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Persistence and seasonal long memory in unemployment in the United States

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ABSTRACT

We investigate the persistence of US unemployment applying seasonal fractional integration (FARISMA) models to assess both seasonal and non-seasonal long-range dependence. The analysis is carried out at three levels of data aggregation: state, regional census division, and national aggregation. Using wavelet multiresolution decomposition, we separate out irregular components to assess changes in persistence in unemployment dynamics. Our findings indicate strong evidence of hysteresis in US unemployment rates, with both seasonal and non-seasonal long memory contributing to the persistence of unemployment. These results are evidence that challenges the NAIRU hypothesis, suggesting that exogenous shocks to unemployment have prolonged effects that do not dissipate within a finite time horizon.

1. Introduction

There are important issues in applied economic analysis for policy planning purposes that require the evaluation of aggregate data over broad time spans. One of these cases is persistence in unemployment rates, understood in economics as rigidity in the labor market (Coakley et al., 2001). The question of persistence has been explored in many macroeconomic and finance areas, such as business cycles, permanent income theory, exchange rates, purchasing power parity, inflation, etc. Renowned references in these areas are Campbell and Mankiw (1987), Diebold and Rudebusch (1989), Cheung and Lai (1993), Baillie and Bollerslev (1994), and Baillie et al. (1996). According to Alogoskoufis et al. (1988), Pissarides (1992) and Pries (2004), persistence in the labor market is understood to be the result of different mechanisms that involve economic growth, technology, institutional benchmarking, and others. Blanchard and Summers (1986) were the first to describe evidence that persistence plays an important role in the analysis of unemployment, with significant implications for policy makers' decisions. These authors studied unemployment in Europe in the 1980's and found that economic shocks have persistent effects that traditional theories cannot fully explain. This persistent behavior in unemployment series was termed "hysteresis" in reference to the physical phenomenon. According to Smith (1994), the phenomenon of hysteresis is the result of a high positive correlation between past and present unemployment. Using physical concepts, Ball and Mankiw (2002) explain that hysteresis in the unemployment rate implies the existence of a stochastic trend in the economic variable. As a consequence, the memory structure of the unemployment series becomes permanently influenced by past shocks. Thus, the Hysteresis Hypothesis (HH) diverges from the concept of a Natural Rate as postulated by Friedman (1968) and Phelps (1968), as well as from the Non-Accelerating Inflation Rate of Unemployment (NAIRU) doctrine. Based on the principles of the Phillips curve, Friedman's theory relates expected inflation to the extent that current unemployment deviates from the long-run NAIRU dynamic. Following the NAIRU concept, the equilibrium dynamic in the unemployment rate is time-reversible (Srinivasan and Mitra, 2012; Caporale et al., 2022).

The NAIRU doctrine predicts that the effects of exogenous shocks on the labor market dissipate in a finite time, leaving little room for active policy. In contrast, HH allows for countercyclical policies; in fact, Caporale and Gil-Alana (2007) argue that under HH monetary policy can combat unemployment without immediately triggering accelerated inflation. The implications for monetary policy are discussed further in Ball (2009). Additional details on series dynamics and theoretical implications can be found in Ayala et al. (2012) and Amable and Mayhew (2011).

This paper offers new perspectives on persistence in US unemployment by focusing on two different sources of persistence. Unlike previous comparative studies, we explore the total persistence by decomposing the seasonal and non-seasonal long-memory components through a fractional integration framework, addressing the gap in the existing literature. State, divisions, and national series were analyzed to consider the effect of regional aggregation on series persistence. We contribute to the literature showing evidence that the hysteresis

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phenomenon in the US unemployment emerges from the interaction of the seasonal and non-seasonal long-memory components.

The remainder of this study is organized as follows: Section 2 explores the econometric approaches used in previous research; Section 3 presents some key concepts related to seasonal long-memory processes and addresses the maximum likelihood estimator; Section 4 provides a brief description of wavelet analysis and Section 5 describes the data and the main results; we conclude the study in Section 6 with final remarks.

2. Econometric approaches

Persistence in unemployment has been tested using several econometric frameworks. For example, Song and Wu (1997) analyzed the HH in US unemployment series using traditional unit root tests on annual data from 48 states and found evidence of stochastic trends. Clemente et al. (2005) revisited these results using unit root tests that allow structural breaks, finding evidence that sometimes contradicts previous research (some series rejecting the unit root hypothesis or the HH only at the 10% significance level). Ewing and Wunnava (2001) included structural breaks in their analysis of monthly, seasonally-adjusted unemployment data for the US, Canada, and Mexico, suggesting that the series are trend-stationary around a breaking trend.

According to Doukhan et al. (2003), long-memory models capture low-frequency information more adequately through the mathematical concept of fractional integration. Persistence in unemployment was one of the first topics explored using long-memory models. For example, Diebold and Rudebusch (1989) examined the persistence of economic shocks on US GDP and unemployment, providing evidence in favor of fractional integration orders consistent with persistent mean-reversible behavior. Using quarterly, seasonally-adjusted unemployment series, they contested the HH based solely on unit root tests. Shimotsu (2010) found that point estimates for the US unemployment rate are mean-reversible, although the HH cannot be rejected at a 5% significance level. In Latin America, Ayala et al. (2012) identified a mean-reverting pattern in most countries after accounting for structural changes. Caporale et al. (2022) have also contributed to this body of research by applying fractional integration methods to seasonally adjusted and unadjusted data of the 27 member states of the EU.

Some economic time series are influenced by periodic phenomena such as temperature changes, rainfall levels, cultural celebrations such as Christmas, Black Friday, Valentine's Day, Easter, Mother's Day, Cyber Monday, Memorial Day (see, e.g. Hylleberg et al., 1990; del Barrio Castro and Rachinger, 2021; Gil-Alana and Poza, 2024). In particular unemployment series are strongly affected by these cyclical or seasonal behaviors originating from different sources that affect industrial and agricultural productions, tourism, and others (Abbring et al., 2001; Yi et al., 2021; Kajal and Yimeng, 2024). The seasonal phenomena have prompted the extension of fractional integration models to account for seasonal long memory. Such seasonal long-memory models have been widely applied in the natural sciences and in climate research (see, e.g., Gil-Alana and Robinson, 2001, Ooms and Franses, 2001; Lohre et al., 2003; Caporale et al., 2021; Koycegiz, 2024; Yoshioka, 2024; Caporale et al., 2025). In economics, these models were used by and Porter-Hudak (1990) and Ray (1993) to analyze aggregate monetary data and forecast product revenues, respectively. Gil-Alana (2008), examined seasonal fractional integration and cointegration in Denmark labor demand series and pointed to the misspecification problem if the seasonal long-memory component were not taken into account in the models. To avoid neglecting these effects, this study evaluates the joint influence of seasonal and regular long memories on US unemployment persistence. Early works such as Carlin et al. (1985) and Carlin and Dempster (1989) highlight both the sensitivity of seasonal adjustments to modeling assumptions and the potential of Bayesian methods to provide a coherent framework for such analyses.

The influences of trends and nonlinearities on US unemployment persistence were also assessed in this study using wavelet analysis. For this purpose, the analytical strategy adopted was to compare the fractional integration parameters estimated from two groups of data: the original series and series whose nonlinear components and trends have been extracted using wavelet decomposition procedures. We follow the procedures used by Craigmile et al. (2004, 2005) in a broad sense. However, instead of choosing the decomposition levels to take just trends relative to low order polynomials into account, they were chosen with reference to the most important asymmetries in the data. Thus, the irregular components were extracted from the data by filtering the coarse-scale component and making the respective phase shift adjustments. Fig. 1 exemplifies the filtering results for the total U.S. unemployment rate. The original series and the scale S4 component calculated using the Maximal Overlap Discrete Wavelet Transform (MODWT) are shown in panel (a), and the filtered series is depicted in panel (b). This non-endogenous procedure of full filtering the irregular components allowed us to assess these components with respect to unemployment persistence and, thus, evaluate their influence on the acceptance of the HH.

3. Seasonal fractionally integrated processes

Fractional integration theory allows for generalizations of long-memory models to capture persistent seasonal movements. Hosking (1981) proposed a generalized long-memory model with a single spectral pole at a nonzero frequency:

$$\left(1 - 2\lambda B + B^2\right)^d X_t = \varepsilon_t, \quad (1)$$

where B is the lag operator, d is the fractional integration parameter, $\lambda = \cos(\omega)$ for $-\pi < \nu < \pi$ and ε_t is white noise with variance σ_ε^2. The process (1), known as the Gegenbauer process, since it can be expanded in Gegenbauer polynomials (Gray et al., 1989). More details can be seen in Hunt et al. (2022). Further studies include Andel (1986), Chung (1996), Giraitis and Leipus (1995), Woodward et al. (1998), Hsu and Tsai (2009).

In our empirical analysis, we use the FARISMA model, closely related to both the ARUMA and Gegenbauer models. Researchers such as Porter-Hudak (1990), Hassler (1994), Arteche and Robinson (2000), Arteche (2002), Reisen et al. (2006a,b) and Diongue et al. (2008) have examined FARISMA models, which extend the autoregressive fractionally integrated moving average (ARFIMA) model to incorporate seasonal effects.

For our study, we use the definition of the FARISMA process as follows. The stochastic process $\{X_t\}$ is defined as a multiplicative FARISMA$(p, d, q) \times (p_s, d_s, q_s)$ model if it satisfies

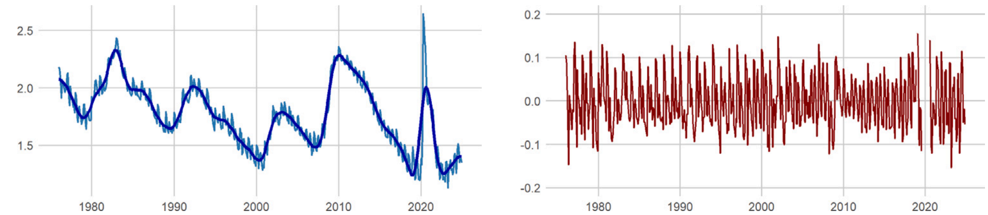
$$\Phi(B)\Phi_s(B^s)(1 - B)^d(1 - B^s)^{d_s}(X_t - \mu) = \Theta(B)\Theta_s(B^s)\epsilon_t, \quad (2)$$

where μ is the mean, ϵ_t is a Gaussian white noise process with zero mean and variance σ^2; $\Phi(B)$ and $\Phi_s(B^s)$ are the autoregressive and seasonal autoregressive polynomials; $\Theta(B)$ and $\Theta_s(B^s)$ are the moving average and seasonal moving average polynomials; and s is the seasonal period. More details of FARISMA models can be seen in Bisognin and Lopes (2009) and Reisen et al. (2006a).

The polynomials in (2) have no common roots, and all characteristic roots lie outside the unit circle. The short-memory components are captured by these polynomials, while the fractional difference operators model long-memory behavior. The non-seasonal fractional difference operator $(1 - B)^d = \Delta^d$ is given by

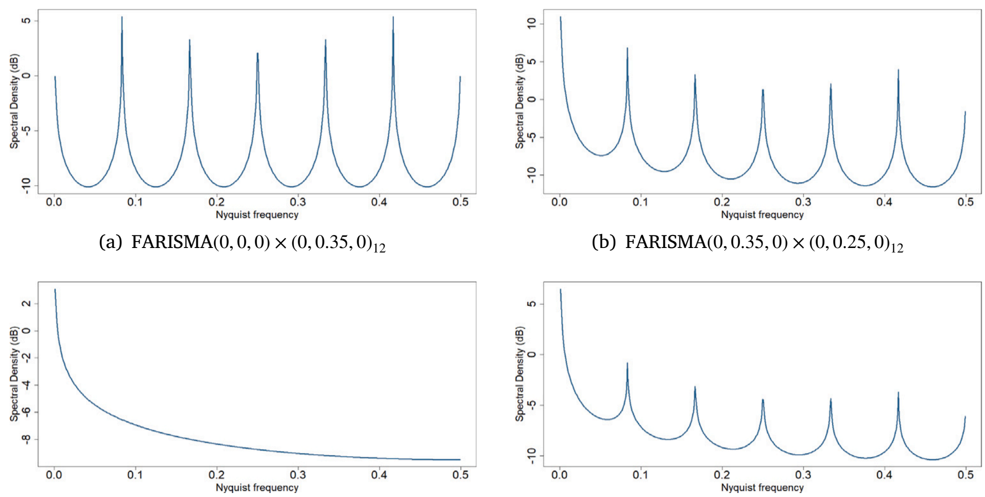
$$(1 - B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k = \sum_{k=0}^{\infty} \frac{\Gamma(-d + k)}{\Gamma(-d)\Gamma(k + 1)} B^k, \quad (3)$$

which is better defined in the frequency domain $\hat{\Delta}^d = (1 - e^{-i\omega})^d$. The analogous seasonal fractional difference operator is $\hat{\Delta}_s^{d_s} = (1 - e^{-i\omega s})^{d_s}$, which applies the filter in time domain to seasonal lags s and zero



(a) US Unemployment vs S4 Component (b) Filtered US Unemployment

Fig. 1. US Total Unemployment rate (log).



(a) FARISMA$(0,0,0)\times(0,0.35,0)_{12}$ (b) FARISMA$(0,0.35,0)\times(0,0.25,0)_{12}$

(c) FARIMA$(0,0.25,0)$ (d) FARISMA$(0,0.25,0)\times(0,0.15,0)_{12}$

Fig. 2. Example of theoretical Spectral Density Function (SDF) of FARISMA models.

otherwise. Bisognin and Lopes (2009) demonstrate that the stationarity conditions for the pure FARISMA processes are that $d + d_s < 0.5$, $d_s < 0.5$.

According to Bisognin and Lopes (2009), the spectral density function (SDF) of the FARISMA model is given by

$$f_X(\omega) = \frac{\sigma_\epsilon^2}{2\pi}\left(|1-e^{-i\omega}|^{-2d}|1-e^{-is\omega}|^{-2d_s}\right)\times f_{\mathrm{ARMA}}(\omega), \tag{4}$$

where $f_{\mathrm{ARMA}}(\omega)$ denotes the transfer filters of the ARMA part of the process, as defined in Brockwell and Davis (2009).

Persistent time series $(d, d_s > 0)$ exhibit a pole at zero frequency for non-seasonal persistence and at seasonal frequencies for seasonal persistence. They are characterized by a pole at their power spectrum (meaning $f_X(\omega) \to \infty, \omega \to 0$) and at frequencies $v_j = 2\pi j/s, j = 1,2,\dots[s/2]$ for seasonally persistent time series. Generally, singularity in the spectral density function at frequency ω_0 represents a cycle of period $2\pi/\omega_0$. However, several peaks at the seasonal frequencies indicate that the time series has a periodic or seasonal component. Fig. 2 depicts four graphic examples of the theoretical spectral density function of the FARISMA process with a seasonal period of $s = 12$ months. The first plot in Fig. 2(a) represents the SDF of a pure seasonal, stationary fractional noise and the plot in Fig. 2(b) represents the SDF of a non-stationary process with both seasonal and non-seasonal long memory. Fig. 2(c) exhibits a pure, non-seasonal long-memory process with $d = 0.25$ with intermediate memory and no seasonal long memory effects. Finally, in Fig. 2(d) displays a stationary FARISMA with $d = 0.25$ and $d_s = 0.15$.

Through factorization of the filter transfer functions, Reisen et al. (2006b) showed that the pure FARISMA process is a particular case of the multifactor ARUMA model, and could also be connected to Gegegenbauer processes.

Under normality assumption, the exact Gaussian maximum likelihood estimator (MLE) maximizes the log-likelihood function

$$\ell(\theta) = -\frac{1}{2}\log|\Sigma(\theta)| - -\frac{1}{2}(X_t-\mu)'\Sigma(\theta)^{-1}(X_t-\mu) - \frac{n}{2}\log(2\pi), \tag{5}$$

where X_t is the vector of observations, θ is the vector of parameters, and Σ is the variance–covariance matrix for X_t. Beran (1994) noted that the ML estimator for θ is obtained by solving $l'(\theta) = 0$, while Haslett and Raftery (1989) provided an ML solution involving extensive computation with many trial values θ_0. Whittle (1953) proposed a maximum likelihood estimator for short-memory processes that avoids these computational difficulties. The resulting Whittle Maximum Likelihood (WML) method minimizes the approximate likelihood computed in the frequency domain:

$$L(\theta) = \int_{-\pi}^{\pi}\left[\log f(\omega;\theta) + \frac{I(\omega)}{f(\omega;\theta)}\right]d\omega, \tag{6}$$

where $f(\omega;\theta)$ is the spectral density function and $I(\omega)$ is the periodogram.

4. Wavelet analysis: A brief description

Wavelet analysis has recently become popular in many scientific areas. A historical and technical perspective on signal processing, from Fourier analysis to wavelets, is found in Meyer (1993), and a non-technical description is provided in Hubbard (1998). It is beyond the scope of this work to provide an extensive description of wavelet theory and methods. Thus, we will focus on the fundamental concepts of wavelet frameworks used in the empirical approach and give the appropriate bibliographical references for the most important remaining issues.

Wavelets are functions $\psi \in L_2$ with $\int_{\mathbb{R}} dt\ \psi(t) = 0$, $\|\psi\| = 1$ and are centered in the neighborhood of $t = 0$, localized in time and scale. It is possible to construct wavelets such that the dilated and translated families form an orthonormal basis of L_2.

The wavelet basis are generated from binary rescales and translations of the wavelet function ψ or scaling function ϕ, also referred to as mother and father wavelets, respectively. This means that, for the wavelet and scaling functions, we set $\psi_{j,k} = 2^{-j/2}\psi(2^{-j}x - k)$ and $\phi_{j,k}(t) = 2^{-j/2}\phi(2^{-j}t - k)$, where $j, k \in \mathbb{Z}$ (Mallat, 2009).

It can be shown that the family of subspaces generated by $\{\psi_{j,k}\}$ and $\{\phi_{j,k}\}$ form a multiresolution analysis (MRA) in the sense of Mallat (1989). A sequence of nested and closed subspaces $V_n, n \in \mathbb{Z}$ in $L_2(\mathbb{R})$ forms an MRA if:

1. $\{0\} \subset \ldots \subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset \ldots \subset L_2$
2. $L_2(\mathbb{R}) = \overline{\bigcup_{j=-\infty}^{\infty} V_j}$
3. $\bigcap_{j=-\infty}^{\infty} V_j = \{0\}$
4. $f \in V_j \Leftrightarrow f(2^j \cdot -k) \in V_0$
5. There exists a scaling function $\phi \in V_0$ such that $\{\phi(x-k), k \in \mathbb{Z}\}$ forms a Riesz basis of V_0.

Whenever ϕ and ψ fulfill the requirements for a MRA, using equations in ψ and ϕ, a multiresolution decomposition can be considered as of the subspaces V_j in L_2 as:

$$V_j = V_J \oplus \bigoplus_{k=0}^{J-j-1} W_{J-k}, \tag{7}$$

where W_j is the subspace generated by $\{\psi_{j,k}\}_{k\in\mathbb{Z}}$ and V_J is the subspace generated by $\{\phi_{J,k}\}_{k\in\mathbb{Z}}$. 1 When these conditions are met, it is possible to generate a wavelet multiresolution decomposition (MRD) of square-integrable signals into different scales which ensure perfect reconstruction under orthonormal wavelet families. Therefore, a time series $\{X_t\}_{t=1}^{T}$ can be decomposed as

$$X_t = c_{J,0}\phi_{J,0}(t) + \sum_{j=1}^{J}\sum_{k=1}^{2^{J-j}-1} d_{j,k}\psi_{j,k}(t) = S_J(t) + \sum_{j=1}^{J} D_j(t), \tag{8}$$

with the coefficients given by the inner product $d_{j,k} = \langle X, \psi_{j,k}\rangle$ and $c_{j,k} = \langle X, \phi_{j,k}\rangle$ (Bruce and Gao, 1996). The signal $S_J(t)$ is commonly referred to as the “smooth signal” of level j, while $D_j(t)$ is known as the “detail signal” of level j.

In practice, the wavelet decomposition of a signal is calculated using standard filterbank theory, where signal x_t is convolved and downsampled by 2 with high-pass $\{g_k\}$ and low-pass $\{h_k\}$ filters, associated, respectively, with $\psi(t)$ and $\phi(t)$, which are obtained as $h_k = \langle \phi, \phi_{1,k}\rangle$ and $g_k = (-1)^k h_{1-k}$ (Vidakovic, 1999). This procedure of convolution and decimation would require that the signal be of length $T = 2^J$ for $J \in \mathbb{Z}$.

In this article, we consider the non-decimated wavelet transform, also referred to as the maximal overlap discrete wavelet transform (MODWT) as described by Percival and Walden (2002), which can be applied to signals of all lengths, since there is no decimation step. The difference is that the filters $\{\tilde{h}_k\}$ and $\{\tilde{g}_k\}$ are periodized on the basis of scaled versions of the original high-pass and low-pass filters. In this case, filters $\tilde{h}$ and $\tilde{g}$ are rescaled by $\tilde{h}_k = h_k/\sqrt{2}$ and $\tilde{g}_k = g_k/\sqrt{2}$ and transforms are performed by circular convolution with no decimation.

The wavelet detail D_j will capture changes in x_t as scales $j < J$ associated with frequencies in the band $(1/2^{j+1}, 1/2^j]$. These components decompose the series deviation from its trend related to its smaller time scales. The smooth component S_0, captures the mean level at the $j = 0$ scale and evaluates the trend of a time series related to its longest time scale.

The MRA provided by the MODWT overcomes some of the limitations of the traditional Discrete Wavelet Transform (DWT). In contrast to the latter, the MODWT is not restricted to sample sizes of a multiple of 2 and it is not translation invariant. The association of the MODWT with zero-phase filters results in detail and smooth components that are perfectly aligned with the original series. Two important features of wavelet MRA in the analysis of time series were highlighted by Kumar et al. (2011). First, wavelet analysis can be performed on non-stationary series without the need for prior transformation and, second, it can isolate any low-frequency nonlinear component, while maintaining all high-frequency details. For the purposes of our analysis, the MRA provides a versatile way to filter time series components associated with specific time scales, as well as polynomial trend and local irregular components. In the next section, the procedure used in our empirical comparative analysis to filter trends is described in detail. The technicalities of filterbank theory and of MODWT in particular can be seen in Nguyen and Strang (1996) and Percival and Walden (2002).

5. Persistence in US unemployment series

The aim of adopting the FARISMA models in this empirical study was to investigate the simultaneous occurrence of seasonal and non-seasonal long-memory processes in the US unemployment rate series and their conjugated effect on series persistence. The integration parameters were estimated using model (2).[1] The maximum likelihood method described in Section 3 was used for the estimations. We were also interested in investigating whether trends and irregular movements in the series, such as breaks, regimes, and high-order polynomial trends, could have any influence on the evaluation of the persistence level. In practice, we have investigated whether the HH would be rejected after all these components have been excluded from the unemployment series. In order to answer this question, we follow a customized version of the algorithm proposed by Bisaglia and Gerolimetto (2009). Instead of using the procedures developed by Bai and Perron (1998, 2003) to detect neglected breaks, we applied wavelet multiresolution decomposition to access the main sources of possible non-linearities and trends.

First, the series were decomposed into four levels using the Daubechies' Least Asymmetric wavelet [LA(8)], also known as Symmlets. As a result of the properties of filterbanks, when Symmlets are used in MODWT, the transform coefficients can be rotated circularly so that they are approximately aligned (in time) with events of the original time series (Nguyen and Strang, 1996; Bruce and Gao, 1996). Therefore, we adjust the wavelet S4 decomposition for zero-phase alignment and they are subtracted from the log-unemployment series. Lastly, the filtered series were generated taking the difference between the original series and their respective S4 components.[2]

We are aware that, when applying this procedure, some of the true low-frequency components of the series might be eliminated and long-memory parameters may be underestimated.[3] However, our argument in favor of this methodology is that, even with this possibility, if we find non-stationary, non-mean-reversible fractional orders of integration, this would be a sign of more robust evidence against the NAIRU hypothesis. When applying procedures like endogenous break identification, researchers cannot be sure if the remaining low-frequency components are an integral part of the series or not, leading to wrong conclusions about series persistence. Thus, it should be clear that our goal in

[1] First estimates have shown the possible presence of short-memory components in the series, as in MA(5). In practice, these components have no influence on long-term estimates given the algorithm we employ. Thus, we avoid the cumbersome short-memory model selections for the 61 series.

[2] The LA(8) wavelet was chosen based on its technical properties, as reported in Percival and Walden (2002)

[3] In our earlier simulation experiments, based on the same procedures used in this study and involving deterministic polynomial trends, the following relation between the long-memory parameters estimated based on the original series (d) and on the filtered series ($d*$) was encountered: $d* \leq d$. The experiment also showed that this result depends primarily on the number of decompositions chosen in the multiresolution analysis.

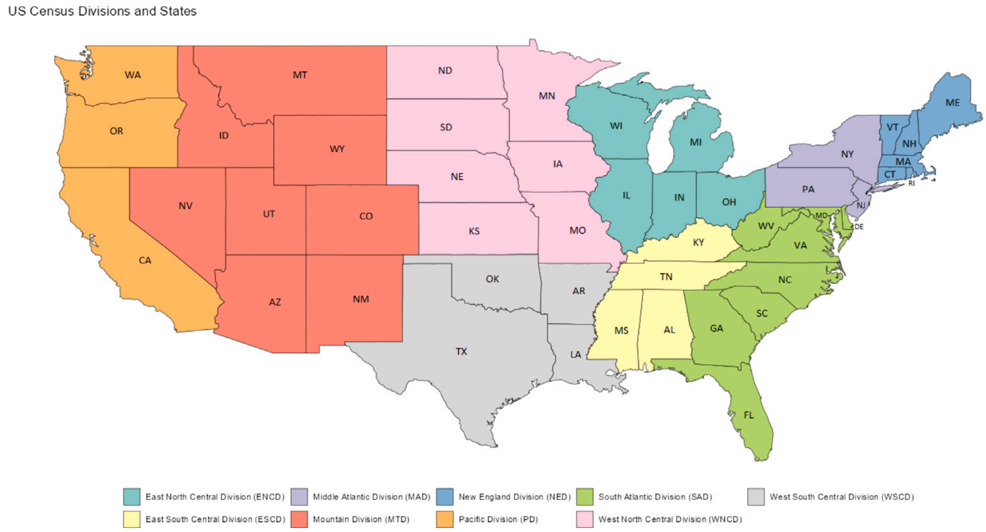


Fig. 3. Map depicting of US Census Divisions and States used in the analysis.

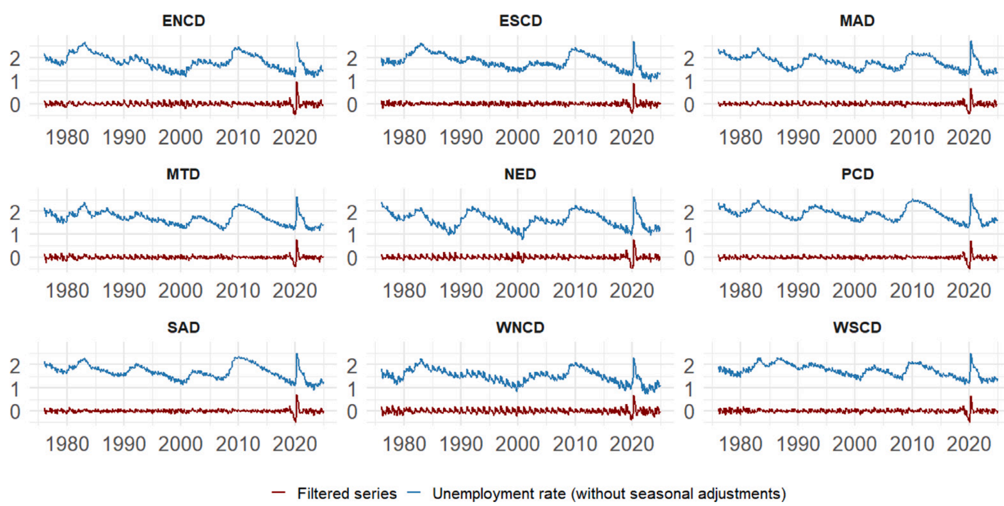


Fig. 4. Unemployment Series of US Census Divisions: Original Series (in log) and Filtered Series.

this empirical study is not to find the true data generating process for the series. Rather, our main goal remains to find solid evidence of persistence in the US unemployment series, excluding all possible sources of breaks and trends.

The dataset comprises 61 series, with $T = 588$ monthly observations at three levels of aggregation: US state (51 series including the District of Columbia), regional (9 series), and the US national unemployment rate.[4] Fig. 3 shows the composition of the nine geographical regions. All data were provided by the US Census Bureau, and the period covered by the study was January 1976 to December 2024.

Fig. 4 depicts the original and filtered series in the nine regions. For the sake of parsimony, we avoid showing the 51 graphs for the state series. The filtered series shown at the bottom of each graph reveal that the wavelet S4 component is responsible for most of the low-frequency information and captures virtually all of the "irregularities" and "irregular low-frequency components" present in the series. Traditional spectral analysis confirmed that information in other frequency bands, including seasonality, was barely affected.

Tables 1 and 2 display the national, regional, and state estimations of d and d_s calculated using the original and filtered series, respectively.

Our analysis focuses on the long-term dynamics of unemployment series, specifically the interaction between seasonal and non-seasonal long-memory components. In practice, short-memory components such as autoregressive (AR) and moving average (MA) terms do not affect long-run behavior. Consequently, the series were modeled as fractional noise processes ($p = 0$, $q = 0$, $p_s = 0$ and $q_s = 0$), with the innovation process ϵ_t in (2) specified either as a stationary SARMA process or, in the simplest case, a white noise.

Fig. 5 displays the superimposed empirical and theoretical spectra for the nine regions in decibels. The empirical spectra were estimated using the periodogram method and the theoretical spectra were calculated using the estimated parameters for the FARISMA model following (4). The continuously slow spectrum decay silhouettes from low to high frequencies indicate the presence of "traditional" or "non-seasonal" long memory, while their decaying singularities in the seasonal frequencies reflect the typical seasonal long memory.

The results using the original series show that 11 of the 61 regions analyzed have $\hat{d} > 1$ and 48 of them are such that $\hat{d} > 0.9$. Taking into account the confidence intervals, 21 of them contain 1. The national unemployment series and 6 of the 9 regional series displayed the same characteristics. The minimum point estimate for non-seasonal long-memory was $\hat{d} = 0.78$ for the state of South Dakota.

The mean of the estimated seasonal long-memory parameter for the states, regions, and national series is around 0.34. Together, the

[4] The District of Columbia was included in the state group to facilitate the presentation of the results.

Table 1
Estimated long-memory parameters using original and filtered series for US regions.

	Original series (log)				Filtered series			
Region	d	d_s	CI(d)	CI(d_s)	d	d_s	CI(d)	CI(d_s)
USA	0.987	0.300	(0.955 ; 1.02)	(0.272 ; 0.329)	0.749	0.319	(0.716 ; 0.781)	(0.29 ; 0.347)
ENCD	0.957	0.299	(0.925 ; 0.99)	(0.271 ; 0.328)	0.717	0.308	(0.684 ; 0.749)	(0.279 ; 0.336)
ESCD	0.881	0.334	(0.848 ; 0.913)	(0.305 ; 0.362)	0.562	0.329	(0.53 ; 0.595)	(0.301 ; 0.358)
MAD	1.001	0.291	(0.968 ; 1.033)	(0.262 ; 0.32)	0.751	0.325	(0.718 ; 0.783)	(0.296 ; 0.354)
MTD	1.007	0.340	(0.974 ; 1.039)	(0.311 ; 0.368)	0.750	0.359	(0.717 ; 0.782)	(0.331 ; 0.388)
NED	0.985	0.329	(0.953 ; 1.018)	(0.3 ; 0.358)	0.715	0.362	(0.683 ; 0.748)	(0.334 ; 0.391)
PCD	1.063	0.302	(1.03 ; 1.095)	(0.273 ; 0.33)	0.857	0.317	(0.825 ; 0.89)	(0.289 ; 0.346)
SAD	0.964	0.274	(0.931 ; 0.996)	(0.245 ; 0.302)	0.728	0.297	(0.696 ; 0.761)	(0.269 ; 0.326)
WNCD	0.936	0.402	(0.904 ; 0.969)	(0.373 ; 0.431)	0.691	0.418	(0.658 ; 0.723)	(0.389 ; 0.446)
WSCD	0.983	0.344	(0.951 ; 1.016)	(0.315 ; 0.372)	0.731	0.347	(0.698 ; 0.763)	(0.318 ; 0.376)

Table 2
Estimated long-memory parameters using original series for US states.

	Original series (log)				Filtered series			
Region	d	d_s	CI(d)	CI(d_s)	d	d_s	CI(d)	CI(d_s)
AK	0.910	0.393	(0.877 ; 0.942)	(0.365 ; 0.422)	0.714	0.428	(0.682 ; 0.747)	(0.399 ; 0.456)
AL	0.909	0.323	(0.877 ; 0.941)	(0.294 ; 0.352)	0.628	0.319	(0.596 ; 0.66)	(0.29 ; 0.347)
AR	0.985	0.391	(0.953 ; 1.018)	(0.362 ; 0.42)	0.811	0.448	(0.778 ; 0.843)	(0.419 ; 0.477)
AZ	0.980	0.276	(0.948 ; 1.013)	(0.248 ; 0.305)	0.707	0.333	(0.674 ; 0.739)	(0.304 ; 0.362)
CA	1.082	0.319	(1.05 ; 1.114)	(0.29 ; 0.348)	0.860	0.332	(0.827 ; 0.892)	(0.304 ; 0.361)
CO	1.062	0.345	(1.029 ; 1.094)	(0.316 ; 0.373)	0.792	0.334	(0.759 ; 0.824)	(0.305 ; 0.363)
CT	0.995	0.394	(0.963 ; 1.027)	(0.365 ; 0.423)	0.742	0.483	(0.709 ; 0.774)	(0.454 ; 0.511)
DC	0.919	0.354	(0.886 ; 0.951)	(0.326 ; 0.383)	0.547	0.365	(0.514 ; 0.579)	(0.337 ; 0.394)
DE	1.002	0.383	(0.969 ; 1.034)	(0.355 ; 0.412)	0.810	0.403	(0.778 ; 0.842)	(0.374 ; 0.432)
FL	1.009	0.247	(0.977 ; 1.042)	(0.218 ; 0.276)	0.817	0.284	(0.784 ; 0.849)	(0.255 ; 0.312)
GA	0.920	0.269	(0.887 ; 0.952)	(0.24 ; 0.298)	0.655	0.290	(0.622 ; 0.687)	(0.262 ; 0.319)
HI	0.897	0.243	(0.864 ; 0.929)	(0.215 ; 0.272)	0.601	0.241	(0.569 ; 0.634)	(0.212 ; 0.269)
IA	0.851	0.387	(0.818 ; 0.883)	(0.358 ; 0.415)	0.566	0.395	(0.533 ; 0.598)	(0.366 ; 0.424)
ID	0.923	0.375	(0.891 ; 0.956)	(0.346 ; 0.403)	0.689	0.390	(0.656 ; 0.721)	(0.362 ; 0.419)
IL	0.990	0.287	(0.958 ; 1.022)	(0.258 ; 0.316)	0.770	0.289	(0.738 ; 0.803)	(0.26 ; 0.318)
IN	0.920	0.293	(0.888 ; 0.953)	(0.264 ; 0.321)	0.666	0.289	(0.633 ; 0.698)	(0.261 ; 0.318)
KS	0.860	0.377	(0.827 ; 0.892)	(0.349 ; 0.406)	0.579	0.365	(0.547 ; 0.612)	(0.337 ; 0.394)
KY	0.807	0.293	(0.774 ; 0.839)	(0.265 ; 0.322)	0.465	0.295	(0.432 ; 0.497)	(0.266 ; 0.324)
LA	0.975	0.375	(0.942 ; 1.007)	(0.347 ; 0.404)	0.773	0.368	(0.741 ; 0.805)	(0.339 ; 0.397)
MA	0.967	0.304	(0.934 ; 0.999)	(0.275 ; 0.333)	0.676	0.325	(0.644 ; 0.709)	(0.296 ; 0.353)
MD	0.950	0.344	(0.918 ; 0.983)	(0.315 ; 0.373)	0.632	0.361	(0.6 ; 0.665)	(0.332 ; 0.389)
ME	0.999	0.333	(0.967 ; 1.032)	(0.305 ; 0.362)	0.821	0.376	(0.789 ; 0.854)	(0.348 ; 0.405)
MI	0.930	0.288	(0.898 ; 0.963)	(0.259 ; 0.317)	0.671	0.295	(0.639 ; 0.704)	(0.266 ; 0.324)
MN	0.910	0.446	(0.878 ; 0.943)	(0.417 ; 0.474)	0.707	0.478	(0.674 ; 0.739)	(0.45 ; 0.507)
MO	0.930	0.370	(0.898 ; 0.963)	(0.341 ; 0.398)	0.624	0.387	(0.591 ; 0.656)	(0.358 ; 0.415)
MS	0.859	0.394	(0.826 ; 0.891)	(0.365 ; 0.423)	0.456	0.387	(0.424 ; 0.488)	(0.358 ; 0.416)
MT	0.869	0.378	(0.837 ; 0.901)	(0.35 ; 0.407)	0.608	0.413	(0.576 ; 0.641)	(0.384 ; 0.441)
NC	0.916	0.296	(0.883 ; 0.948)	(0.267 ; 0.324)	0.643	0.306	(0.611 ; 0.675)	(0.277 ; 0.334)
ND	0.895	0.486	(0.863 ; 0.928)	(0.457 ; 0.515)	0.670	0.491	(0.637 ; 0.702)	(0.463 ; 0.52)
NE	0.828	0.407	(0.796 ; 0.861)	(0.378 ; 0.436)	0.522	0.412	(0.489 ; 0.554)	(0.384 ; 0.441)
NH	0.929	0.287	(0.896 ; 0.961)	(0.258 ; 0.316)	0.616	0.310	(0.584 ; 0.648)	(0.282 ; 0.339)
NJ	1.031	0.306	(0.998 ; 1.063)	(0.277 ; 0.335)	0.802	0.326	(0.77 ; 0.835)	(0.297 ; 0.355)
NM	0.974	0.418	(0.942 ; 1.007)	(0.39 ; 0.447)	0.640	0.476	(0.607 ; 0.672)	(0.448 ; 0.505)
NV	1.038	0.290	(1.006 ; 1.07)	(0.261 ; 0.319)	0.827	0.299	(0.794 ; 0.859)	(0.27 ; 0.327)
NY	1.013	0.234	(0.981 ; 1.046)	(0.205 ; 0.263)	0.788	0.270	(0.756 ; 0.821)	(0.242 ; 0.299)
OH	0.958	0.332	(0.926 ; 0.991)	(0.304 ; 0.361)	0.692	0.333	(0.66 ; 0.725)	(0.304 ; 0.362)
OK	0.942	0.296	(0.91 ; 0.975)	(0.268 ; 0.325)	0.671	0.306	(0.639 ; 0.703)	(0.277 ; 0.334)
OR	1.003	0.331	(0.971 ; 1.036)	(0.303 ; 0.36)	0.795	0.352	(0.763 ; 0.828)	(0.324 ; 0.381)
PA	0.903	0.350	(0.87 ; 0.935)	(0.321 ; 0.378)	0.565	0.379	(0.533 ; 0.597)	(0.35 ; 0.407)
RI	0.953	0.372	(0.921 ; 0.986)	(0.344 ; 0.401)	0.678	0.375	(0.646 ; 0.711)	(0.347 ; 0.404)
SC	0.918	0.294	(0.886 ; 0.951)	(0.266 ; 0.323)	0.650	0.297	(0.617 ; 0.682)	(0.269 ; 0.326)
SD	0.779	0.397	(0.746 ; 0.811)	(0.368 ; 0.426)	0.511	0.409	(0.478 ; 0.543)	(0.38 ; 0.438)
TN	0.897	0.324	(0.865 ; 0.93)	(0.295 ; 0.352)	0.603	0.323	(0.571 ; 0.635)	(0.294 ; 0.352)
TX	0.986	0.341	(0.954 ; 1.019)	(0.313 ; 0.37)	0.723	0.337	(0.691 ; 0.755)	(0.308 ; 0.366)
UT	0.916	0.371	(0.884 ; 0.949)	(0.342 ; 0.399)	0.627	0.378	(0.595 ; 0.66)	(0.35 ; 0.407)
VA	0.911	0.266	(0.879 ; 0.944)	(0.238 ; 0.295)	0.650	0.298	(0.618 ; 0.682)	(0.269 ; 0.327)
VT	0.888	0.357	(0.856 ; 0.921)	(0.329 ; 0.386)	0.570	0.377	(0.537 ; 0.602)	(0.348 ; 0.406)
WA	0.972	0.268	(0.94 ; 1.005)	(0.239 ; 0.297)	0.802	0.284	(0.769 ; 0.834)	(0.255 ; 0.313)
WI	0.889	0.336	(0.857 ; 0.921)	(0.308 ; 0.365)	0.643	0.368	(0.61 ; 0.675)	(0.339 ; 0.396)
WV	0.939	0.355	(0.906 ; 0.971)	(0.326 ; 0.384)	0.681	0.372	(0.648 ; 0.713)	(0.343 ; 0.4)
WY	0.906	0.395	(0.874 ; 0.938)	(0.366 ; 0.424)	0.562	0.417	(0.53 ; 0.594)	(0.388 ; 0.446)

estimated seasonal and non-seasonal parameters imply a divergent non-stationary pattern for all original series. On average, the sum of both $\hat{d}$ and $\hat{d}_s$ is of 1.27. Following Bisognin and Lopes (2009), we know that as $\omega \to 0$, then $f(\omega) \sim C_1|\omega|^{d+d_s}$. Since $d + d_s \approx 1$ for many series, it follows that a more simplistic analysis might suggest unit root behavior when considering only low frequencies of the power spectrum, and this could interfere with typical unit root tests. However, a more careful examination of the spectral density of the signal strongly suggests the

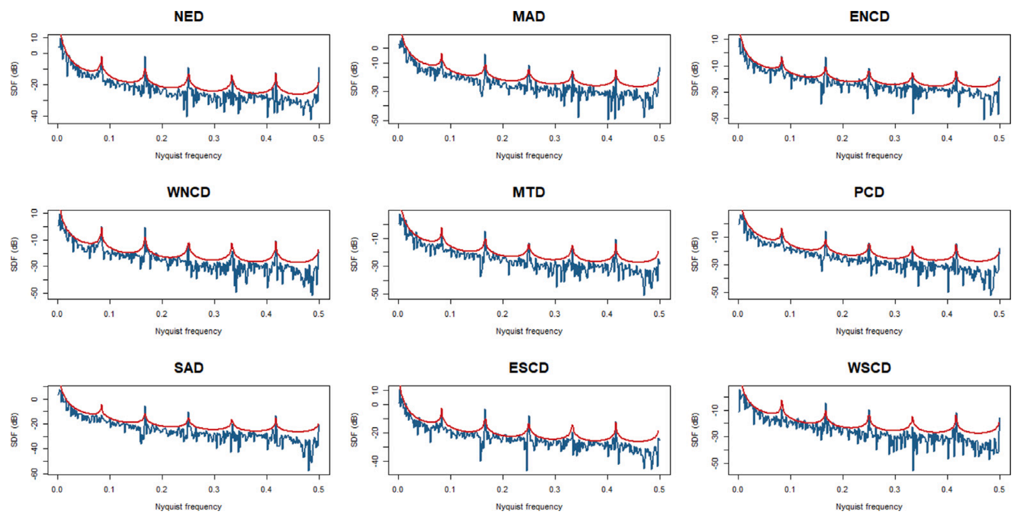


Fig. 5. Spectral density function for all US Census Regions.

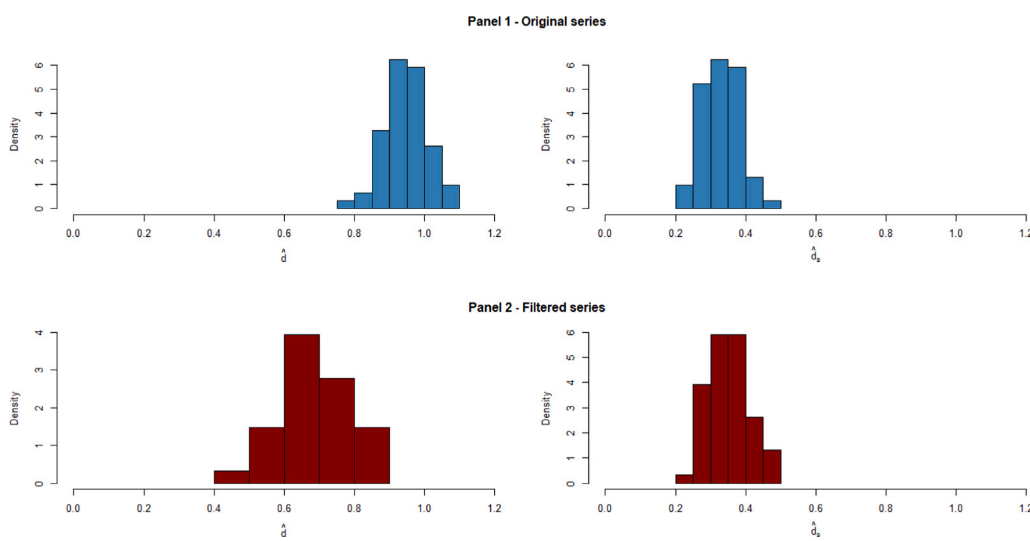


Fig. 6. Histograms of the Long-Memory Parameters for all estimates.

presence of seasonal long memory, as shown in Fig. 5.

On the other hand, almost all (96.7%) estimated non-seasonal parameters from the filtered series lie in the non-stationary mean-reversible interval, with the exception of Kentucky and Missisipi. Fig. 4 shows that the exclusion of S4 components eliminates a large part of the spectral energy in the extreme low frequency bands. Consequently, a significant reduction in values was observed. However, even in this case the combination of seasonal and non-seasonal long memory processes implies $\hat{d} + \hat{d}_s > 1$ for 38 of 61 evaluated series and $\hat{d} + \hat{d}_s > 0.5$ for all series.

The estimated seasonal parameters lie on the non-stationarity threshold for the two types of series. Using the original series, the lowest value $\hat{d} = 0.2405$ was found in the Mississippi series, followed by Kentucky and South Dakota, with 0.4667 and 0.5109 respectively. The highest values were 0.8596, 0.8269 and 0.8213 for the states of California, Nevada, and Maryland, respectively. The $\hat{d}$ estimate for the national series is 0.7489, versus 0.9873 in the unfiltered national series.

The histograms of $\hat{d}$ and $\hat{d}_s$ for the 61 series are shown in Fig. 6. The differences between the parameters estimated using the filtered and original series are clearly seen in the box plot representation presented in Fig. 7. The group of results from the original series shows that the distribution of $\hat{d}$, shows a leptokurtic pattern with values highly concentrated around the mean of 0.9436 with a standard deviation of 0.061. These findings indicate strong non-seasonal persistence in unemployment in states. Using filtered data, the mean and standard deviation of $\hat{d}$ were 0.6815 and 0.09, respectively. Thus, in this case, the empirical distribution of the parameter has higher variability than the distribution based on the original series.

We can draw from this empirical analysis that, for both original and filtered data, the combination of the two long-memory effects implies a non-stationary dynamic in the unemployment series and therefore strong persistence in this variable. Even excluding an important part of the low-frequency components where the non-linearity is located, the joint contribution of the non-seasonal and seasonal long memory effects does not allow us to reject the HH for the national, regional, and state series, with the exception of the District of Columbia.[5]

[5] In order to compare the traditional I(1)-I(0) analysis with our results, we applied the ADF and KPSS unit root tests on the same series analyzed in this study, but seasonally adjusted by the X13 method. Following Enders (1995) algorithm with the ADF test, for all original series, except VT, the unit root hypothesis was not rejected with significance 5%. The inverse result was found for all detrended series. Similar conclusions were obtained using the KPSS test. These results help support our suspicions about seasonal long-memory effects on time series behavior. Tables of results may be obtained upon request.

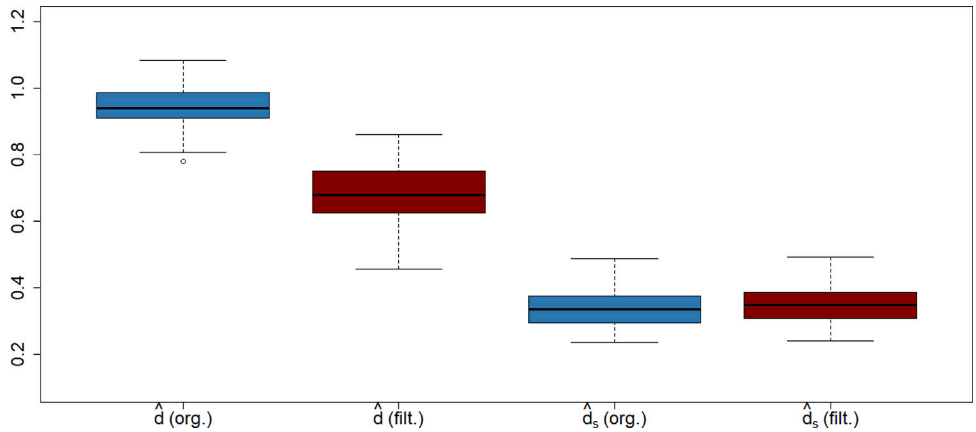


Fig. 7. Boxplot of the Long-Memory Parameters for all estimates.

6. Final remarks

The main objective of our study has been to understand how the conjunction of seasonal and non-seasonal long-memory processes affects the long-term dynamics of US unemployment series. The applied analysis was based on the natural extension of the long-memory concept to the seasonal phenomenon. Using FARISMA models, long-memory parameters were estimated for state, regional, and national geographic areas, providing the basis for the analysis of total persistence in US unemployment.

Previous studies have analyzed persistence in unemployment series based on seasonally adjusted values. Some of them have suggested that US unemployment might follow a long-term mean-reversible dynamic, especially when breaks and regimes are taken into account. However, as in the natural sciences, seasonal phenomena can represent an important source of persistence in economic variables.

Wavelet multiresolution decomposition was used to address potential effects of trends and non-linear changes on series persistence, accessing the specific frequency band where these components are located. The long-memory parameters were estimated with both original and filtered series in order to evaluate the influence of breaks on the persistence of the series. Seasonal long-memory estimations were virtually the same using the filtered and original series. Non-seasonal long-memory estimates based on original data have revealed an extremely persistent non-stationary unemployment dynamic. However, the parameters estimated using filtered series have fallen in the long-term mean-reversible interval, in line with other studies. Surprisingly, however, the average estimated seasonal long-memory parameters lie in the same interval, representing a very high non-stationary level for seasonal persistence. Therefore, even assuming that the wavelet filtering has eliminated a number of low-frequency components, the joint effect of non-seasonal and seasonal long-memory processes still implies strong persistence dynamics in unemployment series across national, regional, and state levels.

The aggregate effects of exogenous shocks from seasonal and non-seasonal dynamics on the labor market are not decreasing within a finite horizon of time, supporting therefore the HH over the NAIRU as the most plausible hypothesis for explaining the unemployment rate behavior in the US. This could open space for sustained countercyclical policies without the immediate threat of inflation acceleration.

In future research, we intend to evaluate the preliminary evidence shown in the results that indicates that states with lower GDPs suffer higher seasonal persistence in their unemployment rates. Based on this analysis further studies can assess the policy implications of hysteresis in a spatially heterogeneous labor market, as well as perform comparative studies with other filtering methods and long memory models. Finally, we intend to extend the analysis to similar models using semiparametric approaches, in which distortions from short-run dynamics could be diminished.

CRediT authorship contribution statement

Guilherme de Oliveira Lima Cagliari Marques: Writing – review & editing, Writing – original draft, Validation, Project administration, Methodology, Investigation, Data curation, Conceptualization. **Mateus Gonzalez de Freitas Pinto:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Data availability

Data will be made available on request.

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