MOLs) of commensurate wavelengths and tunable intensities. The considered MOL confinement is the combination of the four-color optical lattice with an exponential periodic trap, which includes the complete set of the Fourier harmonics. Employing the one-dimensional (1D) extended nonlinear Schrödinger equation (eGPE), the exact analytical form of the wavefunction, mean field/beyond mean field nonlinearities, and MOL trap parameters are described. Utilizing the exact solutions, the formation of supersolid-like spatially periodic matter-wave droplet lattices and superlattices are illustrated under the space-periodic nonlinearity management. The precise positioning of the density maxima/minima of the droplet patterns at the center of the trap and tunable Anderson-like localization are observed by tuning the symmetry and amplitude of the considered MOL trap. The stability of the obtained solution is confirmed using the Vakhitov–Kolokolov criterion.

The paper “Josephson-like Oscillations in Toroidal Spinor Bose–Einstein Condensates: A Prospective Symmetry Probe”, by M.H. Figlioli Donato and S.R. Muniz, studies Josephson junctions that are essential ingredients in the superconducting circuits used in many existing quantum technologies. Additionally, ultra-cold atomic quantum gases have also become essential platforms to study superfluidity. The analogy between superconductivity and superfluidity is discussed concentrating on an intriguing effect caused by a thin finite barrier in a quasi-one-dimensional toroidal spinor Bose–Einstein condensate (BEC). In this system, the atomic current density flowing through the edges of the barrier oscillates similar to the electrical current through a Josephson junction in a superconductor, but in that case there is no current circulation through the barrier. It is also shown how the nontrivial broken-symmetry states of spinor BECs change the structure of this Josephson-like current, creating the possibility to probe the spinor symmetry, solely using measurements of this superfluid current.

In the paper “Spontaneous Symmetry Breaking: The Case of Crazy Clock and Beyond”, by M.C. Pagnacco, J.P. Maksimovic', M. Dakovic', B. Bokic, S.R. Mouchet, T. Verbiest, Y. Caudano and Branko Kolaric, the authors describe the crazy-clock phenomenon involving

the state I (low iodide and iodine concentration) to state II (high iodide and iodine concentration with new iodine phase) transition after a Briggs–Rauscher (BR) oscillatory process. While the BR crazy-clock phenomenon is known, this is the first time that crazy-clock behavior has been linked and explained with the symmetry-breaking phenomenon, highlighting the entire process in a novel way. The presented phenomenon has been thoroughly investigated by running more than 60 experiments, and evaluated by using statistical cluster K-means analysis. The mixing rate, as well as the magnetic bar shape and dimensions, have a strong influence on transition appearance. Although the transition for both mixing and non-mixing conditions take place completely randomly, by using statistical cluster analysis the authors obtain a different number of clusters (showing the time-domains where the transition is most likely to occur). In the case of stirring, clusters are more compact and separated, revealing new hidden details regarding the chemical dynamics of nonlinear processes. The significance of the presented results is beyond oscillatory reaction kinetics since the described example belongs to the small class of chemical systems that show intrinsic randomness in their response and it may be considered a real example of a classical liquid random number generation.

Interesting results are reported in the paper “Exact Solutions for Solitary Waves in a Bose-Einstein Condensate under the Action of a Four-Color Optical Lattice”, by B. Halder, S. Ghosh, P. Basu, J. Bera, B. Malomed and U. Roy who address the dynamics of Bose–Einstein condensates (BECs) loaded into a one-dimensional four-color optical lattice (FOL) potential with commensurate wavelengths and tunable intensities. This configuration lends system-specific symmetry properties. The analysis identifies specific multi-parameter forms of the FOL potential which admits exact solitary-wave solutions. This newly found class of potentials includes more particular species, such as frustrated double-well superlattices and bichromatic and three-color lattices, which are subject to respective symmetry constraints. The exact solutions provide options for the controllable positioning of density maxima of the localized patterns, and tunable Anderson-like localization in the frustrated potential. A numerical analysis is performed to establish dynamic and structural stability of the obtained solutions, making them relevant for experimental realization. The newly found solutions offer applications to the design of schemes for quantum simulations and processing quantum information.

In the paper “Zeroth-Order Nucleation Transition under Nanoscale Phase Separation”, by V.I. Yukalov and E.P. Yukalova, materials with nanoscale phase separation are considered. A system representing a heterophase mixture of ferromagnetic and paramagnetic phases was studied. After averaging the over phase configurations, a renormalized Hamiltonian is derived describing the coexisting phases. The system is characterized by direct and exchange interactions and an external magnetic field. The properties of the system are studied numerically. The stability conditions define the stable state of the system. At a temperature of zero, the system is in a pure ferromagnetic state. However, at a finite temperature, for some interaction parameters, the system exhibits a zeroth-order nucleation transition between the pure ferromagnetic phase and the mixed state with coexisting ferromagnetic and paramagnetic phases. At the nucleation transition, the finite concentration of the paramagnetic phase appears via a jump.

The paper “Small-Angle Scattering from Fractional Brownian Surfaces”, by E.M. Anitas, investigates materials with fractal-like geometry. Recent developments in nanotechnology have allowed the fabrication of a new generation of advanced materials with various fractal-like geometries. Fractional Brownian surfaces (fBs) are often used as models to simulate and characterize these complex geometries, such as the surfaces of particles in dilute particulate systems (e.g., colloids) or the interfaces in non-particulate two-phase systems (e.g., semicrystalline polymers with crystalline and amorphous phases). However, for such systems, a realistic simulation involves parameters averaged over a macroscopic volume. The author proposes a method, based on small-angle scattering technique, to extract the main structural parameters of surfaces/interfaces from experimental data. The method involves the analysis of scattering intensities and the corresponding pair distance

distribution functions. This allows the extraction of information with respect to the overall size, fractal dimension, Hurst and spectral exponents. The method is applied to several classes of fBs, and it is demonstrated that the obtained numerical values of the structural parameters are in good agreement with the theoretical ones.

The paper “Characteristic Length Scale during the Time Evolution of a Turbulent Bose-Einstein Condensate”, by L. Madeira, A.D. Garcí'a-Orozco, M.A. Moreno-Armijos, F.E. Alves dos Santos and V.S. Bagnato, addresses the topic of quantum turbulence that is currently highly studied. Quantum turbulence is characterized by many degrees of freedom interacting non-linearly to produce disordered states, both in space and time. In this work, the decaying regime of quantum turbulence in a trapped Bose-Einstein condensate was investigated. An alternative way of exploring this phenomenon is presented, by defining and computing a characteristic length scale, which possesses the relevant characteristics to study the establishment of the quantum turbulent regime. To reconstruct the three-dimensional momentum distributions, the inverse Abel transform was employed, as has been successfully done in other works. The analysis is presented for both two- and three-dimensional momentum distributions, discussing their similarities and differences. It is argued that the characteristic length allows us to intuitively visualize the time evolution of the turbulent state.

In the paper “Probing Many-Body Systems near Spectral Degeneracies”, K. Ziegler discusses how the employment of the diagonal elements of the time correlation matrix can be used to probe closed quantum systems that are measured at random times. This enables one to extract two distinct parts of the quantum evolution, a recurrent part and an exponentially decaying part. This separation is strongly affected when spectral degeneracies occur, for instance, in the presence of spontaneous symmetry breaking. Moreover, the slowest decay rate is determined by the smallest energy level spacing, and this decay rate diverges at the spectral degeneracies. Probing of the quantum evolution with the diagonal elements of the time correlation matrix is discussed as a general concept and tested in the case of a bosonic Josephson junction. This reveals for the latter, characteristic properties at the transition to Hilbert-space localization.

The paper “Morphology of an Interacting Three-Dimensional Trapped Bose-Einstein Condensate from Many-Particle Variance Anisotropy”, by O.E. Alon, analyzes the characteristic properties of a Bose-Einstein condensate (BEC) through the variances of observable quantities. The variance of the position operator is associated with how wide or narrow a wave-packet is, the momentum variance is similarly correlated with the size of a wave-packet in momentum space, and the angular-momentum variance quantifies to what extent a wave-packet is non-spherically symmetrical. The interacting three-dimensional trapped BEC in the limit of an infinite number of particles was investigated, and its position, momentum, and angular-momentum anisotropies are described. Computing the variances of the three Cartesian components of the position, momentum, and angular-momentum operators, it is shown that there exist scenarios where the anisotropy of a BEC is different at the many-body and mean-field levels of theory, despite having the same many-body and mean-field densities per particle. This suggests a way to classify correlations via the morphology of 100%-condensed bosons in a three-dimensional trap in the limit of an infinite number of particles. Implications are briefly discussed.

The paper “Acoustic Plasmons in Graphene Sandwiched between Two Metallic Slabs”, by L. Salasnich, studied the effect of two metallic slabs on the collective dynamics of electrons in graphene positioned between the two slabs. It is shown that, if the slabs are perfect conductors, the plasmons of graphene display a linear dispersion relationship. The velocity of these acoustic plasmons crucially depends on the distance between the two metal gates and the graphene sheet. In the case of generic slabs, the dispersion relationship of the graphene plasmons is much more complicated, but it was found that acoustic plasmons can still be obtained under specific conditions.

We hope that the papers of this Issue hold useful information and new ideas for the readers.

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