The 11th International Conference Inverse Problems: Modeling and Simulation

May 26 - June 01, 2024, Malta

ABSTRACTS

Editors
Alemdar Hasanov Hasanoglu, Roman Novikov,
Eric Todd Quinto, Otmar Scherzer and Cristiana Sebu

Managing Editor
Burhan Pektaş

THE EURASIAN ASSOCIATION ON INVERSE PROBLEMS

ABSTRACTS

of the 11th International Conference

Inverse Problems: Modeling and Simulation

held on 26 May – June 01, 2024, Malta

Editors:
Alemdar Hasanov Hasanoglu, Roman Novikov, Eric Todd Quinto,
Otmar Scherzer and Cristiana Sebu

Managing Editor: Burhan Pektaş

The Conference is organized by The Eurasian Association on Inverse Problems.

ABSTRACTS
of the 11th International Conference
"Inverse Problems: Modeling and Simulation"
held on 26 May – 01 June 2024, Malta

ABSTRACTS / edited by
A. Hasanov Hasanoglu, R. Novikov, E.T. Quinto, O. Scherzer, C. Sebu

Managing Editor: Burhan Pektaş

ISBN: 978-975-04-0905-9r

M3: Inverse and Control Problems in Vibrating Structures:

Theory, Applications and Computational Aspects

M4: New Trends in Regularization Theory

M5: Inverse Problems in Hybrid Imaging Modalities

M6: Topological Derivatives in Inverse Problems and Shape Optimization

M7: Bayesian, Variational, and Optimization Techniques for Inverse Problems in Stochastic Partial Differential Equations

M8: Inverse Problems with Data-Driven Methods and Deep Learning

M9: Inverse Problems for Fractioal Equations

M10: Inverse and Control Problems for Evolution Equations and Variational Inequalities

M11: Image Restoration Under Poisson Nois

M12: Applied Inverse Problems and Partial Differential Equations

M13: Applications of Rich Tomography

M14: Recent Advances in Inverse Scattering Theory and Applications les

M15: Imaging with Waves in Complex Media

M16: Inverse Problem Theory for Innovation of Detection Methods

M17: In Memoriam: Professor Pierre Célestin Sabatier (1935-2023)

Preface

The Tenth International Conference Inverse Problems: Modeling and Simulation is held during 26 May – 1 June 2024, in the Paradise Bay Resort Hotel, Malta, under the auspices of The Eurasian Association on Inverse Problems (EAIP).

As with the previous nine conferences in this series, the main objective of this conference is to be multidisciplinary and international, by bringing together scientists working on various topics of inverse problems in diverse areas, such as mathematics, statistics, engineering, economics, finance, physics, chemistry, biology, medicine, meteorology and computer science.

The conference brings together more than 200 internationally known experts on inverse problems, and exhibitors from over 30 countries world-wide.

The conference program includes 4 plenary lectures as well as invited lectures given in the framework of 17 minisymposiums. The topics of the conference go through advances in inverse and ill-posed problems over the past 20 years as well as emerging methods in data science.

We hope that this book of abstracts will be useful to those who are interested in inverse and ill-posed problems: theory, numerical implementations and applications.

PLENARY SPEAKERS

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Numerical inverse wave problems for quantitative viscoelastic reconstructions

Florian Faucher[1], Otmar Scherzer[2], Ha Pham[1], Hélène Barucq[1]
[1]) Team Makutu, Inria, Université de Pau et des Pays de l'Adour, TotalEnergies, CNRS, France.
florian.faucher@inria.fr
[2]) Faculty of Mathematics, University of Vienna, Oskar-Morgenstern-Platz, Austria.

We consider the inverse problem for the quantitative reconstruction of the material properties that characterize a viscoelastic medium from wave measurements. Waves in such media display loss of energy (attenuation), and their propagation is mathematically modeled with linear viscoelastic equations, [1]. The time-harmonic formulation of the wave problem is considered to facilitate the investigation of the different attenuation models which are frequency-dependent. For the numerical discretization of the viscoelastic wave equations composed of the equation of motion and constitutive law, the hybridizable discontinuous Galerkin (HDG, [2]) method is employed and allows to reduce the computational cost. For the stabilization of HDG numerical traces, we further construct a hybridized Godunov-upwind flux for anisotropic elasticity, [3]. Then, the nonlinear inversion follows a minimization algorithm to iteratively update the elastic moduli. We carry out reconstructions with attenuation model uncertainty ([4]), that is, using different models of attenuation between the modeling and inversion. We illustrate with ultrasound imaging experiment of synthetic breast sample reconstruction.

References

[1] J. M. Carcione, Wave fields in real media: Wave propagation in anisotropic, anelastic, porous and electromagnetic media, *Elsevier* (2007).

[2] B. Cockburn, J. Gopalakrishnan and R. Lazarov, Unified hybridization of discontinuous Galerkin, mixed, and continuous Galerkin methods for second order elliptic problems, *SIAM Journal on Numerical Analysis* 47 (2009).

[3] H. Pham, F. Faucher and H. Barucq, Numerical investigation of stabilization in the Hybridizable Discontinuous Galerkin method for linear anisotropic elastic equation, *arXiv preprint arXiv:2403.02862* (2024).

[4] F. Faucher and O. Scherzer, Quantitative inverse problem in visco-acoustic media under attenuation model uncertainty, *Journal of Computational Physics* 472 (2023).

New Series Representations and Reconstruction Techniques in Inverse Coefficient Problems

Vladislav V. Kravchenko
Department of Mathematics
Center for Research and Advanced Studies of the National Polytechnic Institute
Queretaro, Mexico
Email: vkravchenko@math.cinvestav.edu.mx

Functional series representations for solutions of Sturm-Liouville differential equations encounter numerous applications in solving direct spectral, scattering and other types of problems. The recently discovered Neumann series of Bessel functions representations, useful for problems on a finite interval, as well as special power series representations for Jost solutions, well suited for infinite intervals (see the monographs [1], [2] and references therein), possess remarkable convergence properties, such as the uniform convergence with respect to $\sqrt{\lambda} \in \mathbb{R}$, where λ is the spectral parameter of the equation. Additionally, the knowledge of the first coefficient of any such series representation is sufficient for recovering the Sturm-Liouville equation. These properties make the representations especially convenient for solving inverse coefficient problems. The corresponding approach is the main object of the talk. We show that difficult and numerically challenging inverse problems are reduced to systems of linear algebraic equations for the coefficients of the representations, and the Sturm-Liouville equation is recovered from the first coefficient.

In particular, we discuss a general inverse coefficient problem for a Sturm-Liouville equation with an unknown complex valued coefficient. Special cases of the problem include the recovery of the potential from a Weyl function, the inverse two-spectra Sturm-Liouville problem, the inverse scattering problem and the inverse transmission eigenvalues problem among others. A variety of inverse coefficient problems for partial differential equations and for quantum graphs are also reduced to the considered problem. The approach leads to a simple and efficient numerical algorithm, that is illustrated by numerical examples.

References

[1] V.V. Kravchenko, Direct and inverse Sturm-Liouville problems: A method of solution, Birkhäuser, Cham, 2020.

[2] A.N. Karapetyants and V.V. Kravchenko, Methods of mathematical physics: classical and modern, Birkhäuser, Cham, 2022.

Correlation-informed ordered dictionary learning for imaging in complex media

Miguel Moscosor[1], Alexei Novikov[2], George Papanicolaou[3] and Chrysoula Tsogka[4]
[1]) Department of Mathematics, Universidad Carlos III de Madrid, Madrid, Spain
moscoso@math.uc3m.es
[2]) Department of Mathematics, Pennsylvania State University, PA, USA
novikov@psu.edu
[3]) Department of Mathematics, Stanford University, Stanford, CA 94305, USA
papanicolaou@stanford.edu
[4]) Department of Applied Mathematics, University of California Merced, CA, USA
ctsogka@ucmerced.edu

We propose an approach for imaging in scattering media when large and diverse data sets are available. We use algorithms that come from computer science and statistics rather than traditional imaging techniques. Our approach has two steps. Using a dictionary learning algorithm, the first step estimates the Green's function vectors as unordered columns of a sensing matrix. The array data comes from many sparse sets of sources whose location and strength are not known to us. In the second step the columns of the estimated sensing matrix are ordered so that imaging is possible, using Multi-Dimensional Scaling with connectivity information derived from cross correlations. Through simulation experiments, we show that the proposed approach is able to provide images in complex media.

References

[1] Miguel Moscoso, Alexei Novikov, George Papanicolaou, and Chrysoula Tsogka, Correlation-informed ordered dictionary learning for imaging in complex media Proc. Nat. Acad. Sci, vol. 121(11), e2314697121, 2024.

[2] Novikov, A., White, S. Dictionary learning for the almost-linear sparsity regime. Algorithmic learning theory, 39 pp. Proc. Mach. Learn. Res. (2023).

The 11th International Conference “Inverse Problems: Modeling and Simulation”
26th May - 1st June 2024, Malta

How Artificial Intelligence is Changing Bayesian Inversion in Imaging

Pierre Weiss[1],[2],[3]
Institut de Recherche en Informatique de Toulouse[1])
pierre.weiss@cnrs.fr
Centre de Biologie Intégrative, Toulouse [2])
Institut de Mathématiques de Toulouse [3])

The need to reconstruct images from incomplete information is widespread in applications (e.g. super-resolution microscopy, magnetic resonance imaging, tomography...). In this talk, I plan to review some core ideas [1, 2] that are currently being explored to learn and use efficient image priors to recover the missing information. In particular, I will show how standard Bayesian estimators (MAP, MMSE, posterior sampling), which have often yielded disappointing results with simple priors (e.g. total variation, sparsity), now shine when computed with carefully trained neural networks. They can provide effective and faithful solutions for inverse problems [3, 4] when trained with appropriate forward models. I will also describe some pitfalls [6, 5] and applications in MRI and microscopy [7, 8]. This shows that while these techniques can considerably improve the reconstruction quality, a fine application specific analysis is still needed.

References

[1] R. Laumont, V. Bortoli, A. Almansa, J. Delon, A. Durmus, M. Pereyra, Bayesian Plug&Play Priors: When Langevin meets Tweedie, SIAM Imaging Science, 2022

[2] J. Song, C. Meng & A. Vahdat, Denoising diffusion models: A Generative Learning Big Bang, CVPR 2023

[3] A. Gossard & P. Weiss, Training Adaptive Reconstruction Networks for Blind Inverse Problems. SIAM Imaging science, 2024

[4] V. Debarnot & P. Weiss, Deep-Blur: Blind Identification and Deblurring with CNN. Biological Imaging, 2024

[5] N. Munier, E. Soubies & P. Weiss, Identifying the Non Identifiable. submitted, 2024

[6] M.H. Nguyen, E. Pauwels & P. Weiss, Blind Deconvolution of Natural Images. Learned Priors Favor Blurry Images. submitted, 2024

[7] A. Gossard, F. de Gournay & P. Weiss, Bayesian Optimization of Sampling Densities, MELBA 2023

[8] C. Lazarus, P. Ciuciu & P. Weiss, Sparkling: Variable Density Filling Curves for MRI. MRM, 2019

MINISYMPOSIUMS

MINISYMPOSIUM
M1. The Radon Transform: Progress and Challenges

The mini-symposium is dedicated to the anniversary of an outstanding expert in inverse problems, Professor of Mathematics at Stockholm University Jan Boman

Organizers:
Pavel Kurasov, Stockholm University, Sweden, kurasov@math.su.se
Roman Novikov, Ecole Polytechnique, France, roman.novikov@polytechnique.edu
Eric Todd Quinto, Tufts University, USA, Todd.Quinto@tufts.edu

The main theme of this mini-symposium is the Radon transform, its generalizations, and applications. The mini-symposium will encompass history, progress and challenges in this domain. We will bring together specialists in pure and applied aspects of the mathematics of tomography and integral geometry.

The mini-symposium celebrates an outstanding expert in the mathematics of tomography and integral geometry, Prof. Jan Boman, on the occasion of his 90th birthday. Prof. Boman has done seminal work in integral geometry that has direct applications to tomography and is beautiful and elegant in its own right.

Domains with algebraic Radon transforms

Mark Agranovsky
Bar-Ilan University
agranovs@math.biu.ac.il

V. Arnold introduced the notion of algebraically integrable domains in $\mathbb{R}^n$, whose cut-off volume functions, evaluating the volumes of portions of the domain on both sides of the secant hyperplane, algebraically depend on the hyperplane. The well known Arnold's problem, inspired by Newton's Lemma about ovals in the plane, is to describe all (smoothly bounded) algebraically integrable domains in Euclidean spaces. In full generality, this problem is still open. In last decade, a variation of Arnold's problem where similar question is addressed the related, section volume, function, i.e., the Radon transform of the characteristic function of a domain, was studied and a series of significant results were obtained in this direction (see survey [1]) The talk will be devoted to recent further results on characterization of domains in terms of the algebraic types of their Radon transforms.

References

[1] M. Agranovsky, J. Boman, A. Koldobsky, V. Vassiliev and V. Yaskin , Algebraically integrable domains and related properties of the Radon transform, in Harmonic Analysis and Convexity, Advances in Analysis and Geometry, v.9, de Gruyter, (2023), 1-36.

The 11th International Conference “Inverse Problems: Modeling and Simulation”
26th May - 1st June 2024, Malta

A medley of range conditions for divergent-beam transforms

R. Clackdoyle[1], H. Nguyen[1], R. Huber[1], J. Lesaint[1], M. Defrise[2], L. Desbat[1]

[1]) TIMC Laboratory, Univ. Grenoble Alpes, France
rolf.clackdoyle@univ-grenoble-alpes.fr

[2]) Vrije Universiteit Brussel, Belgium

The divergent-beam transform $\mathcal{D}$ of a suitable object function f defined on a given compact subset of $\mathbb{R}^n$ ($n = 2$ or $n = 3$ in our context) is given by

$$\mathcal{D}f(a_\lambda, \gamma) = g(a_\lambda, \gamma) = \int_0^\infty f(a_\lambda + t\gamma)\, dt \qquad \gamma \in S^{n-1} \tag{1}$$

where $a_\lambda \in \mathbb{R}^n$ is the vertex point, which we will refer to here as the (x-ray) source location. The source trajectory is parametrized by the real variable λ. For $n = 3$ we call $g(a_\lambda, \cdot)$ a cone-beam projection, and for $n = 2$, a fan-beam projection. Range conditions on the operator $\mathcal{D}$ find practical applications in CT imaging situations. Typically a few necessary conditions are invoked and these are called “data consistency conditions” (DCCs) in the physical context.

Apart from [1], divergent-beam range conditions have usually been presented as reparametrizations of well-known conditions for parallel projection operators, essentially re-writing the parallel conditions using divergent-beam variables. In [1], however, conditions on a finite number of divergent projections were given, provided no three source points were collinear and provided the line connecting any two source points did not traverse the support of the object function f.

Over the last 10 years, new divergent-beam DCCs have been presented in the literature, with a variety of applications. Here we discuss some of these new conditions: the epipolar conditions based heavily on Grangeat’s result; fan-beam conditions for source points along a straight line; conditions for cone-beam projections along a circular trajectory; and, conditions on *truncated* fan-beam projections along a circular trajectory [2].

We describe the notion of DCCs in “projection form”, as being more suitable for applications purposes [3].

References

[1] D.V. Finch, D.C. Solmon. Sums of homogeneous functions and the range of the divergent beam X-ray transform. Num.Func.Anal.Opt. 5:363–419, 1983.

[2] R.Clackdoyle, L. Desbat. Data consistency conditions for truncated fanbeam and parallel projections. Med.Phys. 42:831–845, 2015.

[3] R.Clackdoyle. Necessary and Sufficient Consistency Conditions for Fanbeam Projections Along a Line. IEEE Trans.Nucl.Sci. 60:1560–1569, 2013.

Iterative inversion of the tensor momentum x-ray transform

Aleksander Denisiuk
University of Warmia and Mazury in Olsztyn, Poland
denisiuk@matman.uwm.edu.pl

The momentum x-ray transform maps a symmetric tensor field f of order m in $\mathbb{R}^n$ to its integrals over lines $at + b$ with the weight t^k, $k = 0, \dots, m$. In this article we propose an iterative approach to the reconstruction problem. Namely, a new concept of partial momentum transforms is introduced. These transforms are intermediaries between the momentum and the component-wise x-ray transforms. Moreover, one can use a differential operator to construct a sequence of partial momentum transforms that starts with the momentum and finishes with the component-wise transform. The latter can be inverted with the standard component-wise backprojection. The proposed method was also applied to obtain an analog of the Plancherel formula as well as the stability estimates in the Sobolev spaces that are valid for any m.

The talk is based on the article [1].

References

[1] Aleksander Denisiuk, Iterative inversion of the tensor momentum x-ray transform, Inverse Problems, 39(10)(2023), 105002.

The Range of Projection Pair Operators

Richard Huber[1], Laurent Desbat[1] and Rolf Clackdoyle[1]
[1]) Université Grenoble Alpes, CNRS, Grenoble INP, TIMC, France
richard-martin.huber@univ-grenoble-alpes.fr

Tomographic techniques in radiology are vital in modern medicine, allowing doctors to observe patients' interior features. Modeling the measurement process are (tomographic) projection operators, the most well-known being the Radon transform [1]. These are weighted line integral operators of a rather general nature whose concrete form depends on the specific type of tomography considered. These line integrals are typically grouped together into projections – collections of curves bijectively covering the imaged domain – that relate to specific steps in the measurement process. Knowledge of the range of projection operators has found applications in tomographic approaches, as it allows the verification of data plausibility without the need for reconstruction. Such knowledge can be expressed via data consistency conditions describing an inherent information overlap between projections. In parallel-beam single photon emission computed tomography (SPECT), such approaches were used to correct for noise and align data [2, 3]. However, for more modern pinhole SPECT cameras – modeled via the exponential fanbeam transform $\mathcal{E}_\mu$ – such range conditions remain unknown. To that end, we describe a general method for identifying pairwise data consistency conditions (characterizing the range of projection pair operators involving only two projections), in particular, finding that there is at most one condition for any projection pair. Moreover, we find that the range of any sparse projection operator (consisting of finitely many projections) is closed. An interesting consequence is that information never overlaps between truncated projections. Applying this theory to the exponential fanbeam transform $\mathcal{E}_\mu$, we find that any projection pair data is in the range of $\mathcal{E}_\mu$ – i.e., can be created via the measurement process – meaning there is no inherent information overlap between two pinhole SPECT measurements.

References

[1] F. Natterer. The Mathematics of Computerized Tomography. Society for Industrial and Applied Mathematics, Philadelphia, 2001.

[2] R. G. Wells and R. Clackdoyle. Feasibility of attenuation map alignment in pinhole cardiac SPECT using exponential data consistency conditions. Medical Physics, 48(9), 4955-4965, 2021.

[3] A. Robert, D. Sarrut, A. Etxebeste, J. M. Létang, and S. Rit. Ability of exponential data consistency conditions to detect motion in spect despite other physical effects. In 17th International Meeting on Fully 3D Image Reconstruction in Radiology and Nuclear Medicine, pages 337–340, 2023.

Range description for the free space wave operator

Peter Kuchment[1], Leonid Kunyansky[2]
[1]) Texas A&M University, College Station, TX, USA
[2]) University of Arizona, Tucson, AZ, USA, leonk@math.arizona.edu

The following inverse problem arises in thermo- and photoacoustic tomography, and in several other coupled physics imaging modalities. One considers the Cauchy problem for the whole space wave equation in $\mathbb{R}^d$, $d = 2, 3, ...$:

$$u_{tt}(t,x) = \Delta u(t,x), \quad (t,x) \in (0,\infty) \times \mathbb{R}^d, \tag{1}$$

$$u(0,x) = f(x), \qquad u_t(0,x) = 0, \tag{2}$$

where $f(x)$ is finitely supported inside the unit ball $|x| < 1$. The forward problems consists in finding the trace g of the solution u on the unit sphere $\theta \in S := \mathbb{S}^{d-1}$:

$$g(t,\theta) := u(t,\theta), \quad (t,\theta) \in Z_T, \tag{3}$$

where T is the observation time and $Z_T := (0,T] \times S$ is the cylinder supporting the data. The inverse problem then consists of recovering f from g.

In conjunction with the above inverse problem one may want to answer the following question: given a function g defined on Z_T, what are the necessary and sufficient conditions for g to be expressed as a trace of solution u to the problem (1)-(3)? In other words, we would like to describe the range of the operator mapping f to the trace of u. Since solution of (1)-(3) can be expressed in terms of the spherical means of f, this is equivalent to characterization of the range of the **spherical means operator**. The latter problem has been solved in [2, 3]; results for the wave operator have been obtained in [1].

All of the above results, however, require the measurement time T to be at least 2. On the other hand, it is known that a stable solution of the inverse source problem (1)-(3) is possible if data $g(t,\theta)$ are given on a twice shorter cylinder $Z_1 := (0,1] \times S$. This implies that the data on time interval $[1,\infty)$ are implicitly determined by the data on interval $(0,1]$, and the latter interval should be used for a range description.

In my talk I will present new results on describing the range of the **wave** and the **spherical means operators**, for data given on the cylinder Z_1.

References

[1] D. Finch and Rakesh. The range of the spherical mean value operator for functions supported in a ball. *Inverse Problems*, 22(3) (2006) 923–938.

[2] M. Agranovsky, P. Kuchment, and E. T. Quinto. Range descriptions for the spherical mean Radon transform. *J. Funct. Anal.*, 248(2) (2007() 344–386.

[3] M. Agranovsky, D. Finch, and P. Kuchment. Range conditions for a spherical mean transform. *Inverse Probl. Imaging*, 3(3) (2009) 373–382.

PSWF-Radon approach to super-resolution in multidimensional Fourier analysis

Mikhail Isaev[1], Roman Novikov[2]
[1]) School of Mathematics, Monash University, Clayton, VIC, Australia
[2]) CMAP, CNRS, Ecole polytechnique, Palaiseau, France
novikov@cmap.polytechnique.fr

Reconstructing a compactly supported function from its band-limited Fourier transform is a classical problem of the Fourier analysis. It also arises in studies of inverse scattering problems in the Born approximation. The natural approach that extends the given Fourier data with zero has a well-known diffraction limit: small details are blurred. On the other hand, reconstructions based on analytical extensions theoretically achieve super-resolution, but are severely unstable in practice. In this talk, we present a new approach to super-resolution in this inverse problem that leads to a stable reconstruction significantly beyond the diffraction limit. This approach combines the Radon transform theory with the theory of prolate spheroidal wave functions (PSWFs) offering potential breakthroughs in signal processing and imaging. In particular, we illustrate this approach in the framework of reconstructions from band-limited Hankel transform of integer or half-integer order.

This talk is based on the works [1] – [3].

References

[1] M. Isaev and R.G. Novikov, Reconstruction from the Fourier transform on the ball via prolate spheroidal wave functions, Journal de Mathématiques Pures et Appliquées, 163, 318–333 (2022)

[2] M. Isaev, R.G. Novikov, G.V. Sabinin, Numerical reconstruction from the Fourier transform on the ball using prolate spheroidal wave functions, Inverse Problems 38, 105002 (2022)

[3] M. Isaev, R.G. Novikov, R.V. Zaytsev, PSWF-Radon approach to reconstruction from band-limited Hankel transform, in preparation

Surface of revolution Radon transforms with centers on generalized surfaces in $\mathbb{R}^n$

James Webber,[1] Sean Holman,[2] and
Speaker: Eric Todd Quinto[3]
[1]) Department of Oncology and Gynecology, Brigham and Women's Hospital, Boston, MA, USA
jwebber5@bwh.harvard.edu
[2]) Department of Mathematics, University of Manchester, Manchester, UK
sean.holman@manchester.ac.uk
[3]) Department of Mathematics, Tufts University, Medford, MA, USA
todd.quinto@tufts.edu

We present a novel analysis of a Radon transform, R, which maps an L^2 function of compact support to its integrals over smooth surfaces of revolution with centers on an embedded hypersurface in $\mathbb{R}^n$. Using microlocal analysis, we derive necessary and sufficient conditions relating to R for the Bolker condition to hold, which has implications regarding the existence and location of image artifacts. We present a general inversion framework based on Volterra equation theory and known results on the spherical Radon transform, and we prove injectivity results for R. Several applications of our theory are discussed in the context of, Compton Scatter Tomography (CST) and Ultrasound Reflection Tomography (URT). In addition, using the proposed inversion framework, we show how our microlocal theory is reflected in reconstructions, and present simulated image reconstructions of image phantoms with added noise.

Analytic double fibration transforms

Mikko Salo
University of Jyväskylä, Finland
mikko.j.salo@jyu.fi

We study integral transforms associated with a double fibration. This class includes many transforms encountered in tomography problems, such as (magnetic) geodesic X-ray transforms, generalized Radon transforms, and light ray transforms. If the underlying curve or surface family is real-analytic and a Bolker condition holds, we show that certain analytic singularities of a function can be determined from its transform which is treated as an analytic elliptic Fourier integral operator. This work is based on fundamental ideas introduced by Jan Boman and Todd Quinto in 1987, and leads to local and global uniqueness results and Helgason type support theorems for these transforms.

This is a joint work with Marco Mazzucchelli (ENS Lyon) and Leo Tzou (Amsterdam).

MINISYMPOSIUM
M2. Microlocal Analysis and Waves: Progress and Challenges

Organizers:
Ronny Ramlau, RICAM, Linz, Austria, ronny.ramlau@jku.at
Eric Todd Quinto, Tufts University, USA, Todd.Quinto@tufts.edu
Kamran Sadiq, RICAM, Austria, kamran.sadiq@ricam.oeaw.ac.at
Otmar Scherzer, University of Vienna, Austria, otmar.scherzer@univie.ac.at

In this mini-symposium, we will explore tomographic problems in wave imaging, including Sonar, Radar, seismic imaging, optical tomography, diffraction tomography, and related PDEs. We will bring together young and established researchers from the inverse problems community to present their recent work and to foster discussion among participants.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Inverse scattering problems with internal sources

Yakun Dong[1], Otmar Scherzer[1,2,3], Kamran Sadiq[2]
[1]) Faculty of Mathematics, University of Vienna. Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria
Yakun.Dong@univie.ac.at
[2])Johann Radon Institute for Computational and Applied Mathematics (RICAM), Altenbergerstraße 69, 4040 Linz, Austria
[3]) Christian Doppler Laboratory for Mathematical Modeling and Simulation of Next Generations of Ultrasound Devices (MaMSi), Oskar-Morgenstern-Platz 1, 1090 Vienna, Austria

In this talk, we consider the inverse scattering problems with internal sources[1]. We show that it is possible to reconstruct the dielectric susceptibility of the 2-dimensional and 3-dimensional inhomogeneous medium with subwavelength resolution. The reconstruction algorithm is based on reproducing kernel Hilbert space[1], which is numerically stable and computationally efficient. Applications to photoactivated localization microscopy[3] are described.

References

[1] Gilbert, Anna C et al. "Imaging from the inside out: inverse scattering with photoactivated internal sources." Optics letters vol. 43,12 (2018): 3005-3008.

[2] Pereverzyev, Sergei. "An Introduction to Artificial Intelligence Based on Reproducing Kernel Hilbert Spaces." Birkhäuser, 2022.

[3] Betzig, Eric et al. "Imaging intracellular fluorescent proteins at nanometer resolution." Science (New York, N.Y.) vol. 313,5793 (2006): 1642-5.

Ultrasound Aberration Correction for Layered Media

Simon Hackl[1,2], Simon Hubmer[1], Ronny Ramlau[1,3]

[1]) Johann Radon Institute for Computational and Applied Mathematics, Altenbergerstraße 69, A-4040 Linz, Austria.
[2]) Corresponding author, e-mail: simon.hackl@ricam.oeaw.ac.at
[3]) Johannes Kepler University Linz, Institute of Industrial Mathematics, Altenbergerstraße 69, A-4040 Linz, Austria.

Ultrasound diagnostics is an important, non-invasive examination method in modern medicine. Focused ultrasound enables a deep look inside the human body in real time without causing harm. A crucial assumption in the theory of focused ultrasound imaging is a constant sound speed in the observed medium.

After a short introduction to the currently used one layered focusing algorithm, we will discuss our multi-layer-adapted focusing method. It generalizes the idea of delaying the ultrasound waves' emission for differences in times of flight, but calculates the time of flight in a more precise way using a geometrical acoustics approach. Since in some applications not all necessary medium parameters are known, we present a simple parameter reconstruction method in the second part of the talk. It measures the different times of flight the sound waves take along different paths inside the medium and reconstructs the medium parameters from these measurements.

The effectiveness of both proposed methods is demonstrated through numerical simulation using the k-Wave toolbox for Matlab.

The 11th International Conference ”Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Boundary recovery for the time-harmonic anisotropic Maxwell's equations

Sean Holman

Department of Mathematics, University of Manchester, United Kingdom
sean.holman@manchester.ac.uk

This work concerns inverse boundary value problems for the time-harmonic anisotropic Maxwell's equations on differential 1-forms. We formulate the boundary value problem on a 3-dimensional compact and simply connected Riemannian manifold M with boundary endowed with a Riemannian metric g. In this context, the impedance and admittance maps are pseudodifferential operators and we compute their principal symbols as well as the principal symbols of some related operators. We then show consequences for the recovery of anisotropic electric parameters at the boundary.

References

[1] S. Holman and V. Torega, Boundary Recovery of Anisotropic Electromagnetic Parameters for the Time Harmonic Maxwell's Equations, accepted to appear in *Journal of Geometric Analysis*. Preprint: arXiv:2303.06688.

Vertices classification with topological gradient

Bochra Mejri[1], Peter Gangl[1], Otmar Scherzer[1,2,3]
[1]) Johann Radon Institute for Computational and Applied Mathematics (RICAM)
bochra.mejri@ricam.oeaw.ac.at
[2]) Faculty of Mathematics, University of Vienna
[3]) Christian Doppler Laboratory for Mathematical Modeling and Simulation of Next Generations of Ultrasound Devices (MaMSi), Vienna, Austria

Feature detection consists of transforming an image's intensities into a reconstructed object in terms of vertices, edges, or regions. Vertices contain important information on visual scenes, e.g. 'L-corner' represents objects' corners, T-junction' represents overlapping objects. Our work aims at the detection and localization of vertices of 3d objects in 2d images. The proposed approach is based on the calculation of the second-order of a topological derivative of Mumford-Shah functional with respect to a perturbation of a pixel configuration. Our one-shot detection process provides information on the detected vertices, i.e. their slope, disposition, and number of edges. The novelty in this work is the numerical computation of the weak polarisation matrix for different pixel configurations covering all the vertices classification. Numerical tests are performed to point out the efficiency of the developed approach.

Object reconstruction in diffraction tomography using focused beams

Clemens Kirisits[1], Noemi Naujoks[1], Otmar Scherzer[1]
[1]) Computational Science Center
University of Vienna
noemi.naujoks@univie.ac.at

Diffraction tomography is an inverse scattering technique used to reconstruct the spatial distribution of the material properties of a weakly scattering object. The object is exposed to radiation, typically light or ultrasound, and the scattered waves induced from different incident field angles are recorded. In conventional diffraction tomography, the incident wave is assumed to be a monochromatic plane wave, an unrealistic simplification in practical imaging scenarios. In this talk, we extend conventional diffraction tomography by introducing the concept of Gaussian fields of incidence. Herewith, focused beams are modeled, allowing customization by adjusting the beam waist and focal depth. We present a new forward model that incorporates a Gaussian field of incidence and extends the classical Fourier diffraction theorem to the use of this incident field. This focused illumination approach enables new measurement geometries for data generation, on the basis of which we develop reconstruction methods. These are then comprehensively evaluated through numerical experiments.

Microlocal analysis using shearlets

Swaraj Paul
Department of Mathematics, Faculty of Engineering and Technology, SRM Institute of Science and technology, Kattankulathur-603 203, Tamilnadu, India.
swarajp@srmist.edu.in

Abstract: The wavefront set measures a detailed characterization of the singularities present in a distribution. The classical tool for the analysis of wavefront set is the Fourier transform. In this talk we use continuous shearlet transform (whose underlying systems are discrete affine systems, provide an optimally sparse approximation of a cartoon-type image with discontinuity/singularity across a C2 smooth curve) to characterise different types of wavefront set. We also extend continuous shearlet transform in Sobolev space. Finally we discussed some intertwining property of sherlet transform with Radon transform.

Motion Detection in Diffraction Tomography

Michael Quellmalz[1], Peter Elbau[2], Otmar Scherzer[2,3,4],
Gabriele Steidl[1]
[1]) TU Berlin, Germany
quellmalz@math.tu-berlin.de
[2]) University of Vienna, Austria
[3]) Johann Radon Institute for Computational and Applied Mathematics (RICAM), Linz, Austria
[4]) Christian Doppler Laboratory for Mathematical Modeling and Simulation of Next Generations of Ultrasound Devices (MaMSi), Vienna, Austria

We study the mathematical imaging problem of optical diffraction tomography for the scenario of a rigid particle rotating in a trap created by acoustic or optical forces. Under the influence of the inhomogeneous forces, the particle carries out a time-dependent smooth, but irregular motion. The rotation axis is not fixed, but continuously undergoes some variations, and the rotation angles are not equally spaced, which is in contrast to standard tomographic reconstruction assumptions. Once the time-dependent motion parameters are known, the particle's scattering potential can be reconstructed based on the Fourier diffraction theorem, considering it is compatible with making the first order Born or Rytov approximation.

The aim of this presentation is twofold: We first need to detect the motion parameters from the tomographic data by detecting common circles in the Fourier-transformed data [1]. This can be seen as analogue to method of common lines from cryogenic electron microscopy (cryo-EM), which is based on the assumption that the assumption that the light travels along straight lines. Then we can reconstruct the scattering potential of the object utilizing non-uniform Fourier methods [2].

References

[1] M. Quellmalz, P. Elbau, O. Scherzer and G. Steidl, Motion detection in diffraction tomography by common circle methods, Mathematics of Computation, 93(346)(2024), 747–784.

[2] C. Kirisits, M. Quellmalz, M. Ritsch-Marte, E. Setterqvist, O. Scherzer and G. Steidl, Fourier Reconstruction for diffraction tomography of an object rotated into arbitrary orientations, Inverse Problems, 37(11)(2021).

On a source reconstruction in an absorbing and scattering planar domain from partial boundary data

Kamran Sadiq
Johann Radon Institute of Computational and Applied Mathematics (RICAM)
kamran.sadiq@ricam.oeaw.ac.at

This talk concerns an inverse source problem for the linearized Boltzmann equation in two dimensions. The medium is assumed known. The outgoing radiation is measured on an arc of the boundary. For scattering kernels dependent on the angle of scattering, we show that a source can be recovered in the convex hull of the measuring arc. The method, specific to two dimensional domains, relies on Bukgheim's theory of A-analytic maps and it is joint work with A. Tamasan (UCF) and H. Fujiwara (Kyoto U).

Kernel description of the momentum ray transform

Suman Kumar Sahoo[1]
[1])ETh Zürich
susahoo@ethz.ch

Let I^k, for every non-negative integers k, denote the weighted ray transform acting on compactly supported symmetric m tensor fields defined as:

$$I^k f(x,\xi) := \int_{-\infty}^{\infty} f_{i_1\cdots i_m}(x+t\xi), \xi_{i_1}\cdots\xi_{i_m}\, dt. \tag{1}$$

For $k = 0, I^0$ is the classical ray transform acting on symmetric tensor fields. In this talk, we will discuss kernel of the momentum ray transform using the generalized Saint Venant operators for the Schwartz class of symmetric tensor fields. This talk is based on joint works with Rohit Kumar Mishra, IIT Gandhinagar, India [1, 2].

References

[1] Injectivity and range description of integral moment transforms over m-tensor fields in $\mathbb{R}^n$. SIAM J. Math. Anal. 53 (2021), no. 1, 253–278. 44A12.

[2] he generalized Saint Venant operator and integral moment transforms. Proc. Amer. Math. Soc. 151 (2023), no. 1, 189–199.

Single-stage approach for estimating optical parameters in spectral quantitative photoacoustic tomography

Tanja Tarvainen, Miika Suhonen, Aki Pulkkinen
Department of Technical Physics, University of Eastern Finland, Kuopio, Finland
tanja.tarvainen@uef.fi

We study the inverse problem of spectral quantitative photoacoustic tomography (QPAT) where the aim is to estimate the concentrations of light absorbing molecules (chromophores) utilising the photoacoustic effect induced by an externally introduced light pulse and the resulting photoacoustic wave that can be measured using ultrasound sensors. Conventionally, the inverse problem in QPAT can be seen to consist of two sequentially solved inverse problems. In the first inverse problem, an initial pressure distribution is estimated from the photoacoustic time-series, and in the second inverse problem, the optical parameters are estimated from the initial pressure. In this work, we consider a spectral problem and estimate chromophore concentrations in a single-stage directly from the photoacoustic time-series. The forward model is constructed by combining the models of light and ultrasound propagation and by representing the optical absorption and scattering with their spectral models. The methodology is evaluated with numerical simulations where concentrations of different chromophores, scattering parameters, and the Grüneisen parameter are estimated in different full-view and limited-view imaging scenarios.

Quantification of Optical Parameters in Optical Coherence Tomography

Leopold Veselka[1]

[1]) Faculty of Mathematics, University of Vienna
leopold.veselka@univie.ac.at

Optical coherence tomography(OCT) is a non-invasive imaging technique providing high-resolution images of the inner structure of biological tissues that are based on the interferometric measurement of backscattered light.

In this talk we discuss the related inverse scattering problem which focuses on the extraction of optical properties from experimental OCT measurement data.

In the first part of the talk, a model for the backscattered light that is based on a Gaussian beam model for the illuminating laser light in combination with a medium showing a multi-layered structure is presented.

On the basis of this model, we discuss the corresponding inverse problem in the second part of this talk. The reconstruction of parameters, hereby, is executed via the minimization of a least-squares misfit functional. We discuss the existence of minima and verify the applicability of the method with numerical experiments for simulated and experimental data.

This talk is based on joint work with P. Elbau, L. Mindrinos, L. Krainz and W. Drexler.

An Inverse Problems Approach to Pulse Wave Analysis in the Human Brain

Lukas Weissinger[1], Simon Hubmer[1],
Ronny Ramlau[1,2], Henning U. Voss[3]

[1]) Johann Radon Institute Linz, Altenbergerstraße 69, A-4040 Linz, Austria,
(lukas.weissinger@ricam.oeaw.ac.at)
[2]) Johannes Kepler University Linz, Institute of Industrial Mathematics,
Altenbergerstraße 69, A-4040 Linz, Austria,
[3]) Cornell University - College of Human Ecology, Cornell MRI Facility, Martha Van Rensselaer Hall, Ithaca NY 14870, United States of America

Cardiac pulsations in the human brain have received recent interest due to their possible role in the pathogenesis of neurodegenerative diseases. Further interest stems from their possible application as an endogenous signal source that can be utilized for brain imaging in general. The (pulse-)wave describing the blood flow velocity along an intracranial artery consists of a forward (anterograde) and a backward (retrograde, reflected) part, but measurements of this wave usually consist of a superposition of these components. In this talk, we provide a mathematical framework for the inverse problem of estimating the pulse wave velocity, as well as the forward and backward component of the pulse wave separately from MRI measurements on the middle cerebral artery. After a mathematical analysis of this problem, we consider possible reconstruction approaches, and derive an alternate direction approach for its solution. The resulting methods provide estimates for anterograde/retrograde wave forms and the pulse wave velocity under specified assumptions on a cerebrovascular model system. The proposed method's applicability is demonstrated through numerical experiments using simulation data.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Interface identification and sound speed reconstruction in layered media: Advancing from synthetic to authentic Ultrasound data

Huidong Yang[1,2]

[1]) University of Vienna
huidong.yang@univie.ac.at
[2]) CD laboratory MaMSi

We present a Full Waveform Inversion (FWI) approach designed to simultaneously identify interfaces and reconstruct sound speeds within layered media. This method hinges on synthetic data produced from our forward simulation. By reformulating the inverse problem into a Partial Differential Equation (PDE) constrained optimization, governed by the wave equation, we utilize a gradient descent algorithm to drive optimization. Our results vividly illustrate the efficiency and accuracy of the algorithm in achieving both interface identification and sound speed reconstruction.

In the subsequent section, we will present ongoing progress on peforming our inverse algorithm, leveraging experimental measurements provided by our collaborators at the Medical University of Vienna. This will encompass validating the forward model, retrieving the source, and reconstructing the geometrical and physical parameters of such layered media using authentic data.

This is joint work with Peter Elbau, Michael Figl, Otmar Scherzer, and Lukas Zalka.

Inverting the local transverse and mixed ray transforms

Gunther Uhlmann[1], Jian Zhai[2]
[1]) University of Washington and the Hong Kong University of Science and Technology
gunther@math.washington.edu
[2]) Fudan University
jianzhai@fudan.edu.cn

We consider the transverse and mixed ray transforms on a compact Riemannian manifold with smooth boundary. We show that the transverse ray transform and the mixed ray transform are invertible, up to natural obstructions, near a boundary point. When the manifold admits a strictly convex function, this local invertibility result leads to a global result by a layer stripping argument.

MINISYMPOSIUM
M3. Inverse and Control Problems in Vibrating Structures: Theory, Applications and Computational Aspects

Organizers:
Onur Baysal, University of Malta, Malta, onur.baysal@um.edu.mt
Alexandre Kawano, University of Sao Paulo, Brazil, akawano@usp.br
Antonino Morassi, University of Udine, Italy, antonino.morassi@uniud.it

The mini symposium will provide a forum for researchers to discuss recent developments in this important field of structural dynamics and engineering. Inverse problems, which involve determining the properties or characteristics of a vibrating structure from measured data, and control problems, which involve modifying the vibration characteristics of a structure, are both crucial for understanding and optimizing the behavior of structures under dynamic loads. This forum will feature presentations on various aspects of inverse and control problems, including theoretical developments, computational resources, and practical applications. The symposium is relevant to researchers in applied mathematics, aerospace engineering, civil engineering, mechanical engineering, and other related fields.

A new numerical approach for the determination of shear force in Atomic Force Microscopy

Onur Baysal
Department of Mathematics
Faculty of Science University of Malta
Msida MSD 2080, Malta.
Email: onur.baysal@um.edu.mt

In this study, a new numerical method based on a comprehensive mathematical model of the cone-shaped cantilever tip-sample interaction in Atomic Force Microscopy (AFM) is considered. The importance of such AFMs with cone-shaped cantilevers comes from the ability to provide high-resolution information at the nanoscale. It is a rapidly increasing tool used in a wide range of scientific and industrial fields.

The interaction of the cone-shaped cantilever tip with the surface of the specimen (sample) is modeled by the following damped Euler-Bernoulli beam equation

$$\rho_A(x)u_{tt} + \mu(x)u_t + (r(x)u_{xx} + \kappa(x)u_{xxt})_{xx} = 0, (x,t) \in (0,\ell) \times (0,T),$$

with the homogeneous initial, $u(x,0) = 0$, $u_t(x,0) = 0$ and the following boundary conditions,

$$\begin{aligned} &u(0,t) = u_x(0,t) = 0, (r(x)u_{xx}(x,t) + \kappa(x)u_{xxt})_{x=\ell} = M(t), \\ &(-(r(x)u_{xx} + \kappa(x)u_{xxt})_x)_{x=\ell} = g(t). \end{aligned}$$

Here $M(t) := 2h\cos\theta\, g(t)/\pi$ is the moment generated by the transverse shear force $g(t)$. Unlike many other models in the literature, a variable coefficient equation is considered here, which can include all physical factors. In addition, we propose an inversion algorithm for the reconstruction of an unknown shear force in the AFM that is compatible with this general structure. The measured displacement $\nu(t) := u(\ell,t)$ is used as measured data for the determination of the shear force $g(t)$. The least square functional $J(F) = \frac{1}{2}\|u(\ell,\cdot) - \nu\|^2_{L^2(0,T)}$ is introduced and an explicit gradient formula for the Fréchet derivative of the cost functional is derived via the weak solution of the adjoint problem. This allows us to construct a gradient-based numerical algorithm for the reconstruction of the right-end boundary force from measured output with random noise.

These first step computational experiments, which we believe will form the basis for more advanced engineering applications in the future, indicate that the proposed algorithm is efficient and reliable.

This is the joint work with Alemdar Hasanov and Alexandre Kawano.

On the use of a roving harmonic load to locate cracks in shear deformable beam

Salvatore Caddemi, Francesco Cannizzaro, Ilaria Fiore, Ivo Caliò

Department of Civil and Architectural Engineering, University of Catania

[salvatore.caddemi,francesco.cannizzaro,ilaria.fiore, ivo.calio]@unict.it

The free vibration governing equations of the Timoshenko-Ehrenfest beam model in presence of multiple cracks have been recently formulated by the authors [1]. The equations are formulated over a unique integration domain by embedding the discontinuities arising from the crack localised flexibility model by means of suitable distributional terms. The proposed additional terms allow closed form integration, analogously to intact beams, by making use of the theory of distributions.

In this work the above governing equations are enriched with an external concentrated harmonic load and the closed form expressions of the Green’s functions are presented for the case of multi-cracked beam where both localised bending and shear flexibilities have been accounted for.

In order to devise a procedure to detect the presence of cracks, the authors have also proposed the use of a roving mass with rotatory inertia limited to the Euler-Bernoulli beam [2]. Natural frequencies survey by monitoring a single cross section are proved to be characterised by the appearance of abrupt changes as the roving mass crosses a cracked section due to the concentrated moment arising from the inertial force distribution.

Analogously, the presented Green’s functions, particularised for the case of two out-of-phase harmonic concentrated loads (denoted as doublet load) producing a concentrated external pulsating moment, are here exploited for crack detection purposes. Precisely, when a roving doublet load crosses a cracked section, transversal displacement and rotation response functions at an arbitrary monitored cross section undergo abrupt discontinuities.

References

[1] Cannizzaro, F., Fiore, I., Caddemi, S., Caliò, I.; “The exact distributional model for free vibrations of shear-bending multi-cracked Timoshenko beams”, European Journal of Mechanics/A, 101, 105039, 2023.

[2] Cannizzaro, F., De Los Rios, J., Caddemi, S., Caliò, I., Ilanko, S.; “On the use of a roving body with rotary inertia to locate cracks in beams”, Journal of Sound and Vibration, 425, 275-300, 2018.

Simultaneous Identification of Spatial Load and External Heat Source in Thermoelastic plate

Anjuna Dileep[1], Alemdar Hasanov[2] and Sakthivel Kumarasamy[3]

[1,3]) Indian Institute of Space Science and Technology, Department of Mathematics, Trivandrum- 695 547, India,
dileepanjuna10@gmail.com, sakthivel@iist.ac.in
[2]) Department of Mathematics, Kocaeli University, Turkey, Şehit Ekrem Mah., Ayazma Villalari, 22, Bahčecik, Kocaeli - 41030, Turkey,
alemdar.hasanoglu@gmail.com

In this talk, we discuss the inverse problem of simultaneously identifying the mechanical load $F(x,t)$ and heat source $G(x,t)$ in structurally damped thermoelastic plate equations describing a homogeneous and elastically as well as thermally isotropic plate, from the vertical displacement $u_T(x) = u(x,T)$ measured at the final time $T > 0$. We establish the well-posedness of the initial boundary value problem and corresponding adjoint problem by using Galerkin's approximation method. The inverse problem is reformulated as a minimization problem for the Tikhonov functional using the Tikhonov regularization method. We prove that the regularized Tikhonov functional admits a unique solution in the naturally defined set of admissible sources. Furthermore, the Fréchet differentiability of this functional is proved, and an explicit gradient formula is derived through the weak solution of the corresponding adjoint problem. An upper bound for the final time $T > 0$ is established to derive the stability estimate for the inverse problem by invoking a first-order necessary optimality condition for the minimization problem. This stability result also gives rise to the uniqueness of the solution to the regularized inverse problem. The results presented in this talk help to analyze the influence of thermal and mechanical loading that results in materials deflection, which, in turn, is vital in terms of physical applications.

References

[1] W. Chen, Cauchy problem for thermoelastic plate equations with different damping mechanisms, *Communications in Mathematical Sciences,* **18** (2020), 429-457.

[2] J. E. Lagnese, *Boundary Stabilization of Thin Plates,* SIAM, Philadelphia, PA, 1989.

Nonlinearity imaging in the frequency domain via multiharmonic expansions

Barbara Kaltenbacher[1]
[1]) University of Klagenfurt, Austria
barbara.kaltenbacher@aau.at

This is joint work with Benjamin Rainer, University of Klagenfurt and Bill Rundell, Texas A&M University.
We consider an undetermined coefficient inverse problem for a nonlinear partial differential equation occuring in high intensity ultrasound propagation as used in acoustic tomography. In particular, we investigate the recovery of the nonlinearity coefficient commonly labeled as B/A in the literature, which is part of a space dependent coefficient κ in the Westervelt equation governing nonlinear acoustics. Corresponding to the typical measurement setup, the overposed data consists of time trace measurements on some zero or one dimensional set Σ representing the receiving transducer array. In this talk, we will show some recent results pertaining to the formulation of this problem in frequency domain [1, 3] and numerical reconstruction of piecewise constant coefficients in two space dimensions [2].

References

[1] Barbara Kaltenbacher. Periodic solutions and multiharmonic expansions for the Westervelt equation. *Evolution Equations and Control Theory EECT*, 10:229–247, 2021.

[2] Barbara Kaltenbacher and William Rundell. Nonlinearity parameter imaging in the frequency domain. *Inverse Problems and Imaging*, 2023. see also arXiv:2303.09796 [math.NA].

[3] Benjamin Rainer. The nonlinear periodic Westervelt equation. Existence, uniqueness and numerical approximation of solutions. Master Thesis, University of Klagenfurt, 2024.

Identification of out-of-plane loads over Timoshenko beams

Alexandre Kawano
University of São Paulo
akawano@usp.br

In engineering structural systems, the beam element is virtually ubiquitous, due to its high efficiency in resisting and transmitting moments across the structure. There are two main beam theories, the one known as the Euler-Bernoulli's and the other known as the Timoshenko, or Timoshenko – Ehrenfest beam theory. The second beam theory is intended to be an improvement to the other, taking into account rotational inertia and shear effects. As the slenderness ratio decreases, and as vibration frequencies increase, the Timoshenko – Ehrenfest beam theory offers better agreement with experimental results.

We examine the beam equation formulated by the Timoshenko Beam Theory (TBT). This equation characterizes the displacement field, denoted as u, and the cross-sectional rotation, denoted as ϕ, for an elastic beam. The Timoshenko Beam Theory accounts for inertial rotation and shear effects, aspects not considered in the Euler-Bernoulli theory. In the case it has length $L > 0$, positive material properties G, E, ρ and positive geometric properties A, I, k, it is written as

$$\begin{cases} \rho A \dfrac{\partial^2 u}{\partial t^2} - \dfrac{\partial}{\partial \xi}\left[AG\kappa\left(\frac{\partial u}{\partial \xi} - \varphi\right)\right] = g(t)\ f_1, \text{ in }]0, +\infty[\times]0, L[, \\ \rho I \dfrac{\partial^2 \varphi}{\partial t^2} - \dfrac{\partial}{\partial \xi}\left(EI\frac{\partial \varphi}{\partial \xi}\right) - \kappa AG\left(\frac{\partial u}{\partial \xi} - \varphi\right) = g(t)\ f_2, \text{ in }]0, +\infty[\times]0, L[, \end{cases}$$

when the beam is excited with a load of the form $g(t)(f_1(x), f_2(x))$. We prove that, with known $g \in \mathcal{C}^1$, and given $\omega \subset]0, L[$, there is $T > 0$ such that the knowledge of the set $\{u(t, x) \mid t \in]0, T] \times \Omega\}$ is enough for the determination of the spatial loading $(f_1, f_2) \in \mathrm{H}^{-1}(\Omega) \times \mathrm{H}^{-1}(\Omega)$.

References

[1] A. Hasanov, A. Kawano and O. Baysal, Reconstruction of shear force in Atomic Force Microscopy from measured displacement of the cone-shaped cantilever tip, Mathematics in Engineering, 6(1)(2024), 137–154.

[2] A. Hasanov, A. and A. Kawano, Identification of an unknown spatial load distribution in a vibrating Euler-Bernoulli beam from limited measured data, Inverse Problems, 32(5)(2016), 1–31.

Simple Regularizing Effect of the Kelvin-Voigt Damping in the Determination of Shear Force of the Euler-Bernoulli Beam

K. Sakthivel
Department of Mathematics Indian Institute of Space Science and Technology(IIST)
Trivandrum, Kerala.
sakthivel@iist.ac.in

In this talk, we will explore the inverse source problems of uniquely recovering the shear force in the Euler-Bernoulli beam with Kelvin-Voigt and external dampings from the boundary measurements of deflection and bending moments. We obtain a more regular solution of the direct problem required to solve the boundary inverse problem with less regularity on the flexural rigidity and boundary data through the regularizing effect of the Kelvin-Voigt damping. The inverse problems are solved with less regular admissible input data by appealing to the weak solutionbased method of the quasi-solution approach. By invoking variational methods for the regularized Tikhonov functional, we discuss the Lipschitz-type stability estimates for the unknown transverse shear force in terms of the given measurements by a feasible condition on the Kelvin-Voigt damping coefficient.

References

[1] K. Sakthivel, A. Hasanov and D. Anjuna, Inverse Problems of identifying the unknown transverse shear force in the Euler-Bernoulli beam with Kelvin-Voigt damping, J. Inverse and Ill-posed Problems, https://doi.org/10.1515/jiip-2022-0053.

[2] D. Anjuna, A. Hasanov, K. Sakthivel and C. Sebu, On unique determination of an unknown spatial load in damped Euler-Bernoulli beam equation from final time output, J. Inverse and Ill-posed Problems, 30 (2022), 581-593.

[3] H. T. Banks and D. J. Inman, On damping mechanisms in beams, J. Applied Mech., 58(1991), 716-723.

[4] D. Anjuna, K. Sakthivel and A. Hasanov, Determination of a spatial load in a damped KirchhoffLove plate equation from final time measured data, Inverse Problems, 38 (2022), 015009.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Resonator-Based Mass Detection in Nanostructures

Antonino Morassi[1]
[1] University of Udine, DPIA, Via Cotonificio 114, 33100 Udine, Italy
antonino.morassi@uniud.it

Nanosensors are gathering attention in the last two decades due to the necessity of measuring physical and chemical properties in industrial and biological systems in the sub-micron scale. One of the most representative examples of the advantages of down-scaling in sensing systems is the nanomechanical resonator, which consists of a vibrating nanobeam with remarkable performance in detecting small adherent masses. The mass sensing principle is essentially based on using the resonant frequency shifts caused by additional unknown mass attached on the sensor surface as data to reconstruct the mass change [1].

A key feature of nanostructures is the need of considering size effects when modelling their mechanical response, as their dimensions become comparable to characteristic microstructural distances. To this end, several generalized continuum models succeed in capturing the size effects at the nanoscale, and are currently used to model the mechanical response of the material. From the mathematical point of view, mass identification from few resonant frequencies falls into the class of inverse eigenvalue problems with finite data. These inverse problems typically suffer from the non-uniqueness of the solution and require special strategies for their quantitative analysis.

In this work we present some results obtained for one-dimensional nanosensors undergoing unknown mass distributions [2] and their possible extension to two-dimensional systems such as membranes and nanoplates [3]. Under the main assumption that the mass change is a small perturbation of the reference mass, it is shown that resonant frequency shifts induced by the mass change contain quantitative information on certain generalized Fourier coefficients of the unknown mass variation. In support of the theory, an extensive series of numerical simulations and an experimental application are presented and discussed.

References

[1] M.S. Hanay, S.I. Kelber, C.D. O'Connell, P. Mulvaney, J.E. Sader, M.L. Roukes, Inertial imaging with nanomechanical systems, Nature Nanotechnology, 10 (2015), 339.

[2] M. Dilena, M. Fedele Dell'Oste, J. Fernández-Sáez, A. Morassi, R. Zaera, Hearing distributed mass in nanobeam resonators, International Journal of Solids and Structures, 193-194 (2020) 1-25.

[3] A. Kawano, A. Morassi, R. Zaera, Reconstruction of a general mass density in a rectangular membrane from finite eigenvalue data, Mechanical Systems and Signal Processing 187 (2023) Paper 109901.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

A paraxial approach for the inverse problem of vibroacoustic imaging in frequency domain

Teresa Rauscher[1,*], Barbara Kaltenbacher[1]
[1]) Department of Mathematics, University of Klagenfurt, Klagenfurt, Austria
* Email: teresa.rauscher@aau.at

Ultrasound imaging is a well-established method that uses high frequency sound waves to view inside the body. At higher frequencies better resolutions can be achieved but the sound propagation is affected by scattering and stronger attenuation. To avoid these drawbacks vibroacoustography was developed. In comparison to conventional ultrasound imaging in vibroacoustic imaging two high frequency beams that show a strongly preferred direction of propagation are sent into the medium and focused to a point. There, they interact non-linearly which excites a low frequency field. We make use of a paraxial approach to arrive at a system of PDEs that involve space dependent parameters whose reconstruction can be used for imaging purposes. In this talk, I want to introduce you to this topic, deal with the modeling and talk about the inverse problem for vibroacoustography.

References

[1] M. Fatemi and J.F. Greenleaf, Vibroacoustography: An imaging modality based on ultrasound-stimulated acoustic emission. *Proceedings of the National Academy of Sciences*, **96**(12), 6603–6608, (1999).

[2] B. Kaltenbacher, On the inverse problem of vibro-acoustography. *Meccanica* **58**, 1061–1072 (2023). https://doi.org/10.1007/s11012-022-01485-w

[3] B. Kaltenbacher and T. Rauscher, Simultaneous Reconstruction of Speed of Sound and Nonlinearity Parameter in a Paraxial Model of Vibro-Acoustography in Frequency Domain, *Computational Methods in Applied Mathematics* (2024). https://doi.org/10.1515/cmam-2023-0076

Inverse problems for a wave equation with interface

Alberto Mercado-Saucedo
Universidad Técnica Federico Santa María
alberto.mercado@usm.cl

We are interested in the stability of the inverse problem of recovering a space-dependent coefficient by observing a trace of the corresponding solution on part of the boundary, in a wave equation with discontinuous principal coefficient.

The considered hyperbolic system in a bounded domain $\Omega \subset \mathbb{R}^n$ with a smooth boundary $\partial\Omega$ is given by

$$\begin{cases} u_{tt} - \operatorname{div}(\gamma(x)\nabla u) + p(x)u = h & (x,t) \in \Omega \times (0,T) \\ u = g & (x,t) \in \partial\Omega \times (0,T) \\ u(0) = u_0, \quad u_t(0) = u_1 & x \in \Omega. \end{cases} \tag{1}$$

where $p \in L^\infty(\Omega)$ and the main coefficient $\gamma \in L^\infty(\Omega)$ is regular in each one of the subdomains Ω_1 and Ω_2. Such subdomains compose a partition of Ω, which means that

$$\Omega_1 \cap \Omega_2 = \emptyset \quad \text{and} \quad \overline{\Omega} = \overline{\Omega}_1 \cup \overline{\Omega}_2. \tag{2}$$

The set $\Gamma_* := \overline{\Omega_1} \cap \overline{\Omega_2}$ is called the interface.

Using Carleman estimates and the Bukhgeim–Klibanov method, in [2] it was solved the case where one subdomain is an convex inner domain. This result was recently generalized in [1], where is is constructed special weight functions adapted to the case of an interface that is not necessarily the boundary of a convex set, in the two-dimensional case.

In [3], it is studied the case of a flat interface, and it is provided a precise estimate of the minimum required time for the stability of the inverse problem, as a function of the velocity change and domain size. The main tools are Carleman estimates with a particular weight function adapted to the interface geometry.

In this talk we shall give an overview of these results and other ongoing investigations dealing with non-regular coefficients with more generality.

References

[1] Lucie Baudouin, Pamela Godoy, and Alberto Mercado, Carleman estimates for the wave equation in heterogeneous media with non-convex interface, J. Differential Equations 311 (2022), 1–28.

[2] Lucie Baudouin, Alberto Mercado, and Axel Osses, A global Carleman estimate in a transmission wave equation and application to a one-measurement inverse problem, Inverse Problems 23 (2007), no. 1, 257–278.

[3] Alberto Mercado, An inverse problem for a transmission wave equation with a flat interface in $\mathbb{R}^n$. Inverse Problems, *accepted manuscript* 2024.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Minnaert Frequency and Simultaneous Reconstruction of the Density, Bulk and Source in the Time-Domain Wave Equation

Mourad Sini[1]

[1]) RICAM, Science park 2, Altenbergerstrass 69, A-4040, Linz, Austria
mourad.sini@oeaw.ac.at

We deal with the inverse problem of reconstructing acoustic material properties or/and external sources for the time-domain acoustic wave model. The traditional measurements consist of repeated active (or passive) interrogations, as the Dirichlet-Neumann map, or point sources with source points varying outside of the domain of interest. It is reported in the existing literature, that based on such measurements, one can recover one of the three parameters: mass density, bulk modulus or the external source term. In this work, we first inject isolated small-scales bubbles into the region of interest and then measure the generated pressure field at a *single point* outside, or at the boundary, of this region. Then we repeat such measurements by moving the bubble to scan the region of interest. Using such measurements, we show that

1. If either the mass density or the bulk modulus is known then we can simultaneously reconstruct the other one and the source term.
2. If the source term is known at the initial time, precisely we assume to know its first non vanishing time-derivative, at the initial time, then we reconstruct simultaneously the three parameters, namely the mass density, the bulk modulus and the source function.

This is a joint work with Soumen Senapati. These results are detailed in [3]. They are extensions of those derived in [1] and[2] where we used fluid droplets instead of gas bubbles.

References

[1] M. Sini and H. Wang, The inverse source problem for the wave equation revisited: a new approach. SIAM J. Math. Anal. 54 (2022), no. 5, 5160-5181.

[2] S. Senapati, M. Sini and H. Wang, Recovering both the wave speed and the source function in a time-domain wave equation by injecting contrasting droplets. Discrete and Continuous Dynamical Systems. doi: 10.3934/dcds.2023151

[3] S. Senapati, M. Sini, Minnaert Frequency and Simultaneous Reconstruction of the Density, Bulk and Source in the Time-Domain Wave Equation. arXiv:2311.08114

A sixth order elliptic equation for nanoplates. Holder stability for a Winkler type coefficient

G. Alessandrini[1], A. Morassi[2], E. Rosset[3], E.Sincich[4], S.Vessella[5]
[1]) Università di Trieste
alessang@units.it
[2]) Università di Udine
antonino.morassi@uniud.it
[3]) Università di Trieste
rossedi@units.it
[4]) Università di Trieste
esincich@units.it
[5]) Università di Firenze
sergio.vessella@unifi.it

In this talk, I will discuss the inverse problem of determining a Winkler coefficient of a nanoplate rested on an elastic foundation and clamped at the boundary. The springy effect of the foundation, described by the Winkler coefficient, is recovered by performing interior measurement of the transversal deflection due to the exertion of a load force on the nanoplate. We prove a global Hölder stability by a single measurement. This is based on a joint work with G. Alessandrini, A. Morassi, E. Rosset, S. Vessella.

Topology Optimization of Dynamic Structures via Singular Value Decomposition of the Transfer Matrix

Paolo Venini[1]
[1]) Department of Civil Engineering and Architecture, University of Pavia, Italy
paolo.venini@unipv.it

A new approach unifying static and dynamic topology optimization was recently proposed in [1]. The key idea is to reformulate topology optimization as a minimum norm problem that is based on the Singular Valued Decomposition, see [2], of the transfer matrix of the system.

This focus of this contribution shall be on 3D structures as well as on a proper imposition of the stress constraint. Numerical examples shall be presented to validate the theoretical derivations.

References

[1] P. Venini and M. Pingaro, Static and dynamic topology optimization: an innovative unifying approach, Structural and Multidisciplinary Optimization, 66(85)(2023), https://doi.org/10.1007/s00158-023-03528-6

[2] G. Strang, Linear Algebra and Learning from Data, Cambridge Press, Wellesley, 2019.

MINISYMPOSIUM
M4. New Trends in Regularization Theory

Organizers:

Stefan Kindermann, Johannes Kepler University Linz, Austria, kindermann@indmath.uni-linz.ac.at
Robert Plato, University Siegen, Germany, plato@mathematik.uni-siegen.de
Bernd Hofmann, Technische Universität Chemnitz, Germany, bernd.hofmann@mathematik.tu-chemnitz.de

Due to the ill-posedness of most linear and nonlinear inverse problems one needs regularization techniques for their stable approximate solution. The regularization theory is well developed for problems in Hilbert spaces, and during the last years many results have also been achieved for problems in Banach spaces. However, there are still many challenging questions, and permanently new classes of inverse problems occur, motivated by applications from natural sciences, engineering and finance.

We want to bring together experts and young researches working in this field to discuss about new results in the analysis and numerics of inverse and ill-posed problems.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Regularization with Non-linear Frame Filtering

Andrea Ebner[1], Matthias Schwab[1], Markus Haltmeier[1]
[1]) Department of Mathematics, University of Innsbruck, Technikerstrasse 13, 6020 Innsbruck, Austria
{andrea.ebner, matthias.schwab, markus.haltmeier}@uibk.ac.at

Inverse problems are core issues in several scientific areas, including signal processing and medical imaging. As inverse problems typically suffer from instability with respect to data perturbations, a variety of regularization techniques have been proposed. In particular, the use of filtered diagonal frame decompositions has proven to be effective and computationally efficient. However, the existing convergence analysis is limited to linear filters and a handful of non-linear filters, like soft thresholding [4]. In this talk, we analyze the filtered diagonal frame decomposition with general nonlinear filters. In particular, our results generalize SVD-based spectral filtering from linear to non-linear filters as a special case [1]. We present a comprehensive convergence analysis and illustrate connections between non-linear diagonal frame filtering, variational regularization, and plug-and-play regularization [2]. In addition, we demonstrate that non-linear regularizing filters that are purely learned from data meet the theoretical requirements necessary for convergent regularization.

The talk is based on [3] and ongoing work.

References

[1] C. Arndt, A. Denker, S. Dittmer, N. Heilenkötter, M. Iske, T. Kluth, P. Maass, and J. Nickel, Invertible residual networks in the context of regularization theory for linear inverse problems, In: arXiv:2306.01335, 2023.

[2] A. Ebner and M. Haltmeier, Plug-and-play image reconstruction is a convergent regularization method, In: arXiv:2212.06881, 2022.

[3] A. Ebner and M. Haltmeier, Convergence of non-linear diagonal frame filtering for regularizing inverse problems, In: arXiv:2308.15666, 2023.

[4] J. Frikel and M. Haltmeier, Sparse regularization of inverse problems by operator-adapted frame thresholding, In Mathematics of Wave Phenomena, Springer International Publishing, 163–178, 2020.

On Phase Unwrapping via Digital Wavefront Sensors

Simon Hubmer[1], Victoria Hutterer, Ronny Ramlau,
Ekaterina Sherina, Bernadett Stadler
[1]) Johann Radon Institute (RICAM), Linz, Austria
simon.hubmer@ricam.oeaw.ac.at

In this talk, we consider a new class of methods for phase unwrapping of 2D images based on digital wavefront sensors. Mathematically, many wavefront sensors produce the same measurements of incoming wavefronts regardless of whether they are wrapped or not. Since typical reconstructors for these sensors are optimized to compute smooth wavefronts, it is possible to digitally "propagate" a wrapped phase through such a sensor, resulting in a smooth unwrapped phase. First, we show how this principle can be applied for phase unwrapping using digital Shack-Hartmann and Fourier-type wavefront sensors. Then, we apply our methods to an unwrapping problem appearing in a real-world adaptive optics project currently under development, and compare the results to those obtained with other state-of-the-art algorithms.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Convergence guarantees for Newton type methods in tomographic problems via range invariance

Barbara Kaltenbacher[1]
[1]) University of Klagenfurt, Austria
barbara.kaltenbacher@aau.at

Range invariance is a property that - like the tangential cone condition - enables a proof of convergence of iterative methods for inverse problems. In contrast to the tangential cone condition it can also be verified for some parameter identification problems in partial differential equations PDEs from boundary measurements, as relevant, e.g., in tomographic applications, cf. [1].

The goal of this talk is to highlight some of these examples of coefficient identification from boundary observations in elliptic and parabolic PDEs, among them

- combined diffusion and absorption identification (e.g., in steady-state diffuse optical tomography)
- reconstruction of a boundary coefficient (e.g. in corrosion detection)
- reconstruction of a coefficient in a quasilinear wave equation (for nonlinearity coefficient imaging)

References

[1] Barbara Kaltenbacher. Convergence guarantees for coefficient reconstruction in PDEs from boundary measurements by variational and Newton-type methods via range invariance. *IMA Journal of Numerical Analysis*, page drad044, 07 2023. see also arXiv:2209.12596 [math.NA].

The 11th International Conference "Inverse Problems: Modeling and Simulation"
26th May – 1st June 2024, Malta

Discrepancy principles as parameter choice rules in Tikhonov regularisation

Marek Andrzej Kojdecki
Military University of Technology, 00-908 Warsaw, Poland,
Marek.Kojdecki@wat.edu.pl; m_kojdecki@poczta.onet.pl;

The linear operator equation of the first kind: $Au = f$ with a bounded linear operator $A\colon\ U \to F$ between Hilbert spaces U and F, to which a generalised solution u_+ exists, when only an equation: $A_\eta u = f_\delta$ with inexact data (A_η, f_δ) (a bounded linear operator $A_\eta\colon\ U \to F$, $f_\delta \in F$, together with an estimate $\gamma \equiv (\eta, \delta)$ of error in data: $\|A - A_\eta\| \le \eta$, $\|f - f_\delta\| \le \delta$) is known, is the abstract form for linear ill-posed problems. It can be effectively solved by Tikhonov's regularisation method – then the unique regularised solution $u_{\alpha\gamma}$ can be found as the solution of the linear operator equation of the second kind:

$$(A_\eta^* A_\eta + \alpha I) u_{\alpha\gamma} = A_\eta^* f_\delta \qquad (1)$$

with the regularisation parameter $\alpha > 0$ [1-7]. A question of both theoretical and practical importance is the choice of α enabling the convergence $u_{\alpha\gamma} \underset{\gamma\to 0}{\to} u_+$. Effective (a-posteriori) rules for choosing the regularisation parameter are usually defined in the form of additional equation like (2,3) to be satisfied by a pair $(u_{\alpha_*\gamma}, \alpha_*)$ together with (1). The most usefull are the discrepancy principle [1-6]:

$$\|A_\eta u_{\alpha\gamma} - f_\delta\| = \tau\big(\eta\|u_{\alpha\gamma}\| + \delta\big);\ \tau > 1 \text{ (when } Au_+ = f); \qquad (2)$$

or the second discrepancy principle [7]:

$$\alpha^q \|A_\eta^* A_\eta u_{\alpha\gamma} - A_\eta^* f_\delta\| = \beta\|A_\eta\|(\|u_{\alpha\gamma}\|\eta + \delta);\ \beta > 0,\ q > -\tfrac{1}{2}. \qquad (3)$$

Another rule can be formulated as a combination of these two. The properties of this new method will be considered and commented in the talk.

References

1. V.A. Morozov, Methods for solving incorrectly posed problems, New York – Berlin – Heidelberg, Springer, 1984.

2. A.N. Tikhonov, A.V. Goncharsky, V.V. Stepanov and A.G. Yagola, Numerical methods for the solution of ill-posed problems, Kluwer Academic Publisher, 1995.

3. A.B. Bakushinsky, A.V. Goncharsky, Ill-posed problems: theory and applications, Dordrecht – Boston – London, Kluwer, 1994.

4. V.K. Ivanov, V.V. Vasin, V.P. Tanana, Theory of linear ill-posed problems and its applications, Utrecht, VSP, 2002.

5. G.M. Vainikko, A.Y. Veretennikov, Iteration procedures in ill-posed problems, Moscow, Nauka, 1986, in Russian.

6. Y. Alber, I. Ryazantseva, Nonlinear ill-posed problems of monotone type, Dordrecht, Springer, 2006.

7. M.A. Kojdecki, New criterion of regularisation parameter choice in Tikhonov's method, Biuletyn WAT (Biul. Mil. Univ. Technol.), XLIX (1), 47-126 (2000).

Bi-level iterative regularization for inverse problems in nonlinear PDEs

Tram Nguyen
Max-Planck Institute for Solar System Research, Germany
nguyen@mps.mpg.de

We investigate the ill-posed inverse problem of recovering unknown spatially dependent parameters in nonlinear evolution PDEs from linear measurements. We propose a bi-level Landweber scheme, where the upper-level parameter reconstruction embeds a lower-level state approximation. This can be seen as combining the classical reduced setting and the newer all-at-once setting, allowing us to, respectively, utilize well-posedness of the parameter-to-state map, and to bypass having to solve nonlinear PDEs. Using this, we derive stopping rules for lower- and upper-level iterations and convergence of the bi-level method.

We discuss application to parameter identification for the Landau-Lifshitz-Gilbert equation in magnetic particle imaging.

Non-Uniqueness and reconstructability for the atmospheric tomography problem

Ronny Ramlau[1]
[1]) Industrial Mathematics Institute Johannes Kepler University Linz
ronny.ramlau@jku.at

Earth bound astronomical telescopes rely on Adaptive Optics systems in order to achieve a high imaging quality. In order to observe objects like extended galaxies, a correction for a larger field of view has to be obtained. This can be achieved by the use of the measurements of incoming wavefronts of multiple guide stars that allow a tomography based correction of the incoming light of the scientific object of interest.

The atmospheric tomography operator describes the impact of turbulent atmospheric layers on light passing through the atmosphere. Given wavefronts from different guide stars, measured by an (astronomical) telescope, the inverse problem consists in the reconstruction of the turbulence above the telescope. We show that the collected data is not sufficient to reconstruct the atmosphere uniquely. Additionally, we show that classical regularization methods as Tikhonov regularization or Landweber iteration will always fail to reconstruct a physically meaningful turbulence distribution. On a brighter side, we nevertheless achieve a good correction of the scientific images by using the reconstructed turbulence.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Applications of multiscale hierarchical decomposition to blind deconvolution

Stefan Kindermann[1], Elena Resmerita[2], Luminita Vese[3], and Tobias Wolf[4]
[1]) Johannes Kepler University of Linz, Austria
stefan.kindermann@jku.at
[2]) University of Klagenfurt, Austria
elena.resmerita@aau.at
[3]) University of California at Los Angeles
lvese@math.ucla.edu
[4]) University of Klagenfurt, Austria
tobias.wolf@aau.at

An important problem in image processing is the image restoration one, which aims to remove noise and blur from a degraded image. While this problem governed by linear operators can be approached by a multitude of methods from various perspectives, the notorious blind deconvolution problem still poses significant challenges theoretically and numerically. Recall that blind deconvolution addresses recovering both the true image and the blur kernel, knowing little information about the degradation. This is a highly ill-posed nonlinear inverse problem. One way to alleviate the difficulty in solving this problem is to use single-step variational approaches with regularization.

Our work introduces a more complex technique based on the hierarchical decomposition of images, inspired by the work of Tadmor, Nezzar and Vese on classsical denoising and deblurring (2004, 2008). We analyse the proposed method in the case of Sobolev norm and total variation regularization penalties, and point out its advantages especially when reconstructing images/kernels with features at different scales.

Regularization of inverse problems based on diffusion processes

Bernadette Hahn[1], Gaël Rigaud[2], Richard Schmähl[3]
[1]) Department of Mathematics – IMNG, University of Stuttgart, Pfaffenwaldring 57, D-70569 Stuttgart, Germany
[2]) Department of Mathematics and Stuttgart Center for Simulation Science (SC SimTech), University of Stuttgart, Pfaffenwaldring 57, D-70569 Stuttgart, Germany
[3]) Department of Mathematics – IMNG, University of Stuttgart, Pfaffenwaldring 57, D-70569 Stuttgart, Germany
richard.schmaehl@imng.uni-stuttgart.de

As shown by the rise of data-driven and learning techniques, the use of specific features in datasets is essential to build satisfactory solutions to ill-posed inverse problems suffering limitations, sparsity and large level of noise. Model-based regularization schemes are often based on minimizing functionals which involve appropriate data and penalty terms. Regarding the latter, the well-known total-variation regularization has become a classic choice whenever sharp edges or contrasts are desired. We propose to extend its diffusion properties to a more general nonlinear isotropic diffusion (NID) functional, using some prior information on the different levels of contrast of the target function [1]. This way we gain a better control over edge enhancing, leading to improved reconstructions for different forward problems like computerized tomography (CT), multi-energy CT or deconvolution. Simulation results validate the operator-indepence and potential of the general approach.

References

[1] B.N. Hahn and G. Rigaud and R. Schmähl, A class of regularizations based on nonlinear isotropic diffusion for inverse problems, IMA Journal of Numerical Analysis, 44(1)(2024), 225–261

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Myelin Mapping in the Human Brain Using an Empirical Extension of the Ridge Regression Theorem

Richard G. Spencer[1], Griffin S. Hampton[1], Ryan Neff[1], Zhezheng Song[2], Mustapha Bouhrara[1], and Radu Balan[2]
[1]) National Institute on Aging, National Institutes of Health, Baltimore, MD USA
spencer@helix.nih.gov
[2]) Department of Mathematics, University of Maryland, College Park, MD USA

Myelin mapping in the human brain with magnetic resonance (MR) is a subject of intense research activity, given the central role of myelin in neuropathology and its potential role in cognitive impairment and Alzheimer's disease.

One of the main signal models for myelin mapping assumes the presence of a myelin water fraction (MWF) signal component with a unique transverse signal decay time $T_{2,1}$ along with non-myelin associated water, with a decay time constant of $T_{2,2}$. In general, $T_{2,1} < T_{2,2}$ due to motional restriction of myelin-associated water; the total MR relaxometry signal is then the superposition of the signal from these two components:

$$S(TE; c_1, c_2, T_{21}, T_{22}) = c_1 e^{-\frac{TE}{T_{21}}} + c_2 e^{-\frac{TE}{T_{22}}} + \epsilon. \quad (1)$$

Here, c_1 and c_2 correspond to the myelin and non-myelin water fractions and ϵ represents noise. TE, the echo time, denotes data acquisition times and is the independent variable. Parameter estimation from this model, e.g. with non-linear least-squares (NLLS) analysis, may be extremely noise-sensitive. Indeed, this problem inherits the ill-posedness of the inverse Laplace transform and the Fredholm integral equation of the first kind, of which it is a special case.

The so-called ridge regression theorem proves the existence of a Tikhonov regularization parameter λ that decreases mean square error (MSE) in linear least-squares parameter estimation; the decrease in variance dominates the increase in bias. An analog of this theorem does not exist for non-linear problems; furthermore, Tikhonov regularization has no physical basis for problems of the type described, with parameters of different physical dimensions and that are essentially unrelated. Nevertheless, regularization decreases variance. Thus, we performed extensive simulation studies demonstrating a decrease in MSE through regularization of the biexponential parameter estimation problem. With this, we applied regularization to the inverse problem of MR myelin mapping. An essential element of our analysis is the restriction of regularization to imaging pixels exhibiting biexponential, rather than monoexponential, signal decay, as assessed by the Bayesian information criterion (BIC). As expected, biexponentiality dominates in myelinated white matter, while monoexponentiality is dominant in the largely non-myelinated grey matter. With this pipeline, we find a sizeable decrease in the MSE of MWF estimation, resulting in a substantial improvement in the accuracy of myelin mapping in the brain. Finally, we note that this method should be of general applicability to a wide range of NLLS parameter estimation problems.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Generative modeling for regularization

Christian Wald[1]
[1]) Technische Universität Berlin
wald@math.tu-berlin.de

Generative modeling has emerged as one of the most successful techniques in deep learning in recent years. Various strategies exist for leveraging generative models to address Bayesian inverse problems. Particularly, the discussion will focus on conditional maximum mean discrepancy (MMD) flows and conditional flow matching. For both methods we fix a source measure μ_s and a target measure μ_e. The first method [1, 2] approximates a Wasserstein gradient flow with respect to $MMD(\cdot, \mu_e)$ with start μ_s while the latter [3] estimates a flow vector field associated with a geodesic between μ_s and μ_e with respect to the conditional Wasserstein distance. Both methods are generative models conditioned on observations derived from the forward operator associated with the Bayesian inverse problem. However these methods do not directly enforce data consistency. The proposed method utilizes the outputs of these generative models not as our posterior distribution directly but as a regularizer, guiding samples onto the data manifold. More precisely consider the minimization problem

$$\underset{\mu_x}{\operatorname{argmin}} \, \mathbb{E}_{x\sim\mu_x}[\|Ax-y\|_2^2] + \lambda D\left(\mu_x, G(\cdot, y)_\sharp \mu_s\right)$$

where the minimization is over empirical measures μ_x with a fixed number of particles. Furthermore G is the generative model and D is some metric on the space of probability measures. Important examples of D include the MMD with suitable kernels and the Wasserstein metric. We will show the feasibility of our approach using limited angle CT data and MRI undersampling problems.

This is a joint work with Fabian Altekrüger, Jannis Chemseddine, Paul Hagemann, Johannes Hertrich and Gabriele Steidl.

References

[1] J. Hertrich, C. Wald, F. Altekrüger, P. Hagemann, Generative Sliced MMD Flows with Riesz Kernels, 2023, arXiv:2305.11463

[2] P. Hagemann, J. Hertrich, F. Altekrüger, R. Beinert, J. Chemseddine, G. Steidl, Posterior Sampling Based on Gradient Flows of the MMD with Negative Distance Kernel, 2023, arXiv:2310.03054

[3] J. Chemseddine, P. Hagemann, C. Wald, Y-Diagonal Couplings: Approximating Posteriors with Conditional Wasserstein Distances, 2023, arXiv:2310.1343

Optimal regularized hypothesis testing in statistical inverse problems

Frank Werner[1]

[1] Julius-Maximilians-Universität Würzburg, Germany
frank.werner@uni-wuerzburg.de

In this talk, we propose a regularized approach to hypothesis testing in Inverse Problems in the sense that the underlying estimators or test statistics are allowed to be biased. As one major result we prove that regularized testing is always at least as good as classical unregularized testing. We furthermore provide an adaptive test by maximizing the power functional, which outperforms unregularized tests in numerical simulations by several orders of magnitude.

Solving decomposition problems with nested Bregman iterations

Tobias Wolf[1], Kostas Papafitsoros[2], Elena Resmerita[3], Carola Schönlieb[4], Derek Driggs
[1])University of Klagenfurt
tobias.wolf@aau.at
[2]) Queen Mary University of London
[3]) University of Klagenfurt
[4])University of Cambridge

We focus on solving linear inverse problems, where the solution is a sum of components with different properties. A commonly used method for this is to use a variational approach with an infimal convolution of regularizing functions. While for noise-corrupted data, good approximations of the true solution can be obtained by Bregman iterations, the quality of the single components depends on the proper choice of weights associated with the infimally convoluted functions. In order to overcome the weighting choice, we propose the method of Nested Bregman iterations to improve a decomposition in a structured way. We discuss the convergence behavior and well-definedness of the proposed method, and illustrate its strength numerically for various examples.

MINISYMPOSIUM

M5. Inverse Problems in Hybrid Imaging Modalities

Organizers:
Simon Arridge, University College London, UK, S.Arridge@cs.ucl.ac.uk
Leonid Kunyansky, University of Arizona, USA, leonk@math.arizona.edu

During the last two decades, a plethora of hybrid (or coupled physics) tomography modalities has been invented and developed. All such techniques exploit physical coupling between two different types of waves. Typically, one sort of waves (for example, ultrasound) provides high resolution and stability, while the other type (frequently, of electromagnetic nature) supplies the resulting hybrid modality with high sensitivity to biological properties of interest.

A wide variety of existing coupled physics modalities leads to a large number of new and exciting mathematical problems waiting to be solved. The mini-symposium will bring together experts on theoretical and algorithmic foundations of the photo- and thermoacoustic tomography, ultrasound-modulated optical tomography, acoustoelectric tomography, various magnetoacoustoelectric modalities and other hybrid techniques.

Quantitative Reconstruction for Optical Coherence Tomography

Peter Elbau[1]
[1]) University of Vienna
peter.elbau@univie.ac.at

In optical coherence tomography, a sample is scanned from one direction with low-coherence light and the back-reflected part is interferometrically measured. We want to give in this talk a mathematical model for this device and analyse under what kind of assumptions on the medium a quantitative reconstruction of the optical properties of the object is possible from this sort of scattering data.

References

[1] L. Veselka, P. Elbau, L. Mindrinos, L. Krainz and W. Drexler Quantitative Parameter Reconstruction from Optical Coherence Tomographic Data Inverse Problems, 40(2):025001, 2024

Estimating optical parameters in quantitative photoacoustic tomography utilizing Monte Carlo method for light transport

Niko Hänninen[1], Aki Pulkkinen[1], Simon Arridge[2], Tanja Tarvainen[1,2]
[1]) Department of Technical Physics, University of Eastern Finland
niko.hanninen@uef.fi
[2]) Deparment of Computer Science, University College London

Quantitative photoacoustic tomography (QPAT) aims at estimating distributions of light absorbing molecules utilizing photoacoustic effect [1]. In this work, the optical inverse problem of QPAT, i.e. estimation of absorption and scattering coefficients from the absorbed energy density [2], is studied.

Iterative solving of the related minimization problem requires modeling light propagation and solving search direction of the minimization algorithm on each iteration. A widely accepted method for modeling light propagation in a scattering medium is the Monte Carlo (MC) method for light transport, which is based on simulating paths for photon propagation. Due to its stochastic nature, an approximation of light propagation obtained using the MC is corrupted by random stochastic noise. Consequently, the search direction of the minimization algorithm computed using this approximation is also corrupted by stochastic noise.

In this work, the optical inverse problem of QPAT is approached in the Bayesian framework, and *maximum a posteriori* estimates are computed using the Gauss-Newton method with the MC method as a forward model. The number of simulated photon packets is determined adaptively on each iteration utilizing a so-called norm test [3]. In the norm test, the expected relative error of the gradient of the objective function is evaluated, and the number of simulated photon packets is then adjusted to provide sufficiently accurate minimization direction. The approach is evaluated using numerical simulations.

References

[1] B. Cox, J. Laufer, S. Arridge and P. Beard, Quantitative spectroscopic photoacoustic imaging: A review, Journal of Biomedical Optics, 12 (2012), 061202.

[2] T. Tarvainen, A. Pulkkinen, B. Cox, J. Kaipio and S. Arridge, Bayesian image reconstruction in quantitative photoacoustic tomography, IEEE Transactions on Medical Imaging 32 (2013), 2287–2298.

[3] N. Hänninen, A. Pulkkinen, S. Arridge and T. Tarvainen. Estimating absorption and scattering in quantitative photoacoustic tomography with an adaptive Monte Carlo method for light transport. Inverse problems and Imaging. *Submitted* (2023).

Learned iterative reconstructions in photoacoustic tomography for the acoustic and optical problem

Andreas Hauptmann
University of Oulu, Research Unit of Mathematical Sciences
University College London, Department of Computer Science
andreas.hauptmann@oulu.fi

In photoacoustic tomography biological tissue is illuminated with a short pulse of near infrared light. The absorbed energy creates a local pressure increase that propagates through the tissue, governed by the acoustic wave equation, and we can measure the pressure wave on the boundary. From this measured time-series we first aim to reconstruct the initial pressure in the tissue, providing valuable information on local structures, such as microvasculature. Subsequently, it is possible to recover quantitative absorption and scattering values. Correct recovery of the optical parameters would provide valuable functional and biological information for medical purposes.

In practice, solving both the acoustic and optical inverse problem comes with challenges. Starting from an often encountered limited-view geometry, restricting the measurement surface and resulting in a mild to severely ill-posed linear inverse problem for the acoustic inversion. Reconstruction errors from the acoustic problem will naturally propagate when attempting to solve the optical problem. Classically, iterative model-based reconstruction approaches were employed to mitigate such limitations, but reconstructions can be highly time consuming for high-dimensional problems, especially with non-trivial forward operators. Thus, it is desirable to employ model reduction techniques to speed-up reconstructions in variational approaches as well as to enable training of learned model-based iterative reconstruction techniques [1]. Nevertheless, reduced or approximate models can lead to a degradation of reconstruction quality and need to be accounted for.

In this talk we discuss the possibility of learning model-based and data-driven reconstructions for both, the acoustic and optical inverse problem. We discuss conceptual differences between learning a reconstruction for linear and nonlinear inverse problems. In both cases, we consider the possibility to use approximate models to reduce computational complexity, including a learned model correction in the reconstruction process [2].

References

[1] A. Hauptmann, F. Lucka, M. Betcke, N. Huynh, J. Adler, B. Cox, P. Beard, S. Ourselin, S. Arridge. Model-based learning for accelerated, limited-view 3-D photoacoustic tomography. *IEEE T. Med. Imag.* 37, (2018): 1382-1393.

[2] S. Lunz, A. Hauptmann, T. Tarvainen, C.B. Schönlieb, S. Arridge. "On learned operator correction in inverse problems." *SIAM J. Imag. Sci.* 14, (2021): 92-127.

Acousto-electric Inverse Source Problems

Wei Li[1], John C. Schotland[2], Yang Yang [3], Yimin Zhong[4]
[1]) DePaul University
wei.li@depaul.edu
[2]) Yale University
[3]) Michigan State University
[4]) Auburn University

We propose a method to reconstruct the electrical current density inside a conducting medium from acoustically-modulated boundary measurements of the electric potential. We show that the current can be uniquely reconstructed with Lipschitz stability. We also perform numerical simulations to illustrate the analytical results, and explore the partial data setting when measurements are taken only on part of the boundary.

This method can also be applied to the reconstruction of the electrical current density from acoustically-modulated boundary measurements of time-harmonic electromagnetic fields. This is based on the works [1, 2] with coauthors John C. Schotland, Yang Yang and Yimin Zhong.

References

[1] Wei Li, John C. Schotland, Yang Yang, Yimin Zhong, An Acousto-electric Inverse Source Problem, SIAM Journal on Imaging Sciences, 14(4), 1601-1616(2021).

[2] Wei Li, John C. Schotland, Yang Yang, Yimin Zhong, Inverse Source Problem for AcousticallyModulated Electromagnetic Waves, SIAM Journal on Applied Mathematics, 83(2), 418-435(2023).

Optical tomography with the inverse Rytov series

Manabu Machida
Department of Informatics, Faculty of Engineering, Kindai University,
Higashi-Hiroshima 739-2116, Japan
machida@hiro.kindai.ac.jp

The Rytov approximation is a commonly known approach to determine coefficients of a partial differential equation from boundary measurements [1]. In particular, the Rytov approximation has been used in optical tomography [2]. The Rytov approximation gives better reconstructed images than the Born approximation in many cases.

Although the Rytov approximation is a practical method, it requires linearization of nonlinear inverse problems. In this talk, we will consider the Rytov approximation without linearization. To this end, we will develop the inverse Rytov series.

Let η be an unknown coefficient of the diffusion equation and ψ be a data obtained from boundary measurements. The Rytov series is given by

$$\psi = J_1\eta + J_2\eta \otimes \eta + \cdots . \tag{1}$$

In the conventional Rytov approximation, we obtain $\eta \approx \mathcal{J}_1\psi$ neglecting higher order terms.

The inverse Rytov series can be written as [3]

$$\eta = \mathcal{J}_1\psi + \mathcal{J}_2\psi \otimes \psi + \cdots , \tag{2}$$

where for $j \geq 2$,

$$\mathcal{J}_j = -\left(\sum_{m=1}^{j-1} \mathcal{J}_m \sum_{i_1+\cdots+i_m=j} J_{i_1} \otimes \cdots \otimes J_{i_m} \right) \mathcal{J}_1 \otimes \cdots \otimes \mathcal{J}_1. \tag{3}$$

In addition to stability and error, the reconstruction with the inverse Rytov series is tested both numerically and experimentally.

References

[1] H.W. Engl, M. Hanke, and A. Neubauer, Regularization of Inverse Problems, Springer Netherlands, 1996.

[2] S.R. Arridge, Optical tomography in medical imaging, Inverse Problems 15(2)(1999), R41–R93.

[3] M. Machida, The inverse Rytov series for diffuse optical tomography, Inverse Problems 39(10)(2023), 105012.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Diffuse optical tomography utilizing a nanosecond laser

Meghdoot Mozumder[1,*], Jarkko Leskinen[1], Tanja Tarvainen[1]
[1]Department of Technical Physics, University of Eastern Finland, P.O. Box 1627, 70211 Kuopio, Finland
*meghdoot.mozumder@uef.fi

Diffuse optical tomography (DOT) uses visible or near-infrared light for imaging spatially varying optical parameters in biological tissues [1]. Time-domain diffuse optical tomography (TD-DOT) uses pulsed lasers to illuminate the tissues and measure the time-varying boundary exitance. TD-DOT systems have been based on picosecond light sources using laserdiodes, Ti:Sapphire, and super-continuum lasers. Measurements in TD-DOT have been based on photon counting methods and/or time-gated detectors [2]. Recently, we proposed utilizing nanosecond light sources in TD-DOT [3]. Our motivation was to develop DOT systems that could be implemented with other imaging modalities, for example, with EEG, MRI, ultrasound, and, photoacoustic tomography.

We constructed a TD-DOT system using a nanosecond Nd:YAG laser (model NT352B; Ekspla Uab, Lithuania). Light was measured with an amplified avalanche photodetector (model APD430A/M; Thorlabs), connected to a high-bandwidth oscilloscope (model WavePro 254HD; Teledyne LeCroy, NY, USA). We imaged a liquid phantom inside a black plastic cylindrical tank, with clear glass tube containers for absorption and scattering inclusions. More details are in Ref. [3].

The image reconstruction of TD-DOT, involving the non-linear estimation of absolute absorption and scattering coefficients, was carried out in the Bayesian framework [3]. The location of absorption and scattering inclusions could be distinguished in the reconstructions. The images showed some cross-talk in the scattering image, but the magnitude of the cross-talk was quite low. The results show that nanosecond laser sources and standard digital oscilloscope measurements can provide relatively robust TD-DOT systems, that can be used to image diffuse medium. Furthermore, a nanosecond laser and a digitizer-based signal detection enable compatibility with other techniques such as ultrasound imaging (that uses signal waveform detection) and photoacoustic tomography (which uses signal waveform detection and nanosecond laser sources).

References

[1] Arridge, S. R. (1999). Optical tomography in medical imaging. Inverse problems, 15(2), R41.

[2] Pifferi, A., Contini, D., Mora, A. D., Farina, A., Spinelli, L., Torricelli, A. (2016). New frontiers in time-domain diffuse optics, a review. Journal of biomedical optics, 21(9), 091310-091310.

[3] Mozumder, M., Leskinen, J., Tarvainen, T. (2022). Utilising nanosecond sources in diffuse optical tomography. Measurement Science and Technology, 34(2), 025901.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Full-field Photoacoustic Tomography with Variable Sound Speed and Attenuation

Richard Kowar[1], Ngoc Do[2], Markus Haltmeier[1], Linh Nguyen[3]
[1]) Department of Mathematics, University of Innsbruck, Technikerstrasse 13, 6020 Innsbruck, Austria
richard.kowar@uibk.ac.at, markus.haltmeier@uibk.ac.at
[2]) Department of Mathematics, Missouri State University, Springfield MO 65897 USA
ngocdo@MissouriState.edu
[3]) Department of Mathematics, University of Idaho, Moscow, ID 83844 USA
lnguyen@uidaho.edu

In the standard photoacoustic tomography (PAT) measurement setup, the data used consist of time-dependent signals measured on an observation surface. In contrast, the measurement data of the recently invented full-field detection technique provides the solution of the wave equation in the spatial domain at a single point in time. While reconstruction using classical PAT data has been extensively studied, not much is known about the full-field PAT problem. In this work, we study full-field photoacoustic tomography with spatially variable sound velocity and spatially variable attenuation. In particular, we reconstruct the initial pressure from 2D projections of the full 3D acoustic pressure distribution at a given time. Numerical simulations are presented for both full angle and limited angle data cases.

Mathematical Imaging by Optical Coherence and Photoacoustic Tomography

Otmar Scherzer
University of Vienna, Austria

In this talk we are discussing mathematical models describing the light propagation in Optical Coherence Tomography (OCT) and Photoacoustic Tomography (PAT) and methods for tomographical imaging based on these two modalities. These two imaging modalities allow for visualization of biological specimens of a few millimeters.

We give an overview on recent microscopic and clinical applications, discuss established mathematical models and inversion (imaging) techniques, as well as recent mathematical trends in these fields.

This talk is based on joint work with W. Drexler (Medical University Vienna), P. Elbau, L. Mindrinos, E. Sherina & C. Shi (University of Vienna).

MINISYMPOSIUM
M6. Topological Derivatives in Inverse Problems and Shape Optimization

Organizers:
Peter Gangl, Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austria, peter.gangl@ricam.oeaw.ac.at
María-Luisa Rapún, Universidad Politécnica de Madrid, Spain, marialuisa.rapun@upm.es

The topological derivative of a shape functional measures the sensitivity of such functional to having an infinitesimal perturbation at each point of the explored region. It has many applications in connection with inverse problems such as shape optimization, topology optimization, imaging processing, crack and defect detection in non-destructive testing, to mention a few.

The aim of this mini-symposium is to bring together experts and young researchers working in this field to discuss and review the recent applications, new results and future challenges.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Shape and topology optimization of regions supporting the boundary conditions of a physical problem

Eric Bonnetier[1], Carlos Brito-Pacheco[2], Charles Dapogny[2], Nicolas Lebbe[3], Edouard Oudet [2], Michael Vogelius[4]

[1]) Institut Fourier, Univ. Grenoble-Alpes, 38000 Grenoble, France

[2]) Univ. Grenoble Alpes, CNRS, Grenoble INP, LJK, 38000 Grenoble, France

[3]) LAPLACE, Univ. Toulouse, CNRS, INPT, UPS, 2 Rue Charles Camichel, 31000 Toulouse, France

[4]) Rutgers University, Department of Mathematics, New Brunswick, NJ, USA.

Broadly speaking, shape optimization aims to optimize the design of a $2d$ or $3d$ domain, with respect to an objective and under constraints formulated as functions of the domain. In applications, these functions depend on the shape via to solution to a partial differential equation encoding the physical situation under scrutiny, which is complemented with boundary conditions accounting for the effects the exterior medium. Thus, a mechanical structure is characterized by its displacement, solution to the linear elasticity system, equipped with boundary conditions of homogeneous Dirichlet (modeling the fixation regions of the structure), homogeneous Neumann (for the traction-free boundaries), or inhomogeneous Neumann (boundaries where loads are applied) types.

Most often, only one part of the boundary of the shape is optimized – typically, the traction-free boundary in structural mechanics. The aim of this work is, on the contrary, to consider the optimization of these regions bearing the boundary conditions of the physical problem at play. This question is considered from two complementary viewpoints.

- We investigate the shape derivative of a shape functional in the sense of Hadamard, when the involved deformations do not vanish where the boundary conditions change types: this allows to optimize how the regions bearing these conditions may “slide” along the boundary of the shape.
- We consider the sensitivity of the solution to a physical problem (and that of a related quantity of interest) when a small region bearing a certain type of boundary conditions (typically, of homogeneous Dirichlet type) is nucleated within a region bearing other conditions (e.g. of Neumann type). This paves the way to a notion of “topological derivative” describing change of boundary conditions.

References

[1] E. Bonnetier, C. Dapogny and M.S. Vogelius, Small perturbations in the type of boundary conditions for an elliptic operator, Journal de Mathématiques Pures et Appliquées, 167 (2022), pp. 101–174.

[2] C. Dapogny, N. Lebbe and E. Oudet, Optimization of the shape of regions supporting boundary conditions, Numer. Math., 146, (2020), pp. 51–104

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Lagrangian approach and shape gradient for inverse problem of breaking line identification in solid

Victor A. Kovtunenko[1,2]

[1]) Department of Mathematics and Scientific Computing, Karl-Franzens University of Graz, NAWI Graz, Heinrichstraße 36, 8010 Graz, Austria
victor.kovtunenko@uni-graz.at
[2]) Lavrentyev Institute of Hydrodynamics, Siberian Division of the Russian Academy of Sciences, 630090 Novosibirsk, Russia

A class of inverse identification problems constrained by variational inequalities is studied with respect to its shape differentiability. The specific problem appearing in fracture mechanics describes elastic bodies with a breaking line subject to contact conditions between its faces. Based on the Lagrange multiplier approach and smooth Lavrentiev penalization, a semi-analytic formula for the shape gradient of the Lagrangian linearized on the solution is proved, which contains both primal and adjoint states. It is used for the descent direction in a gradient algorithm for identification of an optimal shape of the breaking line from boundary measurements. The theoretical result is supported by numerical simulation tests of destructive testing in 2D configuration comparing the problems with and without contact.

References

[1] F. Cakoni and V.A. Kovtunenko, Topological optimality condition for the identification of the center of an inhomogeneity, Inverse Probl. 34(3) (2018), 035009.

[2] D. Ghilli, K. Kunisch K and V.A. Kovtunenko, Inverse problem of breaking line identification by shape optimization, J. Inverse Ill-posed Prob. 28 (2020), 119–135.

[3] A.M. Khludnev and V.A. Kovtunenko, Analysis of Cracks in Solids, WIT-Press, Southampton, Boston, 2000.

[4] V.A. Kovtunenko, Lagrangian approach and shape gradient for inverse problem of breaking line identification in solid: contact with adhesion, Inverse Probl. 39(8) (2023), 084004.

[5] V.A. Kovtunenko and K. Kunisch, Shape derivative for penalty-constrained nonsmooth-nonconvex optimization: cohesive crack problem, J. Optim. Theory Appl. 194 (2022), 597–635.

[6] V.A. Kovtunenko and K. Kunisch, Directional differentiability for shape optimization with variational inequalities as constraints, ESAIM: Control Optim. Calc. Var. 29 (2023), 64.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Reconstruction of Voronoi diagrams in inverse problems

Ernesto G. Birgin[1], Antoine Laurain[2], Danilo R. Souza[3]
[1]) Department of Computer Science, Institute of Mathematics and Statistics,
University of São Paulo
egbirgin@ime.usp.br and danilo_souza@ime.usp.br
[2]) Faculty of Mathematics, University of Duisburg-Essen
antoine.laurain@uni-due.de

We propose and analyze a numerical method for the inverse problem of recovering a piecewise constant coefficient with multiple phases from interior data. The unknown function is assumed to be constant in each phase, and the phases are modeled by a Voronoi diagram generated by a set of sites, which are used as control parameters. We first reformulate the inverse problem as an optimization problem with respect to the position of the sites. Combining techniques of non-smooth shape calculus and sensitivity of Voronoi diagrams, we are able to compute the gradient of the cost function, under standard non-degeneracy conditions on the diagram. We provide two different formulas for the gradient, a volumetric and an interface one, and compare them in numerical experiments. We provide several numerical experiments to investigate the dependence of the reconstruction on the problem parameters, such as noise, number of sites and initialization.

The 11th International Conference "Inverse Problems: Modeling and Simulation"
26th May - 1st June 2024, Malta

A study on the topological derivative-based imaging without background information

Seongje Chae[1], Taeyoung Ha[2], Won-Kwang Park[3], Minha Yoo[2]

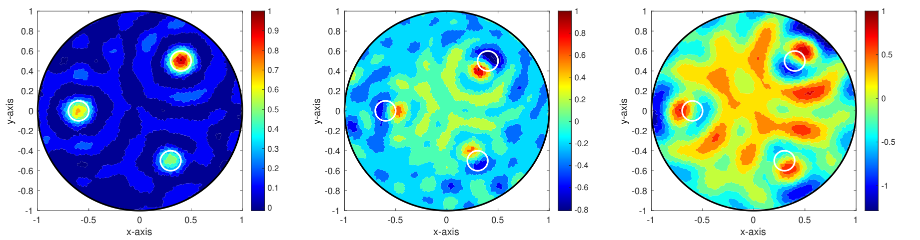
[1)] CJ, Seoul, Korea

[2)] National Institute for Mathematical Sciences, Daejeon, Korea

[3)] Department of Information Security, Cryptology, and Mathematics, Kookmin University, Seoul, Korea

Correspondence: parkwk@kookmin.ac.kr

We consider the application of topological derivative (TD) for a fast imaging of unknown object, which is completely hidden in a homogeneous domain, from boundary measurement data in transverse magnetic (TM) polarization. For a successful application, accurate value of background permittivity must be known. If its accurate value is unknown, inaccurate location and shape of object will be imaged. This phenomenon was examined through the simulation results but theoretical reason has not been revealed yet. To explain this phenomenon, we show that the imaging function of the TD can be expressed by the Bessel function of order zero of the first kind. This result explains why inaccurate location and shape of object is imaged with inaccurate value of background permittivity. Simulation results with noise-corrupted data are exhibited to support theoretical result.



References

[1] W.-K. Park, Topological derivative strategy for one-step iteration imaging of arbitrary shaped thin, curve-like electromagnetic inclusions, Journal of Computational Physics, 231 (2012), 1426–1439.

[2] W.-K. Park, Performance analysis of multi-frequency topological derivative for reconstructing perfectly conducting cracks, Journal of Computational Physics, 335 (2017), 865–884.

Iterative methods based on topological derivative computations for shape reconstruction problems.

F. Le Louër[1], **M.-L. Rapún**[2]
[1] LMAC EA2222, Laboratoire de Mathématiques Appliquées de Compiègne, Sorbonne Université, Université de Technologie de Compiègne, 60203 Compiègne Cedex, France
[2] Dpto. Matemática Aplicada a la Ingeniería Aeroespacial, E.T.S.I. Aeronáutica y del Espacio, Universidad Politécnica de Madrid, 28040 Madrid, Spain
marialuisa.rapun@upm.es

Reconstructing 3D shapes from very limited data is a challenging task. In this work, we propose two automatic algorithms for 3D inverse scattering problems. The first one is fully based on topological derivative computations [1] while the second one synergically combines topological derivatives and regularized Gauss-Newton iterations [2, 3]. Numerical examples dealing with decoding digital holograms and solving inverse acoustic problems will be shown.

References

[1] F. Le Louër and M.L. Rapún. Topological sensitivity analysis revisited for time-harmonic wave scattering problems. Part II: recursive computations by the boundary integral equation method, Engineering Computations, 39(1) (2022), 272–312.

[2] A. Carpio, T.G. Dimiduk, F. Le Louër and M.L. Rapún. When topological derivatives met regularized Gauss-Newton iterations in holographic 3D imaging, Journal of Computational Physics, 388 (2019), 224–251.

[3] F. Le Louër and M.L. Rapún. Topological imaging methods for the iterative detection of multiple impedance obstacles, Journal of Mathematical Imaging and Vision, 64(3) (2022) 321–340.

MINISYMPOSIUM
M7. Bayesian, Variational, and Optimization Techniques for Inverse Problems in Stochastic Partial Differential Equations

Organizers:
Barbara Kaltenbacher, University of Klagenfurt, Austria, Barbara.Kaltenbacher@aau.at
Akhtar A. Khan, Rochester Institute of Technology, USA, aaksma@rit.edu
Hans-Jörg Starkloff, Technische Universität Bergakademie Freiberg, Germany, hjstark@math.tu-freiberg.de
Christiane Tammer, Martin-Luther-University of Halle-Wittenberg, Germany, christiane.tammer@mathematik.uni-halle.de

In recent years, significant attention has been given to developing stochastic models and methods for inverse and control problems associated with stochastic and deterministic equations. Important applications motivate estimating stochastic parameters, source terms, or boundary conditions in stochastic partial differential equations. On the other hand, recent developments in machine learning and related subjects have advocated the utility of stochastic models not only for stochastic inverse problems but also for deterministic identification and control problems. This mini-symposium aims to bring together well-known experts and young researchers in the dynamic and expanding field of stochastic inverse problems.

The main topics include the Bayesian approach, variational and optimization techniques, stochastic approximation, stochastic gradient, neural networks, Kalman filter, and related methods aimed at studying inverse problems associated with stochastic partial differential equations.

Optimal experimental design for correlation data in aeroacoustics

Christian Aarset, Thorsten Hohage
University of Göttingen, Germany
c.aarset@math.uni-goettingen.de

A major problem within the field of aeroacoustics is determining the distribution of an aeroacoustic source, such as an airplane engine, given pressure measurements on external microphone arrays. Taking a Bayesian view and modeling the source as fundamentally random with zero mean leads to the problem of determining the covariance of the random source.

While this can be recovered from correlations of pressure measurements, the consequent dimensionality increase is significant when the number of sensors is large. Accordingly, we study optimal experimental design for correlation data, with the goal of determining the optimal, sparse sensor placement prior to conducting any real-world experiments.

Specifically, we investigate *A-optimal* designs, that is, designs that minimize the average posterior covariance in the reconstruction. Building on the work of [1] for the case of deterministic sources with non-correlated data, we moreover present algorithmic treatment of the aeroacoustics design problem, involving low-rank approximation of correlated forward operators.

References

[1] A. Alexanderian, Optimal experimental design for infinite-dimensional Bayesian inverse problems governed by PDEs: a review, Inverse Problems, 37(2021), 043001.

Stochastic dynamics of influenza infection:Qualitative analysis and numerical results

Ghada Alobaidi
American University of Sharjah
galobaidi@aus.edu

In this paper, a novel influenza SI_NI_RR model with white noise is investigated. According to the research, white noise has a significant impact on the disease. First, we explain that there is global existence and positivity to the solution. Then we show that the stochastic basic reproduction R_r is a threshold that determines whether the disease is cured or persists. When the noise intensity is high, we get $R_r < 1$ and the disease goes away; when the white noise intensity is low, we get $R_r > 1$, and a sufficient condition for the existence of a stationary distribution is obtained, which suggests that the disease is still there. However, the main objective of the study is to produce a stochastic analogue of the deterministic model that we analyze using numerical simulations to get views on the infection dynamics in a stochastic environment that we can relate to the deterministic context.

References

[1] Ming-Zhen Xin, Bin-Guo Wang, Yashi Wang, Stationary distribution and extinction of a stochastic influenza virus model with disease resistance,Mathematical Biosciences and Engineering,19(9)(2022), 9125–9146.

[2] Dynamics of a stochastic SIRS epidemic model with standard incidence and vaccination, Mathematical Biosciences and Engineering,19(10),10618-10636.

Evaluating the accuracy of the posterior probability distribution in an elastic inverse problem

Olalekan Babaniyi[1], Michael Bennett[1]
[1] School of Mathematics and Statistics,
Rochester Institute of Technology,
obsma@rit.edu

In this talk, we present a Bayesian approach to solve inverse problems for the mechanical properties of soft tissue. This is a problem of interest in the elasticity imaging field where the mechanical properties can be used to noninvasively diagnose various disease. The inverse problem is challenging because the dimension of the posterior probability distribution of the parameters (or mechanical properties) depend on its spatial discretization. The standard approach to make this inverse problem tractable is to approximate the posterior as Gaussian. In this work, we evaluate the accuracy of the Gaussian approximation by relaxing the assumption and using sampling based techniques to characterize the true posterior.

A control economic equilibrium problem via inverse stochastic variational inequalities

Annamaria Barbagallo
University of Naples Federico II
Department of Mathematics and Applications "R. Caccioppoli"
via Cintia – Complesso Universitario Monte Sant'Angelo, 80126 Naples, Italy
annamaria.barbagallo@unina.it

The aim of the talk is to study the policy marker's point of view for a random oligopolistic market equilibrium problem. The firms' point of view has been examined in [1]. We are interested to characterize the random optimal control equilibrium conditions by an inverse stochastic variational inequality. In addition, the equivalence with a stochastic variational inequality is presented. Some existence and well-posedness results for optimal regulatory taxes are shown (see [2]). At last, a numerical example is provided.

References

[1] A. Barbagallo, M. Ferrara and P. Mauro, Stochastic variational approach for random Cournot-Nash principle. In: B. Jadamba, A.A. Khan, S. Migórski and M. Sama, Deterministic and Stoichastic Optimal Control and Inverse Problems, CRC Press, Taylor & Francis Group, Boca Raton, (2021), 241–269.

[2] A. Barbagallo, B.A. Pansera, M. Ferrara, Notes on random optimal control equilibrium problem via inverse stochastic variational inequalities, Computational Management Science, in press.

Sampling in Bayesian inversion accelerated by surrogate models

Simona Béresová[1,2], Michal Béreš[1,2]
[1]) Institute of Geonics, Czech Academy of Sciences, Ostrava, Czech republic
simona.beresova@ugn.cas.cz
[2]) Department of Applied Mathematics, FEECS, VŠB-TUO, Ostrava, Czech republic

The motivation for the acceleration of sampling in Bayesian approach stems from the need to solve geotechnical inverse problems with uncertainties: identification of material parameters based on data from a tunnel sealing experiment (TSX), identification of fracture apertures, etc., see [1].

The Bayesian inversion provides the posterior distribution of unknown parameters. Samples from this distribution can be generated using Markov chain Monte Carlo methods. We use a sampling procedure based on the delayed-acceptance Metropolis-Hastings (DAMH) algorithm. The DAMH algorithm works with the true posterior and also with its approximation. Such as the basic Metropolis-Hastings (MH) algorithm, it provides samples from the true posterior, the approximation serves only for the acceleration. Unlike the MH algorithm, the forward mathematical model is not evaluated for each proposed sample. This can lead to a significant increase in efficiency. The better posterior approximation we use, the fewer evaluations we need.

In our sampling procedure, the posterior approximation is constructed using a surrogate of the forward model. There are several sampling processes runing in parallel that share one surrogate model. During the sampling process, the surrogate model is adaptively updated using new snapshots from all of the samplers. Python implementation (SurrDAMH package) is available at [2].

Surrogate models can be constructed using various techniques such as the interpolation using radial basis functions, polynomial approximation, or approximation using values at nearby points. Each of these approaches also allows for further customization. Furthermore, machine learning techniques significantly broaden the spectrum of potential surrogate models. The use of neural networks as surrogate models appears to be a promising approach; however, careful consideration of the network type and learning method is essential. Therefore, selecting a suitable surrogate model for a given forward model can be a complex task. In this talk, the choice of a surrogate model for the TSX problem will be discussed.

References

[1] S. Béresová, Bayesian approach to the identification of parameters of differential equations, PhD thesis, VSB-TUO, Ostrava, 2022.

[2] S. Béresová, SurrDAMH package, github.com/dom0015/surrDAMH, 2022.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Generative modelling with tensor compressed HJB approximations

Martin Eigel[1], Robert Gruhlke[2], Max Kirstein[3], Claudia Schillings[2], David Sommer[1]
[1]) WIAS Berlin
{eigel,sommer}@wias-berlin.de
[2]) Free University Berlin
{gruhlke,schillings}@math.fu-berlin.de
[3]) independent researcher

Sampling from high-dimensional probability densities is a common challenge in fields such as Uncertainty Quantification (UQ) and Generative Modelling (GM). In GM in particular, the use of reverse-time diffusion processes depending on the log-densities of Ornstein-Uhlenbeck forward processes are a popular sampling tool. It is known that these log-densities can be obtained by solution of a Hamilton-Jacobi-Bellman (HJB) equation central to the theory of stochastic optimal control. While this HJB equation is usually treated with indirect methods such as policy iteration and non-supervised training of black-box architectures like Neural Networks, we propose instead to solve the HJB equation by direct time integration. To make this formidable task tractable, we introduce a low-rank tensor train (TT) compression for a polynomial representation of the spatial discretization. Strikingly, our method [1] is sample-free, agnostic to normalization constants and can avoid the curse of dimensionality (under adequate assumptions) due to the TT compression. Numerical experiments demonstrate the performance of the proposed time-step-, rank- and degree-adaptive integration method on nonlinear sampling tasks. We also indicate how the method can be used in the context of Bayesian inverse problems.

References

[1] D. Sommer, R. Gruhlke, M. Kirstein, M. Eigel, C. Schillings, *Generative modelling with tensor train approximations of Hamilton–Jacobi–Bellman equations*, arXiv:2402.15285.

Uncertainty in inverse elasticity problems

Daniel I. Gendin[1], Paul E. Barbone[2]
[1] University at Buffalo
dgendin@buffalo.edu
[2] Boston University

The inverse elasticity problem can be stated thusly: Given one or more measurements of the displacement field within an elastic material, determine the mechanical property distribution within the material. Given adequate displacement data, a corresponding mechanical property distribution can be found. However the inherent noise in the data and the ill-posed nature of inverse problems necessitates evaluation and characterization of the uncertainty in the found solution. We will consider the uncertainty in evaluating both linear and nonlinear elastic properties (a small pilot study found that there may be a connection between malignancy and elastic nonlinearity in human breast tumors [1]).

Our interest in uncertainty naturally leads us to consider the problem in a Bayesian framework. In order to efficiently characterize the posterior probability distribution, we approximate it using a Gaussian distribution (this is called a Laplace approximation). With the mean being given by the point that maximizes the posterior probability (called the MAP point) and the covariance being given by the Hessian of the log-posterior distribution at the MAP point.

We find the MAP point using a Newton-CG approach. We exploit the mathematical structure of the inverse problem and the properties of the conjugate gradient method to approximate the Hessian at the MAP point at no extra computational cost.

In considering the recovery of elastic nonlinearity we use information gain to characterize a trade-off between larger strains with higher noise levels and smaller strains with lower noise levels. These results can be used to inform experimental design.

We compare two different approaches: one where linear and nonlinear elastic properties are evaluated sequentially and another where they are evaluated jointly. We discuss the mathematical implications of both approaches and determine that accurate characterization of the posterior probability distribution requires that the estimates be performed jointly rather than sequentially.

We apply our methods to simulations, phantom data, and *in vivo* measurements.

References

[1] Goenezen, S., Dord, J. F., Sink, Z., Barbone, P. E., Jiang, J., Hall, T. J., & Oberai, A. A. (2012). Linear and nonlinear elastic modulus imaging: an application to breast cancer diagnosis. IEEE transactions on medical imaging, 31(8), 1628-1637.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Optimality of pulse energy for photoacoustic tomography

Barbara Kaltenbacher[1], Phuoc Truong Huynh[1]
[1]) Department of Mathematics, Alpen-Adria-Universität Klagenfurt, Klagenfurt, Austria
Phuoc.Huynh@aau.at

Photoacoustic tomography (PAT) is a rapidly evolving imaging technique that combines high contrast of optical imaging with high resolution of ultrasound imaging. Using typically noisy measurement data, one is interested in identifying some parameters in the governing PDEs for the photoacoustic tomography system. Hence, an essential factor in estimating these parameters is the design of the system, which typically involves multiple factors that can impact the accuracy of reconstruction. In this work, employing a Bayesian approach to a PAT inverse problem we are interested in optimizing the laser pulse of the PAT system in order to minimize the uncertainty of the reconstructed parameter. Additionally, we take into account wave propagation attenuation for the inverse problem of PAT, which is governed by a fractionally damped wave equation. Finally, we illustrate the effectiveness of our proposed method using a numerical simulation.

Computational framework for a distributed parameter identification in PDEs

Basca Jadamba
School of Mathematics and Statistics
Rochester Institute of Technology
85 Lomb Memorial Drive
Rochester, NY 14623

This talk will focus a finite element solution framework for the problem of identifying a distributed parameter in a linear elasticity system. We formulate the problem of estimating the so-called tissue stiffness parameter in the system from measurements of displacement as an optimization problem and present solution framework that uses adaptive mesh refinements. Variations of gradient-based optimization methods and their performances will be discussed.

Identification of random parameters in stochastic variational inequalities

Akhtar A Khan
School of Mathematics and Statistics,
Rochester Institute of Technology,
Rochester, New York, 14623, USA.
E-mail: aaksma@rit.edu

Motivated by the necessity to identify stochastic parameters in a wide variety of stochastic partial differential equations with complicated boundary conditions, an abstract inversion framework is designed for variational inequalities. The stochastic inverse problem is studied in a stochastic optimization framework. The essential properties of the solution maps are derived and used to prove the solvability of the stochastic inverse problem posed as a stochastic optimization problem.

Bayesian Inversion of absorption and scattering coefficients in DOT

Taufiquar Khan
Mathematics and Statistics Department,
University of North Carolina,
9201 University City Blvd. Charlotte, NC 28223.

In this talk, we will discuss an optimal Bayesian estimator to reconstruct both the absorption and scattering coefficient for the Diffuse Optical Tomography (DOT). Mathematically, the reconstruction of the internal absorption or scattering coefficients is a severely ill-posed inverse problem and yields a poor quality image reconstruction. We will present the efficacy of the proposed approach using simulations. This is joint work with Anuj Abhishek (Case Western Reserve University) and Thilo Strauss (Xi'an Jiaotong-Liverpool University).

Textured Image Restoration via Symmetrised Fractional Variation

Antonio LEACI
Dipartimento di Matematica e Fisica "Ennio De Giorgi",
Università del Salento,
73100 Strada Provinciale per Arnesano s.n. Lecce, Italy.

In this talk we introduce and apply a variational model for image analysis based on Riemann-Liouville fractional derivatives. Both the one-dimensional and two-dimensional cases are studied. The model exploits an L^1 fitting data term together with both right and left Riemann-Liouville fractional derivatives as regularizing terms: the aim is to achieve an orientation independent approach.

First numerical experiments in one dimension for signal denoising are shown, by exploiting the Grünwald-Letnikov scheme and aiming to calibrate the fidelity parameter with the fractional order by white noise autocorrelation.

This is a joint research project with Alessandro Lanza (Università di Bologna), Serena Morigi (Università di Bologna) and Franco TOMARELLI (Politecnico di Milano).

Fréchet and Mordukhovich derivatives of the metric projection operator in Banach spaces

Jinlu Li
Department of Mathematics Shawnee State University
Portsmouth, Ohio 45662, USA
jli@shawnee.edu

In this talk, we present the results of several papers finished recently, which include the strict Fréchet differentiability and properties of the metric projection operator onto nonempty closed and convex subsets in Hilbert spaces, and the Fréchet differentiability in uniformly convex and uniformly smooth Banach spaces. When the considered subsets are closed balls, closed and convex cylinders and positives cones in Hilbert spaces, or in uniformly convex and uniformly smooth Banach spaces, we provide the precise solutions of the Fréchet and Mordukhovich derivatives of the metric projection operator.

Uncertainty quantification in residential thermal models

Miguel Sama[1], Jorge Arias [1], Akhtar Khan [2]
[1]) Universidad Nacional Educación a Distancia (UNED), Madrid, Spain
msama@ind.uned.es
[2]) Rochester Insitute of Technology, Rochester, USA.

This work aims to study the thermal behavior of residential buildings by using the data provided by smart thermostats and weather forecast data. For this, we consider an equivalent ODE circuit model depending on four parameters related to the heater power, the solar energy, heat capacity, and the thermal resistance of the building. We consider a random version of the model to overcome natural model uncertainty. More specifically we consider the following Ordinary Differential Equation with random data wich was introduced in [1]

$$\frac{dT}{dt}(t,\omega)+\frac{1}{R(\omega)C(\omega)}T(t,\omega)=\frac{1}{R(\omega)C(\omega)}T^e(t)+\frac{1}{C(\omega)}\left[P(\omega)dU(t)+A(\omega)I^c(t)\right] \text{ a.e } \omega\in\Omega$$

Here:

- $T(t,\omega)$: indoor temperature.
- $T^e(t)$: exterior temperature.
- $I^c(t)$: corrected solar radiation.
- $dU(t)$: heater usage time fraction.

While the random parameters to be identified have the following physical intepretation

- $R(\omega)$: thermal resistance.
- $C(\omega)$: heat capacity.
- $P(\omega)$: effective boiler power.
- $A(\omega)$ factor of solar radiation.

Based on the data available we show how to solve effectively this model.

References

[1] J. Arias, A.A. Khan, J. Rodriguez-Uría, and M. Sama (2022). Analysis of smart thermostat thermal models for residential building. Applied Mathematical Modelling, 110, 241-261.

Stochastic elliptic inverse problems as abstract elliptic inverse problems

Hans-Jörg Starkloff
TU Bergakademie Freiberg (Germany)
hjstark@math.tu-freiberg.de

The investigation of random ordinary or partial differential equations allows a quantification of uncertainties which often occur in real life applications. This is related to direct as well as to inverse problems.

One additional issue in the consideration of random equations is the measurability of desired solutions. Based on the fact that there exist different measurability concepts it is important to use the appropriate measurability concept for each problem. In the talk important measurability concepts are presented and some of the relations between them are discussed. Furthermore it will be shown exemplarily, which stochastic elliptic inverse problems can be treated as abstract elliptic inverse problems and which such stochastic inverse problems require a specific stochastic investigation.

Multiobjective approaches for optimization problems under uncertainty

Christiane Tammer[1], Elisabeth Köbis[2]
[1]) Martin-Luther-University Halle-Wittenberg, Germany
christiane.tammer@mathematik.uni-halle.de
[2]) NTNU Norwegian University of Science and Technology

We consider scalar optimization problems under uncertainty and discuss two general approaches (multiobjective approach and nonlinear scalarization) to robustness and stochastic programming (see [1], [2]). These approaches permit a unified treatment of a large variety of models from robust optimization and stochastic programming, respectively. We review several classical concepts, both from robust optimization and from stochastic programming, and interpret them in the light of multiobjective optimization and using nonlinear scalarization techniques. It turns out that optimal solutions to robust counterpart problems or stochastic programming models are typically obtained as (weakly) efficient solutions of an appropriately formulated deterministic multiobjective counterpart problem. Similarly, nonlinear scalarization functionals which yield (weakly) efficient solutions of the respective multiobjective counterparts can be employed to achieve corresponding results.

The aim of the talk is to show that new optimality conditions for solutions of scalar robust counterpart problems can be obtained using multiobjective optimization and nonlinear scalarization methods (see [3]).

References

[1] A. Göpfert, H. Riahi, C. Tammer, C. Zălinescu, Variational Methods in Partially Ordered Spaces, Second Edition, Springer Nature, Switzerland AG, 2023.

[2] K. Klamroth, E. Köbis, A. Schöbel, A., C. Tammer, A unified approach to uncertain optimization, European J. Oper. Res. 260(2)(2017), 403–420.

[3] E. Köbis, C. Tammer, Optimality conditions in optimization under uncertainty, J. Nonlinear Var. Anal. 7(5)(2023), 769–784.

Inverse problems as a source of vector variational problems

Pablo Pedregal[1]
[1]) Universidad de Castilla-La Mancha
pablo.pedregal@uclm.es

Applied vector variational problems are almost restricted to hyper-elasticity models. Their analysis is anything but straightforward, as they are related to non-linear systems of PDEs and other complicated constraints. Various sophisticated notions of convexity are relevant: poly-convexity, quasi-convexity, and rank-one convexity. It is therefore always interesting to find examples, as explicit as possible, of functionals that may be helpful in illustrating and conveying some of those subtleties of vector variational problems. It turns out that inverse problems in conductivity is one such area where one can find explicit, innocent-looking examples. In this talk, we will explore the interplay of some of those functionals with the inverse problems that motivated them. For the sake of simplicity, we will stay in a 2D setting, and depending on time, we will briefly move to the 3D situation.

Variational approach to pure traction and Signorini problem between linear and finite elasticity

Tomarelli Franco
Politecnico di Milano,
Dipartimento di Matematica,
Piazza Leonardo da Vinci, 20136 Milano, Italy.

An energy functional for the obstacle problem in linear elasticity is obtained as a variational limit of nonlinear elastic energy functionals describing a material body subject to pure traction load under a unilateral constraint representing the rigid obstacle. There exist loads pushing the body against the obstacle, but unfit for the geometry of the whole system body-obstacle, so that the corresponding variational limit turns out to be different from the classical Signorini problem in linear elasticity. However, if the force field acting on the body fulfills an appropriate geometric admissibility condition, we can show coincidence of minima. The analysis developed here provides a rigorous variational justification of the Signorini problem in linear elasticity, together with an accurate analysis of the unilateral constraint.
This is joint research project with Francesco Maddalena (Politecnico di Bari) and Danilo Percivale (Università degli Studi di Genova).

MINISYMPOSIUM
M8. Inverse Problems with Data-Driven Methods and Deep Learning

Organizers:
Tatiana Bubba, University of Bath, UK, tab73@bath.ac.uk
Andreas Hauptmann, University of Oulu, Finland, andreas.hauptmann@oulu.fi
Luca Ratti, University of Bologna, Italy, luca.ratti5@unibo.it

In recent years, the volume and complexity of various data obtained by modern applications has been steadily increasing. Extracting consistent scientific information from these large datasets remains an open challenge for the community, and data-driven approaches such as deep learning have quickly emerged as a potentially powerful solution to some long-term problems. In this context, robust mathematical inversion algorithms, combined with new data science techniques, deliver state-of-the-art results across a wide range of inverse problems.

This mini-symposium aims at bringing together experts in data-driven methods and deep learning for inverse problems and provides an overview of learned image reconstruction approaches, mathematical insights, and real-world applications.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Model-Constrained Uncertainty Quantification For Scientific Deep Learning of Inverse Problems

Russell Philley[1], Hai V. Nguyen[2], and Tan Bui-Thanh[1,2]
[1]) The Oden Institute, the University of Texas at Austin
tanbui@oden.utexas.edu
[2]) The Department of Aerospace Engineering and Engineering Mechanics, the University of Texas at Austin

Principled Uncertainty quantification (UQ) in deep learning is still an unsolved problem. Numerous methods have been developed so far, with Bayesian neural networks (BNNs) as the popular approach. BNNs, while inherently UQ-enabled and resistant to over-fitting, suffer from unnatural and artificial priors over their parameters. This talk develops a model-constrained framework for quantifying the uncertainty in deep neural network inverse solutions. At the heart of our approach is an interpretable and physically meaningful prior over neural network parameters trained through the use of Stein variational gradient descent (SVGD). We provide comprehensive numerical results for a 2D inverse heat conductivity problem and a 2D inverse initial conditions problem for both the time-dependent Burgers' and Navier-Stokes equations.

Optimal transport methods for inverse problems regularization

Marcello Carioni[1]
[1]) University of Twente
m.c.carioni@utwente.nl

In this talk, I will cover recent advancements regarding Optimal Transport methods to learn end-to-end reconstructions and regularizations for ill-posed inverse problems in an unsupervised way. First, I will focus my attention on cycle-based architectures, that use WGAN-type losses as distribution distances during training [1, 2]. Then, I will explain how regularizers can be learnt naturally as the outcome of such training frameworks [3, 1]. Finally, I will demonstrate the value of such methods by presenting numerical reconstructions of relevant inverse problems, such as X-ray computed tomography (CT).

References

[1] S. Mukherjee, M. Carioni, O. Öktem, and C.-B. Schönlieb, End to-end reconstruction meets data-driven regularization for inverse problems, In Advances in Neural Information Processing Systems, volume 34, pages 21413–21425, 2021

[2] B. Sim, G. Oh, J. Kim, C. Jung, and J. C. Ye, Optimal transport driven CycleGAN for unsupervised learning in inverse problems, SIAM Journal on Imaging Sciences, 13(4):2281–2306, 2020.

[3] S. Lunz, O. Öktem, and C.-B. Schönlieb Adversarial regularizers in inverse problems, In Advances in neural information processing systems, volume 31, pages 8507–8516, 2018.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

VAEs with structured image covariance as priors to inverse imaging problems

Margaret A G Duff[1], Ivor J A Simpson[2], Matthias J Ehrhardt[3], Neill D F Campbell[3]
[1]) Science and Technology Facilities Council (UKRI)
margaret.duff@stfc.ac.uk
[2]) University of Sussex
[3]) University of Bath

This talk discusses how generative models can be used as priors for inverse problems, penalizing image reconstructions far from images produced by a generator trained on example high-quality reconstructed images. As in we call this approach *generative regularisers*. The idea is that learned regularization will provide complex data-driven priors to inverse problems while still retaining the control and insight of a variational regularization method.

Consider a linear forward model $A : \mathcal{X} \to \mathcal{Y}$ from observed measurements, $y \in \mathcal{Y} = \mathbb{C}^m$. A trained generator $G : Z \to X$, takes values in a known lower dimensional latent space, Z and outputs images similar to some training set. One common approach for using generative regularisers, introduced by Bora et al. and related work by ; ; , searches through the range of the generator to solve the inverse problem:

$$\arg\min_x d(Ax, y) + \mathcal{R}(x) \tag{1}$$

$$\mathcal{R}(x) = \min_{z \in Z} \iota_{\{0\}}(x - G(z)) \tag{2}$$

where $d : \mathcal{Y} \times \mathcal{Y} \to [0, \infty]$ is a similarity measure, ensuring that the reconstructed image matches the data, and $\mathcal{R} : \mathcal{X} \to [0, \infty]$ is the generative regulariser.

In this talk, based on we utilize variational autoencoders (VAEs) that generate not only an image, $G(z)$ but also a covariance uncertainty matrix, $\Sigma(z)$ for each point in the latent space. The covariance can model changing uncertainty dependencies caused by structure in the image, such as edges or objects, and provides a new distance metric from the manifold of learned images. The resulting regulariser can be written:

$$R(x) = \min_{z \in Z} \left(\log(|\Sigma(z)|) + \frac{\|x - G(z)\|^2_{\Sigma(z)}}{2} + \frac{\|z\|_2^2}{2} \right) \tag{3}$$

where we denote the weighted norm by $\|x\|^2_M := x^T M^{-1} x$ and the determinant of a matrix Σ by $|\Sigma|$.

We evaluate these novel generative regularizers on retrospectively sub-sampled real-valued MRI measurements from the fastMRI dataset. We compare the learned regularization against other unlearned regularization approaches and unsupervised and supervised deep learning methods.

References

Duff, M., Campbell, N. D. F. and Ehrhardt, M. J. (2024) 'Regularising Inverse Problems with Generative Machine Learning Models', J. Math. Imaging Vis., 66(1), pp. 37–56

Bora, A., Jalal, A., Price, E. and Dimakis, A.G., 2017, July. Compressed sensing using generative models. In International conference on machine learning (pp. 537-546). PMLR.

Dhar, M., Grover, A. and Ermon, S., 2018, July. Modeling sparse deviations for compressed sensing using generative models. In International Conference on Machine Learning (pp. 1214-1223). PMLR.

Habring, A. and Holler, M., 2022. A generative variational model for inverse problems in imaging. SIAM Journal on Mathematics of Data Science, 4(1), pp.306-335.

Tripathi, S., Lipton, Z.C. and Nguyen, T.Q., 2018. Correction by projection: Denoising images with generative adversarial networks.

Duff, M.A., Simpson, I.J., Ehrhardt, M.J. and Campbell, N.D., 2023. VAEs with structured image covariance applied to compressed sensing MRI. Physics in Medicine Biology, 68(16), p.165008.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Neural wave-based imaging with amortized uncertainty quantification

Felix J. Herrmann[1], Rafael Orozco[1], Ali Siahkoohi[2], and Ziyi (Francis) Yin[1]
[1]) School of Computational Science and Engineering, Georgia Institute of Technology
felix.herrmann@gatech.com
[2]) Department of Computational Applied Mathematics and Operations Research, Rice University

An overview is given on solving high-dimensional Bayesian Inference with expensive nonlinear (wave-based) forward operators with applications in medical and seismic imaging. After improving MAP estimates with conditional Normalizing Flows (CNFs), a full characterization of the posterior will be given with Bayesian Variational Inference using Simulation-Based Inference. Two complementary techniques to close the amortization gap will be reviewed and evaluated on realistic medical and seismic imaging problems. One is amortized and based on recursive refinements [1] (by improving fiducial points, $\mathbf{x}_0$) of the score-based summary statistics [3, 4], $\bar{\mathbf{y}} = \nabla_{\mathbf{x}} \log p(\mathbf{y}|\mathbf{x})\big|_{\mathbf{x}_0}$, so that $p(\mathbf{x}|\bar{\mathbf{y}}) \approx p(\mathbf{x}|\mathbf{y})$ in

$$\hat{\theta} \ = \arg\min_{\theta} \frac{1}{M} \sum_{m=1}^{M} \left(\|f_\theta(\mathbf{x}^{(m)}; \bar{\mathbf{y}}^{(m)})\|_2^2 - \log|\det \mathbf{J}_{f_\theta}| \right)$$

where $f_\theta(\cdot;\cdot)$ is a CNF with weights θ and Jacobian, $\mathbf{J}_{f_\theta}$, while the other corrects the amortized CNF (obtained by forward KL) by minimizing the reverse KL

$$\underset{\mathbf{x}_{1:N},\boldsymbol{\phi}}{\text{minimize}} \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2\sigma^2} \|\mathcal{F}(\mathbf{x}_i) - \mathbf{y}^{\text{obs}}\|_2^2 + \\ \frac{1}{2\gamma^2} \|\mathbf{x}_i - f_{\hat{\boldsymbol{\theta}}}^{-1}\left(h_{\boldsymbol{\phi}}(\mathbf{z}_i);\bar{\mathbf{y}}^{\text{obs}}\right)\|_2^2 + \frac{1}{2}\|h_{\boldsymbol{\phi}}(\mathbf{z}_i)\|_2^2 - \log\left|\det \mathbf{J}_{h_{\boldsymbol{\phi}}}\right|.$$

with $\mathbf{z}_i \sim \mathcal{N}(0,\mathbf{I})$, $h_{\boldsymbol{\phi}}(\cdot)$, the correction NF, and $\mathbf{y}^{\text{obs}}$ the observed data.

References

[1] Orozco, Rafael, et al. "Refining amortized posterior approximations using gradient-based summary statistics." arXiv preprint arXiv:2305.08733 (2023).

[2] Siahkoohi, Ali, et al. "Reliable amortized variational inference with physics-based latent distribution correction." Geophysics 88.3 (2023): R297-R322.

[3] Orozco, Rafael, et al. "Adjoint operators enable fast and amortized machine learning based Bayesian uncertainty quantification." Medical Imaging 2023: Image Processing. Vol. 12464. SPIE, 2023.

[4] Yin, Ziyi, et al. "WISE: full-Waveform variational Inference via Subsurface Extensions." arXiv preprint arXiv:2401.06230 (2023).

Generative Sliced MMD Flows for Posterior Sampling in Bayesian Inverse Problems

Johannes Hertrich[1]
[1]) University College London
j.hertrich@ucl.ac.uk

We consider gradient flows with respect to the maximum mean discrepancy (MMD) with negative distance kernel, which is also known as energy distance. In order to achieve computational efficiency, we prove that for certain kernels the MMD coincides with its sliced version. Therefore, all computations can be performed in a one-dimensional setting, where the MMD with negative distance kernel can be evaluated by a simple sorting algorithm with improved computational complexity. This enables us to simulate MMD particle flows in high dimensions for a large number of particles. We approximate these particle flows by neural networks and apply them for generative modeling and posterior sampling in Bayesian inverse problems.

This is a joint work with F. Altekrüger, R. Beinert, J. Chemseddine, P. Hagemann, G. Steidl and C. Wald.

Learning Gradually Non-convex Image Priors Using Score Matching

Erich Kobler[1], Thomas Pock[2]
[1]) Department of Neuroradiology
University Hospital Bonn
erich.kobler@ukbonn.de
[2]) Institute of Computer Graphics and Vision
Graz University of Technology

In this talk, we discuss a unified framework of denoising score-based models in the context of graduated non-convex energy minimization. We show that for sufficiently large noise variance, the associated negative log density – the energy – becomes convex. Consequently, denoising score-based models essentially follow a graduated non-convexity heuristic. We apply this framework to learning generalized Fields of Experts image priors that approximate the joint density of noisy images and their associated variance. These priors can be easily incorporated into existing optimization algorithms for solving inverse problems and naturally implement a fast and robust graduated non-convexity mechanism.

Learning Spatio-Temporal Regularization Parameter Maps for TV-Minimization-based Image Reconstruction

A. Kofler[1], F. Altekrüger[2,3], F. A. Ba[3], C. Kolbitsch[1], E. Papoutsellis[4,5], D. Schote[1], C. Sirotenko[6], F. F. Zimmermann[1], K. Papafitsoros[7]
1) Physikalisch-Technische Bundesanstalt (PTB), Braunschweig and Berlin, Germany
andreas.kofler@ptb.de
2) Humboldt-Universität zu Berlin, Department of Mathematics, Berlin, Germany
3) Technische Universität Berlin, Institute of Mathematics, Berlin, Germany
4) Finden Ltd, Rutherford Appleton Laboratory, Harwell Campus, Didcot, UK
5) Science and Technology Facilities Council, Harwell Campus, Didcot, UK
6) Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
7) School of Mathematical Sciences, Queen Mary University of London, UK

We present a recently proposed approach [1] for estimating entire regularization parameter-maps to be used for variational image reconstruction with a particular focus on dynamic imaging problems. We consider a linear problem of the form

$$\mathbf{z} = \mathbf{A}\mathbf{x}_{\text{true}} + \mathbf{e}, \tag{1}$$

where $\mathbf{z}$ is the measured data obtained by applying the forward operator $\mathbf{A}$ to a ground-truth image $\mathbf{x}_{\text{true}}$ and corrupting it with Gaussian noise.
Our method is based on neural networks (NNs) and algorithm unrolling. More precisely, we work with an end-to-end trainable NN consisting of two sub-modules: the first estimates regularization parameter-maps from an input image, i.e. $\mathbf{\Lambda}_\Theta = \text{NET}(\mathbf{x}_0)$, while the second unrolls T iterations of the primal dual hybrid gradient method [2] to (approximately) solve the problem

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{z}\|_2^2 + \|\mathbf{\Lambda}_\Theta \nabla \mathbf{x}\|_1. \tag{2}$$

By training the NN on a set of input-target pairs $(\mathbf{z}^i, \mathbf{x}^i_{\text{true}})$, where $\mathbf{z}^i$ is given as in (1), we are able to learn $\mathbf{\Lambda}_\Theta$ without access to target parameter-maps, which for large-scale problems would be difficult to obtain, e.g. with methods based on bilevel optimization. We show numerical results and comparisons with other methods on different imaging examples, including dynamic MRI, quantitative MRI and dynamic image denoising.

References

[1] Kofler A. Altekrüger F. Antarou F. A. Kolbitsch C. Papoutsellis E. and Schote D. Sirotenko C. Zimmermann F. F. Papafitsoros K. Learning Regularization Parameter-Maps for Variational Image Reconstruction using Deep Neural Networks and Algorithm Unrolling, SIAM Journal on Imaging Sciences, 16(4)(2023), 2202–2246.

[2] Chambolle, A. Pock T. A First-Order Primal-Dual Algorithm for Convex Problems with Applications to Imaging, Springer Journal of Mathematical Imaging and Vision 40(2011), 120–145.

Bayesian Computation with Plug-and-Play priors for inverse problems in imaging sciences.

Rémi Laumont[1], Valentin De Bortoli[2], Andrés Almansa[3], Julie Delon[3], Alain Durmus[4], Marcelo Pereyra[5]
[1]) Technical University of Denmark (DTU)
real@dtu.dk
[2]) Google Deep Mind
[3]) Université Paris-Cité
[4]) Ecole Polytechnique
[5]) Heriot-Watt University

This talk consists in a presentation of Plug-and-Play (PnP) methods applied to inverse problems encountered in image restoration. Since the work of Venkatakrishnan et al. in [1], PnP methods are often applied for image restoration in a Bayesian context. These methods aim at computing Minimum Mean Square Error (MMSE) or Maximum A Posteriori (MAP) for inverse problems in imaging by combining an explicit likelihood and an implicit a-priori defined by a denoising algorithm. In the literature, PnP methods differ mainly in the iterative scheme used for both optimization and sampling. In the case of optimization algorithms, recent works guarantee the convergence to a fixed point of a certain operator, fixed point which is not necessarily the MAP. In the case of sampling algorithms in the literature, there is no evidence of convergence. Moreover, there are still important open questions concerning the correct definition of the underlying Bayesian models or the computed estimators, as well as their regularity properties, necessary to ensure the stability of the numerical scheme. The aim of this talk is to present simple but efficient restoration methods while answering some of these questions. The existence and nature of MAP and MMSE estimators for PnP prior is therefore a first line of study. Three methods with convergence results are then presented, PnP-SGD for MAP estimation and PnP-ULA and PPnP-ULA for sampling. A particular interest is given to denoisers encoded by deep neural networks. The efficiency of these methods is demonstrated on classical image restoration problems such as denoising, deblurring or interpolation. In addition to allowing the estimation of MMSE, sampling makes possible the quantification of uncertainties, which is crucial in domains such as biomedical imaging. Lastly, the influence of the denoiser on the posterior is investigated.

References

[1] Venkatakrishnan, S.V., Bouman, C.A. and Wohlberg, B., 2013, December. Plug-and-play priors for model based reconstruction. In 2013 IEEE global conference on signal and information processing (pp. 945-948). IEEE.

Unsupervised Neural Networks for Image Reconstruction

Dong Liu[1]
[1]) Department of Biomedical Engineering, University of Science and Technology of China
Dong.liu@outlook.com

Neural networks (NNs) have found extensive applications in tomographic imaging, leveraging data-driven training and image processing techniques. However, a significant challenge lies in the demand for substantial volumes of training data, often scarce in clinical settings.

This presentation aims to introduce novel unsupervised deep learning methodologies developed for addressing image reconstruction challenges in electrical impedance tomography (EIT)[1, 2] and positron emission tomography (PET) [3]. By representing the unknowns, such as conductivity in EIT, with neural networks, the image reconstruction task is reframed as a neural network optimization problem. This unsupervised approach offers efficiency and adaptability in learning without relying on labeled training data. The presentation will showcase quantitative state-of-the-art outcomes obtained from both simulated and experimental data.

References

[1] D Liu, J Wang, Q Shan, D Smyl, J Deng and J F Du. DeepEIT: deep image prior enabled electrical impedance tomography. IEEE Transactions on Pattern Analysis and Machine Intelligence, 45(8):9627-9638, 2023.

[2] J Wang, Y Wang, J Deng and D Liu. Unsupervised Coordinate-Based Neural Network for Electrical Impedance Tomography. IEEE Transactions on Computational Imaging, 9:1213-1225, 2023.

[3] Q Shan, J Wang and D Liu. Deep Image Prior Based PET Reconstruction From Partial Data. IEEE Transactions on Radiation and Plasma Medical Sciences, 2023. DOI 10.1109/TRPMS.2023.3280674

The 11th International Conference on "Inverse Problems: Modeling and Simulation"
26th May - 1st June 2024, Malta

Provably convergent plug-and-play quasi-Newton method

Hong Ye Tan[1], Subhadip Mukherjee[2], Junqi Tang[3], and Carola-Bibiane Schönlieb[1]
[1]Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK
[2]Department of Electronics and Electrical Communication Engineering, IIT-Kharagpur, India
[3]School of Mathematics, University of Birmingham, UK

Plug-and-play (PnP) methods, pioneered by Venkatakrishnan et al. [5], are a class of efficient iterative methods that aim to combine data fidelity terms and deep denoisers using classical optimization algorithms, such as ISTA or ADMM, with applications in inverse problems and imaging (see [3] and references therein). Provable PnP methods are a subclass of PnP methods with convergence guarantees, such as fixed point convergence or convergence to critical points of some energy function [1, 4]. Many existing provable PnP methods impose heavy restrictions on the denoiser or fidelity function, such as *nonexpansiveness* or *strict convexity*, respectively. In this work, we propose a novel algorithmic approach incorporating quasi-Newton steps into a provable PnP framework based on proximal denoisers [2], resulting in greatly accelerated convergence while retaining light assumptions on the denoiser. By characterizing the denoiser as the proximal operator of a weakly convex function, we show that the fixed points of the proposed quasi-Newton PnP algorithm are critical points of a weakly convex function. Numerical experiments on image deblurring and super-resolution demonstrate 2–8x faster convergence as compared to other provable PnP methods with similar reconstruction quality.

References

[1] S. H. Chan, X. Wang, and O. A. Elgendy, *Plug-and-play admm for image restoration: Fixed-point convergence and applications*, IEEE Transactions on Computational Imaging, 3 (2016), pp. 84–98.

[2] S. Hurault, A. Leclaire, and N. Papadakis, *Proximal denoiser for convergent plug-and-play optimization with nonconvex regularization*, 2022.

[3] U. S. Kamilov, C. A. Bouman, G. T. Buzzard, and B. Wohlberg, *Plug-and-play methods for integrating physical and learned models in computational imaging: Theory, algorithms, and applications*, IEEE Signal Processing Magazine, 40 (2023), pp. 85–97.

[4] P. Nair, R. G. Gavaskar, and K. N. Chaudhury, *Fixed-point and objective convergence of plug-and-play algorithms*, IEEE Transactions on Computational Imaging, 7 (2021), pp. 337–348.

[5] S. V. Venkatakrishnan, C. A. Bouman, and B. Wohlberg, *Plug-and-play priors for model based reconstruction*, in 2013 IEEE Global Conference on Signal and Information Processing, IEEE, 2013, pp. 945–948.

Learning spatially-adaptive regularization

Sebastian Neumayer[1]
[1]) TU Chemnitz
sebastian.neumayer@mathematik.tu-chemnitz.de

In this talk, I will introduce a framework for the learning of filter-based regularizers for image data. These can be deployed within a variational reconstruction ansatz for solving generic inverse imaging problems (universality of the regularizer). Such a variational reconstruction ansatz ensures data consistency. Moreover, we are able to derive associated stability guarantees. Both are very important when working in critical applications such as medical imaging, since false diagnosis can have fatal consequences. Interestingly, the learned regularizers closely resemble traditional hand-crafted ones.

After introducing the baseline architecture, I will discuss a refinement of this architecture by conditioning the regularizer on the given measurements based on the initial reconstruction. This mechanism allows to compensate for the rather simple fields-of-experty architecture of the regularizer and adapts it to the actually observed measurements. By carefully designing the conditioning mechanism, we can preserve many of the favorable properties of the initial approach. In particular, learning the conditioning networks (which we will identify as strcture extractors) remains independent of the data. In the last part of the talk, I will present numerical results for denoising and MRI. These indicate that even relatively restricted architectures can achieve highly competitive performance. Below is a list of my publications that are covered in this talk.

References

[1] A. Goujon, S. Neumayer, M. Unser, Learning Weakly Convex Regularizers for Convergent Image Reconstruction Algorithms, SIAM J. Imaging Sci., 17(1)(2024), 91–115.

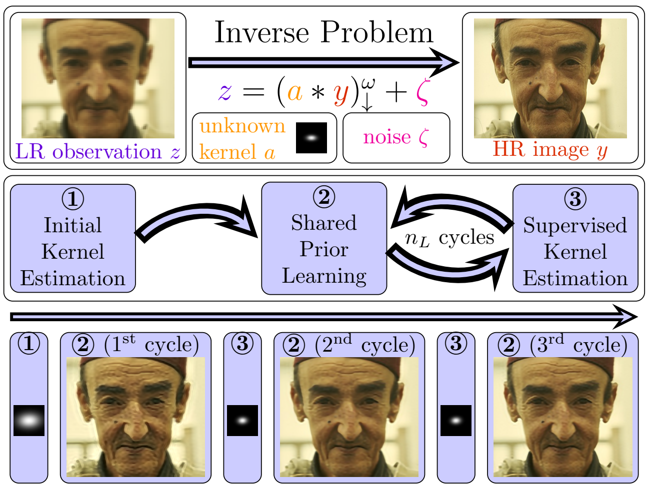
[2] S. Neumayer, M. Pourya, A. Goujon, M. Unser, Boosting Weakly Convex Ridge Regularizers with Spatial Adaptivity, Neurips 2023 Workshop Deep Inverse.

[3] A. Goujon, S. Neumayer, P. Bohra, S. Ducotterd, M. Unser, A Neural-Network-Based Convex Regularizer for Inverse Problems, IEEE Trans. Comput. Imaging, 9(2023), 781–795.

Blind Single Image Super-Resolution via Iterated Shared Prior Learning

Thomas Pinetz[1] , Erich Kobler[2] , Thomas Pock[3] , Alexander Effland[1]
[1]) University of Bonn
pinetz@iam.uni-bonn.de
[2]) Universityclinic Bonn, [3]) University of Graz

This abstract focuses on adapting shared prior learning [1] for blind single image super-resolution (SISR). From a variational standpoint, our objective is to minimize an energy functional comprising a learned data fidelity term and a data-driven prior. The learnable parameters are derived through a mean-field optimal control problem. Our associated loss functional integrates a supervised loss assessed on synthesized observations and an unsupervised Wasserstein loss, comparing local image statistics, for real observations. In shared prior learning, only the parameters of the prior are shared between both loss functions. The kernel estimate is iteratively updated following each step of shared prior learning. A visualization of this process is depicted in the figure below:



Through numerous numerical experiments, we attain state-of-the-art results for blind single image super-resolution (SISR) using a minimal number of learnable parameters and small training sets tailored to real-world applications.

References

[1] Thomas Pinetz, Erich Kobler, Thomas Pock and Alexander Effland, Shared prior learning of energy-based models for image reconstruction, SIAM Journal on Imaging Sciences, 14(4)(2021), 1706–1748.

Learned reconstruction methods for inverse problems: sample error estimates

Luca Ratti
University of Bologna
luca.ratti5@unibo.it

Learning-based and data-driven techniques have recently become a subject of primary interest in the field of reconstruction and regularization of inverse problems. Their key aspect is the possibility to leverage large datasets and statistical learning techniques to encode prior information about the solution and to guide its reconstruction. Besides the development of novel methods, yielding excellent results in several applications, their theoretical investigation has attracted growing interest, e.g., on the topics of reliability, stability, and interpretability.

In this talk, I will describe a general framework allowing us to interpret many of these techniques in the context of (supervised) statistical learning. The main goal of this presentation is then to address the generalization properties of learned reconstruction methods, and specifically to perform their sample error analysis. This task, well-developed in statistical learning, is to estimate the dependence of the learned operators with respect to the data employed for their training. A rather general strategy is proposed, whose assumptions are met for a large class of inverse problems and learned methods, as shown in [1].

I will present two main examples: in the first one, analyzed in [2], we focus on learning the optimal reconstruction operator within the family of (generalized) Tikhonov regularizers, also comparing with an unsupervised technique. The second one, devoted to learning the optimal synthesis operator in sparsity-promoting regularization, is the object of ongoing work.

This is based on a joint project together with G. S. Alberti, E. De Vito, M. Santacesaria (University of Genoa), M. Lassas (University of Helsinki), and T. Helin (LUT).

References

[1] Ratti, Luca. "Learned reconstruction methods for inverse problems: sample error estimates." arXiv preprint arXiv:2312.14078 (2023).

[2] Alberti, Giovanni S., et al. "Learning the optimal Tikhonov regularizer for inverse problems." Advances in Neural Information Processing Systems 34 (2021): 25205-25216.

Deep Inverse Rosenblatt Transport for Bayesian Inverse Problems

Robert Scheichl[1], Tiangang Cui[2], Sergey Dolgov[3], Roland Herzog[1], Karina Koval[1]
[1]) Institute for Mathematics and Interdisciplinary Center for Scientific Computing, Heidelberg University, Germany, r.scheichl@uni-heidelberg.de
[2]) School of Mathematics and Statistics, The University of Sydney, Australia
[3]) Department of Mathematical Sciences, University of Bath, UK

General multivariate distributions are notoriously expensive to sample from, particularly the high-dimensional posterior distributions in PDE-constrained inverse problems. In this talk, I will present a measure transport based approach for Bayesian inverse problems based on low-rank surrogates in the tensor train format, a methodology that has been exploited for many years for scalable, high-dimensional density function approximation in quantum physics and chemistry. We build upon recent developments in the field of cross approximation algorithms in linear algebra to construct a tensor train approximation to the target probability density function using a small number of function evaluations. For sufficiently smooth distributions, the storage required for accurate tensor train approximations is moderate, scaling linearly with dimension. In turn, the structure of the tensor train (TT) surrogate allows sampling by an efficient conditional distribution method since marginal distributions are computable with linear complexity in dimension [1]. Using this generic tool enables conditional sampling, construction of optimal biasing densities or even tractable Bayesian optimal experimental design. In order to keep the arising ranks in the TT approximations manageable, we furthermore propose a 'deep' version of the approach where the transport from reference to target distribution is approximated incrementally via intermediate bridging densities.

The method is demonstrated in the context of complex PDE-constrained Bayesian inverse problems, computing expectations of functionals of the PDE solution with respect to high-dimensional posterior distributions, including rare event probabilities [2] and optimal experimental designs [3].

References

[1] S. Dolgov, K. Anaya-Izquierdo, C. Fox, and R. Scheichl, Approximation and sampling of multivariate probability distributions in the tensor train decomposition, Statistics and Computing, 30 (2020), 603–625.

[2] T. Cui, S. Dolgov, and R. Scheichl, Deep importance sampling using tensor trains with application to a priori and a posteriori rare events, SIAM Journal on Scientific Computing 46(1) (2024), C1–C29.

[3] K. Koval, R. Herzog, and R. Scheichl, Tractable optimal experimental design using transport maps, Preprint, arXiv:2401.07971 (2024).

Specimen reconstruction in atom probe tomography as an inverse problem

Aslam Shaikh[1], Nuutti Hyvönen[2], Mikko Alava[1], Ivan Lomakin[1]
[1]) Aalto University, Dept. of Applied Physics
aslam.shaikh@aalto.fi, mikko.alava@aalto.fi, ivan.lomakin@aalto.fi
[2]) Aalto University, Dept. of Mathematics and System Analysis
nuutti.hyvonen@aalto.fi

Atom probe tomography (APT) is a powerful microanalytical technique that provides quantitative three-dimensional compositional mapping of a wide range of materials. In APT, the tip specimen is irreversibly destroyed as the surface atoms are desorbed gradually and projected onto the time-resolved position-sensitive detector in the process known as field-evaporation. The information collected at the end of the analysis is the 2D detector coordinates of the atom, the evaporation sequence ID of the atoms, time-of-flight (TOF) information to identify the chemical identity of atoms and other various information mostly pertaining to the physics of evaporation. The most important aspect of atom probe analysis is the reconstruction of the specimen so that the microstructural features of the specimen could be visualized and studied. To this end, APT community has, over the years, come up with geometrical-based reconstruction techniques, mostly based on the idea of point projection that works fairly well to some level of accuracy even though there is a lot of room for improvement in terms of lateral precision[1]. The modeling of field-evaporation process (forward problem) is not fully understood yet even though there are models that work well enough for single-element specimens but it gets challenging when two or more elements are involved[2, 4]. However, the detector image shows a clear symmetry in the patterns created which serves as an empirical evidence that field evaporation process adheres to a systematic and predictable set of principles. Moreover, prior information about the crystallography and geometry of the specimen analyzed is easily accessible. Given these information, we have attempted to formulate specimen reconstruction problem as an inverse problem (*within the Bayesian framework*). The principal challenge associated with this problem is a loss of a dimension as the 3D initial system of interest i.e. tip specimen collapses to "*distorted*" 2D coordinates of their imprint on the detector leading to the loss of depth information i.e. z-coordinates. This intrinsic limitation significantly complicates the accurate reconstruction of the specimen. Nonetheless, the ions hitting the detector are also tagged with the evaporation sequence of arrival that could potentially compensate, at least partially, for the loss of that dimension. Even though, APT deals with 3D reconstruction from reduced-dimensional data, it differs from other imaging modalities like X-ray tomography where data from multiple perspectives are fused together to reconstruct the object of interest [3]. Instead, there is only one projection but with spatial correlations captured (with significant noise factor) in all three directions - (x, y) almost linearly mapped to (X, Y) on the detector and z-coordinate correlated to the sequence of arrival. As a next step, the resulting reconstruction is quantitatively assessed for its uncertainty, enabling a comprehensive evaluation of the confidence in the reconstruction.

References

[1] Williams Lefebvre, Francois Vurpillot, and Xavier Sauvage. *Atom probe tomography: put theory into practice*. Academic Press, 2016.

[2] F. Vurpillot and C. Oberdorfer. "Modeling atom probe tomography: A review". *Ultramicroscopy*, vol. 159, pp. 202–216, 2015. Elsevier.

[3] Jari Kaipio and Erkki Somersalo. *Statistical and computational inverse problems*, vol. 160. Springer Science & Business Media, 2006.

[4] Nicolas Rolland, François Vurpillot, Sébastien Duguay, and Didier Blavette. "A meshless algorithm to model field evaporation in atom probe tomography". *Microscopy and Microanalysis*, vol. 21, no. 6, pp. 1649–1656, 2015. Cambridge University Press.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Implicit Regularization in Diagonal Linear Networks

Johan S. Wind
University of Oslo
johanswi@uio.no

Understanding the implicit regularization imposed by neural network architectures and gradient-based optimization methods is a key challenge in deep learning. We study Diagonal Linear Networks (DLNs), a simple model which displays implicit regularization towards sparse solutions. In particular, DLNs can perform sparse recovery by simply fitting the observations with vanilla gradient descent, even when there is no explicit regularization.

We show that the gradient flow of DLNs with tiny initialization approximates minimizers of the basis pursuit optimization problem. Through fine-grained analysis, we obtain new and sharp convergence bounds w.r.t. the initialization size. The sharpness of our results requires us to consider hard instances, such as when the minimizer to the basis pursuit problem is not unique. These hard instances lead to interesting connections to the theory of computability.

This is joint work with Vegard Antun and Anders C. Hansen.

MINISYMPOSIUM
M9. Inverse Problems for Fractional Equations

Organizers:
Giovanni Covi, University of Helsinki, Finland, giovanni.covi@helsinki.fi
Barbara Kaltenbacher, University of Klagenfurt, Austria, Barbara.Kaltenbacher@aau.at
Mikko Salo, University of Jyväskylä, Finland, mikko.j.salo@jyu.fi

Fractional order derivatives and related partial differential equations have been actively studied in various areas of mathematics. Recently they have become popular in the field of inverse problems, due to their relevance in applied scientific modeling and also because of their novel mathematical features. This minisymposium is concerned with the mathematical and computational analysis of inverse problems for fractional partial differential equations.

We aim to bring together experts and young researchers working in this field to discuss new results and advances in inverse problems for fractional equations.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Instability results for the fractional Calderón problem

Hendrik Baers[1,*], Giovanni Covi[2], Angkana Rüland[1]
[1]) Institute for Applied Mathematics, University of Bonn, Endenicher Allee 60, 53115 Bonn, Germany
[2]) University of Helsinki, Department of Mathematics and Statistics, Pietari Kalmin katu 5, 00014 Helsinki
[*]) E-mail: hendrik.baers@uni-bonn.de

The Calderón problem is about determining the conductivity of a medium by making voltage and current measurements on its boundary. We consider the fractional formulation of the problem and prove exponential instability. Doing so, we are able to extend already known instability results for this formulation to a more general setting, involving general geometries and scaling critical regularity of the potentials.

A reduction of the nonlocal Calderón problem to the local Calderón problem

Giovanni Covi[1]
[1]) University of Helsinki
giovanni.covi@helsinki.fi

We relate the anisotropic variable coefficient local and nonlocal Calderón problems by means of the Caffarelli-Silvestre extension. We prove that (partial) Dirichlet-to-Neumann data for the fractional Calderón problem in two and higher dimensions determine the (full) Dirichlet-to-Neumann data for the local Calderón problem. As a consequence, any uniqueness result for the local problem also implies a uniqueness result for the nonlocal problem. We also highlight obstructions for reversing this procedure, which essentially consist of two one-dimensional averaging processes. This is a joint work with Professors Tuhin Ghosh, Angkana Rüland, and Gunther Uhlmann [1].

References

[1] G. Covi, T. Ghosh, A. Rüland and G. Uhlmann, A reduction of the fractional Calderón problem to the local Calderón problem by means of the Caffarelli-Silvestre extension, preprint: arXiv:2305.04227.

A non-local inverse problem with boundary response.

Tuhin Ghosh[1]
[1]Harish-Chandra Research Institute, HBNI, India.
tuhinghosh@hri.res.in

In this talk, we focus on the (nonlocal) inverse problem of recovering a potential based on the boundary measurement associated with the fractional Schrödinger equation. Let $0 < a < 1$, and u solves

$$\begin{cases} ((-\Delta)^a + q)\, u = 0 \text{ in } \Omega \\ supp\, u \subseteq \overline{\Omega} \cup \overline{W} \\ \overline{W} \cap \overline{\Omega} = \emptyset. \end{cases}$$

We show that by making the exterior to boundary measurement as $\left(u|_W, \frac{u(x)}{d(x)^a}\big|_\Sigma\right)$, it is possible to determine q uniquely in Ω, where $\Sigma \subseteq \partial\Omega$ be a non-empty open subset and $d(x) = d(x, \partial\Omega)$ denotes the boundary distance function.

We also discuss local characterization of the large a-harmonic functions in ball and some its applications.

References

[1] Gerd Grubb. Fractional Laplacians on domains, a development of Hörmander's theory of µ-transmission pseudodifferential operators. Adv. Math., 2015.

[2] Xavier Ros-Oton and Joaquim Serra. The Dirichlet problem for the fractional Laplacian: regularity up to the boundary. J. Math. Pures Appl., 2014.

[3] Tuhin Ghosh, Mikko Salo, and Gunther Uhlmann. The Calderón problem for the fractional Schrödinger equation. Anal. PDE, 2020

Inverse problems for some fractional equations with general non-linearity

Pu-Zhao Kow
National Chengchi University
pzkow@g.nccu.edu.tw

In this talk, we will discuss some inverse problems of uniquely identifying coefficients in nonlinear terms from over-determined data. We prove global uniqueness under quite general assumptions of the nonlinearity, including Kerr-type (cubic) nonlinearity, the Ginzburg-Landau (cubic-quintic) nonlinearity as well as Hartree (convolution-type) nonlinearity.

References

[1] Pu-Zhao Kow and Jenn-Nan Wang, Inverse problems for some fractional equations with general nonlinearity, Res. Math. Sci. 10(4)(2023), Paper No. 45, 33 pages. doi:10.1007/s40687-023-00409-8

[2] Pu-Zhao Kow, Shiqi Ma and Suman Kumar Sahoo, An inverse problem for semilinear equations involving the fractional Laplacian, Inverse Problems 39(9) (2023), Paper No. 095006, 27 pages. doi:10.1088/1361-6420/ace9f4

Uniqueness Results for Fractional Inverse Problems

Yi-Hsuan Lin*
*National Yang Ming Chiao Tung University
yihsuanlin3@gmail.com

In this talk, we would like to introduce some progress of inverse problems for fractional equations. We demonstrate several uniqueness results could be established via their exterior measurements.

The talk is based on several joint works from the papers below.

References

[1] T. Ghosh, Y.-H. Lin, J. Xiao, The Caldern problem for variable coefficient nonlocal elliptic operators. Comm. Partial Differential Equations, Vol. 42, No. 12 (2017), 1923-1961.

[2] X. Cao, Y.-H. Lin, H. Liu, Simultaneously recovering potentials and embedded obstacles for anisotropic fractional Schrödinger operators. Inverse Probl. Imaging, Vol. 13, Issue 1 (2019), 197-210.

[3] Y.-H. Lin, H. Liu. Inverse problems for fractional equations with a minimal number of measurements. Comm. Anal. Comp., Vol. 1 (2023), 72-93.

[4] C.-L. Lin, Y.-H. Lin, G. Uhlmann, The Caldern problem for nonlocal parabolic operators: A new reduction from the nonlocal to the local, arXiv:2308.09654.

[5] Y.-H. Lin. Determining both leading coefficient and source in a nonlocal elliptic equation. arXiv:2312.15607.

Recovery of coefficients in parabolic and wave equations from time trace data

William Rundell
Department of Mathematics, Texas A&M University, Texas 77843
rundell@math.tamu.edu

Almost a half century ago, it was shown that $Q(x)$ in $u_t - u_{xx} + Q(x)u = 0$ could be recovered from boundary measurements $u(x_0, t)$ [1]. The solution has the representation $u(x,t) = \sum_{n=1}^{\infty} a_n \phi_n(x) e^{-\lambda_n t}$, $x \in (a,b)$, $t \geq 0$, where $\langle \lambda_n, \phi_n \rangle$ are the eigenvalues/eigenfunctions of $-u_{xx} + Q(x)u = \lambda u$ subject to boundary conditions on a and b, and also an initial condition, $u(x,0) = u_0$. Knowing a time trace $u(x,0) = u_0$ for $t > 0$ allowed the recovery of the spectrum $\{\lambda_n\}$ and "norming constants" for $\phi_n(x)$. In turn it leads to uniqueness of $Q(x)$ and a means of reconstruction. This is the classical inverse Sturm-Liouville problem and dates back to Borg in 1946 and Gel'fand and Levitan in 1951.

Actually, for uniqueness one needs only have the time-values on any open interval as the time trace is an analytic function. However, reconstruction is another matter entirely: only small times measurements are useful since $\lambda_n = cn^2$.

In many diffusion processes this data isn't always obtainable and in fact **only** large time values can be measured; this is the theme of the talk.

The situation under the fractional diffusion operator of Djrbashian type, which utilises the Mittag-Leffler function in place of the exponential, allows a considerable relaxation of the time measurement interval, but the main issue, while ameliorated, remains. We show that both uniqueness and moderately effective reconstructability can be ob- tained through multiple experiments under a *small* number of samples of measurements.

Our spatial operator may well have multiple coefficients; for example $(a(x)u_x)_x + q(x)u$, and both a and q have to be obtained. However, the above formulation will not allow multiple recovery. This due to the fact that a general linear second order equation can be converted into canonical form $-u_{xx} + Q(x)u = \lambda u$ by the Liouville transformation. A similar paradigm holds for the damped wave equation.

However, we show an example where a nonlinearity term annuls the Liouville transform and in fact two spatially-dependent coefficients can be obtained from time trace data.

References

[1] W Rundell, D.L. Colton, Determination of an unknown non-homogeneous term in a linear partial differential equation from overspecified boundary data, *Applicable Analysis*, 10 (3) (1980), 231-242

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

On the fractional Calderón Problem

Angkana Rüland[1]
[1]) University of Bonn
rueland@uni-bonn.de

In this talk I revisit the fractional, anisotropic Calderón problem. Firstly, I will connect it to the local, anisotropic Calderón problem. Secondly, I will revisit the fractional anisotropic Calderón problem studied by Feizmohammadi and Feizmohammadi-Ghosh-Krupchyk-Uhlmann and provide a variable coefficient Caffarelli-Silvestre perspective on it. The talk is based on joint work with G. Covi, T. Ghosh and G. Uhlmann.

Inverse coefficient problems for the heat equation with fractional Laplacian

Durvudkhan Suragan
Department of Mathematics, Nazarbayev University
durvudkhan.suragan@nu.edu.kz

In this talk, we discuss inverse problems related to determining the time-dependent coefficient and unknown source function of fractional heat equations. Our approach shows that having just one set of data at an observation point ensures the existence of a weak solution for the inverse problem. Furthermore, if there is an additional datum at the observation point, it leads to a specific formula for the time-dependent source coefficient. Moreover, we investigate inverse problems involving non-local data and recovering the space-dependent source function of the fractional heat equation.

The talk is based on our recent results from [1] and [2].

References

[1] A. Mamanazarov and D. Suragan, Inverse coefficient problems for the heat equation with fractional Laplacian, preprint, (2024), 1–25.

[2] I. Ismailov, T. Ozawa and D. Suragan, Inverse problems of identifying the time-dependent source coefficient for subelliptic heat equations, Inverse Problems and Imaging, 1(2023), 1–11.

Identification of multi-parameters in a time-fractional diffusion-wave equation

T. Wei[1], R. D. Deng[2]
[1] Lanzhou University
tingwei@lzu.edu.cn
[2] Lanzhou University
120220907830@lzu.edu.cn

I will talk an inverse problem for recovering the order of fractional derivative and a time-dependent potential coefficient in a time-fractional diffusion wave equation by an integral condition or one point measurement on the boundary. The Lipschitz continuity of the forward operators from the unknown order and coefficient to the given data are achieved in terms of the integral equation held by the solution of the direct problem. We also obtain the uniqueness for the considered inverse problems in terms of somewhat general conditions to the given functions. Moreover, we propose a Tikhonov-type regularization method and prove the existence of the regularized solution and its convergence to the exact solution under a suitable regularization parameter choice. Then we use a linearized iteration algorithm to recover numerically the order and time-dependent potential coefficient simultaneously.

Two numerical examples for one- and two-dimensional cases are provided to display the efficient of the proposed method.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Calderón type inverse problem for nonlocal porous medium equations with linear absorption term

Philipp Zimmermann
Departament de Matemàtiques i Informàtica, Universitat de Barcelona
Barcelona, Spain
Email: philipp.zimmermann@ub.edu

In this talk we discuss a Calderón type inverse problem for the nonlocal porous medium equation (NPME) $\rho\partial_t u + L_K(u^m) + qu = 0$, where $m > 1$ and L_K is an integro-differential operator with kernel $K(x,y)/|x-y|^{n+2s}$. The unique determination result presented in this talk depends on two essential ingredients:

1. Unique determination in the elliptic nonlocal inverse problem $L_K u = 0$ (only using non-negative exterior conditions)

2. Suitable comparison principle for the NPME

After reviewing these, we will explain how an asymptotic analysis can be used to solve the Calderón type inverse problem for the NPME. That is, it allows us to uniquely determine the coefficient ρ, the potential q and the kernel K from the related Dirichlet-to-Neumann map $\Lambda_{\rho,K,q}$. We finish the talk by posing some related open questions.

MINISYMPOSIUM
M10. Inverse and Control Problems for Evolution Equations and Variational Inequalities

Organizer:
Marian Slodička, Ghent University, Belgium, marian.slodicka@ugent.be

In the last twenty years, the field of inverse problems has undergone rapid development. The enormous increase in computing power, together with powerful numerical methods, has made it possible to simulate real-world direct problems of growing complexity. Many applications in science and engineering lead to inverse and control problems, which in turn stimulate mathematical research e.g., on uniqueness questions and on developing stable and efficient numerical methods for solving them.

Evolution partial differential equations are frequently used to model transient processes. Variational inequalities are mathematical models used to describe a wide range of dynamic phenomena such as the evolution of physical systems, economic processes, and social dynamics. Solving inverse problems involves formulating an appropriate mathematical model that incorporates available data and unknown parameters. Various computational and mathematical techniques, such as optimization methods, regularization techniques, and numerical algorithms, have been employed to reconstruct unknown quantities and obtain a reliable solution.

The aim of this mini-symposium is the exchange of state-of-the-art knowledge in theoretical and numerical research on inverse, backward, and coefficient identification settings for (non)linear evolution problems. We would like to bring together a wide range of experts in this mini-symposium to discuss the theoretical, practical, and numerical aspects of the subject.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Identification of cavities in a nonlinear model arising from electrophysiology

Andrea Aspri
Department of Mathematics University of Milan
Via Festa del Perdono 7 - 20122 Milano, ITALY
andrea.aspri@unimi.it

Detecting ischemic regions is paramount in preventing potentially fatal ventricular ischemic tachycardia. Traditionally, this involves capturing the heart's electrical activity through noninvasive or minimally invasive methods, such as body surface or intracardiac measurements. Insight into utilizing electrical measurements for ischemia detection can be gained through mathematical and numerical models of cardiac electrophysiology. The ultimate objective is to integrate boundary measurements of potentials with a mathematical model of the heart's electrical activity to pinpoint the location, shape, and size of ischemic regions and/or infarctions. A promising approach involves modeling ischemic regions as electrical insulators using the monodomain model. This model, a semilinear reaction-diffusion system, provides a comprehensive description of cardiac electrical activity.

I will show that perfectly insulating regions can be uniquely determined by partial boundary measurements of the potential.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Stability estimates for some coefficients in a quantitative thermo-acoustic-tomography model.

Michel Cristofol[1]
[1]) Aix Marseille Univ, CNRS, Centrale Marseille, I2M, France
michel.cristofol@univ-amu.fr

This talk is concerned with the determination of coefficients and source term in a strong coupled quantitative thermoacoustic system of equations,

$$\begin{cases} \partial_t^2 p - \rho v_s^2 \mathrm{div}\left(\frac{1}{\rho}\nabla p\right) - \Gamma\partial_t\{\mathrm{div}\,(\kappa\nabla\theta)\} = \Gamma\partial_t\Pi_a, \\ \partial_t\theta - \frac{1}{\rho C_p}\mathrm{div}\,(\kappa\nabla\theta) - \frac{\theta_0\varsigma}{\rho C_p}\partial_t p = \frac{\Pi_a}{\rho C_p}, \end{cases}$$

for the temperature rise θ and the pressure perturbation p.
The methodology used in this paper is based on Carleman estimates [2, 3]. Adapting a Carleman estimate established in [3] we prove stability estimates of Hölder type involving the observation of only one component: the temperature or the pressure [1].

References

[1] M. Cristofol, S. Li and Y. Shang, Carleman estimates and some inverse problems for the coupled quantitative thermoacoustic equations by partial boundary layer data. Part II: Some inverse problems *Math. Meth. Appl. Sci* (2023) 1-16

[2] A. L. Bugkheim and M.V. Klibanov, Global uniqueness of class of multidimentional inverse problems, *Soviet Mathematics Doklady* **24** (1981), 244–247.

[3] S. Li and Y. Shang, Carleman estimates and some inverse problems for the coupled quantitative thermoacoustic equations by boundary data. Part I: Carleman estimates, *Journal of Inverse and Ill-Posed Problems* **30** (2022), 621–658.

Inverse problems for simultaneous determination of source terms and several scalar parameters of fractional diffusion-wave equations

Jaan Janno
Department of Cybernetics, Tallinn University of Technology, Estonia
jaan.janno@taltech.ee

We will consider inverse problems to determine simultaneously unknown source functions and parameters of fractional diffusion-wave equations from measurements in a neighborhood of a final time value.

We will focus on cases when thanks to a certain irregularity the problem can be decomposed into two stages:
1) determination of the parameters;
2) reconstruction of the source.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Stability of the determination of time-dependent coefficients of wave equation in infinite waveguide

Manmohan Vashisth[1], Mandeep Kumar[2], and Rajat Bansal[1]
[1]) Indian Institute of Technology Ropar, Punjab, India, 140001
manmohanvashisth@iitrpr.ac.in
[2]) Indian Institute of Technology Jammu. Jammu and Kashmir, India, 181221
2021rma1029@iitjammu.ac.in

We consider the stability of the inverse problem of determining two time-dependent coefficients of a hyperbolic equation in an infinite cylindrical domain, from input-output map. We prove logarithmic stability in damping coefficients and double logarithmic stability in potentials using geometrical optics solutions specifically designed for hyperbolic waveguides.

.

Adaptive finite element method for electromagnetic coefficient inverse problem in conductive media

Eric Lindström[1], Larisa Beilina[2]
[1]) Chalmers University of Technology, University of Gothenburg
erilinds@chalmers.se
[2]) Chalmers University of Technology, University of Gothenburg
larisa.beilina@chalmers.se

This talk will be an overview of our recent articles concerning a new hybrid method used in medical imaging applications (see [1], [2] and [3]). The aim of this method is to reconstruct the spatial function representing electric permettivity of various tissues in a realistic breast phantom. This is useful since information of interior electromagnetic properties can reveal locations and shapes of tumours.

Mathematically this endeavour is studied as a coefficient inverse problem of a constellation of Maxwell's equations. This in turn is approached as an optimization problem where we introduce the corresponding Tikhonov function as well as the Lagrangian.

The talk will not only review the methods used to solve our optimization practically, but also mention some theoretical results surrounding our system. These theoretical results include stability and a priori estimates of our forward problem and a posteriori estimates concerning the Tikhonov functional.

References

[1] Beilina, L.; Lindström, E. (2022) An Adaptive Finite Element/Finite Difference Domain Decomposition Method for Applications in Microwave Imaging *Electronics 2022, 11, 1359*

[2] Beilina, L.; Lindström, E. (2023) A Posteriori Error Estimates and Adaptive Error Control for Permittivity Reconstruction in Conductive Media *"Gas Dynamics with Applications in Industry and Life Sciences"; Springer: Cham*

[3] Lindström, E.; Beilina, L. (2024) Energy norm error estimates and convergence analysis for a stabilized Maxwell's equations in conductive media *arXiv 2023, arXiv:2312.13049. To appear in Applications of Mathematics, Springer.*

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Determining a space-dependent source in thermoelastic systems

Frederick Maes[1], Karel Van Bockstal[2]

[1]) Research Group NaM[2], Department of Electronics and Information Systems, Ghent University, Krijgslaan 281, B 9000 Ghent, Belgium
frederick.maes@ugent.be
[2]) Ghent Analysis & PDE Center, Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Krijgslaan 281, B 9000 Ghent, Belgium

A thermoelastic system describes the interaction between the changes in the shape of an object and the fluctuation in the temperature. The system of coupled equations governing the displacement $\mathbf{u}(\mathbf{x}, t)$ and temperature $\theta(\mathbf{x}, t)$ of an isotropic thermoelastic system of type-III occupying a bounded domain $\Omega \subset \mathbb{R}^d$ with Lipschitz continuous boundary, is described by

$$\begin{cases} \rho\partial_{tt}\mathbf{u} - \mu\Delta\mathbf{u} - (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + \beta\nabla\theta & = \mathbf{p}, \\ \rho C_s\partial_t\theta - \kappa\Delta\theta - (k * \Delta\theta) + T_0\beta\nabla \cdot \partial_t\mathbf{u} & = h, \end{cases}$$

where $\mathbf{p}$ and h denote the load and heat sources respectively and the coefficients are system and material dependent constants. Our main assumption is that the sources can be decomposed in a time and spatial dependent part, that is, either $\mathbf{p}(\mathbf{x}, t) = g(t)\mathbf{f}(\mathbf{x}) + \widetilde{\mathbf{p}}(\mathbf{x}, t)$ and/or $h(\mathbf{x}, t) = g(t)f(\mathbf{x}) + \widetilde{h}(\mathbf{x}, t)$.

In this contribution we will study and discuss several inverse source problems of determining and finding the spatial component $\mathbf{f}(\mathbf{x})$ of the load source and $f(\mathbf{x})$ of heat source. We will consider the final in time measurement $\mathbf{u}(\mathbf{x}, T)$ (where $T > 0$ is the final time) and the time-average measurement $\int_0^T \mathbf{u}(\mathbf{x}, t)\,\mathrm{d}t$ for the displacement, the time-average measurement $\int_0^T \theta(\mathbf{x}, t)\,\mathrm{d}t$ for the temperature, and a combination of both for finding $\mathbf{f}$ and f simultaneously. The results are formulated under suitable assumptions on the temporal function $g(t)$.

The presented work is based on joint work with Karel Van Bockstal [1].

References

[1] F. Maes and K. Van Bockstal, Uniqueness for inverse source problems of determining a space-dependent source in thermoelastic systems, Journal of Inverse Ill-Posed Problems, 30(6)(2022), 845–856.

Efficient substructured domain-decomposition in inverse problems using Krylov block methods and subspace recycling

Boris Martin[1], Christophe Geuzaine[1]
[1]) University of Liège, Belgium
boris.martin@uliege.com

Full Waveform Inversion in the frequency domain requires the resolution of sequences of Helmholtz-like problems, each one of them with many right-hand sides (i.e. many sources). For large-scale 3D problems, Domain Decomposition Methods are a popular choice, but usual Krylov methods do not handle multiple right-hand sides efficiently.

Coupling Optimized Restrictive Additive Schwarz with Block Krylov Methods (e.g. Block GMRES) and subspace recycling (e.g. GCRO-DR and its block version) has proven to significantly reduce the iteration count[1], but with an arithmetic overhead that mitigates these benefits for large blocks in terms of actual compute time.

In this work, we investigate similar ideas for non-overlapping methods that solve a substructured problem, i.e. with unknowns on the subdomain interfaces.
These methods have a moderately slower convergence, but can handle larger blocks due to the reduced size of the vectors managed by the Krylov method. This property makes substructured non-overlapping methods particularly attractive when many sources are involved. We try to evaluate for both methods the optimal block size and recycling strategy, and determine whether overlapping or non-overlapping methods are the most appropriate for problems with a massive number of sources.

References

[1] Jolivet, Pierre and Tournier, Pierre-Henri. (2016). Block Iterative Methods and Recycling for Improved Scalability of Linear Solvers. 190-203. 10.1109/SC.2016.16.

Numerical Solution to an Inverse Problem of Recovering Source of a Special Type of Parabolic Equation

K.R. Aida-zade[1], A.B. Rahimov[1,2]
[1]) The Institute of Control Systems, Ministry of Science and Education, 68, B. Vahabzadeh str., AZ1141, Baku, Azerbaijan
[2]) Baku State University, 33, Z. Khalilov str., AZ1128, Baku, Azerbaijan
kamil_aydazade@rambler.ru, anar_r@yahoo.com

We consider the following inverse problem of recovering special type sources in the linear parabolic equation:

$$\frac{\partial v(x,t)}{\partial t} = a\left(x,t\right)\frac{\partial^2 v(x,t)}{\partial x^2} + a_1\left(x,t\right)\frac{\partial v(x,t)}{\partial x} + a_2\left(x,t\right)v\left(x,t\right) + f\left(x,t\right) + F\left(x,t\right), (x,t) \in \Omega = \left\{(x,t) : 0 < x < l,\ 0 < t \le T\right\}, \quad (1)$$

where

$$F\left(x,t\right) = \sum_{s=1}^{L} B_s\left(x,t\right)C_s\left(x\right). \quad (2)$$

Initial-boundary and additional conditions are in the following form:

$$v\left(x,0\right) = \varphi_0\left(x\right),\ v\left(0,t\right) = \psi_0\left(t\right),\ v\left(l,t\right) = \psi_1\left(t\right),\ x \in [0,\ l],\ \ t \in [0,\ T], \quad (3)$$

$$v\left(x,\bar{t}_s\right) = \varphi_{1s}\left(x\right), \quad x \in [0,l], \quad \bar{t}_s \in (0,T], \ s = 1,...,L. \quad (4)$$

Here $L > 0$ is a given integer number, $\bar{t}_s \in (0,T]$, $s = 1,...,L$ are given moments of time; The problem (1)-(4) consists in finding the unknown L-dimensional vector-function $C\left(x\right) = \left(C_1\left(x\right),...,C_L\left(x\right)\right)^*$ and the corresponding solution to the boundary-value problem $v\left(x,t\right)$, which satisfy the conditions (1)-(4).

We also consider the case of the inverse source problem when the function $F\left(x,t\right)$ in equation (1) is defined as follows:

$$F\left(x,t\right) = \sum_{s=1}^{L} C_s\left(x,t\right)B_s\left(t\right), \quad (5)$$

where the functions $C_s\left(x,t\right)$ are given, the coefficients $B_s\left(t\right)$ are identified and instead of the conditions in (4), there are the following conditions

$$v\left(\bar{x}_s,t\right) = \psi_{2s}\left(t\right), \quad \bar{x}_s \in (0,l), \quad t \in [0,T], \quad s = 1,...,L.$$

In the work we suggest an approach for a numerical solution to the problem (1)-(4), based on the method of lines. The problem is reduced to a system of ODEs with unknown parameters. We suggest a method to determine the unknown parameters, which does not use any iterative procedures [1].

References

[1] K.R. Aida-zade and A.B. Rahimov, On recovering space or time-dependent source functions for a parabolic equation with nonlocal conditions, Appl. Math. Comp. 419 (2022), 126849 (17 pages).

Inverse problems related to a Fokker-Planck control framework in esophageal cancer

Mesfer Alajmi[1], Zain Khan[1], Lasta Maharjan[1], Souvik Roy[1]
[1]) Department of Mathematics, University of Texas at Arlington, Arlington, Texas-76019, USA
souvik.roy@uta.edu

In this talk, we present an inverse problems framework for controlling aberrant signaling pathways in esophageal cancer. The dynamics of signaling pathways is given by a stochastic process that models the randomness present in the system. The stochastic dynamics is then represented by the Fokker-Planck (FP) partial differential equation that governs the evolution of the associated probability density function. We solve two inverse problems: a FP parameter estimation problem for model fitting and a FP feedback control problem to determine the optimal combination therapies for controlling the signaling pathways. Finally, we demonstrate the efficiency of the proposed framework through numerical results with combination drugs. This work was funded by the US National Science Foundation (award number: DMS 2212938)

On Optimal Regularisation Parameters via Bilevel Learning

Matthias J. Ehrhardt[1], Silvia Gazzola[1], Sebastian J. Scott[1*]
[1]) University of Bath, Bath, BA2 7AY, United Kingdom
[*]) Corresponding author ss2767@bath.ac.uk

Variational regularisation is commonly used to solve linear inverse problems and involves augmenting a data fidelity by a regulariser, weighted by a regularisation parameter. Often the regularisation parameter is assumed to be strictly positive which implicitly assumes the regulariser is a good choice for the given application - but what does it mean for a regulariser to be "good"? One characterisation is offered via bilevel learning, a powerful framework to determine optimal parameters which involves solving a nested optimisation problem. Indeed, by optimising over the regularisation parameter we can determine conditions which guarantee that zero is not an optimal parameter. While existing conditions primarily focus on the denoising application, in this talk a new condition [1] involving Bregman distances will be introduced that offers a better characterisation than existing theory and is applicable to a wide class of inverse problems and regularisers.

References

[1] Matthias J. Ehrhardt, Silvia Gazzola, and Sebastian J. Scott. On Regularization Parameters via Bilevel Learning. arXiv preprint: arXiv:2305.18394, 2023

Stability estimates of time-dependent coefficients for some hyperbolic inverse problems

Soumen Senapati[1]

[1]) RICAM, Austrian Academy of Sciences
soumen.senapati@ricam.oeaw.ac.at

In this talk, our focus will be on exploring stability considerations regarding the inverse problem of determining the lower order perturbations to the standard wave operator. These perturbations are both spatial and temporal in nature and hence capture some time-varying properties of the medium. We employ plane waves (with different time delays) as sources and observe the interaction of these waves with the medium at a fixed (large enough) time-level. From these data, the stability estimates that we derive are of Lipschitz type. We further investigate these questions in the point-source setting. Our analysis is primarily based on the modified Bukhgeĭm-Klibanov approach which itself relies on Carleman estimates.

This talk is based on the works [1] and [2].

References

[1] V. P. Krishnan, Rakesh and S. Senapati, Stability for a formally determined inverse problem for a hyperbolic PDE with space and time dependent coefficients, SIAM J. Math. Anal. 53 (2021), no. 6, 6822–6846.

[2] V. P. Krishnan, Rakesh and S. Senapati, Point sources and stability for an inverse problem for a hyperbolic PDE with space and time dependent coefficients, J. Differential Equations 342 (2023), 622–665.

Inverse source problem for the time-fractional heat equation for positive operators

Daurenbek Serikbaev[1], Michael Ruzhansky[2],
Niyaz Tokmagambetov[1]
[1]) Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
serykbaev@math.kz
[2]) Ghent University, Ghent, Belgium

Let $\mathcal{H}$ be a separable Hilbert space and let $\mathcal{L}$ be operator with a discrete spectrum on $\mathcal{H}$. For

$$\begin{cases} \mathcal{D}_t^{\alpha} u(t) + a(t)\mathcal{L}u(t) = r(t)g \text{ in } \mathcal{H},\ 0 < t \leq T, \\ u(0) = h \text{ in } \mathcal{H}, \end{cases} \tag{1}$$

we study

Inverse source problem. Given $a(t)$, g and h, find a pair of functions (r, u) satisfying the problem (1) and the additional condition

$$F[u(t)] = E(t),\ t \in [0, T], \tag{2}$$

where F is the linear bounded functional.

For this kind of inverse problem for heat equations, see [1, 2] for example. In this work, we prove the well-posedness of the inverse problem (1)–(2) by reducing the problem to an operator equation for the source function.

References

[1] K. Sakamoto and M. Yamamoto, Initial value/boundary value problems for fractional diffusion-wave equations and applications to some inverse problems, Journal of Mathematical Analysis and Applications, 382(1)(2011), 426–447.

[2] A.S. Hendy, K. Van Bockstal, On a reconstruction of a solely time-dependent source in a time-fractional diffusion equation with non-smooth solutions, Journal of Scientific Computing, 90(41)(2022), 33 pp.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Evolutionary PDEs with Volterra operators: direct and inverse source problems

Marian Slodička
Ghent University. Belgium
marian.slodicka@ugent.be

We study the following initial boundary-value problem (Direct problem)

$$\begin{aligned} u_t(t) - \Delta u(t) &= F(t) + E(u, \nabla u, u_t)(t) && \text{in } \Omega \times (0,T) \\ u(t) &= 0 && \text{in } \Gamma \ \times [0,T] \\ u(0) &= u_0 && \text{in } \Omega, \end{aligned} \tag{1}$$

where E is a Volterra operator with weakly singular integral kernel. This includes fractional derivatives with variable order in time. The well-posedness of the setting is obtained using the Banach contraction theorem.

Inverse source problem: Find $u(t,x), h(t)$ obeying

$$\begin{aligned} u_t(t) - \Delta u(t) &= h(t)f(x) + E(u, \nabla u, u_t)(t) && \text{in } \Omega \times (0,T) \\ u(t) &= 0 && \text{in } \Gamma \ \times [0,T] \\ u(0) &= u_0 && \text{in } \Omega \end{aligned} \tag{2}$$

and the local measurement

$$m(t) = \int_\Omega \omega(x) u(x,t) \ \mathrm{d}x. \tag{3}$$

Existence and uniqueness of a solution to this ISP is derived applying a similar proof technique as for direct problem.

References

[1] M. Slodička. Some direct and inverse source problems in nonlinear evolutionary PDEs with Volterra operators. *Inverse Problems*, 38(12):124001, oct 2022.

Theoretical and Numerical Study of Case Reporting Rate with Application to Epidemiology

Alexandra Smirnova[1]
[1]) Department of Mathematics & Statistics, Georgia State University, Atlanta, USA
asmirnova@gsu.edu

A clear understanding of actual infection rate is imperative for control and prevention. In particular, it helps in formulating effective vaccination strategies and in assessing the level of herd immunity required to contain the virus. In this project, we conduct theoretical and numerical study of a novel optimization procedure aimed at stable estimation of incidence reporting rate and time-dependent effective reproduction number from real data on new incidence cases, daily new deaths, and vaccination percentages. The iteratively regularized optimization algorithm can be applied to a broad class of data fitting problems constrained by various biological models, where one has to account for under-reporting of cases. To that end, general nonlinear observation operators in real Hilbert spaces are considered in the proposed convergence analysis. To illustrate theoretical findings, numerical simulations with SVI_uI_vRD compartmental model and real data for Delta variant of COVID-19 pandemic in different states of the US are conducted [1, 2].

References

[1] A. Smirnova, M. Baroonian, Reconstruction of incidence reporting rate for SARS-CoV-2 Delta variant of COVID-19 pandemic in the US, Infectious Disease Modeling, 9, (2024), 70–83.

[2] A. Smirnova, B. Pidgeon, R. Luo, On Stable Parameter Estimation and Short-term Forecasting with Quantified Uncertainty for COVID-19 Transmission, Journal of Inverse and Ill-Posed Problems, 30(6), (2022), 823–844.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Inverse Problems in Image Processing

Filip Sroubek
Institute of Information Theory and Automation, Czech Academy of Sciences
sroubekf@utia.cas.cz

Inverse problems in image processing comprise a spectrum of tasks, ranging from traditional denoising to more intricate challenges like (non-)blind deconvolution and superresolution, culminating in comprehensive scene understanding through tasks like motion estimation and 3D object reconstruction. This talk aims to provide a comprehensive overview of the mathematical methodologies employed to tackle these strongly ill-posed inverse problems.

Through the lens of image processing tasks, we explore various forms of loss functions and regularization techniques. Proximal methods, notably the alternating direction method of multipliers and primal-dual method [1], stand out as popular choices for efficiently solving convex problems. However, the limitation posed by convexity has been surmounted by stochastic gradient methods derived from deep learning, which exhibit remarkable performance on numerous non-convex problems encountered in image processing.

The conventional approach of representing discrete unknown images by pixels, commonly employed in optimization, is currently being challenged by neural representations like deep image priors [2] and neural fields [3]. Rather than optimizing in the space of pixels, contemporary methodologies optimize in the higher dimensional space of network parameters. Here, the network architecture not only provides the necessary regularization but also introduces novel ways to navigate the solution space effectively.

This presentation illustrates the evolution of mathematical approaches in addressing inverse problems within image processing, emphasizing the transition from traditional optimization methods to the integration of deep learning frameworks. By exploring examples across various image processing tasks, attendees will gain insights into the efficacy of different methodologies and the transformative impact of neural representations on modern image processing techniques.

References

[1] A. Chambolle, T. Pock, A First-Order Primal-Dual Algorithm for Convex Problems with Applications to Imaging, Journal of Mathematical Imaging and Vision, 40(1)(2011), 120—145.

[2] D. Ulyanov, A. Vedaldi and V. Lempitsky, Deep Image Prior, International Journal of Computer Vision 128(2020), 1867–1888.

[3] V. Sitzmann, J. Martel, A. Bergman, D. Lindell and G. Wetzstein, Implicit Neural Representations with Periodic Activation Functions, Advances in Neural Information Processing Systems, 33 (2020), 7462–7473

MINISYMPOSIUM
M11. Image Restoration Under Poisson Noise

Organizers:
Federico Benvenuto, University of Genoa, Italy, benvenuto@dima.unige.it
Yifei Lou, University of North Carolina at Chapel Hill, USA, yflou@unc.edu
Frank Werner, Julius-Maximilians-Universität Würzburg, Germany, frank.werner@mathematik.uni-wuerzburg.de

Poisson noise is a very common noise statistic in imaging problems, specifically when data is obtained by photon counting. Poisson noise arises in a wide range of applications, such as phase retrieval, computed tomography (CT), fluorescence microscopy, and astronomical imaging. Inverse problems with Poisson noise, such as the recovery of an image from a blurred and noisy version, requires a regularization to model the underlying image as well as a data fidelity term taking into account the Poisson noise statistics. Some popular regularizations for imaging applications include total variation (TV), fractional order TV (FOTV), wavelet thresholding, and low-rank models.

The data fidelity term for Poisson noise is well known to be the Kullback-Leibler divergence, which suffers from a nonlinearity, making it more complicated than the least-squares term for Gaussian noise. It can be avoided by applying a variance stabilizing transform that converts the Poissonian data into (approximately) Gaussian data, or by approximating the Poisson fidelity term by a weighted least-squares model. Otherwise, the nonlinearity causes difficulties in designing efficient algorithms and analyzing the convergence properties. Some optimization techniques used for Poisson noises include trust region algorithms, augmented Lagrangian methods, primal-dual majorization-minimization, and iteratively reweighted algorithms. In addition, recent advances in deep learning can be borrowed to deal with Poisson noise including neural networks, self-supervised denoising models, and dictionary learning.

In this minisymposium, we aim to bring together a wide range of experts in this field to share ideas and findings in theoretical, numerical, and practical aspects. Topics include real-data applications, algorithmic approaches, the usage of neural networks for image restoration, Bayesian approaches, as well as semi-blind and blind problems. Of interest will also be mixed noise models, which occur in many practical applications. The objective is to have a thorough understanding of the behaviours and properties of Poisson noises and to brainstorm in-depth discussions on the numerical methods to effectively remove Poisson noise.

Bilinear decomposition based splitting algorithms for curvature driven image restoration

Huibin Chang
Tianjin Normal University
changhuibin@gmail.com

The Euler Elastica (EE) model [1] with surface curvature can generate artifact-free results compared with the traditional total variation regularization model in image processing. However, strong nonlinearity and singularity due to the curvature term in the EE model pose a great challenge for one to design fast and stable algorithms for the EE model. In this paper, we propose a new, fast, hybrid alternating minimization (HALM) algorithm for the EE model based on a bilinear decomposition of the gradient of the underlying image and prove the global convergence of the minimizing sequence generated by the algorithm under mild conditions. The HALM algorithm comprises three sub-minimization problems and each is either solved in the closed form or approximated by fast solvers making the new algorithm highly accurate and efficient. We also discuss the extension of the HALM strategy to deal with general curvature-based variational models, especially with a Lipschitz smooth functional of the curvature. A host of numerical experiments are conducted to show that the new algorithm produces good results with much-improved efficiency compared to other state-of-the-art algorithms for the EE model. As one of the benchmarks, we show that the average running time of the HALM algorithm is at most one-quarter of that of the fast operator-splitting-based Deng-Glowinski-Tai algorithm [2].

References

[1] T. F. Chan, S. H. Kang, and J. Shen, Euler’s elastica and curvature-based inpainting, SIAM Journal on Applied Mathematics, 63 (2002), pp. 564–592

[2] L.-J. Deng, R. Glowinski, and X.-C. Tai, A new operator splitting method for the Euler elastica model for image smoothing, SIAM Journal on Imaging Sciences, 12 (2019), pp. 1190–1230

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

The Balancing Principle for automatic restoration of Poissonian images

Germana Landi[1], Marco Viola[2], Fabiana Zama[3]
[1]) Department of Mathematics, University of Bologna, Bologna, Italy
germana.landi@unibo.it
[2]) School of Mathematics and Statistics, University College Dublin, Dublin, Ireland
[3]) Department of Mathematics, University of Bologna, Bologna, Italy

In this contribute, we consider second-order Total Generalized Variation (TGV) regularization [1] for the restoration of images corrupted by Poisson noise. TGV regularization is a generalization of Total Variation regularization where the first and second-order derivatives of the image are balanced by two positive weights α_0 and α_1. If the weights values are properly selected, TGV regularization can remove blur and noise from an image while avoiding the staircase effect as well as preserving the sharp edges. The quality of the restored images strongly depends on the weights values, therefore it is necessary to accurately chose α_0 and α_1. The Balancing Principle (BP) is a criterium proposed for the selection of regularization parameters in multi-penalty regularization [2, 3]. Here, by extending the BP to TGV regularization, we propose to select the weights α_0 and α_1 such that data fidelity and penalties terms are balanced. We use the two-block ADMM method developed in [4] for the efficient solution of the TGV-based optimization problem. The numerical results show that the BP principle can give restored images which are comparable to those obtained by an optimal choice of the weights.

References

[1] K. Bredies and K. Kunisch and T. Pock, Total generalized variation, SIAM J. Imaging Sci., 3 (2010), 492–526.

[2] K. Ito and B. Jin and T. Takeuchi, Multi-parameter Tikhonov regularization, Methods and Applications of Analysis, 18(1)(2011), 031–046.

[3] V. Bortolotti and G. Landi and F. Zama, Uniform multi-penalty regularization for linear ill-posed inverse problems, arXiv (2023), 2309.14163.

[4] D. di Serafino and G. Landi and M. Viola, Directional TGV-based image restoration under Poisson nois, Journal of Imaging 7(6)(2021), 99.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Novel criteria for automatic solution of inverse problems under low photon-count Poisson noise corruption

A. Lanza[1], F. Bevilacqua[1], M. Pragliola[2], F. Sgallari[1]
[1]) Department of Mathematics, University of Bologna, IT
{alessandro.lanza2, francesca.bevilacqu8, fiorella.sgallari}@unibo.it
[2]) Department of Mathematics and Applications, University of Naples Federico II, IT
monica.pragliola@unina.it

Poisson noise is a pervasive cause of data degradation in many inverse imaging problems. Typical applications where Poisson noise removal is a crucial issue are astronomical and medical imaging. Both scenarios can in fact be characterized by a "low-light" condition, which is intrinsically related to the acquisition set-up in the former case, while in the latter it is somehow preferable so as to preserve the specimen of interest (microscopy) or keep the patient safer by irradiating lower electromagnetic doses (CT). However, the weaker the light intensity, the lower the signal-to-Poisson noise ratio in the acquired images and the more difficult the reconstruction problem.

Variational methods are an effective model-based approach for reconstructing images corrupted by Poisson noise. However, their performance strongly depends on a suitable selection of the regularization parameter balancing the effect of the regulation term(s) and the data fidelity term. One of the approaches still most used today for choosing the parameter is the discrepancy principle proposed in [1]. It relies on imposing a value of the data term approximately equal to its expected value and works quite well for mid- and high-photon count scenarios. However, the approximation used in the theoretical derivation of the expected value leads to poor performance for low-count Poisson noise.

The talk will illustrate three novel parameter selection strategies which are demonstrated to outperform the state-of-the-art discrepancy principle in [1], especially in the low-count regime. The three approaches rely on decreasing the approximation error in [1] by means of a suitable Montecarlo simulation [2], on applying a so-called Poisson whiteness principle [3] and on suitably masking the data used for the parameter selection [4], respectively. Extensive experiments are presented which prove the effectiveness of the three novel methods.

References

[1] M. Bertero, P. Boccacci, G. Talenti, R. Zanella, L. Zanni. A discrepancy principle for Poisson data. Inverse Problems, 26(10)(2010), 105004.

[2] F. Bevilacqua, A. Lanza, M. Pragliola, F. Sgallari, Nearly exact discrepancy principle for low-count Poisson image restoration, Journal of Imaging, 8(1)(2022), 1–35.

[3] F. Bevilacqua, A. Lanza, M. Pragliola, F. Sgallari, Whiteness-based parameter selection for Poisson data in variational image processing, Applied Mathematical Modelling, 117(2023), 197–218.

[4] F. Bevilacqua, A. Lanza, M. Pragliola, F. Sgallari, Masked unbiased principles for parameter selection in variational image restoration under Poisson noise, Inverse Problems, 39(3)(2023), 034002.

Predictive risk estimation for inverse problems with Poisson data

Paolo Massa[1], Federico Benvenuto[2]

[1]) University of Applied Sciences and Arts Northwestern Switzerland,
Bahnhofstrasse 6, 5210 Windisch, Switzerland
paolo.massa@fhnw.ch
[2]) MIDA, Dipartimento di Matematica, Università degli Studi di Genova, Via
Dodecaneso 35, 16146 Genova, Italy

Predictive risk minimization has been widely studied in inverse problems as a technique for determining the optimal value of the regularization parameter in variational approaches, or as a stopping rule for iterative maximum likelihood approaches [1]. Several strategies for predictive risk estimation have been proposed in the case of data corrupted by Poisson noise. However, they either consider the square ℓ^2 norm as a loss, which is not suited for the Poisson case due to the heteroscedasticity of the noise, or they consider the Kullback-Leibler (KL) divergence as a loss, but they lack of a complete theoretical justification [2].
In this talk we describe a recently proposed estimator of the predictive KL risk in the case of Poisson data [3] . This estimator is asymptotically unbiased with increasing number of measured counts under mild assumptions on the regularization method. We show that the iterative Expectation Maximization (EM) algorithm with Poisson data satisfies these conditions. Therefore, the proposed estimator can be used for selecting the optimal EM iterate, as shown in the cases of image deconvolution and image reconstruction from data recorded by the Spectrometer/Telescope for Imaging X-rays (STIX; [4]).

References

[1] F. Lucka, et al., Risk estimators for choosing regularization parameters in ill-posed problems-properties and limitations, Inverse Problems and Imaging, 12(5) (2018), 1121–1155.

[2] C.-A. Deledalle, Estimation of Kullback–Leibler losses for noisy recovery problems within the exponential family, Electron. J. Statist., 11(2) (2017), 3141–3164.

[3] P. Massa and F. Benvenuto, Predictive risk estimation for the expectation maximization algorithm with Poisson data, Inverse Problems, 37 (4) (2021), 045013.

[4] S. Krucker, et al., The Spectrometer/Telescope for Imaging X-rays (STIX), Astronomy & Astrophysics 642(A15) (2020).

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Pre-conditioned dual algorithm for TV regularization in Compton camera image reconstruction

Voichiţa Maxim[1], Vincent Lequertier[1], Élie Bretin[2]
[1]) Laboratoire CREATIS, Institut National des Sciences Appliquées, Lyon, France
voichita.maxim@creatis.insa-lyon.fr
[2]) Institut Camille Jordan, Institut National des Sciences Appliquées, Lyon, France

Total variation (TV) regularization has proven to be effective in noise reduction in emission tomography, where data representing measured γ-rays follow a Poisson distribution. Computing time is a critical concern in medical imaging, and utilization of preconditioning facilitates its reduction. An example is the preconditioned primal-dual hybrid gradient (PDHG) algorithm [1], instantiated for tomographic reconstruction in [2].

In SPECT applications where the Compton camera could have a significant impact, such as proton or radionuclide therapy monitoring, only a low number of coincidences may be acquired. Each coincidence is mathematically represented as an integral of the unknown function on conical surfaces. The projection and backprojection operators cannot be implemented as efficiently as their counterparts in conventional X-ray or γ-ray imaging.

We proposed a MAP-EM algorithm that solves the TV-denoising step through convex duality [3]. Its advantage over the PDHG algorithm lies in a reduced number of projection/backprojection operations, albeit with a more computationally expensive TV denoising step. While the proposed algorithm may be slightly slower when the direct and inverse operators are fast to compute, it outperforms PDHG in Compton camera imaging. In this work, we show that pre-conditioning can significantly accelerate the convergence of the MAP-EM algorithm which could in turn foster its adoption in routine clinical care.

Acknowledgment: This work was partially funded by the French National Research Agency (ANR) under grant ANR-20-CE45-0020 and by the LABEX PRIMES (ANR-11-LABX-0063) of Université de Lyon within the program "Investissements d'Avenir" (ANR-11-IDEX-0007).

References

[1] T. Pock and A. Chambolle, Diagonal preconditioning for first order primal-dual algorithms in convex optimization, 2011 Int. Conf. on Computer Vision (2011), 1762–1769.

[2] E.Y. Sidky, J. Jørgensen, and X. Pan, Convex optimization problem prototyping for image reconstruction in computed tomography with the Chambolle–Pock algorithm, Phys. Med. Biol., 57(10) (2012), 3065–3091.

[3] V. Maxim, Y. Feng, H. Banjak and E. Bretin, Tomographic reconstruction from Poisson distributed data: a fast and convergent EM-TV dual approach, Numerical Algorithms, 94 (2023), 701–731.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Whiteness-based parameter selection for Poisson data in variational image processing

Francesca Bevilacqua[1], Alessandro Lanza[1], Monica Pragliola[2], Fiorella Sgallari[1]

[1]) Department of Mathematics, University of Bologna, Italy
[2]) Department of Mathematics and Applications, University of Naples Federico II, Italy
monica.pragliola@unina.it

We propose a novel parameter selection strategy for variational imaging problems under Poisson noise corruption. The selection of a suitable value of the regularization parameter, which is crucial for achieving high quality reconstructions, is known to be a particularly hard task in low photon-counting regimes. In this talk, we extend the so-called residual whiteness principle originally proposed in [1] for additive white noise to Poisson data. The designed strategy relies on exploiting the whiteness property of a suitably standardized Poisson noise process. After deriving the theoretical properties underlying our proposal, we solve the target optimization problem by the alternating direction method of multipliers, in its standard two-blocks version or in a semi-linearized version depending on the imaging problem. Our strategy is extensively tested on image restoration and computed tomography reconstruction problems.

References

[1] A. Lanza, M. Pragliola and F. Sgallari, Residual whiteness principle for parameter-free image restoration, Electronic Transactions on Numerical Analysis, 53 (2020), 329–351.

Weighted discrepancy principle and adaptivity in Poisson inverse problems

Zbigniew Szkutnik
AGH University of Krakow, Poland
szkutnik@agh.edu.pl

A new weighted form of the Morozov discrepancy principle for Poisson inverse problems with compact folding operators will be discussed. In particular, for a broad class of spectral filters the resulting estimators are not only consistent (in probability) but also essentially L^2-rate-minimax under source conditions on the estimated signal, if the degree of smoothing of the residuals is properly related to the smoothing properties of the folding operator. This is an improvement over results recently published in [2] for discrepancy principle in Poisson inverse problems pre-conditioned in a standard way, where not only the rates were sub-optimal but also the operator was additionally assumed Hilbert-Schmidt. The theoretical results will be illustrated with a numerical study of two stereological problems: the Wicksell's problem (with a non-Hilbert-Schmidt Abel operator) and the Spektor, Lord and Willis problem (with the Hilbert-Schmidt integration operator). Theoretical results will also be discussed in a broader context of white noise inverse problems.

The general idea of using weighted or smoothed residuals is taken from [3]. To prove the main theoretical results, we adapt some ideas and techniques introduced in [1] and [2]. Additionally, we introduce and use some special concepts like, e.g., γ-boundedness of the filter.

References

[1] B. Harrach, T. Jahn and R. Potthast, Beyond the Bakushinskii veto: Regularising linear inverse problems without knowing the noise distribution, Numerische Mathematik,145(2020), 581–603.

[2] G. Mika and Z. Szkutnik, Towards adaptivity via a new discrepancy principle for Poisson inverse problems, Electronic Journal of Statistics, 15(2021), 2029-2059.

[3] B. Stankewitz, Smoothed residual stopping for statistical inverse problems via truncated SVD estimation, Electronic Journal of Statistics, 14(2020), 3396-3428.

MINISYMPOSIUM
M12. Applied Inverse Problems and Partial Differential Equations

Organizers:
Karel Van Bockstal, Ghent University, Belgium, karel.vanbockstal@ugent.be
Michael Ruzhansky, Ghent University, Belgium, Michael.Ruzhansky@ugent.be
Berikbol T. Torebek, Ghent University, Belgium; Institute of Mathematics and Mathematical Modeling Almaty, Kazakhstan, berikbol.torebek@ugent.be

The vast majority of mathematical models related to inverse problems are governed by partial differential equations. Therefore, the methods of the theory of partial differential equations have a decisive role in the study of inverse problems. This mini-symposium aims to present a discussion of the main ideas and methods for examining inverse problems for PDEs that arise in various mathematical models. Most of these models are governed by initial and boundary value problems for PDEs, and inverse problems governed by these equations occur naturally in nearly all branches of science and engineering.

We hope to bring together well-known experts and young researchers working on inverse problems governed by partial differential equations and their applications.

Inverse problem for the subdiffusion equation with fractional Caputo derivative

R. R. Ashurov
Institute of Mathematics
of the Academy of Sciences of the Republic of Uzbekistan
University street 9, 100174, Tashkent
ashurovr@gmail.com

This is a joint work with M. Shakarova, PhD student at the Institute of Mathematics, Uzbekistan (see [1], [2], [3]).

In the Hilbert space H consider the subdiffusion equation $D_t^\rho u(t) + Au(t) = fg(t)$, $\rho \in (0, 1]$, $t \in (0, T]$. In this equation $g(t) \in C[0, T]$, $f \in H$, A is an unbounded positive self-adjoint operator and D_t^ρ stands for the Caputo fractional derivative. We assume that A has a compact inverse A^{-1}.

Let us consider two types of the initial conditions (a) $u(0) = \varphi$, (b) $u(0) = u(T)$, where $\varphi, \psi \in H$ and obtain two (let us call them (a), (b), respectively) forward problems. First, we prove the existence and uniqueness of a solution to both forward problems. Next we consider three inverse problems (IP1 -IP3) of determining a pair $\{u(t), f\}$. IP1 := Problem (a) + additional condition: $u(t_0) = \Psi$, $t_0 \in (0, T]$, IP2 := Problem (b) + additional condition: $u(t_0) = \Psi$, $t_0 \in (0, T)$, and IP3 := Problem (a) + additional condition: $\int_0^T u(t)dt = \psi$, where ψ and Ψ are given elements of H.

We briefly present the results obtained for the above inverse problems. A criterion for the uniqueness of a solution to the inverse problem is found. It is interesting to note that this criterion is completely different for each problem. But if function $g(t)$ retains its sign, then all the criteria are satisfied and we were able to prove the existence and uniqueness of a solution to inverse problems in this case. For all inverse problems, examples of functions $g(t)$ have been constructed (different for each problem), changing sign and, as a result, there is no unique solution to the inverse problem. When $g(t)$ changes sign, then in some cases, the existence and uniqueness of the solution was proved (for all 3 problems), while in other cases, the necessary and sufficient condition for the existence of the solution was found (different for each problem). We note that, almost all the results obtained here are also new for the classical diffusion equation.

References

[1] R. Ashurov, M. Shakarova, Inverse problem for the subdiffusion equation with fractional Caputo derivative, doi.org/10.48550/arXiv.2211.00081, 2023.

[2] M. Shakarova, Inverse problem for fractional order subdiffusion equation, doi.org/10.48550/arXiv.2309.04852, 2023.

[3] R. Ashurov, M. Shakarova, Inverse problem of determining the source function for the subdiffusion equation with a non-local condition, doi.org/10.48550/arXiv.2308.05356, 2023.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

The reconstruction of a time-dependent source in the time-fractional subdiffusion equation with time-dependent piecewise constant order

Karel Van Bockstal[1]
[1]) Ghent Analysis & PDE center, Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Krijgslaan 281, 9000 Ghent, Belgium
karel.vanbockstal@ugent.be

Let $0 < T < \infty$ and let $\Omega \subset \mathbb{R}^d$, $d \in \mathbb{N}$, be an open bounded domain with Lipschitz boundary $\partial\Omega$. For $\alpha : (0, T) \to (0, 1)$, we consider the Caputo variable-order derivative in the form

$$\partial_t^{\alpha(t)} u(t) = \frac{1}{\Gamma(1-\alpha(t))} \int_0^t (t-s)^{-\alpha(t)} u'(s) ds, \qquad t > 0,$$

where $\alpha(t)$ is a piecewise constant function with a finite number of jumps. In this talk, we consider the inverse source problem of determining a solely time-dependent source $h(t)$ -next to the function u- in the following initial-boundary value problem

$$\begin{cases} \left(\partial_t^{\alpha(t)} u\right)(x,t) - \Delta u(x,t) & = h(t) f(x), \qquad (x,t) \in \Omega \times (0,T), \\ u(x,t) & = 0, \qquad (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) & = u_0(x), \qquad x \in \Omega. \end{cases}$$

More specifically, we will discuss under which type of measurement we can obtain the existence and uniqueness of a solution to this problem. The approach will be based on results from constant-order fractional subdiffusion equations, see e.g. [1].

References

[1] K. Sakamoto and M. Yamamoto, Initial value/boundary value problems for fractional diffusion-wave equations and applications to some inverse problems, Journal of Mathematical Analysis and Applications, 382(1)(2011), 426–447.

Real-time solutions of source inverse problems using the asymptotic expansions

Chaikovskii D.[1], Zhang Y.[2]
[1]) Shenzhen MSU-BIT University, Shenzhen 518172, China.
dmitriich@smbu.edu.cn
[2]) School of Mathematics and Statistics, Beijing Institute of Technology, Beijing, 100081, China.
ye.zhang@smbu.edu.cn

This article extends the research initially presented in [1] regarding the one-dimensional source function determination in the reaction-diffusion-advection equation, by exploring the problem in a three-dimensional contexts. The results of this work can be found in [2].

Reaction-diffusion-advection models are pivotal in simulating various physical phenomena, particularly in constructing autowave models. We employ the asymptotic expansions method to address both the direct and inverse singularly perturbed reaction-diffusion-advection problems, establishing conditions for a smooth solution with a clearly defined transitional layer inside the considered domain. The effectiveness of the asymptotic approach is significantly improved by introducing local coordinates in the vicinity of the inner transitional layer, which assists in accurately identifying its precise location. Subsequently, we demonstrate the uniqueness and existence of the obtained solution.

Additionally, asymptotic analysis streamlines solving the inverse problem of identifying the source function. It does so by deriving a simplified model of the original problem, which yields precise solutions throughout the domain, barring the narrow transitional layer area. By employing asymptotic analysis, we can disregard the noisy data from the transitional layer, smoothen the remaining data, and, upon integrating it into the simplified model, achieve highly accurate inverse problem solutions. The article concludes with the presentation of a numerical experiment that validates the effectiveness of our proposed methodology.

The work has been supported by the National Natural Science Foundation of China (No. 12350410359).

References

[1] *Dmitrii Chaikovskii* and *Ye Zhang*. Convergence analysis for forward and inverse problems in singularly perturbed time-dependent reaction-advection-diffusion equations. Journal of Computational Physics, 470:111609, 2022.

[2] *Dmitrii Chaikovskii* and *Ye Zhang*. Solving forward and inverse problems in a non-linear 3D PDE via an asymptotic expansion based approach. IMA Journal of Applied Mathematics (2023) 00, 1–33.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

A convolutional neural network-based reconstruction framework in magnetic resonance elastography

Anwesa Dey[1], Johann Rudi[2], Elena Cherkaev[1]
[1]) Department of Mathematics, University of Utah, Salt Lake City, Utah-84112, USA
anwesary@math.utah.edu, deyanwesa@gmail.com
[2]) Department of Mathematics, Virginia Tech, Blacksburg, Virginia-24061, USA

An inverse problem in magnetic resonance elastography (MRE) is formulated where the forward problem is governed by a modified stationary Stokes system; we solve the inverse problem using convolutional neural networks (CNN). In MRE, low frequency acoustic pressure waves are sent through the body. These waves propagate rapidly in stiff unhealthy tissues compared to the healthy tissues. The dynamic measurement of the acoustic wave velocity in the tissue is performed using magnetic resonance imaging (MRI), which provides characterization of the tissue's stiffness. A mathematical inverse problem of reconstructing the viscoelastic modulus is formulated as a map from the acoustic wave displacement vector field. This presentation investigates the applicability of CNNs for constructing a direct inverse map that is accurate and computationally efficient. The wave propagation is described by a modified Stokes equation with piecewise constant coefficients, which are the unknown parameters, to model a layered viscoelastic medium. We discuss analytic results concerning the resolution and sensitivity of recovering the viscoelastic modulus. Numerical simulations demonstrate accurate reconstruction of the complex-valued properties for the case of two-layered, two-dimensional domain.

The 11th International Conference “Inverse Problems: Modelling and Simulation”
26th May – 1st June 2024, Malta

Mikusiński’s Operational Calculus: Algebraic Inversion to Solve Fractional Partial Differential Equations

Arran Fernandez[1], Noosheza Rani[1]
[1]) Department of Mathematics, Eastern Mediterranean University, 99628 Famagusta, Northern Cyprus, via Mersin-10, Turkey
arran.fernandez@emu.edu.tr

The operational calculus approach pioneered by Jan Mikusiński gives a new perspective on inversion in the context of integro-differential operators and equations. In standard calculus, integrals and derivatives are not exact inverses of each other; in operational calculus, integral operators are understood as elements of an algebraic rng under convolution, with exact inverses constructed formally in the field of fractions, and then relations are constructed to connect derivative operators with those abstract algebraic inverses.

Mikusiński [1] developed this theory for ordinary differential equations; Gutterman [2] extended it to partial differential equations; Luchko [3] developed the corresponding theory for fractional ordinary differential equations; and here, for the first time, we develop a theory of Mikusiński’s operational calculus for fractional partial differential equations. This requires new spaces of functions of several variables, and some abstract algebra manipulation to understand both one-dimensional and multi-dimensional integral operators (and their inverses) acting on these spaces. So far, the tools that we have developed enable us to solve fractional partial differential equations on infinite sector regions, such as the first quadrant in two dimensions.

References

[1] J. Mikusiński, Operational Calculus, Pergamon Press, Oxford, 1959.

[2] M. Gutterman, An operational method in partial differential equations, SIAM Journal of Applied Mathematics, 17(2)(1969), 468–493.

[3] Y.F. Luchko, Operational method in fractional calculus, Fractional Calculus and Applied Analysis, 2(4)(1999), 463–488.

Inverse problems for pseudoparabolic equations with fractional time derivative

Kh. Khompysh[1,2], M. Ruzhansky[2]
[1]) Al-Farabi Kazakh National University, Kazakhstan
khompysh.khonatbek@gmail.com
[2]) Ghent University, Belgium

This work devoted to study the following time dependent inverse source problem for a fractional derivative pseudoparabolic equation with p-Laplacian and nonlinear damping term

$$\mathcal{D}_t^{\alpha}(u-\Delta u)-\operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)=\gamma|u|^{\sigma-2}u+f(t)g(x,t)\ \text{ in }\ Q_T, \tag{1}$$

which is supplemented with the initial condition

$$u(x,0)=u_0(x) \text{ in } \Omega, \tag{2}$$

the boundary condition

$$u(x,t)=0\ \text{ on }\ \Gamma_T. \tag{3}$$

and the given integral measurement

$$\int\limits_{\Omega} u(x,t)\omega(x)dx=e(t),\ \ t\in[0,T]. \tag{4}$$

Here $Q_T=\{(x,t):\ x\in\Omega,\ 0<t\leq T\}$ is a bounded cylinder and $\Omega\subset\mathbb{R}^d$ $d\geq 2$, is a bounded domain with a smooth boundary $\partial\Omega$, $\Gamma_T=\partial\Omega\times[0,T]$, $T<\infty$ is a finite number. $\mathcal{D}_t^{\alpha}$ denotes the usual Caputo fractional derivative of order $\alpha\in(0,1)$. The functions $g(x,t),\ u_0(x),\ \omega(x)$, and $e(t)$ in the system are given, while $u(x,t)$ and $f(t)$ are unknowns. The coefficient γ is a given number with the sign that might be positive $\gamma\geq 0$ either negative $\gamma\leq 0$. The exponents p and σ are also given numbers, such that

$$1<p,\ \sigma<\infty. \tag{5}$$

Under suitable assumptions on the data, we establish global and local int time existence and uniqueness of weak generalized solutions of the inverse problem (1)-(4).

Acknowledgments. First author supported by the Grant No. AP19676624 of the Ministry of Higher Education and Science of the Republic of Kazakhstan.

An inverse source problem for convective Brinkman-Forchheimer equations with the final overdetermination

Pardeep Kumar[1], Manil T. Mohan[2]
[1,2]Department of Mathematics, Indian Institute of Technology Roorkee-IIT Roorkee, Haridwar Highway, Roorkee, Uttarakhand 247667, INDIA.
[1]pkumar3@ma.iitr.ac.in

In this talk, we consider an inverse problem for the following convective Brinkman-Forchheimer (CBF) equations or damped Navier-Stokes equations:

$$\boldsymbol{u}_t - \mu\Delta\boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \alpha\boldsymbol{u} + \beta|\boldsymbol{u}|^{r-1}\boldsymbol{u} + \nabla p = \boldsymbol{f}g, \quad \nabla\cdot\boldsymbol{u} = 0,$$

on a torus $\mathbb{T}^d$, $d = 2, 3$, where $\alpha, \beta, \mu > 0$ and $r \in [1, \infty)$. The CBF equations describe the motion of incompressible fluid flows in a saturated porous medium. This inverse problem aims to reconstruct the vector-valued velocity function $\boldsymbol{v}$, the pressure gradient ∇p and the vector-valued forcing function $\boldsymbol{f}$. Using the Tikhonov fixed point theorem, we prove the existence of a solution for the inverse problem for 2D and 3D CBF equations with the final overdetermination data for the divergence free initial data in the energy space $\mathbb{L}^2(\mathbb{T}^d)$. Moreover, we overcome the technical difficulties while proving the uniqueness and Hölder type stability results by using the regularity results available for the direct problem for CBF equations. The well-posedness results hold in two dimensions for $r \geq 1$ and three dimensions for $r \geq 3$ for appropriate values of α, β and μ. The nonlinear damping term $|\boldsymbol{v}|^{r-1}\boldsymbol{v}$ plays a crucial role in obtaining the required results. In the case of supercritical growth ($r > 3$), we obtain better results than that are available in the literature for 2D Navier-Stokes equations.

References

[1] A.I. Prilepko, D.G. Orlovsky and I.A. Vasin, Methods for Solving Inverse Problems in Mathematical Physics, Marcel Dekker, New York, 2000.

[2] J. Fan and G. Nakamura, Well-posedness of an inverse problem of Navier–Stokes equations with the final overdetermination, Journal of Inverse and Ill-Posed Problems, 17(2009), 565–584.

[3] V.K. Kalantarov and S. Zelik, Smooth attractors for the Brinkman-Forchheimer equations with fast growing nonlinearities, Communications on Pure and Applied Mathematics, 11(2012), 2037–2054.

[4] P. Kumar and M.T. Mohan, Well-posedness of an inverse problem for two- and three-dimensional convective Brinkman-Forchheimer equations with the final overdetermination, Inverse Problems and Imaging, 16(5)(2022), 1255–1298.

Local data inverse problem for the polyharmonic operator with anisotropic perturbations

Pranav Kumar

Indian Institute of Science Education and Research, Bhopal, India
pranav19@iiserb.ac.in

In this talk, we study an inverse problem with local data for a linear polyharmonic operator with several lower order tensorial perturbations. We consider our domain to have an inaccessible portion of the boundary where neither the input can be prescribed nor the output can be measured. We prove the unique determination of all the tensorial coefficients of the operator from the knowledge of the Dirichlet and Neumann map on the accessible part of the boundary, under suitable geometric assumptions on the domain. This is based on a joint work with Dr. Sombuddha Bhattacharyya.

Determination of the time-dependent blood perfusion coefficient in the thermal-wave model of bio-heat transfer

Daniel Lesnic[1], Moataz Alosaimi[2]
[1]) Department of Applied Mathematics, University of Leeds, UK
D.Lesnic@leeds.ac.uk
[2]) Department of Mathematics and Statistics, Taif University, Saudi Arabia

We aim to retrieve the time-dependent blood perfusion coefficient in the thermal-wave model of bio-heat transfer from non-intrusive boundary temperature measurements that may be taken with an infrared scanner. This non-linear and ill-posed problem is reformulated as a non-linear minimization problem of a Tikhonov regularization functional subject to lower and upper simple bounds on the unknown coefficient. For the numerical discretisation, an unconditionally stable direct solver based on the Crank-Nicolson finite-difference scheme is developed. The Tikhonov regularization functional is minimized iteratively by the built-in routine *lsqnonlin* from the MATLAB optimization toolbox. The reconstruction of the unknown blood perfusion coefficient for three benchmark numerical examples is illustrated and discussed to verify the proposed numerical procedure. Moreover, the proposed algorithm is tested on a physical example which consists of identifying the blood perfusion rate of a biological tissue subjected to an external source of laser irradiation.

The support of the EPSRC grant EP/W000873/1 is acknowledged.

An accelerated inexact Newton regularization scheme with a learned feature-selection rule for non-linear inverse problems

Haie Long[1], Ye Zhang[2], Guangyu Gao[3]
[1]) MSU-BIT-SMBU Joint Research Center of Applied Mathematics, Shenzhen MSU-BIT University, Guangdong Province, 518172, PR China
haie_long@smbu.edu.cn
[2]) School of Mathematics and Statistics, Beijing Institute of Technology, Beijing, 100081, PR China
[3]) Department of Mathematics, Harbin Institute of Technology, Heilongjiang Province, 150001, PR China

With computational inverse problems, it is desirable to develop an efficient inversion algorithm to find a solution from measurement data through a mathematical model connecting the unknown solution and measurable quantity based on the *first principles*. However, most of mathematical models represent only a few aspects of the physical quantity of interest, and some of them are even incomplete in the sense that one measurement corresponds to many solutions satisfying the forward model. In this paper, in light of the recently developed iNETT method in [1], we propose a novel iterative regularization method for efficiently solving non-linear ill-posed inverse problems with potentially non-injective forward mappings and (locally) non-stable inversion mappings. Our approach integrates the inexact Newton iteration, the non-stationary iterated Tikhonov regularization, the two-point gradient acceleration method, and the structure-free feature-selection rule. The main difficulty in the regularization technique is how to design an appropriate regularization penalty, capturing the key feature of the unknown solution. To overcome this difficulty, we replace the traditional regularization penalty with a deep neural network, which is structure-free and can identify the correct solution in a huge null space. A comprehensive convergence analysis of the proposed algorithm is performed under standard assumptions of regularization theory. Numerical experiments with comparisons with other state-of-the-art methods for two model problems are presented to show the efficiency of the proposed approach.

References

[1] Davide Bianchi, Guanghao Lai, and Wenbin Li. Uniformly convex neural networks and non-stationary iterated network Tikhonov (iNETT) method. Inverse Problems, 39(5):055002, 2023.

Analysis of wave equations describing ultrasound propagation in complex media

Mostafa Meliani
Department of Mathematics, Radboud University, The Netherlands
m.meliani@math.ru.nl

To be able to successfully image an object, one needs adequate data collection tools and a good representation of the physics of wave propagation often called governing equations. There exists a number of models in the literature describing ultrasound propagation in complex media. In this talk, we will relate fractionally damped models to inviscid and strongly damped ones. To achieve this goal, we will prove the well-posedness of the models uniformly with respect to some small involved parameter on which the memory kernels depend and which can be physically interpreted as the sound diffusivity or the thermal relaxation time. We then analyze the behavior of solutions as this parameter vanishes and establish convergence rates in appropriate norms.

The talk is based on [1, 2].

References

[1] Barbara Kaltenbacher, Mostafa Meliani, and Vanja Nikolić. "Limiting behavior of quasilinear wave equations with fractional-type dissipation." arXiv preprint arXiv:2206.15245 (2022).

[2] Barbara Kaltenbacher, Mostafa Meliani, and Vanja Nikolić. "The Kuznetsov and Blackstock equations of nonlinear acoustics with nonlocal-in-time dissipation." arXiv preprint arXiv:2308.10580 (2023).

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Regularization of Ill-Posed Stochastic Problems

Irina V. Melnikova
Ural Federal University, Russia
irina.melnikova@urfu.ru

A wide class of processes arising in various areas of natural science, economics and social phenomena can be mathematically described by stochastic differential equations. The paper is devoted to the new direction in the study of stochastic problems: regularization of infinite-dimensional stochastic problems

$$du(t)/dt = Au(t) + B\mathbb{W}(t),\ t > 0, \quad u(0) = \xi, \tag{1}$$

with A not generating a strongly continuous semigroup, just a regularized semigroup (R-semigroup) in a Hilbert space H. The problem (1) in stochastic analysis usually understood as the integral problem with the Ito integral over Wiener process $\{W(t), t \geq 0\}$, "antiderivative" from the white noise $\mathbb{W}$, and operator B acting from a space $\mathbb{H}$, where W is defined in the form of series with respect to independent Brownian motions, into H. Regularization of this ill-posed problem is constructed in the form of sum of a regularized solution for the corresponding homogeneous problem and stochastic convolution.

Following to [1] regularized solutions for the homogeneous problem are constructed by the quasi-reversibility method, by the method of boundary-value problems, and by R-semigroup method based on the connection established between regularizing operators $\mathbf{R}_{\alpha,t}$ and regularized semigroups $\{S_\alpha(t), t \geq 0\}$. Estimates are obtained for the error of the regularized solutions for the homogeneous and stochastic problems. For a somewhat wider class of operators A, error estimates similar to those given in the paper by the quasi-reversibility method for the homogeneous problem are obtained in [2].

Let us make some comments regarding the possibility of extending the results obtained. Firstly, constructing a regularized semigroup with an arbitrarily given A is far from an easy task. We constructed $\mathbf{R}_{\alpha,t}$ via regularized semigroups for the Cauchy problem with differential operators $A = A\left(i\frac{\partial}{\partial x}\right)$ depending on A belonging to different classes in the Gelfand–Shilov classification. Secondly, regularized solutions to the stochastic Cauchy problem are constructed under fairly strong conditions for the stochastic inhomogeneity. Relaxation of conditions on the integral term with Wiener process W and more general Levy processes requires additional research on existence of strong and mild solutions to the stochastic Cauchy problem.

References

[1] I. V. Melnikova, V. A. Bovkun, Correctness and regularization of stochastic problems, *J. Inverse and Ill-posed Probl.* (2023). Accepted July 29, 2023.

[2] D. Chen, B. Hofmann, and J. Zou, Regularization and convergence for ill-posed backward evolution equations in Banach spaces, *J. Differential Equations* **265** (2018), 3533–3566.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Unique identifiability of a passive inverse parameter problem: The example of helioseismology

Björn Müller[1], Thorsten Hohage[1,2]
[1] Max-Planck-Institute for Solar System Research, Göttingen
muellerb@mps.mpg.de
[2] Institute for Numerical and Applied Mathematics, Georg August University, Göttingen

In passive imaging, one attempts to reconstruct some coefficients in a wave equation from correlations of observed randomly excited solutions to this wave equation. While there has been some work on reconstruction, addressing challenges like the considerable computational costs [1], the uniqueness is only established in very specific situations. An example includes situations where the cross-correlation is assumed to equal the imaginary part of the Green's function [2, 3]. This relation arises from a particular choice of the source covariance. In this talk, we prove that measurements at two distinct boundaries uniquely determine the scalar potential and advection term up to a gauge transformation by simultaneously determining the source strength within a comprehensive source model. This gauge transformation can be resolved for measurements at two different frequencies in helioseismology. The fundamental idea of the proof is to relate the surface cross-correlation to the Dirichlet-to-Neumann map, where the result follows from the well-established uniqueness theory regarding the Calderón problem (e.g. [4]).

References

[1] B. Müller, T. Hohage, L. Gizon and D.Fournier, Quantitative passive imaging by iterative holography: The example of helioseismic holography, submitted to Inverse Problems, 2023.

[2] A. Agaltsov, T. Hohage and R. Novikov, Monochromatic identities for the Green function and uniqueness results for passive imaging, SIAM J. Appl. Math., 78(5)(2018), 2865–2890.

[3] A. Agaltsov, T. Hohage and R. Novikov, Global uniqueness in a passive inverse problem of helioseismology, Inverse Problems, 36(5)(2020).

[4] M. Salo, Inverse Problems for Nonsmooth First Order Perturbations of the Laplacian, Doctoral Thesis, University of Helsinki, Department of Mathematics and Statistics, 2004.

Linear Statistical Inverse Problems for Hilbert Space Processes in Hilbert Scales

Mihaela Pricop-Jeckstadt[1]
[1]) POLITEHNICA University of Bucharest
mihaela.pricop@upb.ro

In this talk, we introduce and characterize Hilbert space processes in Hilbert scales (s-HSP) and we study linear statistical inverse problems between s-HSPs (see e.g. [1, 2]). The smoothness of the exact solution and of the noise as well as the ill posedness of the linear operator are expressed with the help of indeces of Hilbert spaces in Hilbert scales. Finally, conditions for the well-posedness of the inverse problem are presented.

References

[1] M.S. Lehtinen and L. Paivarinta and E. Somersalo, Linear inverse problems for generalised random variables, Inverse Problems, 5(4)(1989).

[2] J. Angulo and V. Anh and M. Ruiz-Medina, Fractional Generalized Random Fields on Bounded Domains, Stochastic Analysis and Applications, 21(1)(2003).

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

A linear data completion problem in inverse electrocardiography

Jorge Vicente[1,2], Ismael Hernández-Romero[2,3], Ernesto Zacur[2], Andreu M. Climent[2,3], Maria S. Guillem[2,3], Judit Chamorro-Servent[1]
[1]) Universitat Autònoma de Barcelona, Mathematics Department
[2]) Corify Care S.L., jorge.vicente@corifycare.com
[3]) Universitat Politècnica de València, ITACA Institute

We consider the inverse problem of electrocardiography, which consists of reconstructing the cardiac electrical activity from the electric potential measured at the torso surface. We assume that the quasi-static approximation of Maxwell's equations holds for the electro-magnetic fields considered. Then, the inverse problem can be formulated as a linear data completion problem aiming to estimate the electric potential at the inaccessible pericardial heart's surface (or outer layer of the heart).

There are different approaches that can be used to construct the linear operator used in the data completion problems. One of the most common approaches in clinical scenarios is based on the Boundary Integral Equation (BIE) formulation [1], where single and double layer operators are utilized to establish the relationship between torso and pericardial potentials.
However, few studies have explored alternatives such as the Rayleigh-Ritz-Galerkin (RRG) methodology [2].
For both formulations, the operators are then discretized over the torso and heart triangulations to formulate a linear discrete inverse problem. Tikhonov regularization is applied to achieve a stable approximation of the solution.

In this study, using simulated data of various cardiac pathologies, we perform numerical experiments for both the BIE and RRG formulations, employing Tikhonov regularization with the L-curve criteria to achieve a stable approximation. The results indicate superior performance of the RRG method in the tested dataset.

References

[1] Gander, L., Krause, R., Multerer, M., & Pezzuto, S. (2021). Space–time shape uncertainties in the forward and inverse problem of electrocardiography. International journal for numerical methods in biomedical engineering, 37(10), e3522.

[2] Sundnes, J., Lines, G. T., Cai, X., Nielsen, B. F., Mardal, K. A., & Tveito, A. (2007). Computing the electrical activity in the heart (Vol. 1). Springer Science & Business Media.

Nonlinear Inverse Problem of Moisture Transfer in the Soil

Nazerke Rysbayeva[1], Bolatbek Rysbaiuly[2]
[1]) Kazakh-British Technical University, Almaty, Kazakhstan
n.rysbayeva@mail.ru
[2]) International Information Technology University, Almaty, Kazakhstan

Mathematical models of most natural processes are based on the concept of continuous medium. The concept of "continuous medium" does not necessarily mean that it cannot incorporate pores or cracks that contain liquids, gases or a suspension of fine particles. The solid phase of a continuous phase can be non-porous (slightly porous), porous or capillary.

Currently, methods of mathematical modeling of hydraulic conductivity are becoming widespread [1]. And note that experimental methods and approaches require improvements [1].

The equation of fluid motion when soil is saturated can be written by Darcy's law. In this case, the speed of fluid movement is proportional to the pressure gradient

$$\frac{\partial W}{\partial t} = div(K(W) * grad(H)) \tag{1}$$

where $K(W)-$ hydraulic conductivity that depends on coordinates x,y, z; $W-$ volumetric soil moisture; $H-$ pressure; $t-$ time.

Depending on movement, soil moisture can change differently. If at the initial period of time the soil has an uneven distribution of moisture in depth, then over time the moisture will increase in the drier layers according to the diffusion law. This phenomenon is called the Hallare effect, which uses the concept of fractured porous soil to describe the noted fact. A correction term is introduced into the moisture transfer equation, which considers moisture transfer in soils. Hallare's model has the following form:

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial x}\left(D(W)\frac{\partial W}{\partial x} + A\frac{\partial^2 W}{\partial x \partial t}\right) \tag{2}$$

where $W-$ moisture, $A-$ proportionality coefficient; $D(W)-$ capillary diffusion coefficient. This work proposes iterative methods of finding parameters $D(W)$ and A.

References

[1] Rysbaiuly B.,Rysbayeva N, An Iteration method for solving the Inverse problem of freezing soil. The 10th International Conference "Inverse problem: Modeling and Simulation", 2022, 22-28.

[2] S. Alpar, B. Rysbaiuly, Determination of thermophysical characteristics in a nonlinear inverse heat transfer problem, Applied Mathematics and Computation, 2023, p. 20.

Numerical methods for solving the initial boundary problems for subdiffusion equations with nonlocal boundary conditions

M. A. Sultanov[1], M. A. Sadybekov[2], V. E. Misilov[3,4]
[1]) Khoja Akhmet Yassawi International Kazakh-Turkish University, Turkistan, Kazakhstan
murat.sultanov@ayu.edu.kz
[2]) Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
sadybekov@math.kz
[3]) Krasovskii Institute of Mathematics and Mechanics, Ekaterinburg, Russia
[4]) Ural Federal University, Ekaterinburg, Russia
v.e.misilov@urfu.ru

In this work we consider the numerical algorithms for solving the initial boundary problem for the subdiffusion equation with nonlocal boundary conditions. Consider the equation

$$D_t^\alpha u(x,t) = u_{xx}(x,t) + q(x)u(x,t) + f(x,t), \tag{1}$$

with initial conditions

$$u(x,0) = \tau(x), \quad 0 \le x \le 1, \tag{2}$$

and nonlocal boundary condition in the form

$$\begin{cases} u_x(0,t) - u_x(1,t) + \beta u(1,t) = 0, \\ u(0,t) = 0, \end{cases} \quad 0 \le t \le T. \tag{3}$$

Assume that $q(x) = q(1-x)$.

We use the method [1] for reducing problem (1)–(3) to solving two problems for functions $c(x,t)$ and $s(x,t)$, such as $u(x,t) = c(x,t) + s(x,t)$:

$$\begin{cases} D_t^\alpha c(x,t) = c_{xx}(x,t) + q(x)c(x,t) + f_0(x,t), \\ c(x,0) = \tau_0(x), \quad 0 \le x \le 1, \\ 2c_x(0,t) + \beta c(0,t) = 0, \quad 2c_x(1,t) + \beta c(1,t) = 0, \quad 0 \le t \le T; \end{cases} \tag{4}$$

$$\begin{cases} D_t^\alpha c(x,t) = s_{xx}(x,t) + q(x)s(x,t) + f_1(x,t), \\ s(x,0) = \tau_1(x), \quad 0 \le x \le 1, \\ s_x(0,t) = 0, \quad s_x(1,t) = 0, \quad 0 \le t \le T; \end{cases} \tag{5}$$

Problems (4) and (5) are easier to solve since their boundary conditions are of the Sturm type. To solve them, we implement the Fourier method.

References

[1] M.A. Sadybekov, Initial-Boundary Value Problem for a Heat Equation with not Strongly Regular Boundary Conditions, Springer Proceedings in Mathematics & Statistics, 216 (2017), 330–348.

Inverse problems of dynamics in the presence of random perturbations

Marat Tleubergenov[1], Gulmira Vassilina[1,2]
[1]) Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
marat207@mail.ru
[2]) Almaty University of Power Engineering and Telecommunication named Gumarbek Daukeev, Almaty, Kazakhstan

Galiullin proposes a classification of the main types of inverse problems of dynamics and methods for their solution in the class of ODEs in [1, 2]. Later, in the class of SDEs, inverse problems of dynamics were considered in [3, 4, and others]. The main inverse problem, the reconstruction problem and the closure problem were studied for the second order stochastic equations $\ddot{x} = f(x, \dot{x}, t) + +\sigma(x, \dot{x}, t)\dot{\xi},\ x \in R^n,\ \xi \in R^k$, in the presence of random perturbations, where a) ξ is a vector Wiener process, b) ξ is a vector process with independent increments. At present they study inverse problems in the class of SDEs with degenerate diffusion

$$\begin{cases} \dot{x} = g_1(x, y, t)dt,\ x \in R^{n_1},\ y \in R^{n_2},\ n_1 + n_2 = n,\ n_1 \geq 0,\ n_2 \geq 0, \\ \dot{y} = g_2(x, y, t)dt + \sigma(x, y, t)\dot{\xi},\ \xi \in R^k. \end{cases} \quad (1)$$

In the present paper we consider the main inverse problem: Let the set

$$\Lambda(t) : \lambda(x, \dot{x}, t) = 0,\ \lambda = \lambda(x, \dot{x}, t) \in C^{121}_{x\dot{x},t},\ \lambda \in R^m, \quad (2)$$

be given. It is required to construct a set of SDE systems with degenerate diffusion (1) such that the set (2) is an integral manifold of (1). This problem is solved by the quasi-inversion method [4].

This research has been funded by the Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP19677693).

References

[1] A.S. Galiullin, Methods of solving inverse problems of dynamics, Moscow, 1986.

[2] A.S. Galiullin, Selected works in two volumes. Vol. I and Vol. II. Moscow: RUDN, 2009.

[3] M. I. Tleubergenov, G. K. Vassilina, and D.S. Kulakhmetova, Stochastic Helmholtz Problem with Constraints Linearly Depending on Velocities, Lobachevskii Journal of Mathematics, 43(11) (2022), 3292–3297.

[4] Marat Tleubergenov, Gulmira Vassilina and Gulmira Ibraeva, On the separation method in stochastic reconstruction problem, Open Mathematics, 19 (2021), 1486–1492.

Surface of revolution Radon transforms with centers on generalized surfaces in $\mathbb{R}^n$

James W. Webber[1], Sean Holman[2] and Eric Todd Quinto[3]
[1]) Department of Oncology and Gynecology, Brigham and Women's Hospital, 221 Longwood Ave., Boston, MA 02115
jwebber5@bwh.harvard.edu
[2]) Department of Mathematics, The University of Manchester, Alan Turing Building, Oxford Road, Manchester M13 9PY
[3]) Department of Mathematics, Tufts University, 177 College Ave, Medford, MA 02155

We present a novel analysis of a Radon transform, R, which maps an L^2 function of compact support to its integrals over smooth surfaces of revolution with centers on an embedded hypersurface in $\mathbb{R}^n$. Using microlocal analysis, we derive necessary and sufficient conditions relating to R for the Bolker condition to hold, which has implications regarding the existence and location of image artifacts. We present a general inversion framework based on Volterra equation theory and known results on the spherical Radon transform, and we prove injectivity results for R. Several example applications of our theory are discussed in the context of, e.g., Compton Scatter Tomography (CST) and Ultrasound Reflection Tomography (URT). In addition, using the proposed inversion framework, we validate our microlocal theory via simulation, and present simulated image reconstructions of image phantoms with added noise.

The inverse problem of thermal conductivity in a curved region

Aigul Satybaldina[1], Aizhan Ydyrys[2], Korlan Rysbaeva[3]
[1]) L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
a.satybaldina@iitu.edu.kz
[2]) International Information Technology University, Almaty, Kazakhstan
[3]) National engineering academy of the Republic of Kazakhstan, Almaty, Kazakhstan

Linear models of thermal conductivity are used to describe real physical phenomena. Such an equation, in particular, describes the filtration of an ideal polytropic gas in a porous medium; in the English-language literature, it was given the name "the porous medium equation". Of considerable interest is the study of various initial and boundary value problems for the nonlinear heat equation, such as the Cauchy problem, the Dirichlet problem, the Neumann problem, and mixed boundary value problems. Despite the numerous publications of direct problems for the equation of thermal conductivity, the number of studies devoted to solving inverse problems of thermal conductivity is quite small. Therefore, the present work is devoted to the study of the inverse problem of thermal conductivity.

The problem of heat propagation in a cylindrical region with three different layers is considered. To verify the reliability of the theoretical results, each layer of the cylindrical area is filled with various soils and measurement work is carried out. In order to solve the inverse problem, we use the equation of thermal conductivity in a cylindrical coordinate system:

$$C(r)\rho(r)\frac{\partial(u)}{\partial\bar{t}} = \frac{1}{r}\frac{\partial}{\partial\bar{r}}\left(rk(r)\frac{\partial u}{\partial r}\right) + k(r)\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2} + k(r)\frac{\partial^2 u}{\partial z^2}. \tag{1}$$

In this paper, methods for finding the parameters $C(r), \rho(r)$ and $k(r)$ are proposed. First, the analytical solution of the boundary value problem for equation (1) is determined. Based on the analytical solution of the thermal conductivity equation, the thermophysical parameters of the soil or soil are found. It should be noted that some methods for solving the inverse problem of thermal conductivity in a rectangular coordinate system have been studied in [1]-[2].

References

[1] S. Alpar, B. Rysbaiuly, Determination of thermophysical characteristics in a nonlinear inverse heat transfer problem, Applied Mathematics and Computation, 2023, 20 p.

[2] Karashbayeva Zh., Berger J., Orlande H.R.B., Rysbaiuly B. Estimation of ground thermal diffusivity using the conjugate gradient method with adjoint problem formulation. -2023, -Vol. 52, Article 101676, (quartile: Q1, impact factor: 6.4). https://doi.org/10.1016/j.uclim.2023.101676

Inverse Cauchy problems: revisit and a new approach

Qin Huang[2], Ye Zhang[1,2]
[1]) MSU-BIT-SMBU Joint Research center of Applied Mathematics, Shenzhen MSU-BIT University, Shenzhen, 518172, China
[2]) School of Mathematics and Statistics, Beijing Institute of Technology, Beijing, 100081, China
ye.zhang@smbu.edu.cn

In this talk, we consider a Cauchy problem of finding the unknown Cauchy data on a part of the boundary (unaccessible boundary) from the knowledge of the Cauchy data on the rest of the boundary (accessible boundary). Specifically, let $\Omega \subset \mathbb{R}^d$ ($d \leq 3$: space dimension) be an open bounded set with Lipschitz boundary $\Gamma := \partial\Omega$, which is split into two measurable subsets: $\Gamma = \Gamma_a \bigcup \Gamma_u$ with $\Gamma_a \bigcap \Gamma_u = \emptyset$, and ν be the unit outward normal to Γ. Then the Cauchy problem to be discussed as follows: Given f in Ω, and Cauchy data (Φ, T) on Γ_a, find (ϕ, t) on Γ_u such that the following relations hold:

$$\begin{cases} -\Delta u = f & \text{in } \Omega, \\ \partial_\nu u = \Phi, \quad u = T & \text{on } \Gamma_a, \\ \partial_\nu u = \phi, \quad u = t & \text{on } \Gamma_u. \end{cases} \tag{1}$$

In this talk, we will discuss the following two new methods with the reformulations of original problem (1).

- The coupled complex boundary method [1]: With (Φ, T) on Γ_a, find (φ, t) on Γ_u such that $u_2 = 0$ in Ω, where $\mathbf{u} = u_1 + i\, u_2$ solves

$$\begin{cases} -\Delta \mathbf{u} = 0, & \text{in } \Omega, \\ \partial_n \mathbf{u} + i\, \mathbf{u} = \Phi + i\, T, & \text{on } \Gamma_a, \\ \partial_n \mathbf{u} + i\, \mathbf{u} = \varphi + i\, t, & \text{on } \Gamma_u. \end{cases}$$

- A relaxation model [2]: Given f in Ω, and Cauchy data (Φ, T) on Γ_a, find (ϕ^α, t^α) on Γ_u such that $u_1^\alpha = u_2^\alpha$ in Ω, where

$$\begin{cases} -\Delta u_1^\alpha = 0 & \text{in } \Omega, \\ \alpha\partial_\nu u_1^\alpha + u_1^\alpha = T & \text{on } \Gamma_a, \\ \partial_\nu u_1^\alpha = \varphi & \text{on } \Gamma_u, \end{cases} \quad \text{and} \quad \begin{cases} -\Delta u_2^\alpha = 0 & \text{in } \Omega, \\ \partial_\nu u_2^\alpha = \Phi & \text{on } \Gamma_a, \\ \alpha\partial_\nu u_2^\alpha + u_2^\alpha = t & \text{on } \Gamma_u. \end{cases}$$

References

[1] Gong R., Wang M., Huang Q., Zhang Y., A CCBM-based generalized GKB iterative regularization algorithm for inverse Cauchy problems, Journal of Computational and Applied Mathematics, 432(1)(2023), 115282 (26pp).

[2] Huang Q., Gong R., Jin Q., Zhang Y., A Tikhonov regularization method for Cauchy problem based on a new relaxation model, Nonlinear Analysis: Real World Applications, 74(2023), 103935 (26pp).

MINISYMPOSIUM
M13. Applications of Rich Tomography

Organizers:
Sean Holman, The University of Manchester, UK, sean.holman@manchester.ac.uk
William Robert Breckon Lionheart, The University of Manchester, UK
bill.lionheart@manchester.ac.uk

Traditionally, tomography involves the reconstruction of a scalar function, i.e. an image, from its integrals along lines, or possibly surfaces, passing through a region. Rich tomography is a broad term, referring to a family of methods aiming to reconstruct a more complicated object, such as a vector or tensor field, using additional data, such as travel time, energy, or frequency, along curves or curved surfaces. Many problems with important applications fit under this umbrella including spectral CT, Neutron polarimetric and Bragg edge tomography, various types of emission tomography including that arising in range verification for proton therapy, and in synthetic aperture radar.

This mini-symposium will examine such applications, looking at the similar mathematical techniques which unite them.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Computational Uncertainty Quantification for Inverse problems in Python (CUQIpy)

Nicolai A B Riis[1,4], **Amal M A Alghamdi**[1], Felipe Uribe[21], Silja L Christensen[1], Babak M Afkham[1], Per Christian Hansen[1] and Jakob S Jørgensen[1,3]
[1]) Department of Applied Mathematics and Computer Science, Technical University of Denmark. Richard Petersens Plads, Building 324, 2800 Kongens Lyngby, Denmark.
[2]) School of Engineering Science, Lappeenranta-Lahti University of Technology (LUT). Yliopistonkatu 34, 53850 Lappeenranta, Finland.
[3]) Department of Mathematics, The University of Manchester, Oxford Road, Alan Turing Building, Manchester M13 9PL, UK
[4]) Copenhagen Imaging ApS, Herlev, Denmark

Inverse problems are prevalent in various scientific and engineering applications, and uncertainty quantification (UQ) of solutions to these problems is essential for informed decision-making. In this talk, we present CUQIpy [1, 2] (pronounced "cookie pie"), a Python software package for computational UQ in inverse problems using a Bayesian framework. CUQIpy offers concise syntax that closely matches mathematical expressions, streamlining the modeling process and enhancing the user experience. The versatility and applicability of CUQIpy to many Bayesian inverse problems are demonstrated in various test cases including, but not limited to, computed tomography, electric impedance tomography, and characterization of ear aqueduct varying-in-space diffusivity. These examples showcase the software's efficiency, consistency, and intuitive interface. Our comprehensive approach to UQ in inverse problems provides accessibility for non-experts and advanced features for experts. CUQIpy is developed as part of the CUQI project at the Technical University of Denmark and is available at https://github.com/CUQI- DTU/CUQIpy.

References

[1] Riis N A B, Alghamdi A M A, Uribe F, Christensen S L, Afkham B M, Hansen P C and Jørgensen J S 2024 CUQIpy: I. Computational uncertainty quantification for inverse problems in Python Inverse Problems 40 045009

[2] Alghamdi A M A, Riis N A B, Afkham B M, Uribe F, Christensen S L, Hansen P C and Jørgensen J S 2024 CUQIpy: II. Computational uncertainty quantification for PDE-based inverse problems in Python Inverse Problems 40 045010

[1]Part of the work by F.U. was done while employed at the Technical University of Denmark.

The 11th International Conference "Inverse Problems: Modeling and Simulation"
26th May - 1st June 2024, Malta

Rich and Non-Rich Tomography - a multidisciplinary approach using the Core Imaging Library

Casper da Costa-Luis[1], Margaret A. G. Duff*[1], Gemma Fardell[1], Jakob S. Jørgensen[4,5], Laura Murgatroyd[1], Evangelos Papoutsellis[2,5], Edoardo Pasca[1], Hannah Robarts[1], Danica Sugic[1], and Franck P. Vidal[1,3]

[1] *Scientific Computing Department, Science & Technology Facilities Council, Rutherford Appleton Laboratory, Harwell Campus, Didcot OX11 0QX, UK*
[2] *Finden Ltd, Rutherford Appleton Laboratory, Harwell Campus, Didcot OX11 0QX, United Kingdom*
[3] *School of Computer Science & Engineering, Bangor University, Dean Street, LL57 1UT, UK*
[4] *Department of Applied Mathematics and Computer Science, Technical University of Denmark. Richard Petersens Plads, Building 324, 2800 Kgs. Lyngby, Denmark.*
[5] *Department of Mathematics, The University of Manchester, Oxford Road, Alan Turing Building, Manchester M13 9PL, UK*

Contact: margaret.duff@stfc.ac.uk

In recent years, a multitude of complex tomography challenges have emerged, demanding collaborative efforts across mathematics, algorithm design, and numerical software development.To support this collaboration, we introduced the Core Imaging Library (CIL) [1, 2], an open source Python package designed for the solution of both rich and "non-rich" tomographic and other inverse problems (https://ccpi.ac.uk/cil/).

In this presentation, we illustrate the versatility of CIL through two distinct case studies. Firstly, we showcase our work on hyperspectral neutron tomography [3], where CIL offers a robust solution for spatially and spectrally resolving materials based on Bragg edges in energy-resolved neutron data. Secondly, considering "non-rich" tomography, we demonstrate a directional total variation reconstruction method implemented in CIL. This method, used to achieve a prize at the Helsinki Tomography Challenge 2022 for limited-angle X-ray CT reconstruction [4], underscores CIL's adaptability to diverse imaging scenarios.

Finally, we will demonstrate some of the recent work in CIL to provide a framework for stochastic algorithms which have the potential to increase speed and efficiency with the increasingly large datasets encountered with modern imaging methods.

References

[1] Jørgensen, J.S., Ametova, E., Burca, G., Fardell, G., Papoutsellis, E., Pasca, E., Thielemans, K., Turner, M., Warr, R., Lionheart, W.R. and Withers, P.J., 2021. Core Imaging Library-Part I: a versatile Python framework for tomographic imaging. Philosophical Transactions of the Royal Society A, 379(2204), p.20200192.

[2] Papoutsellis, E., Ametova, E., Delplancke, C., Fardell, G., Jørgensen, J.S., Pasca, E., Turner, M., Warr, R., Lionheart, W.R. and Withers, P.J., 2021. Core Imaging Library-Part II: multichannel reconstruction for dynamic and

spectral tomography. Philosophical Transactions of the Royal Society A, 379(2204), p.20200193.

[3] Ametova, E., Burca, G., Chilingaryan, S., Fardell, G., Jørgensen, J.S., Papoutsellis, E., Pasca, E., Warr, R., Turner, M., Lionheart, W.R. and Withers, P.J., 2021. Crystalline phase discriminating neutron tomography using advanced reconstruction methods. Journal of Physics D: Applied Physics, 54(32), p.325502.

[4] Jørgensen, J.S., Papoutsellis, E., Murgatroyd, L., Fardell, G. and Pasca, E., 2023. A directional regularization method for the limited-angle Helsinki Tomography Challenge using the Core Imaging Library (CIL). Applied Mathematics for Modern Challenges, 1(2), pp.143-169.

Normal operators for momentum ray transforms

Shubham R. Jathar[12]

[1])Lappeenranta-Lahti University of Technology LUT, Finland
[2]) Indian Institute of Science Education and Research (IISER) Bhopal, India
shubham18@iiserb.ac.in

The inversion of the ray transform serves as an important mathematical tool for investigating the properties of objects from external measurements, with extensive applications spanning from medical imaging to geophysics. However, the inversion of the ray transform on symmetric tensor fields is constrained by the presence of an infinite-dimensional null space. A natural question arises: Can we utilize supplementary data in the form of higher-order moments of the ray transform for the explicit recovery of the entire tensor field?

The momentum ray transform, I_m^k, integrates a rank m symmetric tensor field f on $\mathbb{R}^n$ over lines with the weight t^k, given by

$$I_m^k f(x,\xi) = \int_{-\infty}^{\infty} t^k \langle f(x+t\xi), \xi^m \rangle \, dt.$$

In this talk, we compute the normal operator $N_m^k = (I_m^k)^* I_m^k$ and present an inversion formula for recovering a rank m tensor field f from the data $(N_m^0 f, \dots, N_m^m f)$ [1].

Another pertinent question is: What local information about a symmetric m-tensor field can be obtained if, instead of information from the first $m+1$ momentum ray transforms, only partial information, say $I_m^0, \cdots, I_m^k$, where $0 \leq k < m$, is available? To address this, Sharafutdinov introduced the generalized Saint Venant operator $W^{k,m}$ [2]. We next present the explicit recovery of $W^{k,m} f$ for $f \in \mathcal{S}(S^m)$ from the data $(N_m^0 f, \dots, N_m^k f)$ for an arbitrary $0 \leq k \leq m$ [3].

This talk is based on my joint works [1, 3] with Manas Kar (IISER Bhopal), Venkateswaran P. Krishnan (TIFR-CAM), and Vladimir A. Sharafutdinov (Sobolev Institute of Mathematics).

References

[1] S. R. Jathar, M. Kar, V. P. Krishnan and V. A. Sharafutdinov, Normal operators for momentum ray transforms, I: The inversion formula, arXiv preprint arXiv:2401.00791.

[2] V. A. Sharafutdinov, Integral geometry of tensor fields, Inverse and Ill-posed Problems Series, VSP, Utrecht, 1994.

[3] S. R. Jathar, M. Kar, V. P. Krishnan and V. A. Sharafutdinov, Normal operators for momentum ray transforms, II: Generalized Saint Venant operators, Preprint

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Object Characterisation and Bayesian Classification in Metal Detection

P.D. Ledger[1], W.R.B. Lionheart[2] and J. Elgy[1]
[1]) School of Computer Science & Mathematics, Keele University, U.K
p.d.ledger@keele.ac.uk
[2]) Department of Mathematics, The University of Manchester, U.K.

Magnetic polarizability tensors (MPTs) provide an economical characterisation of conducting metallic objects and can aid in the solution of metal detection inverse problems, such as scrap metal sorting, searching for unexploded ordnance in areas of former conflict, and security screening at event venues and transport hubs. Previous work has established explicit formulae for their coefficients, and a rigorous theory for the characterisation they provide (see [1] and references therein). Our open-source `MPT-Calculator` software uses *hp*-finite elements and a proper orthogonal decomposition reduced order modelling (ROM) approach for computing MPT spectral signatures [2, 3] for different object geometries. MPT invariants have been used as features in machine learning (ML) classification of metallic targets [3].

For objects with a high conductivity, and/or objects that have a high magnetic permeability, the electromagnetic skin depth becomes very small compared to the size of the object, and requires careful numerical treatment for accurate solutions. We will present an approach for designing the number and thickness of prismatic layers combined with *p*-refinement for this task. By combining with an adaptive ROM, we will show that MPT characterisations of realistic metal detection targets can be efficiently and accurately computed. Using a dictionary of characterisations obtained from this approach, we will also present results of a Bayesian classification technique for classifying objects in metal detection using MPT spectral signatures, which improves on our previous ML classification, and includes probabilistic outputs and uncertainty measures.

References

[1] P.D. Ledger and W.R.B. Lionheart, The spectral properties of the magnetic polarizability tensor for metallic object characterisation. Mathematical Methods in the Applied Sciences, 43 (1) (2020), 78–113.

[2] J. Elgy, P.D. Ledger, J.L. Davidson, T. Özdeger and A.J. Peyton, Computations and measurements of the magnetic polarizability tensor characterisation of highly conducting and magnetic objects. Engineering Computations,. 40 (7/8) (2023), 1778–1806.

[3] B.A. Wilson, P.D. Ledger and W.R.B. Lionheart, Identification of metallic objects using spectral magnetic polarizability tensor signatures: object classification, International Journal for Numerical Methods in Engineering, 123 (9) (2022), 2076–2111.

Neutron strain tomography

William Lionheart[1]
[1]) Department of Mathematics, University of Manchester
bill.lionheart@manchester.ac.uk

One important example of Rich Tomography is the imaging of strain from Bragg Edge Neutron tomography. For each detector pixel the time-of-flight data is recorded, effectively a transmission spectrum of the neutrons of different energy or wavelength. If the material under investigation is polycrystaline, with crystals below the resolution of the imaging technique, we can interpret this spectrum in terms of the orientation distribution function. This is a scalar function $SO(3)$ bundle of the domain under investigation. This gives the probability density function of orientation of crystals in a two scale approximation. In the case where this can be assumed to be uniform, but the material has been subjected to a strain, certain sharp changes (Bragg edges) in the spectrum can be interpreted as the cumulative density function of the component of the strain in the direction of the beam. In [1] we described a method to recover the second moment of this distribution. In this work we concentrate on the mean strain, which corresponds to the longitudinal ray transform (LRT) of a symmetric tensor field as defined by Sharafutdinov [2]. In this case we take the total strain ε, a rank 2 symmetric tensor field supported on a compact set $\Omega \subset \mathbb{R}^3$ The LRT of the strain, for a line going through $x \in \mathbb{R}^3$ in direction $\xi \in S^2$, is

$$I\varepsilon(x, \xi) = \int_{-\infty}^{\infty} \varepsilon(x + t\xi)_{ij} \xi_i \xi_j.$$

Unfortunately the LRT has a nullspace consisting of tensor fields that are the symmetric derivative of vector fields. We will discuss what can be recovered when the Bragg edge neutron data are combined with *a priori* data from Hook's law and the equilibrium equations in two relevant two-dimensional cases, including experimental and theoretical work reported in [3]

References

[1] Fogarty, K., Ametova, E., Burca, G., Korsunsky, A. M., Schmidt, S., Withers, P. J., & Lionheart, W. R. (2022). Recovering the second moment of the strain distribution from neutron Bragg edge data. Applied Physics Letters, 120(16).

[2] Sharafutdinov, V.A. Integral geometry of tensor fields. Walter de Gruyter, 2012.

[3] Wensrich CM, Holman S, Courdurier M, Lionheart WR, Polyalova A, Svetov I. Direct inversion of the Longitudinal Ray Transform for 2D residual elastic strain fields. arXiv preprint arXiv:2309.02440. 2023 Jul 21.

The 11th International Conference ”Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Magnetic and thermostatic nonabelian ray transforms

Jesse Railo
Lappeenranta–Lahti University of Technology LUT, Finland
jesse.railo@lut.fi

We discuss recent advances and open problems related to the attenuated and nonabelian ray transforms for generalized geodesic flows.

The talk is based on my recent work with Shubham R. Jathar and Manas Kar from IISER Bhopal, India.

MINISYMPOSIUM
M14. Recent Advances in Inverse Scattering Theory and Applications

Organizers:
Thorsten Hohage, Georg-August-Universität Göttingen, Germany, hohage@math.uni-goettingen.de
Roman Novikov, Ecole Polytechnique, France, roman.novikov@polytechnique.edu

Many developing and emerging fields in physics and engineering, including medical and biological applications, are rich sources of the inverse scattering problems of waves and particle propagation. The aim of this minisymposium is to bring together experienced and young researchers interested in different aspects of inverse scattering and to discuss the recent advances, challenges, and applications of these studies. Among other things, we focus on uniqueness, reconstruction algorithms, problems with partial data, and simulations.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Inverse Scattering Problem for Dissipative Equations

Elena Cherkaev[1]

[1]) University of Utah
elena@math.utah.edu

Wave propagation in lossy electromagnetic or viscoelastic media is characterized by attenuation and dispersion effects which are absent in the case of lossless materials. Applications of the inverse dissipative problems for lossy media can be found in medical imaging and geophysical exploration; in biomedical applications, changes in viscoelasticity of soft tissues often serve as an important biomarker associated with specific malignancies and have a diagnostic value for the detection, characterization, and monitoring of arteriosclerosis, osteoporosis, various cancers.

Different efficient methods have been developed for solution of the inverse scattering problems, for example, based on reduced order model finite-difference embedding procedure (Krein embedding) [4] in a 1d dissipative problem or the reduced order method for Lippman-Schwinger integral equation [1].

The talk discusses an approach to imaging of lossy media based on representing the scattering problem with complex coefficients using decomplexification that results in a problem for a J-self-adjoint (or J-symmetric) operator [3]. We present an extension of the data driven reduced order method for the Lippman-Schwinger equation [1] to the case of the lossy dissipative scattering problem with complex coefficients by formulating the problem for a J-symmetric operator and using a J-symmetric Lanczos algorithm [2].

References

[1] J. Baker, E. Cherkaev, V. Druskin, S. Moscow, M. Zaslavsky, Regularized reduced order Lippmann-Schwinger-Lanczos method for inverse scattering problems in the frequency domain, https://arxiv.org/abs/2311.16367

[2] J. Baker, E. Cherkaev, V. Druskin, Reduced order model Lippman-Schwinger approach to imaging of lossy media, in preparation.

[3] E. Cherkaev, A new inversion method for dissipating electromagnetic problem, Mathematical and Numerical Aspects of Wave Propagation, A. Bermudez et al., eds., SIAM, 2000, 637-642.

[4] J. Zimmerling, V. Druskin, M. Guddati, E. Cherkaev, R. Remis, Solving inverse scattering problems via reduced-order model embedding procedures, Inverse Problems 40 (2024) 025002.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Data-driven computation of the interior solutions for inverse scattering problems and beyond

Vladimir Druskin[1]
[1] Worcester Polytechnic Institute
vdruskin@wpi.edu

Data-driven reduced order models (ROMs) are utilized primarily in cases where a full-scale model is inaccessible and can only be approximated through observed data, hence the name. A novel extension of this methodology has recently emerged, particularly in the context of wave-based imaging applications such as seismic exploration and radar systems [1, 2, 3]. This extension involves construing physically meaningful network representations of ROMs that can be constructed as finite-volume discretizations of an underlying partial differential equation (PDE). Consequently, observed data can be embedded into the state-space of the PDE, even when the coefficients therein are unknown. This framework enables the estimation of both the state solution and the properties of the medium in regions where direct measurements are unavailable. We present the theoretical underpinnings of this approach using a 1D wave example in the frequency domain, show its connections to established model order reduction (MOR) techniques, and, time permitting, explore its extensions to multidimensional settings and its applications in seismic and synthetic aperture radar domains in presence of strong multiple-scattering effect, where the inverse problem becomes strongly nonlinear.

Contributions from Liliana Borcea, Alex Mamonov, Shari Moskow, Rob Remis, Mikhail Zaslavskiy, and Joern Zimmerling have been instrumental at various stages of this research.

References

[1] V. Druskin, A. Mamonov, A. Thaler, and M. Zaslavsky, Direct, nonlinear inversion algorithm for hyperbolic problems via projection-based model reduction, SIAM Journal on Imaging Sciences, vol. 9(2), pp.684–747, 2016.

[2] V. Druskin, A. Mamonov, and M. Zaslavsky, A nonlinear method for imaging with acoustic waves via reduced order model backprojection, SIAM Journal on Imaging Sciences, vol. 11(1), pp. 164–196, 2018.

[3] V. Druskin, S. Moskow and M. Zaslavsky, Reduced order modeling inversion of monostatic data in a multi-scattering environment, SIAM Journal on Imaging Sciences, vol. 17(1), pp. 334–350, 2024.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Seismic resonance tomography: inverse resonance problem in surface-wave tomography

Alexei Iantchenko
Department of Materials Science and Applied Mathematics
Faculty of Technology and Society, Malmö University SE-205 06 Malmö, Sweden
alexei.iantchenko@mau.se

Semiclassical analysis can be employed to describe surface waves in an elastic half space which is quasi-stratified near its boundary. In case of isotropic medium, the surface wave decouples up to principal parts into Love and Rayleigh waves associated to scalar and matrix spectral problems, respectively. Since the mathematical features (such as spectrum, resonances or leaking modes) of these problems can be extracted from the seismograms, we are interested in recovering the Lamé parameters from these data.

The occurrence of leaking modes was noted more than 60 years ago. However, usual exploratory methods were mostly numerical and did not take advantage of modern achievements in inverse spectral, scattering, and resonance theories. The progress in detecting and exploiting them in studying Earth's intrerior has been very limited, in part due to absence of comprehensive analysis of leaking modes, which we completed in [1], showing that the frequencies of leaking modes (wavenumber resonances) for the Rayleigh waves are the poles of the resolvent and specifying the location of these poles.

Note that similar analysis for Love waves is essentially well known, as scalar Love equation can be reduced to the Schrödinger equation, which is of type studied in [2], via simple Liouville or calibration transforms, while no such simple reduction is possible for the non-Schrödinger type matrix Rayleigh equation. The inverse problems for Schrödinger operators are generally well studied, while similar studies for non-Schrödinger type matrix-valued Rayleigh operator are essentially unknown.

In this contribution, based on the solution of the direct problem in [1] and the solution of the inverse problem for the Neumann-to-Dirichlet map (Weyl function) in [3], we consider the inverse Rayleigh problem with discrete data characterizing the reflection matrix and including the wavenumber resonances.

This is a part of the joint project with Maarten V. de Hoop.

References

[1] M.V.de Hoop and A. Iantchenko, Analysis of wavenumber resonances for the Rayleigh system in half space, Proc. R. Soc. A 479:20220845 (2023).

[2] E. Korotyaev, Inverse resonance scattering on the half line, Asymptot. Anal. 37, 215–226 (2004).

[3] M.V.de Hoop and A. Iantchenko, Inverse problem for the Rayleigh system with spectral data, J. Math. Phys. 63, 031505 (2022).

The temporal domain derivative and an application in inverse acoustic scattering

Marvin Knöller[1], Jörg Nick[2]
[1]) Karlsruhe Institute of Technology, Karlsruhe, Germany
marvin.knoeller@kit.edu
[2]) ETH Zürich, Zürich, Switzerland

The concept of domain derivatives is a significant tool in shape optimization, particularly within the context of inverse scattering problems. Domain derivatives that arise from solving a time-harmonic scattering problem have been studied for various geometrical setups, boundary conditions and partial differential equations. In this talk we focus on establishing the temporal domain derivative for the acoustic wave equation in the presence of a sound-soft scattering object. We proceed through the Laplace domain, wherein we establish bounds for the frequency-domain shape derivative expressed in terms of powers of the frequency. Performing an inverse Laplace transform, this provides the temporal domain derivative. In our inverse problem, the aim is the reconstruction of the scattering object based on time-domain measurements of the scattered wave at a limited number of observation points located away from the scattering object. In the application of a Gauss-Newton algorithm to reconstruct the scattering object, the convolution quadrature method is employed to ensure an efficient time-integration. This is crucial for evaluating both the forward map and the temporal domain derivative. The efficacy of our approach is emphasized by numerical examples.

Passive inverse obstacle scattering problems

Thorsten Hohage[1], Meng Liu[2]
[1]) University of Göttingen, Germany
[2]) Max Planck Institute for Solar System Research, Germany

In this talk, we are concerned with the passive inverses obstacle scattering problems associated with the Helmholtz equation in $\mathbb{R}^d$ $(d = 2, 3)$. The random source is modelled by an uncorrelated centered Gaussian process. We formulate three continuous, infinite dimensional mathematical models to describe passive source, obstacle and source-obstacle problems, respectively. Uniqueness results are then established for the corresponding inverse problems in determining the source strength, shape and location of an obstacle and both of them simultaneously, by using the near-field correlation measurements. Finally, we present several efficient inversion methods to reconstruct them numerically.

An accelerated level-set method for inverse scattering problems

Lorenzo Audibert[1], Houssem Haddar[2], Xiaoli Liu[3]
[1]) EDF R&D PRISME, 78400, Chatou, France
[2]) INRIA, Center of Saclay Ile de France and UMA, ENSTA Paris Tech, Palaiseau Cedex, France
[3]) School of Mathematical Sciences, Beihang University, Beijing, 100191, China
xiaoli_liu@buaa.edu.cn

In this talk, we propose a rapid and robust iterative algorithm to solve inverse acoustic scattering problems formulated as a PDE constrained shape optimization problem. We use a level-set method to represent the obstacle geometry and propose a new scheme for updating the geometry based on an adaptation of the celebrated Nesterov accelerated gradient descent method. The resulting algorithm aims at reducing the number of iterations and improving the accuracy of reconstructions. To cope with regularization issues, we apply a moderate smoothing to the shape gradient using a single layer potential associated with ik where k is the wave number. Numerical experiments are given for several data types (full aperture, backscattering, phaseless, multiple frequencies) to show the effectiveness of our method comparatively to a non accelerated level set method.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Stability and Instability for Random Inverse Source Problems

Philipp Mickan[1,*], Thorsten Hohage[1,2]
[1]) Institute for Numerical and Applied Mathematics, Georg-August University Göttingen, Germany
[2]) Max Planck Inistute for Solar System Research, Göttingen, Germany
[*])p.mickan@math.uni-goettingen.de

We investigate the inverse source problem to determine the strength of a random acoustic source from correlation data. Specifically, the data to this inverse problem consists of correlations of time-harmonic acoustic waves acquired on a surface surrounding a region of random and uncorrelated sources. Uniqueness to this inverse problem has been established in previous studies [1, 4]. A natural extension of uniqueness for an inverse problem is to study its stability. Such an analysis is important in the context of regularisation as it enables to deduce convergence rates for regularisation methods. There are two bounds to be establish the upper bound usually referred to as stability estimate and the lower bound called to as instability estimate. We show both bounds to be of logarithmic type under some Sobolev-type smoothness assumptions on the exact source strength of the random source. The stability result is based on the method to verify variational source conditions for inverse problems [3]. Such a variational source condition can also be used in general settings to give convergence analysis and stability and fit well into the framework of variational methods in regularisation theory. The instability argument is based on the general setting of an entropy argument presented for operators of the type $X \to \mathcal{L}(H, H')$ with X some metric space and H some separable Hilbert space [2]. As a third and last part we show numerical experiments that support our theoretical results.

References

[1] A. J. Devaney. The inverse problem for random sources. In: *Journal of Mathematical Physics* **20.8** (1979), pp. 1687–1691.

[2] M. Di Cristo and L. Rondi. Examples of exponential instability for inverse inclusion and scattering problems. In: *Inverse Problems*, **19** (2003), p. 685.

[3] T. Hohage and F. Weidling. Characterizations of variational source conditions, converse results, and maxisets of spectral regularization methods. In: *SIAM Journal on Numerical Analysis*, **55.2** (2017), pp. 598–620.

[4] T. Hohage, H.-G. Raumer, and C. Spehr. Uniqueness of an inverse source problem in experimental aeroacoustics. In: *Inverse Problems* **36(7)** (2020), p. 075012.

Field extrapolation and denoising in the inverse magnetisation problem

Dmitry Ponomarev[1]
[1]) Centre Inria d'Université Côte d'Azur
dmitry.ponomarev@inria.fr

Motivated by a concrete experimental setting in the Paleomagnetism lab at EAPS Department of MIT (USA), we consider an instance of the inverse magnetisation problem when one component of the magnetic field is measured on the plane above a thin slice of a magnetic rock sample. More precisely, the vertical component of the field B_3 measured at distance $h > 0$ from the sample and its unknown magnetisation distribution $\vec{M} \in \left[W^{1,2}(Q)\right]^3$ supported on a bounded region $Q \subset \mathbb{R}^2$ are related as follows [1]:

$$B_3(\mathbf{x}, h) = \frac{h}{4\pi} \iint_Q \frac{\partial_{t_1} M_1(\mathbf{t}) + \partial_{t_2} M_2(\mathbf{t})}{\left(|\mathbf{x}-\mathbf{t}|^2 + h^2\right)^{3/2}} \mathrm{d}^2 t - \frac{1}{4\pi} \frac{\partial}{\partial h} \iint_Q \frac{h M_3(\mathbf{t})}{\left(|\mathbf{x}-\mathbf{t}|^2 + h^2\right)^{3/2}} \mathrm{d}^2 t, \quad \mathbf{x} \in \mathbb{R}^2,$$

where $\mathbf{x} \equiv (x_1, x_2)^T$. The values of $B_3(\cdot, h)$ are typically available over the region Q (or a slightly larger area). In practice, when the magnetisation distribution is spreaded inside a rock sample, this size of the measurement area is not sufficient for performance of most inversion procedures (see, e.g., [2, 3]). On top of that, magnetic fields in the paleomagnetic context are of very weak intensity and hence subject to a significant contamination by noise. This motivates the problem of the field extrapolation and denoising. We attempt to address both issues at once by proposing an approach for constructing a stable field extrapolant from measured data based on an auxiliary spectral problem for a matrix integral operator. The proposed method is illustrated numerically.

References

[1] L. Baratchart, D.P. Hardin, E.A. Lima, E.B. Saff, B.P. Weiss, Characterizing kernels of operators related to thin-plate magnetizations via generalizations of Hodge decompositions, *Inverse Problems* **29** (2013).

[2] L. Baratchart, S. Chevillard, D.P. Hardin, J. Leblond, E.A. Lima, J.-P. Marmorat, Magnetic moment estimation and bounded extremal problems, *Inverse Problems and Imaging* **13**(1) (2019).

[3] D. Ponomarev, Magnetisation moment of a bounded 3D sample: asymptotic recovery from planar measurements on a large disk using Fourier analysis, *arXiv:2205.14776* (2022).

Completed-data-driven ROMs for SAR imaging

Mikhail Zaslavsky[1], Vladimir Druskin[2], Shari Moskow [3]
[1])Southern Methodist University, Department of Mathematics
mzaslavskiy@smu.edu
[2]) Worcester Polytechnic Institute, Department of Mathematical Sciences
[3]) Drexel University, Department of Mathematics

In this talk we discuss Lippmann–Schwinger–Lanczos (LSL) algorithm that has recently been shown to provide an efficient tool for imaging and direct inversion of synthetic aperture radar data in multi-scattering environments [1], where the dataset is limited to the monostatic a.k.a. single input/single output (SISO) measurements. The approach is based on interpolatory reduced-order models (ROMs) that allow to learn the internal solutions using only data-driven Grammians of the time-domain snapshots. In particular, we construct internal solutions for each SISO channel separately and then couple them together via Lippmann-Schwinger integral equation. We also discuss ROM-based data completion algorithm to populate the missing off diagonal elements of the multiple input/multiple output (MIMO) matrix valued transfer function and, consequently, to improve the approximation of internal solutions and reduce possible echoes in the image. In particular, first, we apply the LSL algorithm to the SISO data as in [1] to obtain approximate reconstructions as well as the estimate of internal field. Next, we use these to calculate a forward Lippmann-Schwinger integral to populate the missing off diagonal data. Finally, we solve Lippmann-Schwinger equation for the original SISO data where the internal fields are approximated via ROM for partially approximate MIMO data we have obtained. Efficiency of the proposed approach is demonstrated on 2D and 2.5D numerical examples, where we see reconstructions are improved substantially.

References

[1] V. Druskin, S. Moskow, and M. Zaslavsky, Reduced order modelling inversion of monostatic data in a multi-scattering environment, SIAM Journal on Imaging Sciences, 17(1)(2024), 334–350

Reduced order modeling approach to inverse scattering

Jörn Zimmerling[1]
[1]) Uppsala University, Sweden, jorn.zimmerling@it.uu.se

Full waveform inversion is a technique used to estimate the properties of a complex medium characterized by variable coefficients in wave equations. Sources emit probing signals into a penetrable medium and receivers record the resulting waves. Despite being a well-established and extensively studied inverse problem with broad applications, existing inversion methods remain unsatisfactory. The conventional approach involves nonlinear least squares data fit optimization which faces challenges due to the non-convex nature of the objective function.

This behavior has at least three etiologies: (1) The mapping from the unknown coefficients to the wave field is nonlinear and complicated; (2) The sources and receivers often lie on a single side of the medium, so only backscattered waves are measured unless large offsets capture diving waves; (3) The probing signals are band-limited and lacking low-frequency content.

There is extensive research in the computational science and engineering communities into mitigating the difficulty of estimating the medium via data fitting. In this talk, we present a different method based on reduced order models (ROMs) of two operators that control the wave propagation. The ROMs are called data-driven because they are computed directly from the measurements, without any knowledge of the wave field inside the inaccessible medium. The resulting ROMs capture features of the physics of wave propagation in a complementary way and have surprisingly good approximation properties that facilitate waveform inversion.

Acknowledgements

The presented work is in collaboration with Liliana Borcea, Josselin Garnier and Alexander V. Mamonov. For details see [1, 2].

References

[1] Liliana Borcea, Josselin Garnier, Alexander V. Mamonov, and Jörn Zimmerling, When data driven reduced order modeling meets full waveform inversion, arXiv:2302.05988

[2] Liliana Borcea, Josselin Garnier Alexander V. Mamonov, and Jörn Zimmerling, Waveform inversion with a data driven estimate of the internal wave, SIAM Journal on Imaging Sciences, 16:1, 280 - 312, (2023),

MINISYMPOSIUM
M15. Imaging with Waves in Complex Media

Organizers:
Arnold Kim, University of California, Merced, USA, adkim@ucmerced.edu
Chrysoula Tsogka, University of California, Merced, USA, ctsogka@ucmerced.edu

The aim of this minisymposium is to bring together scientists working on imaging problems involving wave propagation in complex media. Examples of these problems include media with random fluctuations due to turbulence, randomly distributed discrete particles, and rough interfaces and boundaries. This research area finds applications in diverse fields ranging from underwater acoustics, exploration seismology and geophysics to medical imaging and radar.

This minisymposium will present some of the latest advances in this research area.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Seismic Imaging of Dam-Rock Interface using Full-Waveform Inversion

Mohamed Aziz Boukraa[1], Lorenzo Audibert[2], Marcella Bonazzoli[1], Houssem Haddar[1], Denis Vautrin[2]

[1]) UMA, INRIA, ENSTA Paris, Institut Polytechnique de Paris, 91120 Palaiseau, France
mohamed.boukraa@inria.fr
[2]) EDF R&D, département PRISME, bâtiment U, 6 quai Watier, BP 49, 78401 Chatou Cedex, France

In the stability analysis of hydroelectric dams, comprehending the shape of the contact between the dam structure made of concrete and the underlying rock holds paramount importance. Although conventional geotechnical techniques such as core drilling are utilized, they often prove invasive and offer restricted insights. Challenges emerge due to the minimal parameter contrast between concrete and rock, alongside the distinctive dam geometry, diverging from the stratified mediums typically encountered in geological and geophysical investigations.

In our research, we propose an advanced approach to analyze seismic data for generating high-resolution images using geophysical measurement techniques. Our method is based on Full-Waveform Inversion (FWI) coupled with a shape optimization strategy. We employ a numerical optimization algorithm that integrates finite element simulations of wave equations alongside perimeter regularization. The model accommodates diverse measurement types, including elastic waves originating from the dam wall and acoustic waves from the water. Perfectly Matched Layer (PML) techniques are employed to truncate the computational domain in the rock and water media. High-performance computing techniques are used to ensures the efficiency of simulations, while accurately modeling small-sized sources and receivers relative to the dam's extensive structure.

To validate our approach, we present numerical results from simulated seismic data generated by Code_Aster—an established engineering software developed by EDF (Électricité de France), recognized for its reliability and effectiveness in industrial contexts. The dataset encompasses diverse interface geometries, enabling a thorough evaluation of our approach. Our method excels in accurately reconstructing the interface, surpassing the limitations of traditional reverse time migration techniques. Furthermore, we conduct extensive sensitivity analyses, exploring variables such as source and receiver positions and quantities, wave frequency span and sampling, as well as uncertainties in material properties.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Estimation of the effective sound speed in an acoustic medium

Josselin Garnier[1], Laure Giovangigli[2,⋆], Quentin Goepfert [1,2] and Pierre Millien [3]

[1] CMAP, CNRS, Ecole polytechnique, Institut Polytechnique de Paris, 91120 Palaiseau, France

[2] POEMS, CNRS, Inria, ENSTA Paris, Institut Polytechnique de Paris, 91120 Palaiseau, France

[3] Institut Langevin, ESPCI Paris, PSL University, CNRS, 1 rue Jussieu, F-75005 Paris, France

⋆ Corresponding author: laure.giovangigli@ensta-paris.fr

The emergence of quantitative medical imaging techniques constitutes a major shift of paradigm for the theory of inverse problems. Imaging modalities are now expected not only to produce an image but quantitatively reconstruct parameters of interest. In this work, we focus on medical ultrasound imaging. We aim at constructing an estimator of the effective velocity in a complex medium where echoes come from numerous unresolved scatterers randomly distributed throughout the medium [1]. By analyzing the dependence of the point spread function (PSF) with respect to the backpropagation speed, we build an estimator of the sound speed in the medium. We perform a quantitative sensitivity analysis and confront our result with experimental measurements [2].

References

[1] Josselin Garnier, Laure Giovangigli, Quentin Goepfert and Pierre Millien. Scattered wavefield in the stochastic homogenization regime. arXiv:2309.07777, 2023.

[2] Flavien Bureau PhD Thesis - Multi-dimensional analysis of the reflection matrix for quantitative ultrasound imaging. Université Paris sciences et lettres, 2023.

Inverse problems with anomalous diffusion

Pedro González-Rodríguez[1],
[1]) Universidad Carlos III de Madrid
pgonzale@ing.uc3m.es

During the last decades fractional calculus has gained a lot of popularity due to its widespread applications for modeling processes with memory and processes with spatial non-locality. Among these, transient-anomalous diffusion has been observed in biological tissues, as demonstrated by different single-particle-tracking optical experiments [1].

This have consequences in some imaging techniques, for example diffusion magnetic resonance imaging (DMRI) [2]. In this talk I would like to show what are the differences that I have found when solving inverse problems associated with anomalous diffusion with methods such as L1 minimization or MUSIC with respect to the same problems using conventional diffusion.

References

[1] C. Manzo and M.F. Garcia-Parajo, A review of progress in single particle tracking: from methods to biophysical insights, Rep Prog Phys. 78 (2015)

[2] S. Caupani and M. Palombo, Mini review on anomalous diffusion by MRI: Potential advantages, pitfalls, limitations, nomenclature, and correct interpretation of literature, Frontiers in Physics, 7 (2020) 248.

Ground-penetrating synthetic aperture radar imaging of dispersive targets

Arnold D. Kim[1] and Chrysoula Tsogka[2]
Department of Applied Mathematics, University of California, Merced, 5200 North Lake Road, Merced, CA 95343, USA
Email: [1]adkim@ucmerced.edu and [2]ctsogka@ucmerced.edu

We consider ground-penetrating synthetic aperture radar (SAR) imaging problems related to applications of buried landmine detection using unmanned aerial vehicles (UAV). For this application, a UAV flies along a prescribed flight path. Along that flight path, the UAV emits multi-frequency signals down onto an interface separating air from soil and records the resulting echoes that include both ground bounce signals from the interface and scattered signals from subsurface targets.

For this problem the synthetic aperture corresponds to a relatively short flight path above the surface. Moreover, for monostatic SAR imaging, which we consider here, measurements are restricted to retroreflected or backscattered signals only. These limitations lead to a partial aperture imaging problem with a relatively small aperture. Because of this limited aperture, it is unlikely that one can recover geometrical features such as the support of an extended target.

In light of this partial aperture imaging problem, we introduce a dispersive point target model [1]. Rather than using geometrical features of the target, this model seeks to distinguish different targets through their frequency-dependent reflectivities. Using this dispersive point target model, we extend the UAV-based ground-penetrating SAR imaging problem to (i) identify and locate these targets and (ii) recover quantities related to their frequency-dependent reflectivities.

We show that Kirchhoff migration (KM) is able to identify dispersive point targets in a subsurface imaging region, even when the interface is not known [2]. However, KM predicts target locations that are shifted in range from their true locations due to their dispersive reflectivities [1]. Nonetheless, we seek to recover the frequency-dependent radar cross-section (RCS) for targets identified using KM. The RCS is proportional to the modulus-squared of the reflectivity and provides a means for classifying different targets. We demonstrate our method using two-dimensional simulations using the method of fundamental solutions.

References

[1] A. D. Kim and C. Tsogka, Synthetic aperture imaging of dispersive targets, IEEE Transactions on Computational Imaging 9 (2023), 954-962.

[2] A. D. Kim and C. Tsogka, Synthetic aperture radar imaging below a random rough surface, Radio Science 58(12) (2023), e2023RS007712.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Near Field sampling and Resolution in linear inverse scattering problems

Maria Antonia Maisto[1], Raffaele Solimene[1]
[1])Department of Engineering, Universitá degli Studi della Campania "Luigi Vanvitelli"
mariaantonia.maisto@unicampania.it.

In this contribution we are concerned with linear inverse scattering problems. As is well known, they require the inversion of a compact operator. Under the assumption that the targets have bounded energy and they are a few wavelengths far from the observation domain, the kernel function of the scattering operator behaves nearly as an entire function of exponential type. This leads its singular values to nearly exhibit a step-like behaviour, i.e., they abruptly decay beyond a certain critical index called Number of Degree of Freedom NDF mainly dependent on geometrical parameters. This results into two main implications: on one hand, the inverse problem is severally ill-posed; on the other one, the Range of the operator (the set of all possible scattered fields) can be approximated by a finite dimensional subspace. As concern the first issue, it can be overcame by regularization schemes that lead to approximate solutions resulting from a trade-off between accuracy and stability. According to the above-mentioned considerations, the best regularized solutions that can be obtained is the projection of the actual unknown onto the space spanned by the fist NDF singular functions. This entails that the achievable resolution is limited and mainly dependent on the configuration parameters. Instead, the second issue entitles us to look for finite dimensional sampling representation of the scattered field and its size mainly depends geometrical parameters. In this paper, the focus is establishing a theoretically clear link between the configuration parameters (i.e., measurement aperture, frequency band and background medium electromagnetic features) the number of data required to represent the field and the achievable performance in terms of resolution. To achieve such a task, the mathematical features of the scattering operator, and how they depend on the background medium, are studied. The study is developed for a 2D scalar near field configuration. Since, in these cases far-field approximation does not usually work, many nice results concerning convolution kernels and Fourier transformations cannot directly be applied. Therefore, here we introduce suitable variable transformations that allow to highlight the kernel of the scattering operator as a pseudo-differential operator. As a consequence, a data sampling criterion is derived, and resolution estimation obtained [1].

References

[1] M. A. Maisto, R. Solimene, and R. Pierri "Transverse Resolution in Microwave Imaging for Strip Objects Buried in a Half-Space Medium", Progress In Electromagnetics Research M, vol. 88, pp. 145-157, 2020

Phase- and absorption-contrast imaging

Miguel Moscoso[1], Alexei Novikov[2], George Papanicolaou[3], Chrysoula Tsogka[4]
[1]) Universidad Carlos III de Madrid, 28911 Leganes, Madrid, SPAIN
moscoso@math.uc3m.es
[2]) Penn State University, University Park, PA 16802, USA
novikov@psu.edu
[3]) Stanford University, Stanford, CA 94305, USA
papanicolaou@stanford.edu
[4]) University of California, Merced, Merced, CA 95343, USA
ctsogka@ucmerced.edu

I will discuss situations in which phase-contrast imaging provides a means to distinguish between structures of similar transparency that cannot be seen in images based on absorption differences. The proposed approach uses coherent illumination and records the intensity of the distorted waves that pass through the object under inspection but not the phases. It takes advantage of the fact that these structures have different refractive indices and, thus, they not only attenuate the waves but also delay, bend or refract the waves that go through them by different amounts. Contrary to absorption changes, phase changes cannot be visualized directly with the naked-eye, so they must be transformed into intensity differences that are observable in the images. This has applications in, for example, biology where one wants to image unstained samples to distinguish between structures that are of similar transparency and, hence, cannot be visualized in absorption contrast.

References

[1] M. Moscoso, A. Novikov, G. Papanicolaou and C. Tsogka, "Quantitative Phase and Absorption Contrast Imaging," in IEEE Transactions on Computational Imaging, vol. 8, pp. 784-794, 2022, doi: 10.1109/TCI.2022.3204401.

Dictionary learning for imaging in complex media

Alexei Novikov[1]
novikov@psu.edu

The purpose of this talk is to introduce and develop a new high-resolution imaging approach in Heterogeneous Media. The idea is to learn the exact realization of media's heterogeneities by capitalizing on the diversity of illuminations, measurements, and positions of point-like objects. This allows to bring new methods from computational geometry and computer science to sensing.

More specifically, we explain how the Dictionary Learning and the Multidimensional Scaling algorithms could be used for high-resolution imaging.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Synthetic aperture imaging using physically informed convolutional neural networks

Symeon Papadimitropoulos[1], Mohamed Hasan[2], Chrysoula Tsogka[3]

[1]) Applied Mathematics Department, University of California, Merced, CA 95343
symeon@ucmerced.edu

[2]) Computer Science Department, University of California, Merced, CA 95343

[3]) Applied Mathematics Department, University of California, Merced, CA 95343

We propose a physically informed neural network (PINN) approach to address the problem of locating scatterers using synthetic aperture radar (SAR). The proposed approach uses the physical properties of wave propagation to aid the neural network in learning to detect and localize scatterers from SAR data. This methodology is especially efficient in super-resolution cases, *i.e.* when the scatterers are located close to each other, at distances smaller than the nominal resolution of the imaging system. Using the proposed approach, we are able to detect an a priorly unknown number of scatterers and localize them with high accuracy.

References

[1] Kahana, A., Papadimitropoulos, S., Turkel, E., Batenkov, D.; A physically informed deep-learning approach for locating sources in a waveguide. J. Acoust. Soc. Am. 2023; 154 (4): 2553–2563

[2] Cheney, M., Borden, B., 2009. Fundamentals of radar imaging. SIAM.

[3] Alvarez, J., Kim, A., Marcia, R.F. and Tsogka, C.. Synthetic aperture radar inverse scattering reconstruction using convolutional neural networks. In Applications of Machine Learning 2022 (Vol. 12227, p. 1222703). SPIE.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Source imaging through a complex section

Knut Solna
Mathematics Department, School of Physical Sciences
University of California, Irvine
340 Rowland Hall, 3875, Irvine, CA 92697
Email: ksolna@uci.edu

An interesting phenomenon in optics is that it is possible to see a person behind a shower curtain better than that person can see us. This effect has been referred to as the shower curtain effect [1].

We address the challenge of giving a precise mathematical description of this phenomenon. In addition, we identify what governs the effect and discuss how imaging algorithms can be designed and analyzed when the objective is to image a source hidden behind a complex section.

References

[1] J. Garnier, K. Sølna, Shower curtain effect and source imaging, *Inverse Problems and Imaging*, Accepted (2024).

MINISYMPOSIUM
M16. Inverse Problem Theory for Innovation of Detection Methods

Organizers:
Jérôme Fehrenbach, Université Paul Sabatier (Toulouse 3), France, jerome.fehrenbach@math.univ-toulouse.fr
Hiromichi Itou, Tokyo University of Science, Japan, h-itou@rs.tus.ac.jp
Yavar Kian, Université de Rouen Normandie, France, yavar.kian@univ-rouen.fr
Victor A. Kovtunenko, University of Graz, Austria, and Lavrentyev Institute of Hydrodynamics SB RAS, Novosibirsk, Russia, victor.kovtunenko@uni-graz.at

This mini-symposium aims to consider an inverse problem theory for innovations in detection technology and to develop numerical analysis schemes backed by theoretical reconstruction methods, which are connected with several important applications appearing in problems of medical imaging, environmental and industrial problems. More precisely, we concentrate on inverse source and inverse obstacle problems in connection with practical research topics such as detecting defects in elastic structures, localization of (static or moving) contaminant sources, detection of areas of abnormal electrical and mechanical characteristics inside the human body, etc.

We expect to bring together young researchers and experts working in such directions and to have deep discussions, interactions with each other and further developments.

Carleman estimates and inverse problems for first-order hyperbolic equations

Giuseppe Floridia[1]
[1]) Sapienza Università di Roma
giuseppe.floridia@uniroma1.it

In this talk we present some results about inverse problems for first-order hyperbolic equations via Carleman estimates. In particular, following the approach of [1] and [2] we study the case of transport equations with singular potential terms.

References

[1] G. Floridia and H. Takase, *Inverse problems for first-order hyperbolic equations with time-dependent coefficients*, J. Differential Equations (Elsevier), 2021, 45-71, DOI: https://doi.org/10.1016/j.jde.2021.10.007.

[2] G. Floridia, H. Takase, M. Yamamoto, *A Carleman estimate and an energy method for a first-order symmetric hyperbolic system*, Inverse Problems and Imaging (IPI, AIMS), 2022, **Volume 16**, Issue 5:1163-1178. doi: 10.3934/ipi.2022016.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

An iterated sensitivity equation to reconstruct perturbations with microwave imaging

Jérémy Heleine[1]
[1]) Institut de Mathématiques de Toulouse - Toulouse, France
jeremy.heleine@math.univ-toulouse.fr

We are interested in an inverse problem consisting in emitting an electromagnetic wave towards an object and then, from surface measurements, reconstructing the complex refractive index of the medium inside the object.

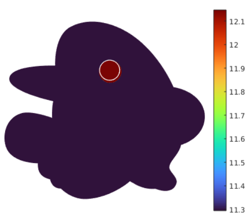
Working in a 2D or 3D domain Ω, we denote its boundary $\Gamma = \partial\Omega$ and its outward unit normal $\mathbf{n}$. The wavenumber $k > 0$ is fixed. The direct problem consists in determining the intensity of the electric field propagating inside the medium, for a given refractive index κ (which is a function of the position). This field is the solution of Maxwell's equations:

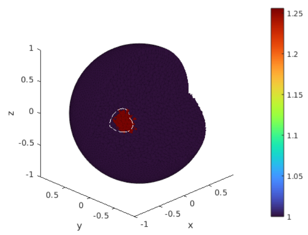
$$\left\{ \begin{array}{rcll} \mathbf{curl}\,\mathbf{curl}\,\mathbf{E} - k^2\kappa\mathbf{E} & = & 0, & \text{in } \Omega, \\ \mathbf{curl}\,\mathbf{E} \times \mathbf{n} & = & \mathbf{g}, & \text{on } \Gamma, \end{array} \right.$$

where $\mathbf{g}$ is the trace of the incident wave.

The inverse problem consists in reconstructing κ from the knowledge of $\mathbf{E}\times\mathbf{n}$ on at least a part Γ_0 of the boundary Γ. More precisely, we make here the assumption that κ is a perturbation of a background refractice index κ_0 which is assumed to be known: $\kappa = \kappa_0(1 + a\varrho)$, where a is the amplitude of the perturbation and ϱ is the characteristic function of its support. We then want to reconstruct both a and ϱ.

To this end, we introduce the iterated sensitivity equation, which consists in computing the derivatives of the electric field with respect to the amplitude of the perturbation. This allows us to define a cost functional that can then be minimized to reconstruct the perturbations, leading to good recsontructions in both 2D and 3D configurations, as illustrated below.





References

[1] J. Heleine, Uniqueness of an inverse electromagnetic coefficient problem with partial boundary data and its numerical resolution through an iterated sensitivity equation, Submitted. https://hal.science/hal-04210800.

On inverse and forward problems of some viscoelastic models

Hiromichi Itou[1]
[1]) Department of Mathematics, Tokyo University of Science, JAPAN
h-itou@rs.tus.ac.jp

Viscoelastic materials have the property involving both viscous and elastic characteristics at the same time when undergoing deformation, and exhibit time-dependent strain. There are many examples of viscoelastic materials in a wide range of fields: engineering, biology, geophysics, etc., such as synthetic polymers and concrete. In this talk, some mathematical models of viscoelastic materials within the context of infinitesimal strain theory are introduced.

Firstly, we treat the classic Kelvin-Voigt model whose constitutive relation between the stress σ and the linearized strain ε can be described $C_1\varepsilon + C_2\dot{\varepsilon} = \sigma$ with the time rate of the linearized strain $\dot{\varepsilon}$ and constant tensors C_1, C_2. In this model, reconstruction of a cavity is considered by means of the *enclosure method* for dynamic observation data on the boundary of the body (cf. [1]).

Next, we generalize the linear Kelvin-Voigt model to a quasi-linear viscoelastic relation given by the integral form:

$$\varepsilon(t) = J(t)\mathcal{F}(\sigma(0)) + \int_0^t J(t-s)\frac{d}{ds}\mathcal{F}(\sigma(s))\,ds, \tag{1}$$

where J is a so-called creep function and $\mathcal{F}$ stands for a material response function. (1) is used for biological models such as a human patellar tendon. Under suitable conditions of $\mathcal{F}$ and J, the existence of a solution to the quasi-static crack problem is examined (cf. [2]).

Lastly, we investigate more general model $\varepsilon(t) = [\mathcal{I}(t) \circ \mathcal{F}]\sigma$. Here the time-dependent function $\mathcal{I}$ represents the creep relaxation due to viscosity. We discuss the solvability of the boundary value problem, and example models expressed by Volterra convolution and aging type memory kernels described isotropic and anisotropic creep tensors, and by the Caputo derivative and the Liouville-Weyl integral equation stemming from fractional derivatives (cf. [3]).

References

[1] M. Ikehata and H. Itou, On reconstruction of a cavity in a linearized viscoelastic body from infinitely many transient boundary data, Inverse Problems, 28(12)(2012), 125003.

[2] H. Itou, V.A. Kovtunenko and K.R. Rajagopal, Crack problem within the context of implicitly constituted quasi-linear viscoelasticity, Mathematical Models and Methods in Applied Sciences, 29(2)(2019), 355–372.

[3] H. Itou, V.A. Kovtunenko and G. Nakamura, Forward and inverse problems for creep models in viscoelasticity, submitted.

Spectral analysis of the Neumann–Poincaré operator on touching disks

Mikyoung Lim
Korea Advanced Institute of Science and Technology
mklim@kaist.ac.kr

We present the spectral analysis of the Neumann–Poincaré (NP) operator on touching disks in two dimensions. This domain has a cusp at the touching point of two circles. For a $C^{1,\alpha}$ domain, the NP operator admits only discrete real eigenvalues as its spectrum, where zero is the only possible accumulation point. The operator admits a continuous spectrum and eigenvalues for a Lipschitz domain with corners. We show that the NP operator on touching disks has only absolutely continuous spectrum on the closed interval $[-1/2, 1/2]$, by analyzing the operator via the Fourier transform on the boundary circles of the touching disks.

This work is in collaboration with Younghoon Jung.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Unique determination and numerical reconstruction of orders in coupled time-fractional diffusion equations

Yikan Liu[1]
[1]) Department of Mathematics, Kyoto University
liu.yikan.8z@kyoto-u.ac.jp

Let $T > 0$ be a constant and $\Omega \subset \mathbb{R}^d$ ($d \in \mathbb{N}$) be a bounded domain with a smooth boundary $\partial\Omega$. For a constant $K \in \mathbb{N}$, let $\alpha_1, \dots, \alpha_K$ be constants satisfying $1 > \alpha_1 \ge \cdots \ge \alpha_K > 0$. In this talk, we consider an initial-boundary value problem for a coupled system of time-fractional diffusion equations

$$\begin{cases} \partial_t^{\alpha_k} u_k = \operatorname{div}(\boldsymbol{A}_k(\boldsymbol{x})\nabla u_k) + \displaystyle\sum_{\ell=1}^K c_{k\ell}(\boldsymbol{x}) u_\ell & \text{in } \Omega \times (0,T), \\ u_k = u_k^{(0)} & \text{in } \Omega \times \{0\}, \\ u_k = 0 & \text{on } \partial\Omega \times (0,T), \end{cases} \qquad k = 1, \dots, K. \tag{1}$$

Here $\partial_t^{\alpha_k}$ denotes the Caputo derivative of order α_k and $\boldsymbol{A}_k = (a_{ij}^{(k)})_{1 \le i,j \le d}$ ($k = 1, \dots, K$) are symmetric and strictly positive-definite matrices on $\overline{\Omega}$.

We are concerned with the following inverse problem.

Problem 1. Let $(u_1, \dots, u_K)$ satisfy (1) and fix $T > 0$, $\boldsymbol{x}_0 \in \Omega$, $k_0 \in \{1, \dots, K\}$ arbitrarily. Determine the orders $\alpha_1, \dots, \alpha_K$ of Caputo derivatives in (1) by the single point observation of the k_0-th component u_{k_0} at $\{\boldsymbol{x}_0\} \times (0,T)$.

Owing to the coupling effect, we can obtain the uniqueness for Problem 1.

Theorem 1 (see [1, Theorem 3]). *Let $d = 1,2,3$, $u_0^{(k)} \in L^2(\Omega)$ satisfy $u_0^{(k)} \ge 0, \not\equiv 0$ for all $k = 1, \dots, K$ and $c_{k\ell} \in L^\infty(\Omega)$ satisfy*

$$c_{k\ell} \ge 0, \not\equiv 0 \quad \text{in } \Omega, \quad k, \ell = 1, \dots, K,\ k \ne \ell,$$

$$\sum_{\ell=1}^K c_{k\ell} \le 0 \quad \text{in } \Omega, \quad k = 1, \dots, K.$$

Further, let $(u_1, \dots, u_K)$ and $(v_1, \dots, v_K)$ be the solutions to (1) with orders $(\alpha_1, \dots, \alpha_K)$ and $(\beta_1, \dots, \beta_K)$, respectively. Then for any $T > 0$, $\boldsymbol{x}_0 \in \Omega$ and $k_0 \in \{1, \dots, K\}$, $u_{k_0} = v_{k_0}$ at $\{\boldsymbol{x}_0\} \times (0,T)$ implies $\alpha_k = \beta_k$ for all $k = 1, \dots, K$.

For the numerical reconstruction, we reformulate Problem 1 as a minimization problem and develop a Gauss-Newton method to approximate the minimizer iteratively. Finally, we provide several examples to illustrate the satisfactory numerical performance of the proposed method.

References

[1] Z. Li, X. Huang and Y. Liu, Well-posedness for coupled systems of time-fractional diffusion equations, *Fract. Calc. Appl. Anal.*, **26**, 2023, 533–566.

Analysis for Viscoelastic Models

Gen Nakamura
Department of Mathematics
and Research Institute for Electronic Science
Hokkaido University, Japan

In this talk, I mainly focus on providing two methods for deriving the anisotropic relaxation tensor and its exponentially decaying property for the extended Burgers model (EBM), which is an important model in Earth and planetary sciences. Also, among the spring-dashpot models, the EBM is the most difficult model to derive the relaxation tensor and analyze its property.

Upon having this tensor, the EBM can be converted to a Boltzmann-type viscoelastic system of equations (abbreviated by BVS) and can show that its solution to the initial boundary value problem for the EBM has the possibility to decay exponentially in time (see [3] for the sufficient condition to have this property). Also, knowing the properties of the relaxation tensor is important for considering the inverse problem for the BVS.

The first method (abbreviated by I-method) is the conventional way using the Laplace transform to obtain the tensor ([1]). It needs to assume the commutativity of tensors under the Voigt notation. The I-method uses the common spectral representation of elastic tensor and its abstract version, which is the common spectral measure. The second method (abbreviated by II-method) is for the case this commutativity assumption is not available. There are two approaches. One manipulates the formulae of the system of equations describing the EBM to obtain the tensor by the integral equation method, which could be numerically practical. The other uses the multiplication semigroup theory which gives the most general result for the II-method ([4]).

If I have time, I would also like to mention some results of the inverse problem for the BVS.

The collaborators of this study are M.de Hoop, C-L. Lin, M. Kimura, K. Tanuma.

References

[1] R. Cristensen, Theory of Viscoelasticity, 2nd Edition, Academic Press, 2012.

[2] M. de Hoop, M. Kimura, C-L. Lin and G. Nakamura, Resolvent estimates for viscoelastic systems of extended Maxwell Type and their applications, arXiv 2308.1622v (2023).

[3] M. de Hoop, C-L. Lin and G. Nakamura, Uniform decaying property of solutions for anisotropic viscoelastic systems, arXiv 2308.03988 (2023).

[4] B. Heymann, Eigenvalues and stability properties of multiplication operators and multiplication semigroups, Math. Nachr., 287 (2014) pp. 574-584.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Identification of the number of dipoles for a biomagnetic inverse problem based on a reproducing kernel

Takaaki Nara[1], Yuichi Oosawa[1]
[1]) The University of Tokyo
takaakinara@g.ecc.u-tokyo.ac.jp

We consider a biomagnetic inverse problem to reconstruct the neural current source in the human brain from measurements of the magnetic field outside the head, which is applied to the identification of epileptic foci. This problem can be formulated as an inverse source problem for the Poisson equation where the source is modeled as multiple dipoles. The difficulty of this inverse problem exists in the identification of the number of dipoles because the magnetic field is blurred rapidly as the distance between the source and the sensor increases. For example, the magnetic field generated by two dipoles aligned along a line looks like a dipolar pattern created by a single dipole.

In this talk, we develop a method to reconstruct the magnetic field close to the head surface with sufficient high spatial frequency from the measured magnetic field in order to clearly observe two distinct dipoles. Given the magnetic field B_i^{data} at a sensor position $\boldsymbol{r}_i$ directed to $\boldsymbol{n}_i$ for $i = 1, 2, ..., N$, consider a minimization problem for the magnetic field $B(\boldsymbol{r}, \boldsymbol{n}) \in \mathcal{H} := \{f : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R} \,|\, f \in C^2, \Delta f(\cdot, \boldsymbol{n}) = 0\}$ at an arbitrary position $\boldsymbol{r}$ with an arbitrary direction $\boldsymbol{n}$ given by

$$\sum_{i=1}^{N} |B(\boldsymbol{r}_i, \boldsymbol{n}_i) - B_i^{data}|^2 + \lambda ||B||_{\mathcal{H}}^2 \to \min \tag{1}$$

Here we define the norm by $||B||_{\mathcal{H}} := (\int_S |\Delta_{\theta,\phi} B(\boldsymbol{r}, \boldsymbol{r}/||\boldsymbol{r}||)|^2 dS)^{\frac{1}{2}}$ where S is a sphere with radius r_0 and $\Delta_{\theta,\phi}$ is the surface Laplacian. The solution is given by the representer theorem as $B(\boldsymbol{r}, \boldsymbol{n}) = \sum_{i=1}^{N} c_i K(\boldsymbol{r}, \boldsymbol{n}, \boldsymbol{r}_i, \boldsymbol{n}_i)$, where $K(\boldsymbol{r}, \boldsymbol{n}, \boldsymbol{r}_i, \boldsymbol{n}_i)$ is the reproducing kernel of $\mathcal{H}$ with the norm $|| \cdot ||_{\mathcal{H}}$ given by[1]

$$K(\boldsymbol{r}, \boldsymbol{n}, \boldsymbol{r}_i, \boldsymbol{n}_i) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{r_0^{2l+6}}{l^2(l+1)^4} \boldsymbol{n}_i \cdot \frac{\hat{Y}_{lm}(\theta_i, \phi_i)}{r_i^{l+1}} \boldsymbol{n} \cdot \frac{\hat{Y}_{lm}^*(\theta, \phi)}{r^{l+1}}. \tag{2}$$

The coefficients c_i are expressed in terms of $K(\boldsymbol{r}_i, \boldsymbol{r}_j, \boldsymbol{n}_i, \boldsymbol{n}_j)$ and the data. We will show numerically that two dipoles are discriminated more easily when using the computed magnetic field close to the head surface with the reproducing kernel than using the magnetic field measured with the real sensors.

References

[1] K. Kabashima, T. Wu, and T. Nara, Reconstruction of current dipoles based on tensor decomposition of multipole coefficients, JSIAM Letters, 11(2019), 29-32.

Global Lipschitz stability for inverse problems of waves on Lorentzian manifolds

Hiroshi Takase
Institute of Mathematics for Industry, Kyushu University
htakase@imi.kyushu-u.ac.jp

We consider wave equations on Lorentzian manifolds whose coefficients depend on both space and time variables and prove global Lipschitz stability for an inverse source problem [1]. Global Lipschitz stability for hyperbolic equations has been proved, starting from the Euclidean wave equations with constant coefficients, and also for wave equations on Riemannian manifolds whose coefficients depend on space variables. The basic tool is called the Bukhgeim–Klibanov method, which is based on a Carleman estimate with boundary terms and an energy estimate, and this is compatible with analysis on manifolds. However, applying this method to the Lorentzian wave equations requires a more detailed analysis than in the above cases, where the coefficients are independent of time. In this presentation, these analytical methods will be presented.

References

[1] H. Takase, Global Lipschitz stability for inverse problems of wave equations on Lorentzian manifolds, *J. Differential Equations*, **372** (2023), 564–590.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

A remark on the reconstruction formula of the support function for the magnetic Schrödinger operator

Ryusei Yamashita[1]
[1]) Polytechnic University of Japan, Tokyo Metropolitan University
yamashita@uitec.ac.jp

This study is based on the paper [1]. We show a reconstruction formula of the convex hull of the defect D from the Dirichlet to Neumann map associated with the magnetic Schrödinger operator

$$D_A^2 u + qu = 0 \quad \text{in} \quad \Omega \setminus \overline{D} \quad (\overline{D} \subset \Omega)$$

under the Robin condition on the boundary ∂D. For $A = (A_1, A_2, \dots, A_n)$, let

$$D_A^2 u := \sum_{j=1}^{n} D_{A,j}(D_{A,j} u), \quad D_{A,j} := \frac{1}{i}\partial_j + A_j.$$

We assume that $\Omega \subset \mathbb{R}^n (n = 2, 3)$ is a bounded domain with smooth boundary $\partial\Omega$, and let D be an open set satisfying $\overline{D} \subset \Omega$ and $\Omega \setminus \overline{D}$ is connected. The defect D consists of the union of disjoint bounded domains $\{D_j\}_{j=1}^m$, where the boundary of D is C^2. In [2] Ikehata gave the formula to reconstruct the defect D for the Helmholtz equation under the Dirichlet boundary condition on ∂D, and in [3] gave the formula to reconstruct the defect D for the Helmholtz equation under the Robin condition on ∂D. We extended this result for the magnetic Schrödinger operator under the Dirichlet and Robin condition on ∂D in the two and three dimensional case in [4]. Recently in [5], Ikehata established the new estimates of the stationary Schrödinger equation without magnetic field. Using an idea in [5], we improve the result for the magnetic Schrödinger operator in [4] for the Robin case.

References

[1] R. Yamashita, *A remark on the reconstruction formula of the support function for the magnetic Schrödinger operator,* J. Math. Anal. Appl., **528** (2023), doi.org/10.1016/j.jmaa.2023.127598.

[2] M. Ikehata, *Reconstruction of the support function for inclusion from boundary measurements,* J. Inv. Ill-Posed Problems, **8** (2000), 367-378.

[3] M. Ikehata, Two sides of probe method and obstacle with impedance boundary condition, Hokkaido Math. J., **35** (2006), 659-681.

[4] K. Kurata and R. Yamashita, Reconstruction of the defect by the enclosure method for inverse problems of the magnetic Schrödinger operator, Tokyo J. Math., **45**, (2022), 547-577.

[5] M. Ikehata, Revisiting the probe and enclosure methods, Inverse Problems, **38** (2022), 075009(33pp).

MINISYMPOSIUM
M17. In Memoriam: Professor Pierre Célestin Sabatier (1935-2023)

Organizers:
Cristiana Sebu, University of Malta, Malta, cristiana.sebu@um.edu.mt

This minisymposium is in honour of Professor Pierre Sabatier who passed away in July 2023. Professor Sabatier was the Founding Editor of the journal Inverse Problems, initiated the RCP264 workshops in Montpellier, and is one of the foremost exponents of the field of inverse problems. During his career, Professor Sabatier had been trying to unify and extend the domain of interdisciplinary studies of inverse problems, involving theorists and mathematicians as well as practitioners in a great synergy which in line with the aims of the IPMS Conference series.

The scope of this minisymposium is to gather scientists working on different aspects of inverse problems who knew and met Professor Sabatier as well as scholars from the younger generations who followed their mentors contemporary with him.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

The Interior Problem for the Radon transform

Jan Boman[1]
[1]) Stockholm University
jabo@math.su.se

Let D and $D_0 \subset D$ be two concentric and not identical disks in the plane. It is well known that the restriction to D_0 of a function f supported in D cannot be determined from the restriction of the Radon transform Rf to the set of lines that intersect D_0. The corresponding statement when D_0 and D are replaced by arbitrary bounded convex sets with $\bar{D_0} \subset D$ was conjectured in [1]. A natural idea to prove the conjecture would be to construct a distribution $f_0 \neq 0$ whose Radon transform is supported on the set of tangents to a bounded, convex, smooth curve Γ contained in $D \setminus \bar{D_0}$ and then replace f_0 by a regularization $f = f_0 * \phi$, where ϕ is a smooth function with small support. However, at IPMS in Malta in 2018 I reported that this idea cannot work, because such distributions f_0 exist only if Γ is an ellipse. The latter fact turned out to have independent interest, which has led to two more publications, [2], [3]. Finally I will report on my recent work on the conjecture, although my proof is not complete at the time of writing.

References

[1] *A hypersurface containing the support of a Radon transform must be an ellipsoid. I: The symmetric case*, J. Geom. Analysis 2020, and *II: The general case*, J. Inverse Ill-Posed Probl. 2021.

[2] *Regularity of a distribution and of the boundary of its support*, J. Geom. Anal. 2022.

[3] *Algebraically integrable bodies and related properties of the Radon transform* (joint work with M. Agranovsky, A. Koldobsky, V. Vassiliev, and V. Yaskin), in A. Koldobsky and A. Volberg (eds), Harmonic Analysis and Convexity, De Gruyter 2023.

Calculating the number of electrodes for uniqueness and global convergence in an inverse coefficient problem

Andrej Brojatsch[1], Bastian Harrach[2]
[1]) Goehthe University, Frankfurt, Germany
brojatsc@math.uni-frankfurt.de
[2])Goehthe University, Frankfurt, Germany

Inverse coefficient problems hold the challenge of non convexity in the natural data fitting approach. We consider a problem that is motivated by impedance-based corrosion detection, where an unknown transmission function is determined from a finite number of measurements on attached electrodes. We are able to characterise mathematically how many electrodes are required to uniquely determine the unknown coefficient function with a finite resolution. By rewriting the problem as a concex semidefinite optimisation problem we overcome the non convexity and so that we able to provide a globally convergent reconstruction algorithm.

Detection-Identification of disturbances in transmission networks

Jean-Guy Caputo
Laboratoire de Mathématiques, INSA Rouen Normandie, Normandie Université
76801 Saint-Etienne du Rouvray, France
jean-guy.caputo@insa-rouen.fr

In memory of Pierre Sabatier who introduced me to Inverse Problems

We address the inverse problem of detecting and identifying time-dependent unknown disturbances in transmission networks modeled by a non oriented graph. Disruptions can occur in many applications of miscible flows over networks, for instance, electromechanical waves on the electrical grid, a leak in a fluid or gas network, etc... The mathematical model governing the time evolution of the network state at its N nodes labeled $n = 1, ..., N$ is the following discrete graph wave equation:

$$X''(t) + VX'(t) - \Delta X(t) = F^H(t) + F^D(t) \quad (E),$$

where $X(t) = (x_1(t), ..., x_N(t))$ is the global network state at time t, $x_n(t)$ is the local state at node n, Δ is the $N \times N$ graph Laplacian matrix and V is a damping. In the right hand side of (E), F^H is a given forcing term and F^D is an unknown disturbance.

For detection-identification purposes, the network is observed by a set $\mathcal{S}$ of sensor nodes. We assume disturbances F^D occur away from $\mathcal{S}$, otherwise they would be detected immediately. Two important questions arise:
How many observation nodes do we need and how they should be placed to ensure uniqueness of detection of a disturbance as well as uniqueness of identification of the detected disturbance?

To answer these questions, a first condition on the observation nodes is that they form a so-called *strategic set* i.e. they contain information about all the spectrum of Δ. Then, we prove that, during a non-disturbance time period, the state of the network $X(t)$ can be uniquely recovered from the observations of the nodes of $\mathcal{S}$. Then to detect disturbances, we consider a "healthy" network where $F^D = 0$ (no disturbances) and the same initial conditions as equation (E). We detect disturbances by comparing the solution $X^H(t)$ of the healthy network with the state $X(t)$ at the nodes of $\mathcal{S}$. If $X^H(t) \neq X(t)$ at one node of $\mathcal{S}$ then there is a disturbance.

Remark that $F^D(t) \neq 0$ does not imply $X^H(t) = X(t)$ on the nodes of $\mathcal{S}$. For this to hold, we need a a *strategically-dominant* network and in particular $|\mathcal{S}| > N/2$. Then we can detect disturbances and identify them in real time.

For non *strategically-dominant* networks, a disturbance will be detected with some delay. If $\mathcal{S}$ is an absorber for the graph, then identification is possible. In general it must be done a posteriori; we provide a method for this identification.

This is joint work with Adel Hamdi.

Quantum-inspired classification algorithms

Christine De Mol
Department of Mathematics and ECARES, Université libre de Bruxelles,
Brussels, Belgium
christine.de.mol@ulb.be

In this talk, we present some algorithms for supervised classification tasks in machine learning, algorithms that are inspired by quantum mechanics. Although their implementation on quantum computers could be envisaged, we focus on their behaviour as classical algorithms, whence the appellation "quantum-inspired".

More specifically, they rely on the so-called problem of quantum state discrimination, which consists in identifying which one of a known set of quantum states has been prepared based on the outcome of a quantum measurement on the state. After building the quantum states associated with the training data, which are encoded as density matrices, we explore various quantum measurement strategies suited for the classification task, including those based on semidefinite programming and the so-called Pretty Good Measurement.

We also consider how the measurement performs on multiple copies of the quantum state, since this in principle improves quantum state discrimination at the price of a higher computational cost. Nevertheless, for the Pretty Good Measurement classifier, we devise an analogue of the well-known Kernel Trick in learning theory, and we show that the cost scales only with the number of examples in the training dataset. This allows us to study the performance of this classifier on tensor products of the quantum state.

Finally, we benchmark the different classifiers on the MNIST and MNIST-1D datasets and compare their performance also to that of classical classifiers based on Ridge Regression and Logistic Regression.

This is joint work with Emmanuel Zambrini Cruzeiro, Serge Massar and Stefano Pironio (see [1] where relevant references can be found).

References

[1] E. Zambrini Cruzeiro, C. De Mol, S. Massar and S. Pironio, Quantum-inspired classification based on quantum state discrimination, ArXiv preprint https://doi.org/10.48550/arXiv.2303.15353

Regularization of the inverse Laplace transform by mollification

Pierre Maréchal[1], Faouzi Triki[2], Walter C. Simo Tao Lee[3]
[1]) Université Paul Sabatier, Toulouse, France
pr.marechal@gmail.com
[2]) Université Grenoble Alpe, Grenoble, France
faouzi.triki@univ-grenoble-alpes.fr
[3]) Université Toulouse Capitole, Toulouse, France
wsimotao@gmail.com

In this talk, we consider the inversion of the real Laplace transform. We first derive a new global logarithmic stability estimate that shows that the inversion is severely ill-posed. Then we propose a regularization method to compute the inverse Laplace transform using the concept of mollification. Taking into account the exponential instability we derive a criterion for selection of the regularization parameter. We show that by taking the optimal value of this parameter we improve significantly the convergence of the method. Finally, making use of the holomorphic extension of the Laplace transform, we suggest a new PDEs based numerical method for the computation of the solution. The effectiveness of the proposed regularization method is demonstrated through several numerical examples.

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Transparent scatterers and transmission eigenvalues

Piotr Grinevich[1], Roman Novikov[2]
[1]) Steklov Mathematical Institute of Russian Academy of Sciences
[2]) CMAP, CNRS, Ecole polytechnique, Palaiseau, France
novikov@cmap.polytechnique.fr

We give a short review of old and recent results on scatterers with transmission eigenvalues of infinite multiplicity, including transparent scatterers. Historically, these studies go back to the works of Tulio Regge (1959), Roger Newton (1962), and Pierre Sabatier (1966).

This talk is based on the publications [1] – [4], where our examples include potentials from the Schwartz class and multipoint potentials of Bethe - Peierls type.

References

[1] P.G. Grinevich, R.G. Novikov, Transparent potentials at fixed energy in dimension two. Fixed-energy dispersion relations for the fast decaying potentials, Communications in Mathematical Physics, 174, 409–446 (1995)

[2] P.G. Grinevich, R.G. Novikov, Transmission eigenvalues for multipoint scatterers, Eurasian Journal of Mathematical and Computer Applications, 9(4),17–25 (2021)

[3] P.G. Grinevich, R.G. Novikov, Spectral inequality for Schrödinger's equation with multipoint potential, Russian Mathematical Surveys, 77(6), 1021–1028 (2022)

[4] P.G. Grinevich, R.G. Novikov, Transparent scatterers, transmission eigenvalues, and multipoint potentials of Bethe–Peierls type, https://hal.science/hal-04243756

Inverse problems for screens

Lassi Päivärinta
Department of Cybernetics, Division of Mathematics
Tallinn University of Technology
Estonia
lassi-paivarinta@ttu.ee

In the talk we discuss the inverse scattering and inverse boundary value problems for screens with passive measurements. A screen in an n-dimensional space is an objects modelled by an open (n-1) dimensional surface. A passive measurement means that the transmitter has a fixed position but the scattered field is still measured in all directions. The conjecture here is that such a data for fixed energy is enough to determine the screen uniquely. In the limit case when the wave length is zero we prove this exactly in the case n=2. Note that the data in this case is one dimensional. The proof is based on the theory of the Hilbert transform defined for functions defined on finite smooth curves in the complex plane. This is classically well understood (Carleman, Tricomi) if the curve is an interval (a piece of a line).

This is a joint work with Emilia Blåsten, LUT University, Petri Ola, University of Helsinki and Sadia Sadique, Taltech University.

The 11th International Conference "Inverse Problems: Modeling and Simulation"
26th May - 1st June 2024, Malta

Fully discretized reconstruction for the inverse conductivity problem

Luca Rondi
Università di Pavia, Italy
luca.rondi@unipv.it

We address the reconstruction issue for the inverse conductivity problem with discontinuous conductivities. We discuss the choice of the regularization and the effect of the discretization on the reconstruction. We construct a completely discrete minimization problem whose solution is a good approximation of a solution to the inverse problem.

This is a joint work with Alessandro Felisi (Università di Genova, Italy).

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

Recent developments in Electrical Impedance Mammography

Cristiana Sebu[1]
[1]) Department of Mathematics, University of Malta, Malta
cristiana.sebu@um.edu.mt

The talk is focused on recent developments of reconstruction algorithms that can be used to approximate admittivity distributions in Electrical Impedance Tomography with direct applications to breast cancer screening. The algorithms are non-iterative and are based on linearized integral equation formulations [1, 2] which have been extended to reconstruct the conductivity and/or permittivity distributions of two and three-dimensional domains from boundary measurements of both low and high-frequency alternating input currents and induced potentials [3]. The linearized approaches rely on the solutions to the Laplace equation on a disk and a hemispherical domain subject to appropriate idealized Neumann boundary conditions corresponding to applied spatial varying trigonometric current patterns. Reconstructions from noisy simulated data are obtained from single-time, time-difference and multiple-times data. Moreover, a proposed design of a prototype for a novel integrated circuit based electrical impedance mammographic system embedded in a brassiere will be presented [4].

References

[1] C. Hähnlein, K. Schilcher, C. Sebu and H. Spiesberger, Conductivity imaging with interior potential measurements, Inverse Problems in Science and Engineering, 19(5)(2011), 729–750.

[2] K-H. Georgi, C. Hähnlein, K. Schilcher, C. Sebu and H. Spiesberger, Conductivity Reconstructions Using Real Data from a New Planar Electrical Impedance Device, Inverse Problems in Science and Engineering, Inverse Problems in Science and Engineering, 21(5)(2013), 801–822.

[3] C. Sebu, A. Amaira and J. Curmi, A linearized integral equation reconstruction method of admittivity distributions for Electrical Impedance Tomography, Engineering Analysis with Boundary Elements 150(2023), 103–110.

[4] Andrei Amaira, Reconstruction algorithms for Electrical Impedance Mammography. MSc thesis. University of Malta (2023).

The 11th International Conference "Inverse Problems: Modeling and Simulation
26th May - 1st June 2024, Malta

A Strong Unique Continuation Property for the Wave Equation

Sergio Vessella
University of Florence, Italy
sergio.vessella@unifi.it

In this talk we consider the following hyperbolic equation with time–independent coefficients

$$\partial_t^2 u + a(x)\partial_t u - \operatorname{div}(A(x)\nabla u) = 0, \quad \text{in } B_R \times (-T,T) \tag{1}$$

where B_1 is the ball of $\mathbb{R}^n$ centered at 0 and with radius $R > 0$, $A(x)$ is a symmetric real matrix which satisfies a uniform ellipticity condition and whose entries are Lipschitz continuous, $a(x)$ is bounded (and measurable) function. We prove the following **Strong Unique Continuation Property (SUCP)**: If u is any solution to equation (1) which satisfies

$$\int_{-T}^{T}\int_{B_r} u^2(x,t)dxdt = \mathcal{O}\left(r^N\right) \text{ as } r \to 0, \text{ for every } N \in \mathbb{N}$$

then u vanishes in a neighborhood of $\{0\} \times (-T,T)$.

The above **SUCP**, especially its related quantitative estimate, is a crucial tool to prove sharp stability results for some inverse problems with unknown boundaries [1].

References

[1] Vessella, S.: Stability estimates for an inverse hyperbolic initial boundary value problem with unknown boundaries. SIAM J. Math. Anal. 47(2), 1419 – 1457 (2015)