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# GLUONS IN THE STARS

DAVID A. FOGAÇA, BRUNO FRANZON and FERNANDO S. NAVARRA

*Instituto de Física, Universidade de São Paulo*
*C.P. 66318, 05315-970 São Paulo, SP, Brazil*
*david@if.usp.br*
*bfranzon@if.usp.br*
*navarra@if.usp.br*

Using an equation of state based on a mean-field approximation for QCD to describe the cold quark gluon plasma we study stellar structures of compact stars which are compatible with recent astrophysical data.[1]

*Keywords*: QCD; Dense Star; Super-conducting Phase.

## 1. Introduction

The QGP at zero temperature and high baryon number is a system that may exists in the core of dense stars. This cold QGP has a richer phase structure and at high enough chemical potential we may have a color super-conducting phase. There are limitations of lattice calculations in this domain and also because of the lack of experimental information, the cold QGP is less known than the hot QGP. Nevertheless it is quite possible that it shares some features with the hot plasma, being also a strongly interacting and semi-classical system. We shall recall the EOS[2] and apply it to stellar structure determination.

Recently[1] the pulsar PSR J1614-2230 was determined to possess mass $1.97 \pm 0.04\, M_{\odot}$ which was by far the highest yet measured. In the next sections we show the EOS and stellar mass compatible with this measure.

In this work we employ the natural units $\hbar = 1$ and $c = 1$.

## 2. The Equation of State for Cold sQGP

In a previous work,[2] we imposed a gluon field decomposition in low ("soft") and high ("hard") momentum components in the QCD Lagrangian. After such decomposition we performed a mean-field approximation (MFA) for the hard gluons and considered the matrix elements of the soft gluon fields in the plasma. The latter are related to the condensates of dimension two and four. With these approximations we derived an analytical expression for the equation of state (EOS), that we call the MQCD description. We noticed that the effect of the condensates is to soften the equation of state whereas the hard gluons significantly increase the energy density and the pressure. In this work we recall to this EOS for the cold sQGP to study stellar structure.

In the MQCD the energy density is given by:

$$\varepsilon = \left(\frac{27g^2}{2{m_G}^2}\right) {\rho_B}^2 + \mathcal{B}_{QCD} + 3\frac{\gamma_Q}{2\pi^2}\left\{\frac{{k_F}^3\sqrt{{k_F}^2+m^2}}{4}\right\}$$
$$+\frac{3m^2}{8}\frac{\gamma_Q}{2\pi^2}\left[k_F\sqrt{{k_F}^2+m^2} - m^2 ln\left(\frac{k_F+\sqrt{{k_F}^2+m^2}}{m}\right)\right] \tag{1}$$

and pressure by:

$$p = \left(\frac{27g^2}{2{m_G}^2}\right) {\rho_B}^2 - \mathcal{B}_{QCD} + \frac{\gamma_Q}{2\pi^2}\left\{\frac{{k_F}^3\sqrt{{k_F}^2+m^2}}{4}\right\}$$
$$-\frac{3m^2}{8}\frac{\gamma_Q}{2\pi^2}\left[k_F\sqrt{{k_F}^2+m^2} - m^2 ln\left(\frac{k_F+\sqrt{{k_F}^2+m^2}}{m}\right)\right] \tag{2}$$

where $\gamma_Q$ is the quark degeneracy factor $\gamma_Q = 2(\text{spin}) \times 3(\text{flavor})$, $m_G$ is the dynamical gluon mass, $k_F$ is the Fermi momentum defined by the baryon density $\rho_B$:

$$\rho_B = \frac{\gamma_Q}{6\pi^2}{k_F}^3 \tag{3}$$

and the bag constant for QCD is given in terms of the gluon condensate:

$$\mathcal{B}_{QCD} = \left\langle \frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu}\right\rangle \tag{4}$$

## 3. The Tolman-Oppenheimer-Volkoff Equation

To describe a compact star we solve the Einstein's equation:[3,4]

$$G^{\mu\nu} = -8\pi G T^{\mu\nu} \tag{5}$$

for an isotropic, static, general relativistic and ideal fluid spherical star in hydrostatic equilibrium. This solution gives the Tolman-Oppenheimer-Volkoff (TOV)[5,6] equation for the pressure $p(r)$:[3,4]

$$\frac{dp(r)}{dr} = -\frac{G\,\varepsilon(r)\mathcal{M}(r)}{r^2}\left[1+\frac{p(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]\left[1-\frac{2G\,\mathcal{M}(r)}{r}\right]^{-1} \tag{6}$$

where G is the Newtons gravitational constant. In classical mechanics we have:[7]

$$\frac{dp(r)}{dr} = -\frac{G\,\varepsilon(r)\mathcal{M}(r)}{r^2} \tag{7}$$

In Eq.(6) the first two factors in the square brackets represent the special relativity corrections and the last set of brackets is the general relativistic correction to Eq.(7). For a detailed study we strongly recommend the references.[7,8]

The enclosed mass $\mathcal{M}(r)$ of the compact star is given by the mass continuity equation:

$$\frac{d\mathcal{M}(r)}{dr} = 4\pi r^2 \varepsilon(r) \tag{8}$$

## 4. Numerical Results

We numerically solve the Eq.(6) and Eq.(8) which are coupled nonlinear equations for $p(r)$ and $\mathcal{M}(r)$ to obtain stellar structure. The energy density and pressure in Eq.(6) and Eq.(8) are given by Eq.(1) and Eq.(2) respectively.

We impose the central energy density $\varepsilon(r=0) = \varepsilon_c$ and then we integrate from $r = 0$ to $r = R$ where the pressure on the surface equals to zero: $p(r = R) = 0$ . This gives the stellar radius $R$ and the gravitational mass $M$ from Eq.(8):

$$M \equiv \mathcal{M}(r) = 4\pi \int_0^R dr\, r^2 \varepsilon(r) \tag{9}$$

In Fig.1 we compare the EOS from MQCD given by Eq.(1) and Eq.(2) with the EOS from MIT.[2] We have considered $B_{MIT} = B_{QCD} = 110\, MeV/fm^3$, $g = 0.35$ and $m_G = 290$ MeV. In this situation we noticed that MQCD has more pressure and so we obtain higher stellar mass and radius.

In Fig.2 we compare different values for the dynamical gluon mass for the EOS of MQCD given by Eq.(1) and Eq.(2) with fixed $B_{QCD} = 110\, MeV/fm^3$ As the gluon mass increases we obtain lower stellar mass and radius.

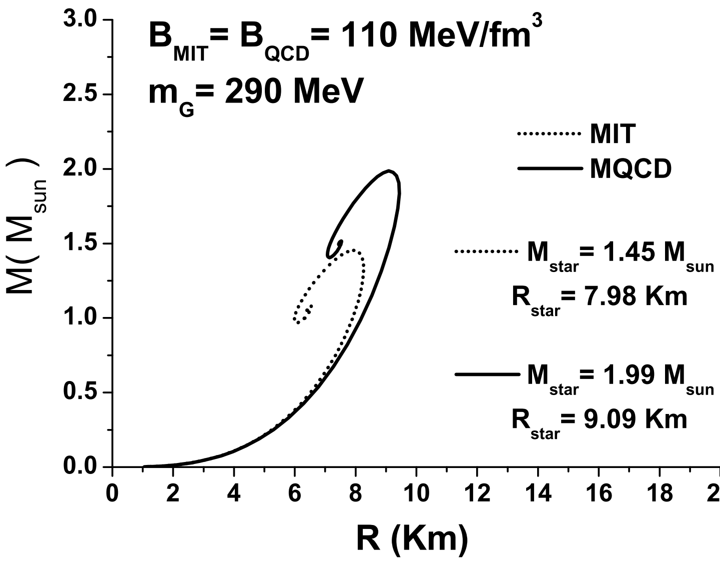


Fig. 1. A comparision between MIT and MQCD.

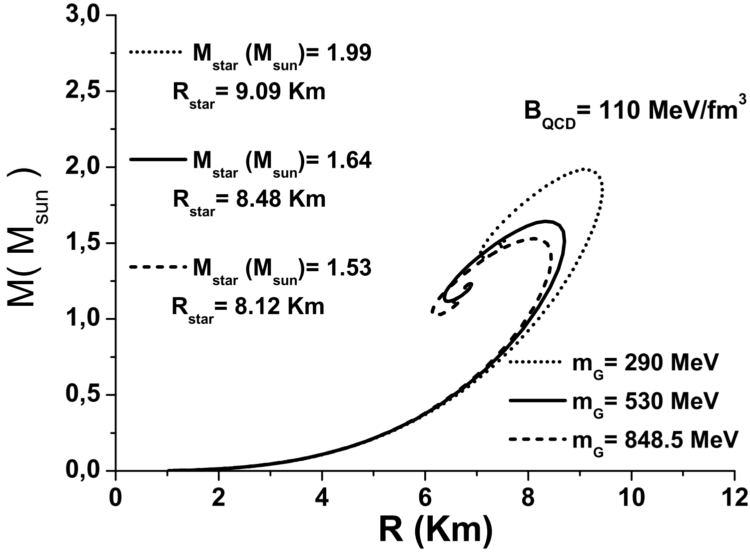


Fig. 2. Stellar structure as function of the dynamical gluon mass variation.

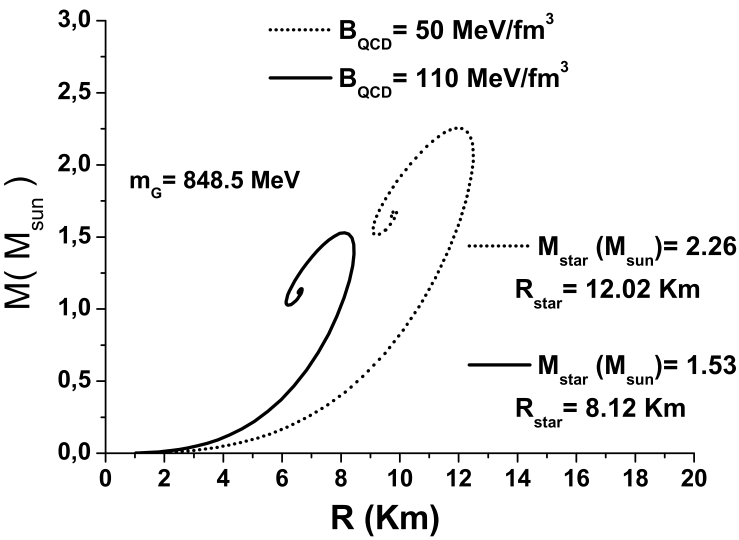


Fig. 3. Stellar structure as function of the $B_{QCD}$ variation.

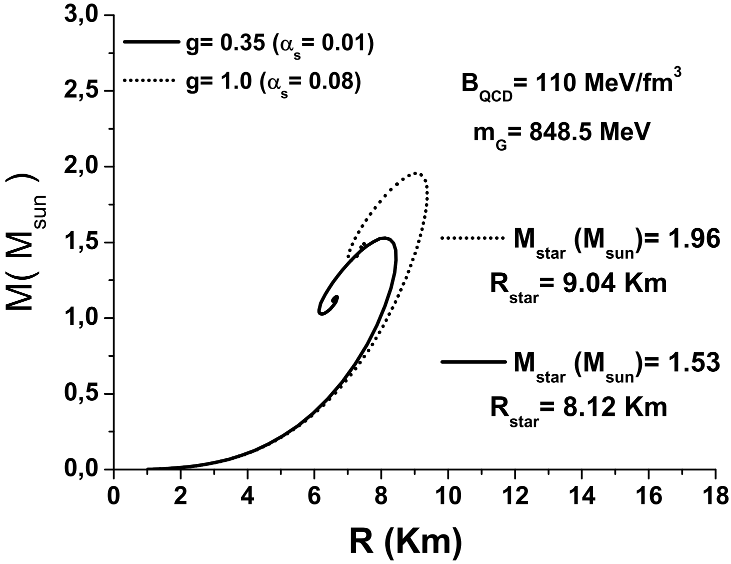


Fig. 4. Stellar structure as function of the coupling $g$ variation.

In Fig.3 we perform variation in $B_{QCD}$ for the fixed $m_G = 848.5\,MeV$. As $B_{QCD}$ decreases we obtain much more pressure and stellar mass and radius increases.

In Fig.4 we perform variation in the coupling constant $g$ for the fixed $m_G = 848.5\,MeV$ and $B_{QCD} = 110\,MeV/fm^3$. As $g$ increases we obtain more pressure and stellar mass and radius increases.

## 5. Conclusions

We propose a simple approach to dense and cold sQGP based on a gluon field decomposition in soft and hard components. This approach gives an EOS that can be considered a richer version of the MIT bag model. In our description the condensates make the EOS softer, the hard gluons increase the pressure and so we obtain more massive stars by TOV equations.

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