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GLOBAL AND LOCAL INSTABILITY OF CONCRETE TALL BUILDINGS

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ABSTRACT

The CEB-FIP Model Code MC-90, 1993, in its section 6.6, "ULTIMATE LIMIT STATE OF BUCKLING", classifies structures and structural elements as "braced" or "unbraced" and as "sway" or "non-sway"; the entire treatment of second order problems in that Code is based upon the said double classification.

In this paper both classification criteria are initially discussed, and it will be shown that the first one is related exclusively to the chosen structural model, and that the second one is merely conventional, depending only upon the limit under which global second order effects may be neglected; in fact, this limit is not the same in all codes.

Approximate design stiffness values for columns, beams and slabs to be considered in a global analysis will be presented. Simple criteria that allow to classify a structure as "sway" or "non-sway" will be proposed. These criteria will be included in the new issue of the Brazilian Code for the design of concrete structures, NB-1, to be published in 1995.

Second order global analysis of building structures will be briefly discussed; a correction that allows to take into account with good approximation the flexural stiffness reduction of compressed members will be presented.

It will be further shown that the treatment of second order effects in buildings must always start by a global analysis, either with (for sway structures) or without (for non-sway structures) the consideration of second order effects due to the lateral displacement of the connections; a local analysis should follow. The analysis of isolated elements will therefore be possible only after the global analysis has been performed, when the final design end moments and axial force in each element will therefore be known; only the design moment at the critical intermediate section of each element will be determined by the local analysis, which must be performed in the same manner for sway and non-sway structures. In particular, it will be shown that the equivalent length ℓ_o of the element and its slenderness bound λ_l do not depend upon the structure being sway or non-sway, contrarily to what is said in MC-90 and other codes such as REBAP, 1984 and ACI 318/89, 1992.

Expressions for the slenderness bound λ_l of columns are presented; for $\lambda \leq \lambda_l$ no local second order analysis is needed. The model column method is briefly rewiewed.

1. INTRODUCTION.

Quoting Levi, 1992, the subject of instability "concerns a complex topic, demanding both a rigorous conceptual analysis and a truly practical approach". In fact, under the general label of "instability problems" a number of related - although conceptually different - problems are studied. We may distinguish basically the following situations related to instability which are of concern to the structural engineer:

a) **Buckling**, or loss of stability with bifurcation of equilibrium.

b) **Limit point with snap-through**, which is the loss of stability without bifurcation of equilibrium. This case may occur for instanee in shallow arches and domes;

c) **Limit point without snap-through**, which is the loss of stability that occurs whenever, as the loading increases, the increase of the internal resistent capacity of the structure is lower than the increase of the external actions. This type of stability failure may occur in slender elements of non-linear material (e.g. reinforced concrete) with initial geometric imperfections, such as columns or, rarely, in-plane loaded slabs (Ghoneim, 1994), whenever second order effects are important; failure may then occur before the critical cross sections are loaded to the limits of their strength.

d) **Second order problem**, in which equilibrium is analyzed in the deformed configuration of the structure. It concerns structures either of linear or non-linear materials and is not necessarily associated with loss of stability.

In the structural design of tall buildings we are mainly concerned about cases c) and d). In fact, the ratio *global buckling load / characteristic load* of a properly designed building is usually in the order of 7 or more. The author has found ratios of 15 to 20 in apparently slender frame structures. As for "snap through" problems, they are confined to very particular structural systems, seldom found in tall buildings. In the present paper we will discuss only second order problems.

2. BRACING.

The CEB-FIP MC-90, 1993, in 6.6.1.2, **Classification of structures and structural elements**, initially classifies structures *"as braced and unbraced, depending on the provision of bracing elements or not"*. This is curious enough, because every structure should have such a provision, in order to safely withstand lateral loads. It would seem that the meaning of this classification is merely to state that a structure may or may not have a clearly defined bracing sub-structure designed to resist all lateral forces; if not, we must understand that the entire structure should be designed to withstand such forces, and will therefore be, as a whole, the bracing structure of the building. Elements that belong to the bracing sub-structure are therefore affected by the lateral loading, and will be called **bracing elements**, whereas those which do not belong to that sub-structure are the **braced elements**, and, as such, are unaffected by lateral loads. It can be seen therefore that the concept often used in MC-90 and other codes of "unbraced element" has no meaning at all: an element does or does not belong to the bracing sub-structure, but in no case it will be "unbraced".

Such a classification clearly depends upon the model chosen by the structural engineer. In the past, when the analysis of highly hyperstatical structures was extremely cumbersome - if not altogether unfeasible - it was necessary to simplify the analysis for lateral loads by defining a rather elementary bracing structure consisting of plane frames or shear-walls, or plane associations of both, the remaining structural elements being therefore considered as braced. Nowadays, with the enormous progress of structural analysis provided by computer-supported matrix analysis, is has become clear that the consideration of a great number of elements, slender as they might be, can dramatically increase the efficiency and economy of the bracing system.

In the mid sixties, Franco, 1967, developed a method of analysis of simple, symmetrical space frames by a non-matricial continuous elastic medium method. The application of this method permitted to show that **spatial action is of paramount importance** and should not be neglected in major buildings. A very slender (9,25 m wide, 90 m tall) parking structure

(CTUBH, 1981), was designed using the mentioned method; initially, a conventional plane analysis had shown the impossibility of designing the foundation of the building for the great upward concentrated forces due to wind action, whereas the consideration of the complete spatial model permitted to distribute these forces throughout the entire structure and thus obtain a fairly uniform foundation loading.

At present, bracing sub-structures should be modeled in such a way as to include the greatest possible number of elements, in order to increase their efficiency and economy. The author has found cases in which the use of a comprehensive spatial model that included not only shear-walls (elevators, staircases), but also slender frames, some of which formed by columns tied to slab strips, increased the overall lateral stiffness of the building by a factor in the order of up to 4 as compared with the stiffness of the shear-walls alone. Recent experimental research by Zalka, 1993, has shown similar results. Only quite secondary elements, which clearly cannot contribute significantly to the overall behavior of the building, should therefore be excluded from the bracing structure and treated as braced elements.

3. FIRST ORDER GLOBAL ANALYSIS.

Once the bracing structure has been defined, a global first order spatial analysis at design level must be performed for the worst combinations of vertical and horizontal factored loads and taking into account geometrical imperfections. The stiffness of the reinforced concrete elements should be appropriately defined in order to account for the constitutive equations of the materials. However, at this stage of analysis the reinforcement of the structural elements has not yet be determined, and therefore we must assume approximate design $(EI)_d = E_d \cdot I_d$ stiffnesses, E_d being the initial tangent deformation modulus of concrete and I_d being a fraction β of the gross section moment of inertia I_o of the element: $I_d = \beta \cdot I_o$. The following values for β may be used with sufficient approximation and are considered for the next edition of the Brazilian Code NB-1:

- columns $\beta = 0,8$ (1)
- symmetrically reinforced beams $\beta = 0,5$ (2)
- one-side reinforced beams $\beta = 0,4$ (3)
- slabs $\beta = 0,3$ (4)

MacGregor, 1993, has recently proposed, for the next revision of the ACI-318 Code, the values:

- columns $\beta = 0,70$ (1a)
- beams $\beta = 0,35$ (2a)
- slabs $\beta = 0,25$ (3a)

Our proposed values (1)(2)(3)(4) are based upon experimental and analytical evidence by several authors, such as Kordina, 1972, Hage, 1974, MacGregor, 1977, Furlong, 1980, MacDonald, 1986. The strength reduction coefficient $\varphi = 0,875$ proposed by MacGregor, 1993, to account for the variability in the predicted lateral deflections determined by experience is consistent with the general formulation of the ACI 318/89 Code but this further reduction seems to be excessively conservative and was discarded in eqs. (1) to (4).

The author is preparing a further checking of the proposed values by using a complete second order program developed by Pimenta, 1990, which permits to accurately take into account physical nonlinearity through the constitutive equations of concrete and steel; geometrical nonlinearity will be considered through the general exact matrix formulation.

4. NON-SWAY AND SWAY STRUCTURES.

Section 6.6.1.2 of the MC-90 further classifies structures as *"non-sway or sway, depending on the sensitivity to second order effects due to lateral displacements with respect to the direction of compressive forces"*. Section 6.6.1.3 "in fine" states that *"slenderness bounds below which second order effects may generally be neglected should be related to reduction in bearing capacity, with respect to the ultimate limit state for bending and longitudinal force according to first order theory, of not more than 10%"*. This amounts to say (Section 6.6.3.1.3 of MC-90) that *"a building frame can be considered as **non-sway** if the displacements of the connections would result in a 10% increase of the **relevant** first order bending moments"*. The ACI 318/89 Code uses the more conservative limit of 5%.

It is therefore of the utmost relevance to establish criteria to decide "a priori" whether a structure can or cannot be considered as non-sway, without performing, at first, a global second-order analysis.

One such criterion was proposed by Beck, 1966, and is applicable to tall uniform structures whose bracing consists of one or more shear-walls of constant characteristic stiffness $E_c \cdot I_c$, height H and total characteristic vertical load N_c. The instability parameter is:

$$\alpha = H\sqrt{N_c / E_c \cdot I_c} \qquad (5)$$

and the non-sway condition is, for tall buildings:

$$\alpha \leq 0{,}6 \qquad (6)$$

Later Franco, 1985, showed that the concept of instability parameter could be applied to a generalized regular structure by assuming an "equivalent" $E_c \cdot I_c$ product, and that the 0,6 limit, appropriate to usual buildings braced by shear-walls and frames, can be increased to 0,7 in the case of a bracing formed exclusively by shear-walls, and must be reduced to 0,5 in the case of a bracing consisting only of frames. The equivalence consists in assuming equal first order displacements at the top for an equivalent prismatic structure of stiffness $E_c \cdot I_c$ and for the actual structure, both affected by the same characteristic lateral load. The above mentioned values of α were determined assuming $\gamma_f = 1{,}4$, as prescribed by the Brazilian Code; they should be increased by ~ 4% for $\gamma_f = 1{,}5$.

A better approach consists in calculating in an approximate way the second order magnification coefficient γ_z for the relevant global member forces with respect to first order values. This can be done, according to Franco, 1991, for fairly regular structures, as follows:

$$\gamma_z = \frac{1}{1 - \frac{\Delta M_{tot,d}}{M_{1,tot,d}}} \qquad (7)$$

where:

$M_{1,tot,d}$ is the overturning moment of the design lateral forces;

$\Delta M_{tot,d}$ is the increase of the overturning moment caused by the displacement of the resultant of the design vertical forces, obtained in a first order analysis, as described in Section 3.

The non-sway condition is:

$$\gamma_z \leq 1{,}1 \qquad (8)$$

If condition (8) is met a second order global analysis is not necessary.

It can be shown that for a regular structure with uniformly distributed vertical and horizontal loads and with a linear lateral displacement along its vertical axis, the coefficient γ_z as calculated by eq (7) is **exactly** the amplification global second order coefficient for all deflections and member forces of the structure. In practical cases of fairly regular buildings with a non-uniform wind load, the deformed axis will still follow approximately a straight line, and therefore γ_z **represents with good approximation the said amplification coefficient,** provided it does not exceed 1,2. The author verified this fact in a number of major tall structures.

MC-90 frequently refers to "sway" and "non-sway" **elements**, meaning elements included in sway or non-sway structures. This definition is quite objectionable, because the behaviour of a particular element is affected by the sway/non-sway condition of the structure only insofar as its end moments and member forces are or are not affected by the lateral displacements of the connections. Once those displacements have been duly taken into account through a second order global analysis, and final end moments and axial forces are correctly assessed, no difference whatsoever should exist in the further treatment of an isolated element. There should not be, for instance, a slenderness bound λ_l for sway elements (eqs. (6.6-2) and (6.6-3), page 194 of MC-90) and a different one for non-sway elements (eqs. (6.6-4) and (6.6-5) in the same page). Also, the equivalent length ℓ_o of a column should always be smaller than its geometrical length ℓ, and not greater (as stated by Cranston, 1972, and ACI 318/89, 1992). This distinction between sway and non-sway elements (other authors say "braced" and "unbraced" elements with the same meaning) apparently comes from a time when it was believed to be possible the design a column belonging to a sway structure without performing a global second order analysis; however, this concept should be discarded. The terms "sway" and "non sway" should be used only referring to the structure as a whole.

5. SECOND ORDER GLOBAL ANALYSIS.

If $\gamma_z > 1,1$ and, in general, for major tall building structures, a global second order analysis must be performed in order to determine the final deflections and design member forces. This is usually done by a $P-\Delta$ analysis, using the reduced stiffnesses discussed in section 3. The flexural stiffness reduction of compressed members due to the presence of a constant axial force is not captured by the classical $P-\Delta$ routine. The said reduction can be of importance, as shown by Lai, 1983, who calls it "C and S effect"; in extreme cases it can reduce the column stiffness by as much as 18%. A sufficient approximation consists in reducing the flexural rigidity $E_d \cdot I_d$ of a column of equivalent length ℓ_o accordingly to the design axial force N_d determined by a first order analysis, as follows:

$$(E_d \cdot I_d)_{red.} = \psi \cdot E_d \cdot I_d \tag{9}$$

$$\psi = \frac{u^2 \cdot tgu}{3(tgu - u)} \tag{10}$$

$$u = \frac{\ell_o}{2}\sqrt{\frac{N_d}{E_d \cdot I_d}} \tag{11}$$

If the effect of the lateral loading upon N_d is great, the factor ψ should be calculated after each iteration. Otherwise it is sufficient to retain its initial first order value. Proof of (9), (10) and (11) is given in Appendix A.

Some available structural analysis programs, such as SAP-90 (Wilson, 1988), automatically perform a $P-\Delta$ analysis taking into account the above mentioned reduction of the flexural stiffness members due to the presence of an axial force, by assuming, as an approximation, that the column deforms according to a cubic parabola instead of the accurate, trascendental curve. The author tested SAP-90 results against the accurate ones and found that the approximation is satisfactory.

Once the reinforcement of the structural members is determined, a still more accurate analysis should in principle be performed, instead of the $P-\Delta$ analysis such as described above, whose approximation consists mainly in the simplified consideration of the physical nonlinearity through eq. (1) to (4) and does not take into account either the actual reinforcement nor the real constitutive equations of concrete and steel. Although, as mentioned before, several programs are available which allow such a refined, general analysis, at present they are not economically applicable for large tall building structures, because of the enormous amount of additional computational effort needed. This may not be the case in the near future, when such programs are expected to become available in a user-friendly format. A quite elegant general design method was developed more than 20 years ago by Aas-Jacobsen, 1973, in which cross sections and reinforcement are directly determined starting from the constitutive equations of concrete and steel and using as an additional condition a minimum-cost criterion.

6. ANALYSIS OF ISOLATED ELEMENTS.

6.1 Only after a global analysis of the structure, either of first order or, if necessary, of second order, structural elements belonging to the bracing sub-structure can be isolated. Information will then be available, for each member, about its design axial force N_d and end moments M_{Ad} and M_{Bd} (we will assume $|M_{Ad}| \geq |M_{Bd}|$). No data will be known, however, about bending moments at intermediate sections of the isolated member. These will depend upon the **lateral deflections along the member**, which, because of the presence of the axial force, will introduce second order **local** moments, to be added to the moments previously determined by the global analysis. It is important to remember that these local second order moments **vanish at the ends of each element.**

6.2 It becomes therefore necessary, at this stage, to define a method by which to calculate the **local second order moments**. However, since compressed members are usually designed with symmetrical constant reinforcement along their axis, it may suffice in most cases to find, in the first place, whether it exists an intermediate section C whose absolute moment $|M_{Cd}|$ is greater than $|M_{Ad}|$. If not, M_{Ad} will govern the reinforcement design of the member.

6.3 To further discuss local second order moments it is necessary, at first, to define some geometrical data.

- ℓ_o (effective length): the length of a pin-ended column which will lead to the same second order behaviour and load carrying capacity as the real column; it will be calculated using the Cranston formulas for braced columns (Cranston, 1973), as shown in section 6.4.
- i_g (radius of gyration): calculated for the concrete cross-section (gross section) in the plane considered;
- $\lambda = \ell_o / i_g$ (slenderness)
- h (section height) : measured in the plane considered.

MacGregor, 1993, starting from Galambo's equation (Galambos, 1968), shows that in order to have $\delta = |M_{Cd}| / |M_{Ad}| \leq 1{,}05$ (using the 5% criterion adopted by the ACI-318) the following condition must be satisfied:

$$\lambda_{5\%} \leq \frac{35}{\sqrt{\nu_k}} = \lambda_l \qquad (12)$$

where

$$\nu_k = \frac{N_d}{A_c \cdot f_{ck}} \qquad (13)$$

Eq.(12) was derived for $f_{ck} \cong 55MPa$ and an end moment ratio $\mu = M_{Bd} / M_{Ad} = -0,5$; it has been submitted for the next revision of ACI-318. It can be shown (see Appendix B) that the said equation can be usefully generalized for other values of μ and f_{ck} (the latter varying from 20 MPa to 60 MPa) and assuming, on the safe side, $\delta = 1$.

$$\lambda_1 = \frac{42}{\sqrt{\nu_k}} \sqrt{0,5-\mu}\ \sqrt[4]{25/f_{ck}} \quad \text{for } -0,8 \le \mu \le 0 \tag{14}$$

$$\lambda_1 = \frac{30}{\sqrt{\nu_k}} \sqrt{1,0-\mu}\ \sqrt[4]{25/f_{ck}} \quad \text{for } 0 < \mu \tag{15}$$

For μ between $-0,8$ and $-1,0$, it is safe to make $\mu = -0,8$ in eq. (14).

Substituting in eq.(14) $f_{ck} = 55MPa$ and $\mu = -0,5$ we find $\lambda_1 = 34,5 / \sqrt{\nu_k}$, slightly smaller than the value of MacGregor's eq. 12.

An extensive parametrical analysis by França, 1991, has shown that, in order to have in the column a second order decrease of load-bearing capacity smaller than 10%, the slenderness bound λ_1 is:

$$\lambda_1 = (1,5 - 0,5\mu) \cdot \left[12 \cdot \left(\frac{e}{h} - 0,10 \right) + 35 \right] \le 140 \tag{16}$$

with

$$\frac{e}{h} = \frac{M_{Ad}}{N_d \cdot h} \ge 0,10 \tag{17}$$

Eqs. (16) and (17) are being considered for inclusion in the revision of the Brazilian Code NB-1.

6.4 Whenever $\lambda > \lambda_1$, a local second order analysis is necessary to determine the governing intermediate moment M_{Cd}. To this purpose the classical model column approach can be used as follows:

As stated above, ℓ_o will be calculated using Cranston's expressions for braced columns:

$$\ell_o = \eta \cdot \ell \tag{18}$$

in which η is the smaller of the following expressions:

$$\eta = 0,7 + 0,05(\alpha_A + \alpha_B) \le 1 \tag{19}$$

$$\eta = 0,85 + 0,05 \cdot \alpha_{min} \tag{20}$$

where: α_A is the ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at end A of the column;

α_B is the analogous ratio at end B;

α_{min} is the minimum of α_A and α_B.

It is advisable in any case to consider:

$$\eta \ge 0,85 \tag{21}$$

The moment M_{Cd} at the intermediate critical section C may be calculated as follows:

$$M_{Cd} = M_{1d} + M_a + M_{2d} \tag{22}$$

where the first order moment at the critical section is approximately:

$$M_{1d} = 0,6 \cdot M_A + 0,4 \cdot M_B \tag{23}$$

M_a is the moment due to local geometrical imperfections

M_{2d} is the second order local moment at C, given approximately by:

$$M_{2d} = N_d \cdot (\ell_o^2 / 10)(1/r) \tag{24}$$

The curvature $1/r$ can be approximately written:

$$1/r = \frac{0,0035 + f_{yd} / E_s}{(\nu + 0,5) \cdot h} \qquad (25)$$

$$\nu + 0,5 \geq 1 \qquad (26)$$

$$\nu = N_d / A_c \cdot f_{cd} \qquad (27)$$

When $\lambda > 80$ creep must be accounted for.

Eqs. (18) to (27) define the classical model column approach. A refined model column method can be used. Of course the best results are obtained by using the real moment-curvature (M, $1/r$) relationships for the element. An approach using alternatively the secant stiffness κ as a function of N and M was used by França, 1991, who has shown that for any particular value of N the curve $M, 1/r$ can be accurately linearized in the interval of interest, thus greatly simplifying the second order local analysis of the element.

7. CONCLUSIONS.

A review of the instability problems affecting tall reinforced concrete buildings was presented. It was shown that the concept of "unbraced" structures and elements is pointless: every building must have a **bracing spatial substructure,** whose definition depends entirely on the designer. It is recommended that this substructure includes most, if not all, the structural elements, thus resulting an efficient and economic solution. Elements that belong to the bracing substructure are affected by lateral loads and are called **bracing elements**. The others are the **braced** elements and are therefore unaffected by lateral loads.

It was shown that initially a first order spatial analysis of the bracing structure must be performed at design level with recommended reduced flexural stiffnesses for columns, beams and slabs. A simple criterion was proposed in order to assess, after the first order analysis, whether a second order global analysis is mandatory. If such is the case, before performing a $P - \Delta$ analysis, column stiffnesses must be further reduced to take into account the so-called "C and S" effects (loss of stiffness due to the presence of the axial force).

A **local** second order analysis of isolated members will follow the **global** (first order or second order) analysis and will be performed using design end moments and axial forces as determined by that global analysis. In the local analysis the effective length of each column ℓ_o will be determined according to its end elastic restrictions and will always be smaller than the total length ℓ, **either for sway or for non-sway structures**. Criteria were presented to verify whether a second order local analysis of the compressed element is necessary, depending upon its slenderness $\lambda = \ell_o / i$. No local second order analysis should be performed prior to the global analysis. The classical model column was briefly presented.

APPENDIX A. Proof of eqs. (9), (10) and (11).

Let us consider the column of fig. A-1 with length $\ell = 2a$ and stiffness product EI, simultaneously loaded by the vertical force P and by the horizontal force F.

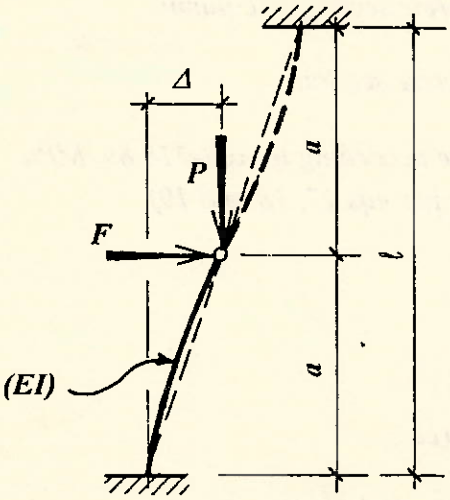

Fig. A-1

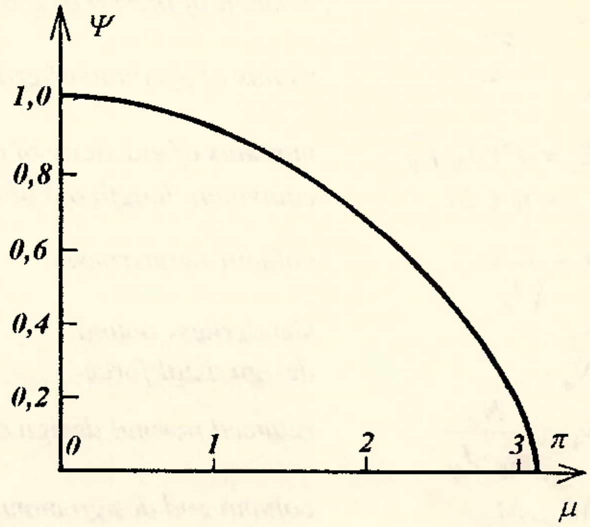

Fig. A-2

The mid-point horizontal deflection Δ, calculated using the $P-\Delta$, simplified, method is:

$$\Delta = (F + \frac{P \cdot \Delta}{a}) \frac{h^3}{3EI} \tag{A-1}$$

It results that the apparent stiffness k_o of the half column is:

$$k_o = \frac{F}{\Delta} = (\frac{3EI}{a^3} - \frac{P}{a}) \tag{A-2}$$

The exact stiffness k is:

$$k = \frac{3EI}{a^3} \cdot \frac{u^3}{3(tgu - u)} \tag{A-3}$$

with

$$u = a\sqrt{\frac{P}{EI}} \tag{A-4}$$

If we now multiply the stiffness EI of the simplified model by a reduction factor ψ so that the stiffness k_o of the $P-\Delta$ model equals the exact stiffness k as given by eq.(A-3) we may write:

$$\frac{3EI \cdot \psi}{a^3} - \frac{P}{a} = \frac{3EI}{a^3} \cdot \frac{u^3}{3(tgu - u)} \tag{A-5}$$

and finally (see fig. B-1):

$$\psi = \frac{u^2 tgu}{3(tgu - u)} \tag{A-6}$$

APPENDIX B. Proof. of eqs.(14) and (15).

Notation

A_g	*area of gross concrete section of column*
I_g	*moment of inertia of gross concrete section of column*
$i_g = \sqrt{\frac{I_g}{A_g}}$	*radius of gyration of gross concrete section*
$E_c = 4700\sqrt{f_{ck}}$	*modulus of elasticity of concrete according to ACI-318/89, MPa*
$\ell_o = \eta \cdot \ell \le \ell$	*equivalent length of column (as per eqs.17, 18 and 19)*
$\lambda = \sqrt{\frac{\ell_o}{i_g}}$	*column slenderness*
λ_1	*slenderness bound*
N_d	*design axial force*
$\nu_k = \frac{N_d}{A_g \cdot f_{ck}}$	*reduced normal design axial force*
M_{Ad}, M_{Bd}	*column end design moments* $\left(\lvert M_{Ad}\rvert \ge \lvert M_{Bd}\rvert\right)$
M_{Cd}	*maximum span design moment*
$\mu = \frac{M_{Bd}}{M_{Ad}}$	*ratio of end moments*
$\delta = \frac{\lvert M_{Cd}\rvert}{\lvert M_{Ad}\rvert}$	
$\alpha = \ell_o\sqrt{\frac{N_d}{EI}}$	*instability coefficient of column*

According to Galambos, 1968:

$$\delta = \frac{\sqrt{1+\mu^2-2\mu\cos\alpha}}{\sin\alpha} \qquad \text{(B-1)}$$

Solving for $\delta = 1$:

$$\mu = \cos\alpha \qquad \text{(B-2)}$$

Equation (B-2) can be accurately represented by the bi-linear diagram defined by:

$$\mu = 0{,}5 - 0{,}21\alpha^2 \quad \text{for} \quad -0{,}8 \le \mu \le 0 \qquad \text{(B-3)}$$

$$\mu = 1{,}0 - 0{,}42\alpha^2 \quad \text{for} \quad 0 < \mu \qquad \text{(B-4)}$$

as shown in Fig. B-1.

According to ACI-318/89 it can be safely assumed:

$$EI = 0{,}4E_c \cdot I_g = 0{,}4E_c \cdot A_g \cdot i_g^2 \qquad \text{(B-5)}$$

Therefore:

$$\alpha^2 = \frac{N_d}{EI} = \frac{N_d \cdot \ell_o^2}{0{,}4E_c \cdot A_g \cdot i_g^2} = \frac{\nu_k \cdot \lambda_1^2}{0{,}4\varepsilon} \qquad \text{(B-6)}$$

Substituting (B-6) in eq. (B-3):

$$\lambda_1 = \sqrt{\frac{0,5-\mu}{\nu_k}}\sqrt{\frac{\varepsilon}{0,525}} = \frac{k_1}{\sqrt{\nu_k}}\sqrt{0,5-\mu} \tag{B-7}$$

with

$$k_1 = \sqrt{\frac{\varepsilon}{0,525}} \tag{B-8}$$

Substituting (B-6) in eq. (B-4):

$$\lambda_1 = \sqrt{\frac{1,0-\mu}{\nu_k}}\sqrt{\frac{\varepsilon}{1,05}} = \frac{k_2}{\sqrt{\nu_k}}\sqrt{1,0-\mu} \tag{B-9}$$

with

$$k_2 = \sqrt{\frac{\varepsilon}{1,05}} \tag{B-10}$$

It can be seen from Table B-1 that k_1 and k_2 can be respectively approximated (with an error smaller than 1%) to:

$$k_1 = 42\ \sqrt[4]{25/f_{ck}} \tag{B-11}$$

$$k_2 = 30\ \sqrt[4]{25/f_{ck}} \tag{B-12}$$

TABLE B-1

f_{ck} (MPa)	$\varepsilon = \frac{4700\sqrt{f_{ck}}}{f_{ck}}$	$k_1 = \sqrt{\frac{\varepsilon}{0,525}}$	$42\ \sqrt[4]{\frac{25}{f_{ck}}}$	$k_2 = \sqrt{\frac{\varepsilon}{1,05}}$	$30\ \sqrt[4]{\frac{25}{f_{ck}}}$
20	1051	44,7	44,4	31,6	31,7
25	940	42,3	42,0	29,9	30,0
30	858	40,4	40,1	28,6	28,7
35	794	38,9	38,6	27,5	27,6
40	743	37,6	37,3	26,6	26,7
45	701	36,5	36,3	25,8	25,9
50	665	35,6	35,3	25,2	25,2
55	634	34,8	34,5	24,6	24,6
60	607	34,0	33,7	24,0	24,1

We have, finally:

$$\lambda_1 = \frac{42}{\sqrt{\nu_k}}\sqrt{0,5-\mu}\ \sqrt[4]{25/f_{ck}} \quad \text{for} \quad -0,8 \le \mu \le 0 \tag{B-13}$$

$$\lambda_1 = \frac{30}{\sqrt{\nu_k}}\sqrt{1,0-\mu}\ \sqrt[4]{25/f_{ck}} \quad \text{for} \quad 0 < \mu \tag{B-14}$$

For μ between $-0,8$ and $-1,0$ it is safe to make $\mu = -0,8$ in eq. (B-13).

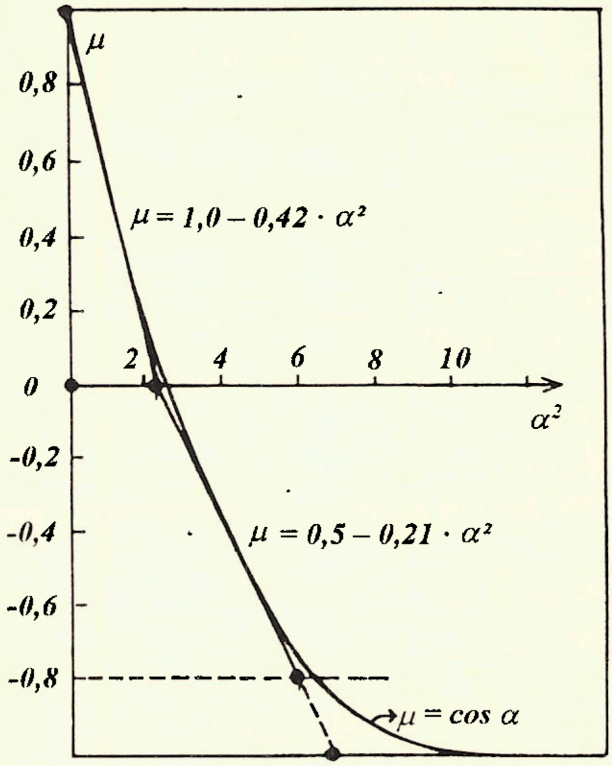

Fig. B-1

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