International Journal of Modern Physics D
Vol. 24, No. 6 (2015) 1550038 (9 pages)

DOI: 10.1142/S0218271815500388

Holographic superconductors in Hořava–Lifshitz gravity

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Received 6 February 2015
Revised 23 February 2015
Accepted 24 February 2015
Published 23 March 2015

We consider holographic superconductors related to the Schwarzschild black hole in the low energy limit of Hořava–Lifshitz spacetime. The nonrelativistic electromagnetic and scalar fields are introduced to construct a holographic superconductor model in Hořava–Lifshitz gravity and the results show that the α_2 term plays an important role, modifying the conductivity curve line by means of an attenuation of the conductivity.

Keywords: Hořava–Lifshitz gravity; holographic superconductor.

PACS Number(s): 11.25.Tq, 04.70.Bw, 74.20.−z

1. Introduction

Quantization of gravity is a key issue in modern theoretical gravitational theory, since as a quantum field theory Einstein's general relativity with Lorentz symmetry is unrenormalizable. A renormalizable candidate of quantum gravity has been recently proposed by Hořava,[1] who assumed that the Lorentz symmetry is broken in the ultraviolet, so that the anisotropic scalings between space and time are given by

$$\mathbf{x} \to \ell \mathbf{x}, \quad t \to \ell^z t. \tag{1.1}$$

A power-counting renormalizable gravity theory must satisfy $z \geq 3$ in (3+1)-dimensional spacetime. Such a theory is called the Hořava–Lifshitz gravity.

Hořava–Lifshitz theory has attracted the attention of many theoretical physicists. However, it also faces several problems, in particular arising from the spin-0 graviton. In order to solve such shortcomings, a local U(1) gauge field A is introduced.[2] Meanwhile some recent works prove that the problems from spin-0 graviton can be cancelled in Hořava–Lifshtiz gravity by means of the U(1) gauge field A,[3] and the post-newtonian approximation is also satisfied.[4]

On the other hand, recently, Hartnoll, Herzog and Horowitz considered the AdS/CFT (Anti de Sitter/Conformal Field Theory) correspondence principle to study the strongly correlated condensed matter physics with the gravitational dual. They found a correspondence between the instability of black string and the second-order phase transition from normal to superconductor state.

Subsequently, several authors generalized the idea to investigate holographic superconductors of various black hole solutions.[6] However, the holographic superconductor models are under the framework of spacetime with Lorentz symmetry. In this paper, we build a holographic superconductor model with nonrelativistic matter in static Hořava–Lifshtiz spacetime. The present research could help understanding the stability properties in Hořava–Lifshtiz spacetime.

We plan this paper as follows. In Sec. 2, we generalize the model in general relativity to build a holographic superconductor action in Hořava–Lifshitz spacetime, and focus on discussing the correction from nonrelativistic terms in Sec. 3. Then, in Sec. 4, the conductivity will be calculated and Sec. 5 includes some conclusions and a summary.

2. Superconductor Action in Hořava–Lifshitz Gravity

In this section, we build a holographic superconductor model in Hořava–Lifshitz gravity. We first analyze the holographic superconductor in general relativity. In Ref. 5, Hartnoll, Herzog and Horowitz proposed the model with the Lagrangian density

$$\mathcal{L}_G = -\frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} - |\nabla\bar{\Psi} - iq\mathcal{A}\bar{\Psi}|^2 - V(|\bar{\Psi}|), \tag{2.1}$$

where $\bar{\Psi}$ is the scalar field, $\mathcal{A}_\nu$ is electromagnetic four-potential in general relativity, and $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$. The holographic superconductor model also requests

$$V(|\bar{\Psi}|) = m^2|\bar{\Psi}|^2. \tag{2.2}$$

Rewriting the scalar field $\bar{\Psi}$ as Ψe^{ip},[7] with real Ψ and p, Eq. (2.1) becomes

$$\begin{aligned}\mathcal{L}_G = {} & -\frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} - \nabla_\mu\Psi\nabla^\mu\Psi - V(|\Psi|) \\ & - (q\mathcal{A}_\mu - \partial_\mu p)(q\mathcal{A}^\mu - \partial^\mu p)\Psi^2.\end{aligned} \tag{2.3}$$

We choose the gauge $p = 0$ and get the action of the simplest models in general relativity,

$$S_G = \int d^4x\sqrt{-g^{(4)}}\left(\frac{1}{4}\mathcal{L}_G^E + 2\mathcal{L}_G^S - \mathcal{L}_G^C\right), \tag{2.4}$$

where $g^{(4)}$ is the determinant of the four-dimensional metric $g^{(4)}_{\mu\nu}$, while the electromagnetic, scalar and coupling parts of the Lagrangian are, respectively, given by

$$\begin{aligned}\mathcal{L}_G^E &= \mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu},\\ \mathcal{L}_G^S &= -\frac{1}{2}\nabla_\mu\Psi\nabla^\mu\Psi - \frac{1}{2}V(|\Psi|),\\ \mathcal{L}_G^C &= q^2 g^{(4)\mu\nu}\mathcal{A}_\mu\mathcal{A}_\nu\Psi^2.\end{aligned} \tag{2.5}$$

Next, we generalize the action in Hořava–Lifshitz theory, with the Arnowitt–Deser–Misner metric

$$ds^2 = -N^2dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt). \tag{2.6}$$

The nonrelativistic matter in Hořava–Lifshitz gravity was proposed in Ref. 8 and the Lagrangians of complex scalar and electromagnetic fields are

$$\begin{aligned}\mathcal{L}_H^E &= \frac{2}{N^2}g^{ij}(F_{0i} - F_{ki}N^k)(F_{0j} - F_{lj}N^l)\\ &\quad - F_{ij}F^{ij} - \beta_0 - \beta_1 a_i B^i - \beta_2 B_i B^i - \mathcal{G}_E,\\ \mathcal{L}_H^S &= \frac{1}{2N^2}|\partial_t\Psi - N^i\partial_i\Psi|^2 - \frac{1}{2}|\partial\Psi|^2\\ &\quad - \frac{1}{2}V(|\Psi|) + \alpha_2|\partial\Psi|^2 - \mathcal{H}_S,\end{aligned} \tag{2.7}$$

where $F_{ij} = \partial_j A_i - \partial_i A_j$. The Hořava–Lifshitz higher order corrections $\mathcal{G}_E$ and $\mathcal{H}_S$ are given by

$$\begin{aligned}\mathcal{G}_E &= \beta_3(B_iB^i)^2 + \beta_4(B_iB^i)^3 + \beta_5(\nabla_iB_j)(\nabla^iB^j)\\ &\quad + \beta_6(B_iB^i)(\nabla_kB_j)(\nabla^kB^j)\\ &\quad + \beta_7(\nabla_iB_j)(\nabla^iB^k)(\nabla^jB_j)\\ &\quad + \beta_8(\nabla_i\nabla_jB_k)(\nabla^i\nabla^jB^k),\\ \mathcal{H}_S &= \alpha_3(\Psi\Delta\Psi)^2 + \alpha_4(\Psi\Delta\Psi)^3 + \alpha_5\Psi\Delta^2\Psi\\ &\quad + \alpha_6(\Psi\Delta\Psi)(\Psi\Delta^2\Psi) + \alpha_7\Psi\Delta^3\Psi,\end{aligned} \tag{2.8}$$

where α_i are arbitrary functions of Ψ and β_i arbitrary functions of A_iA^i. Nonetheless, we consider α_i and β_i as constants in this paper, what can be seen as a weak field approximation. Moreover, $B^i = \frac{1}{2}\frac{\epsilon^{ijk}}{\sqrt{g}}F_{jk}$ with the Levi-Civita symbol ϵ^{ijk}. What we want to consider is just the lower order terms of above equations, the higher order terms $\mathcal{G}_E$ and $\mathcal{H}_S$ are ignored in this paper.

Now, let's construct the coupling between electromagnetic field and scalar field. The simplest transformations are given by

$$\begin{aligned}p_i &\to p_i - qA_i,\\ p_0 &\to p_0 - qA_0,\end{aligned} \tag{2.9}$$

where A_i and A_0 satisfy the gauge invariant

$$\begin{aligned} A_i &\to A_i + \nabla_i \chi, \\ A_0 &\to A_0 - \partial_t \chi. \end{aligned} \tag{2.10}$$

Therefore, we make the replacement

$$\begin{aligned} \nabla_i &\to \nabla_i - iqA_i, \\ \partial_0 &\to \partial_0 - iqA_0, \end{aligned} \tag{2.11}$$

and the complex scalar field is rewritten as

$$\begin{aligned} \tilde{\mathcal{L}}_H^S = {} & \frac{1}{2N^2}|\partial_t \Psi - iqA_0\Psi - N^i(\partial_i \Psi - iqA_i\Psi)|^2 \\ & - \left(\frac{1}{2} - \alpha_2\right)|\partial\Psi - iqA_i\Psi|^2 - \frac{1}{2}V(|\Psi|) - \tilde{\mathcal{H}}_S, \end{aligned} \tag{2.12}$$

where $\mathcal{H}_S$ is replaced by $\tilde{\mathcal{H}}_S$ with $\partial_i \to \partial_i - iqA_i$.

Therefore, we build a holographic superconductor model in Hořava–Lifshitz gravity,

$$S_H = \int dtd^3xN\sqrt{g}\left(\frac{1}{4}\mathcal{L}_H^E + 2\tilde{\mathcal{L}}_H^S\right). \tag{2.13}$$

Considering the relationship[8]

$$\begin{aligned} g^{(4)00} &= -\frac{1}{N^2}, \quad g^{(4)0i} = \frac{N^i}{N^2}, \\ g^{(4)ij} &= g^{ij} - \frac{N^iN^j}{N^2}, \end{aligned} \tag{2.14}$$

Eq. (2.13) reduces into Eq. (2.1) when $\alpha_i = \beta_i = 0$.

3. Numerical Results

In this paper, we aim at the holographic superconductor in the low energy limit of Hořava–Lifshitz gravity, while Schwarzschild spacetime is one of the solutions of the low energy limit in Hořava–Lifshitz gravity.[9] Thus, let us consider the Schwarzschild spacetime

$$ds^2 = -N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2), \tag{3.1}$$

with

$$N^2(r) = f(r) = r^2 - \frac{r_0^3}{r}, \tag{3.2}$$

where we have chosen the AdS radius $L = 1$. In the low energy case, we consider the temperature of black hole as given by $T_h = \frac{3r_0}{4\pi}$. We also set the simplest form for the mass parameter $m^2 = -2 + 4\alpha_2$. Therefore, we derive the field equations in this Hořava–Lifshitz spacetime,

$$\Psi'' + \left(\frac{2}{r} + \frac{f'}{f}\right)\Psi' + \left[\frac{2}{f} + \frac{\Phi^2}{(1-2\alpha_2)f^2}\right]\Psi = 0,$$
$$\Phi'' + \frac{2}{r}\Phi' - \frac{2\Psi^2}{f}\Phi = 0. \tag{3.3}$$

At infinity, because $f(r) \to r^2$, we can get the boundary condition for Ψ and Φ

$$\Psi = \frac{\sqrt{2}\langle\mathcal{O}_1\rangle}{r} + \frac{\sqrt{2}\langle\mathcal{O}_2\rangle}{r^2} + \cdots,$$
$$\Phi = \mu - \frac{\rho}{r} + \cdots. \tag{3.4}$$

Substituting the boundary condition into the main equations of the holographic superconductor, we can use the shooting method to calculate Eq. (3.3) numerically. Note that the correction from Hořava–Lifshitz gravity in Eq. (3.3) is α_2, so we focus on studying the effect of α_2.

We set $r_0 = 1$ in the definition of $f(r)$ and the charge of test particle as unit, $q = 1$. The critical temperature T_c in $\mathcal{O}_1$ and $\mathcal{O}_2$ is given in Table 1.

The results show that the effect of α_2 is to increase the critical temperature, while the effect of $\langle\mathcal{O}_2\rangle$ is more obvious than the effect of $\langle\mathcal{O}_1\rangle$.

Then, we draw the transition curve in Fig. 1, and we find that, as the α_2 increases, the curved line gets higher.

Table 1. Critical temperature T_c with $\mathcal{O}_1$ and $\mathcal{O}_2$ respectively.

α_2	$\langle\mathcal{O}_1\rangle$	$\langle\mathcal{O}_2\rangle$
0	$0.2255\rho^{1/2}$	$0.1184\rho^{1/2}$
0.1	$0.2385\rho^{1/2}$	$0.1252\rho^{1/2}$
0.2	$0.2563\rho^{1/2}$	$0.1346\rho^{1/2}$
0.3	$0.2836\rho^{1/2}$	$0.1489\rho^{1/2}$
0.4	$0.3373\rho^{1/2}$	$0.1771\rho^{1/2}$

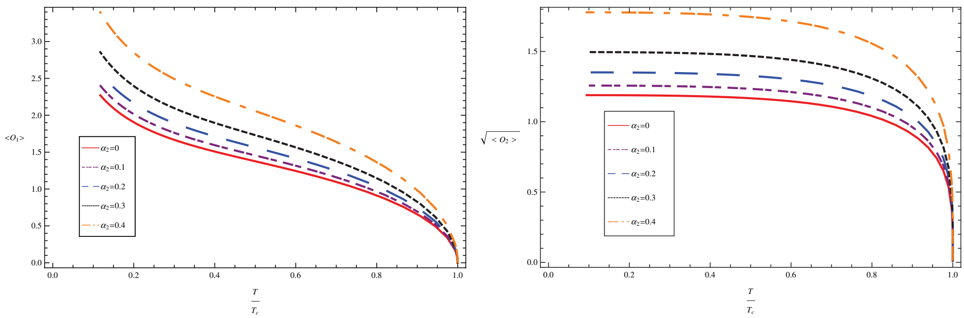

Fig. 1. The condensate as a function of the temperature for the operators $\mathcal{O}_1$ and $\mathcal{O}_2$.

4. Conductivity

Here, we discuss the conductivity. Considering the perturbed Maxwell field $A_i = \delta_i^x e^{-i\omega t} A_x(r)$ while $A_0 = 0$, we obtain the equation

$$A_x''(r) + \frac{8r^3\beta_2 + f'(r)}{2r^4\beta_2 + f(r)} A_x'(r) + \frac{\omega^2 - 2(1-2\alpha_2)f(r)\Psi^2}{(2r^4\beta_2 + f(r))f(r)} A_x(r) = 0, \tag{4.1}$$

but the case in which we are interested is $\beta_2 = 0$, so that the above equation is rewritten as

$$A_x'' + \frac{f'(r)}{f(r)} A_x' + \left[\frac{\omega^2}{f(r)^2} - \frac{2(1-2\alpha_2)\Psi^2}{f(r)}\right] A_x = 0. \tag{4.2}$$

We consider the low energy scale case, when the boundary condition at $r = r_0$ requires

$$A_x(r) \sim f(r)^{-\frac{\omega}{3r_0}}, \tag{4.3}$$

while the behavior of A_x in the asymptotic AdS region is given by

$$A_x(r) = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \cdots, \tag{4.4}$$

and the definition of conductivity is[5]

$$\sigma = -\frac{iA_x^{(1)}}{\omega A_x^{(0)}}. \tag{4.5}$$

Using the above formulas, we draw the relation between $\mathrm{Re}(\sigma)$ and ω in Fig. 2. We find that the real part of conductivity curved lines are lower as α_2 increases. On the other hand, from the relations between $-\mathrm{Im}(\sigma)$ and ω in Fig. 3, we find that the imaginary part of the conductivity lines are higher as α_2 increases.

Finally, we plot $\frac{\Theta(\omega)}{\Theta(0)}$ with small ω (where $\Theta \equiv -\omega\,\mathrm{Im}\,\sigma$), and Fig. 4 shows that the $\frac{\Theta(\omega)}{\Theta(0)}$ goes to a constant as ω is enough small, but the curves are lower as α_2 increases. The values of $\Theta(0)$ are given in Table 2.

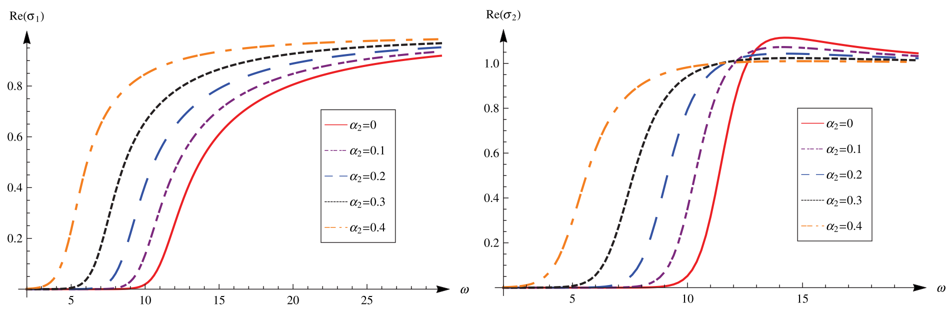

Fig. 2. The real part of conductivity for the two operators $\mathcal{O}_1$ and $\mathcal{O}_2$.

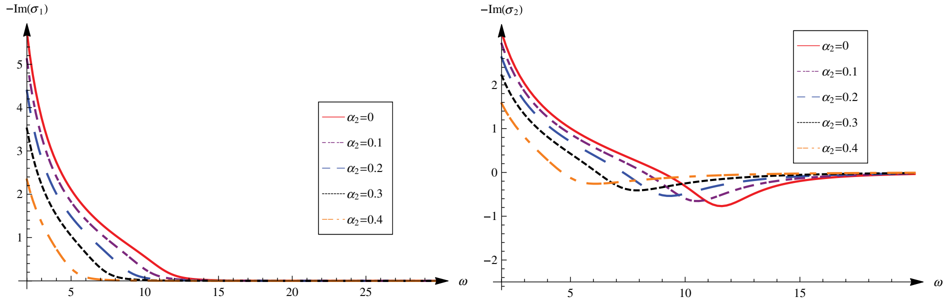

Fig. 3. The imaginary part of conductivity for the two operators $\mathcal{O}_1$ and $\mathcal{O}_2$.

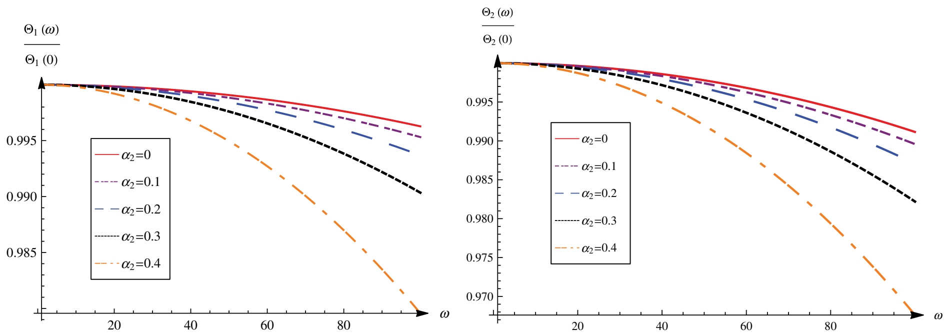

Fig. 4. $\frac{\Theta(\omega)}{\Theta(0)}$ for the two operators $\mathcal{O}_1$ and $\mathcal{O}_2$ in rotating spacetime, where $\Theta(\omega) \equiv -\omega\,\mathrm{Im}\,\sigma$.

Table 2. $\Theta_1(0)$ and $\Theta_2(0)$ with α_2 as $r_0\Psi(r_0) = 5.8$.

α_2	$\Theta_1(0)$	$\Theta_2(0)$
0	11.6798	6.6225
0.1	10.4294	6.1340
0.2	9.0088	5.5426
0.3	7.3211	4.7786
0.4	5.1131	3.6468

5. Conclusion

We built the simplest holographic superconductor model in the low energy limit of Hořava–Lifshitz gravity and studied the property of transition near the critical temperature T_c in static Hořava–Lifshitz spacetime. We found that the correction comes from the $\alpha_2\Psi\Delta\Psi$ term in (3+1)-dimensional static spacetime. The study also shows that the correction of $\langle\mathcal{O}_2\rangle$ is more obvious than the effect on $\langle\mathcal{O}_1\rangle$. From Eq. (3.3), if we make the transformation $\frac{\Phi}{\sqrt{1-2\alpha_2}} \to \tilde{\Phi}$, we find the equations of superconductors will give the same results for any constant α_2, but α_2 can modify the conductivity in Sec. 4.

What we considered is just the simplest case, but it is possible that the correction comes from $\mathcal{H}_S$ and $\mathcal{G}_E$ as the black hole is charged or rotated. On the other hand, our work proves that it should introduce a $U(1)$ symmetrical field to avoid the difficulties from spin-0 graviton in Hořava–Lifshitz theory,[3,4] so it is more meaningful to research the holographic superconductor in general case with a $U(1)$ symmetric field.

The effect of the constant α_2 is to decrease the value of conductivity and lower the superconductor effect (see Table 2). However, we did not see the insulator effect, a case in which we have to consider large values of α_2, which then might be field dependent.

Acknowledgments

This work is supported in part by FAPESP No. 2012/08934-0 (EA, KL); CNPq (EA, KL); DOE Grant, DE-FG02-10ER41692 (AW); Ciência Sem Fronteiras, No. 004/2013 - DRI/CAPES (AW) and NSFC No. 11375153 (AW).

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