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DYNAMIC SIGNATURES OF A COHERENT SYSTEM

by

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Palavras-Chave: System signatures, dynamic system signature, coherent systems, mixed systems, point processes martingales.

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Dynamic signatures of a coherent system

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Abstract. In this paper we define dynamic signatures of a coherent system observing the ordered components lifetimes, as they appear on time until system failure. The dynamic signatures actualizes itself on time and does not depend of the particular distribution F of the components lifetimes and of the particular time t. Its also recover the system signature at infinity.

Keywords: System signature; Dynamic system signature; Coherent systems; Mixed systems; Point processes martingales.

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1.Introduction. As in Barlow and Proschan [3] a complex engineering system is completely characterized by its structure function Φ which relate its lifetime T and its components lifetimes T_i, $1 \leq i \leq n$, defined in a complete probability space $(\Omega, \Im, P)$

$$T = \Phi(\mathbf{T}), \mathbf{T} = (T_1, ..., T_n).$$

A physical system would be quit unusual (or perhaps poorly designed) if improving the performance of a component (that is, replacing a failed component by a functioning component) caused the system to deteriorate (that is, to change from the functioning state to the failed state). Thus we consider structure functions which are monotonically increasing in each coordinate. Also to avoid trivialities we will eliminate consideration of any system whose state does not depend on the state of its components. A system is said to be coherent if its structure function Φ is increasing and each component is relevant, that is, there exist a time t and a configuration of $\mathbf{T}$ in t such that the system works if, and only if, the component works.

The performance of a coherent system can be measured from this structural relationship and the distribution function of its components lifetimes. The structure functions offer a way of indexing the class of coherent system but such representations make the distribution function of the system lifetime analytically very complicated (mainly in the dependent case). An alternative representation for the coherent system distribution function is through the system signatures, as in Samaniego [10], that, while narrower in scope than the structure function, is substantially more useful.

Samaniego [8] consider the order statistics of the independent and identically distributed components lifetimes of a coherent system of order n with absolutely continuous distribution. Clearly $\{T = T_{(i)}\}\ \ 1 \leq i \leq n\}$ is a (P-a.s.) partition of the probability space and

$$P(T \leq t) = \sum_{i=1}^{n} P(T \leq t, T = T_{(i)}) = \sum_{i=1}^{n} P(T = T_{(i)}) P(T \leq t | T = T_{(i)}) =$$

$$\sum_{i=1}^{n} P(T = T_{(i)}) P(T_{(i)} \leq t | T = T_{(i)}) = \sum_{i=1}^{n} P(T = T_{(i)}) P(T_{(i)} \leq t) =$$

$$\sum_{i=1}^{n} \alpha_i P(T_{(i)} \leq t).$$

In the above context Samaniego [10] defines

Definition 1.1 Let T be the lifetime of a coherent system of order n, with components lifetimes $T_1, ..., T_n$ which are independent and identically distributed random variables with absolutely continuous distribution F. Then the signature vector α is defined as

$$\alpha = (\alpha_1, ..., \alpha_n)$$

where $\alpha_i = P(T = T_{(i)})$ and the $\{T_{(i)}, 1 \leq i \leq n\}$ are the order statistics of $\{T_i, 1 \leq i \leq n\}$.

The key feature of system signatures that makes them broadly useful in reliability analysis is the fact that, in the context of independent and identically distributed (i.i.d.) absolutely continuous components lifetimes, they are distribution free measures of system quality, depending solely on the design characteristics of the system and independent of the behavior of the systems components .

A detailed treatment of the theory and applications of system signatures may be found in Samaniego [10]. This reference gives detailed justification for the i.i.d. assumption used in the definition of system signatures. By the way there are a host of applications in which the i.i.d. assumption is appropriate, and in such case, the use of system signatures for comparisons among systems is wholly appropriate; such applications range from batteries in lighting, to wafers or chips in a digital computer to the subsystem of spark plugs in an automobile engine.

The utility of signatures in gauging the performance of systems in i.i.d. components derives largely from representation and preservation results. Some of then link the characteristics of system signatures with system performance.

Before stating these results, we first recall the definitions of three standard forms of stochastic relations between random variables.

Definition 1.2 Let T_1 and T_2 random variables. Then:

a) T_1 is stochastically smaller than T_2 ($T_1 \leq_{st} T_2$) if, and only if, $P(T_1 > t) \leq P(T_2 > t), \forall t$;

b) T_1 is stochastically smaller than T_2 in the hazard rate ordering ($T_1 \leq_{hr} T_2$) if, and only if, $\frac{P(T_1>t)}{P(T_2>t)}$ is nonincreasing in $t, \forall t$;

c) in the case where T_1 and T_2 have absolutely continuous distributions, with densities f_1 and f_2, respectively, T_1 is stochastically smaller than T_2 in the likelihood rate ordering ($T_1 \leq_{lr} T_2$) if, and only if, $\frac{f_1(t)}{f_2(t)}$ are nonincreasing in $t, \forall t$.

The following result shows that certain relationships between two (discrete) signatures ensure that a similar relationship holds between the corresponding (continuous) system lifetimes.

Theorem 1.3(Kochar et al. [6]) Let α_1 and α_2 be the signatures of two coherent systems of order n, both based on n components with i.i.d. lifetimes with common continuous distribution F. Let S_1 and S_2 be their respective lifetimes. Then:

a) if

$$\alpha_1 \leq_{st} \alpha_2 \Longrightarrow S_1 \leq_{st} S_2;$$

b) if

$$\alpha_1 \leq_{hr} \alpha_2 \Longrightarrow S_1 \leq_{hr} S_2;$$

c) if F is absolutely continuous and

$$\alpha_1 \leq_{lr} \alpha_2 \Longrightarrow S_1 \leq_{lr} S_2.$$

The applications of system signature can be extended to mixed system. A mixed system of order n is a stochastic mixture of coherent systems of order n and can be realized in practice via randomization which selects a system at random according to a fixed probability distribution on the class of coherent systems of order n (see [5]). The mixed system that selects among n-component systems with signatures vectors $\alpha_1, \alpha_2, ..., \alpha_n$ according to the distribution $\mathbf{p} = (p_1, ..., p_n)$ will have signature $\sum_{i=1}^{n} p_i \alpha_i$. We note that the representation and preservation theorem above is applicable for mixed systems.

One further important issue is the fact that we will, at times, be interested in comparing systems of different sizes. Although such a comparison might arise in general, it is special relevant when comparisons involve new and used systems. Theorem 1.3 is not immediately applicable to this problem. However, the exact relationship has been characterized between the signature of a given system with a system of any larger order, which has an equivalent lifetime distribution under the assumption of i.i.d. component lifetimes. The following theorem is an example.

Theorem 1.4 (Samaniego [9]) Let $\alpha = (\alpha_1, ..., \alpha_n)$ be the signature of a mixed system in n i.i.d. components lifetimes with continuous distribution F. Then the mixed system with

$(n+1)$ i.i.d. components lifetimes with continuous distribution F and corresponding to the system signature

$$\alpha^{*} = (\frac{n\alpha_1}{n+1}, \frac{\alpha_1 + (n-1)\alpha_2}{n+1}, \frac{2\alpha_2 + (n-2)\alpha_3}{n+1}, ..., \frac{(n-1)\alpha_{n-1} + \alpha_n}{n+1}, \frac{n\alpha_n}{n+1})$$

has the same distribution lifetime as the n-component system with signature $\dot{\alpha}$.

Samaniego [8], Kochar, et al. [6] and Shaked and Suarez-Llorens ([12] extended the signature concept to the case where the components lifetimes $T_1, ..., T_n$, of a system are exchangeable (i.e. the joint distribution function, $F(t_1, ..., t_n)$, of $(T_1, ..., T_n)$ is the same for any permutation of $t_1, ..., t_n$), an interesting and practical situation in reliability theory.

Navarro et al. [7] and Samaniego et al. [11] consider dynamic (conditioned) signatures and their use in comparing the reliability of new and used systems. Their procedures consider the system lifetime conditioned in an event on time. Navarro et al. [7] consider either the event $\{T > t\}$ and $\{T_{(i)} \leq t\} \cap \{T > t\}$ with system signature $P(T = T_{(i)} | T > t)$ and $P(T = T_{(i)} | \{T_{(i)} \leq t\} \cap \{T > t\})$ respectively. A systems signature has proven to be quite a useful proxy for a systems design, as it is a distribution-free measure (i.e., not depending on F) that efficiently captures the precise features of a systems design which influence it performance but unhappiness, in both Navarros above situations, the system signatures does depend on $F(t)$.

Samaniego et all. [11] consider the event in time $\{T_{(i)} \leq t < T_{(i+1)}\} \cap \{T > t\}$ and in this case the system signature $P(T = T_{(i)} | \{T_{(i)} \leq t < T_{(i+1)}\} \cap \{T > t\})$ does not depend on t and on $F(t)$ and have the usual signatures properties. Samaniego et all. [11] extend Theorem 1.3, however their conditioned signature does not capture the dynamics aspects of the problem.

Theorem 1.5 (Samaniego et al [[11]) Consider two used mixed systems with lifetimes S_1 and S_2, based on n original components with i.i.d. lifetimes having the common continuous distribution F. Suppose both systems are working and have exactly i and j failed component, respectively, at time t. Let $\alpha_1(\mathbf{n}-\mathbf{i})$ and $\alpha_2(\mathbf{n}-\mathbf{j})$ be their dynamics signatures, as in [11]. Then

a) if

$$\alpha_1(\mathbf{n}-\mathbf{i}) \leq_{st} \alpha_2(\mathbf{n}-\mathbf{j}) \Longrightarrow$$

$$(S_1 | \{T_{(i+1)} \leq t < T_{(i+2)}\} \cap \{S_1 > t\}) \leq_{st} (S_2 | \{T_{(j+1)} \leq t < T_{(j+2)}\} \cap \{S_2 > t\});$$

b) if

$$\alpha_1(\mathbf{n}-\mathbf{i}) \leq_{hr} \alpha_2(\mathbf{n}-\mathbf{j}) \Longrightarrow$$

$$(S_1 | \{T_{(i+1)} \leq t < T_{(i+2)}\} \cap \{S_1 > t\}) \leq_{hr} (S_2 | \{T_{(j+1)} \leq t < T_{(j+2)}\} \cap \{S_2 > t\});$$

c) if F is absolutely continuous and if

$$\alpha_1(\mathbf{n}-\mathbf{i}) \leq_{lr} \alpha_2(\mathbf{n}-\mathbf{j}) \Longrightarrow$$

$$(S_1 | \{T_{(i+1)} \le t < T_{(i+2)}\} \cap \{S_1 > t\}) \le_{lr} (S_2 | \{T_{(j+1)} \le t < T_{(j+2)}\} \cap \{S_2 > t\});$$

In this paper we consider the system evolution on time under a complete information level. The natural tool to consider the increasing information on time through a family of sub-σalgebras is the martingale theory and, in our case, the point process martingale theory. To make the exposition understandable, in the Subsection 2.1 of the Section 2 we develop the independent case which is a natural approach for systems signatures theory. We give some examples at the end of this Subsection. In Subsection 2.2 we give the mathematical details.

2. Dynamic signatures

We intend to give a new approach to dynamic systems signatures. We consider the system evolution on time under a complete information level. In this fashion, the expected dynamic system signature enjoy the special property that they are independent of both the distribution F and the time t. This fact has significance beyond the mere simplicity and tractability of the signature vector, reflect only characteristics of the corresponding system design and may be used as proxies for system designs in the comparison of system performance. Also the dynamic system signature actualizes itself under the system evolution on time recovering the dynamical system signature in the set $\{T_{(i)} \le t < T_{(i+1)}\} \cap \{T > t\}$, as in [11] and the original coherent system signature in the set $\{T_{(n)} \le t\}$ as in [10].

In our general setup, we consider the vector $(T_1, ..., T_n)$ of n component lifetimes which are finite and positive random variables defined in a complete probability space $(\Omega, \Im, P)$, with $P(T_i \ne T_j) = 1$, for all $i \ne j, i, j$ in $E = \{1, ..., n\}$, the index set of components. The lifetimes can be dependent but simultaneous failure are ruled out.

In what follows, to simplify the notation, we assume that relations such as $\subset, =, \le, <, \ne$ between random variables and measurable sets, respectively, always hold with probability one, which means that the term P-a.s., is suppressed.

The evolution of components in time define a marked point process given through the failure times and the corresponding marks.

We denote by $T_{(1)} < T_{(2)} < ... < T_{(n)}$ the ordered lifetimes $T_1, T_2, ..., T_n$, as they appear in time and by $X_i = \{j : T_{(i)} = T_j\}$ the corresponding marks. As a convention we set $T_{(n+1)} = T_{(n+2)} = ... = \infty$ and $X_{n+1} = X_{n+2} = ... = e$ where e is a fictitious mark not in E. Therefore the sequence $(T_n, X_n)_{n \ge 1}$ defines a marked point process.

The mathematical formulation of our observations is given by a family of sub σ-algebras of $\Im$, denoted by $(\Im_t)_{t \ge 0}$, where

$$\Im_t = \sigma\{1_{\{T_{(i)} > s\}}, X_i = j, 1 \le j \le n, j \in E, 0 < s \le t\},$$

satisfies the Dellacherie conditions of right continuity and completeness .

Intuitively, at each time t the observer knows if the events $\{T_{(i)} \le t, X_i = j\}$ have either occurred or not and if they have, he knows exactly the value $T_{(i)}$ and the mark X_i. We assumed that $T_i, 1 \le i \le n$ are totally inaccessible $\Im_t$-stopping time.

We observe that, in the original concept of signature, Samaniego [10] does not use any information on time which we represent by the trivial σ- algebra $\Im_t = \{\Omega, \emptyset\}, \quad \forall t$.

An extended and positive random variable τ is an $\Im_t$-stopping time if, and only if, $\{\tau \leq t\} \in \Im_t$, for all $t \geq 0$; an $\Im_t$-stopping time τ is called predictable if an increasing sequence $(\tau_n)_{n \geq 0}$ of $\Im_t$-stopping time, $\tau_n < \tau$, exists such that $\lim_{n \to \infty} \tau_n = \tau$; an $\Im_t$-stopping time τ is totally inaccessible if $P(\tau = \sigma < \infty) = 0$ for all predictable $\Im_t$-stopping time σ. For a mathematical basis of stochastic processes applied to reliability theory see the book of Aven and Jensen [2]. In a practical sense we can think of a totally inaccessible $\Im_t$-stopping time as an absolutely continuous lifetime.

2.1. Dynamic signatures with independent and identically distributed lifetimes.

In this Subsection the following remark is of fundamental importance:

Remark 2.1.1 Under the assumption that the components lifetimes are i.i.d. with absolutely continuous distribution F, $P(\cup_{i=1}^{n}\{T = T_{(i)}\}) = 1$ and the information in $\Im_t$ is of the kind $\{T_{(i)} \leq t < T_{(i+1)}\}$, $i = 1, 2, ..., n$ with $T_{(0)} = 0$ and $T_{(n+1)} = \infty$. Follows that:

$P(T \leq t|\Im_t) = 0$ in the set $\{T_{(1)} > t\}$;

$P(T \leq t|\Im_t) = \frac{P(T=T_{(i)})}{P(T \geq T_{(i)})}$ in the set $\{T_{(i)} \leq t < T_{(i+1)}\}$;

$P(T \leq t|\Im_t) = 1$ in the set $\{t \geq T_{(n)}\}$.

Therefore

$$\int_A P(T \leq t|\Im_t) dP = P(A \cap \{T \leq t\})$$

$\forall A \in \Im_t$ and we have

$$P(T \leq t|\Im_t) = \Sigma_{i=1}^{n} \frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} 1_{\{T_{(i)} \leq t < T_{(i+1)}\}}.$$

We are going to use Remark 2.1.1 in the next Theorem and we will prove it, rigorously, in the next Subsection.

Theorem 2.1.2 Let T be the lifetime of a coherent system of order n, with component lifetimes $T_1, ..., T_n$ which are independent and identically distributed with continuous distribution F. Then,

$$P(T \leq t|\Im_t) = \Sigma_{i=1}^{n} \beta_i 1_{\{T_{(i)} \leq t\}}$$

where

$$\beta_i = \frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} - \frac{P(T = T_{(i-1)})}{P(T \geq T_{(i-1)})},$$

with $T_{(0)} = 0$, $T_{(n+1)} = \infty$, $\beta_i \geq 0$ and $\Sigma_{i=1}^{n} \beta_i = 1$.

Proof Firstly, as the order statistics have the likelihood ratio order property (Shaked and Shanthikumar [13], p 54), they also have the reversed hazard order property and are independent of the events of the type $\{T = T_{(i)}\}$ the β_i are positives. Also

$$\Sigma_{i=1}^n \beta_i = \Sigma_{i=1}^n [\frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} - \frac{P(T = T_{(i-1)})}{P(T \geq T_{(i-1)})}] = \frac{P(T = T_{(n)})}{P(T \geq T_{(n)})} = 1$$

if $P(T = T_{(n)}) \neq 0$ and equal to $\frac{P(T=T_{(n-1)})}{P(T \geq T_{(n-1)})} = 1$, if $P(T = T_{(n)}) = 0$ and so successively.

From Remark 2.1.1, we have

$$P(T \leq t | \Im_t) = \Sigma_{i=1}^n \frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} 1_{\{T_{(i)} \leq t < T_{(i+1)}\}} =$$

$$\Sigma_{i=1}^n \frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} [1_{\{T_{(i+1)} > t\}} - 1_{\{T_{(i)} > t\}}] =$$

$$1 - \Sigma_{i=1}^n [\frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} - \frac{P(T = T_{(i-1)})}{P(T \geq T_{(i-1)})}] 1_{\{T_{(i)} > t\}} =$$

$$\Sigma_{i=1}^n \beta_i - \Sigma_{i=1}^n \beta_i 1_{\{T_{(i)} > t\}} = \Sigma_{i=1}^n \beta_i 1_{\{T_{(i)} \leq t\}}.$$

Definition 2.1.3 Let T be the lifetime of a coherent system of order n, with component lifetimes $T_1, ..., T_n$ which are independent and identically distributed random variables with absolutely continuous distribution F. Then the dynamic signature vector β is defined as

$$\beta = (\beta_1, ..., \beta_n)$$

where $\beta_i = \frac{P(T=T_{(i)})}{P(T \geq T_{(i)})}$ and the $T_{(i)}$ are the order statistics of $T_i, 1 \leq i \leq n$.

Remarks 2.1.4: Given the information $\Im_t$ we knows that, in the set $\{T_{(i)} \leq t < T_{i+1)}\}$,

$$P(T \leq t | \{T_{(i)} \leq t < T_{i+1)}\}) = \sum_{j=1}^{i} \beta_i = \sum_{j=1}^{i} [\frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} - \frac{P(T = T_{(i-1)})}{P(T \geq T_{(i-1)})}] = \frac{P(T = T_{(i)})}{P(T \geq T_{(i)})},$$

as we have in Remark 2.1.1.

The next Corollary shows that how the dynamic signature actualizes itself on time and how we recover the Samaniego [10] signature vector at infinity.

Corollary 2.1.5 Let T be the lifetime of a coherent system of order n, with component lifetimes $T_1, ..., T_n$ which are independent and identically distributed with absolutely continuous distribution F. Then, in the set $\{T_{(i)} \leq t < T_{(i+1)}\} \cap \{T > t\}$, the system signature actualizes in time with

$$P(T \leq t + x | \{T_{(i)} \leq t < T_{i+1)}\} \cap \{T > t\}) =$$

$$\sum_{j=i+1}^{n} \beta_j P(T_{(j)} \leq t + x | \{T_{(i)} \leq t < T_{i+1)}\} \cap \{T > t\}),$$

and $\sum_{j=i+1}^{n} \beta_j = 1$. Also, its restores the Samaniego [10] system signature in the set $\{t \geq T_{(n)}\}$.

Proof In the set $\{T > t\} \cap \{T_{(i)} \leq t < T_{(i+1)}\}$, $P(T = T_{(j)}) = 0$ if $j \leq i$ and the coherent system signature actualizes to

$$P(T \leq t + x | \{T_{(i)} \leq t < T_{i+1)}\} \cap \{T > t\}) =$$

$$\sum_{j=i+1}^{n} \beta_j P(T_{(j)} \leq t + x | \{T_{(i)} \leq t < T_{i+1)}\} \cap \{T > t\}),$$

with

$$\sum_{j=i+1}^{n} \beta_j = \sum_{j=i+1}^{n} [\frac{P(T = T_{(j)})}{P(T \geq T_{(j)})} - \frac{P(T = T_{(j-1)})}{P(T \geq T_{(j-1)})}] = \frac{P(T = T_{(n)})}{P(T \geq T_{(n)})} = 1,$$

if $P(T = T_{(n)}) \neq 0$ and equal to $\frac{P(T=T_{(n-1)})}{P(T \geq T_{(n-1)})} = 1$, if $P(T = T_{(n)}) = 0$ and so successively.

Also, as $P(T = T_{(i)}) = 0$ and $P(T \geq T_{(i+1)}) = 1$ we have

$$\beta_{i+1} = \frac{P(T = T_{(i+1)})}{P(T \geq T_{(i+1)})} - \frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} = \frac{P(T = T_{(i+1)})}{P(T \geq T_{(i+1)})} = P(T = T_{(i+1)}) = \alpha_{i+1}$$

and the signatures actualizes itself in the set $\{T > t\} \cap \{T_{(i)} \leq t < T_{(i+1)}\}$.

As $\beta_1 = \alpha_1$ and $\{T_i \leq t\}$ occurs successively in time for $i = 1, 2, 3, ...,$ in the set $\{t \geq T_{(n)}\}$ we have:

$$P(T \leq t | \Im_t) = \sum_{i=1}^{n} \alpha_i 1_{\{T_{(i)} \leq t\}}.$$

Taking expected values we get

$$P(T \leq t) = \sum_{i=1}^{n} \alpha_i P(T_{(i)} \leq t)$$

recovering the Samaniego signature decomposition as in [10].

Examples 2.1.6 i) If T_1, T_2, T_3 are independent and identically distributed component's lifetimes of the system with lifetime $T = T_1 \wedge (T_2 \vee T_3)$.

The Samaniego [9] system signatures are:

$\alpha_1 = P(T = T_{(1)}) = \frac{1}{3}$, $\alpha_2 = P(T = T_{(2)}) = \frac{2}{3}$ and $\alpha_3 = P(T = T_{(3)}) = 0$ and the signature system distribution lifetime decomposition is

$$P(T \leq t) = \frac{1}{3}P(T_{(1)} \leq t) + \frac{2}{3}P(T_{(2)} \leq t).$$

However $\frac{P(T=T_{(1)})}{P(T \geq T_{(1)})} = \frac{1}{3}$, $\frac{P(T=T_{(2)})}{P(T \geq T_{(2)})} = 1$ and $\frac{P(T=T_{(3)})}{P(T \geq T_{(3)})} = 0$ and therefore, the dynamical signature are

$\beta_1 = \frac{1}{3}$, $\beta_2 = \frac{2}{3}$, $\beta_3 = 0$ and the dynamic signature system distribution lifetime decomposition is

$$P(T \leq t|\Im_t) = \frac{1}{3}1_{\{T_{(1)} \leq t\}} + \frac{2}{3}1_{\{T_{(2)} \leq t\}}.$$

Taking expected values we get

$$P(T \leq t) = \frac{1}{3}P(T_{(1)} \leq t) + \frac{2}{3}P(T_{(2)} \leq t).$$

and note that $\alpha_i = \beta_i, \quad i = 1, 2$ in the set $\{T_{(2)} < t\}$ recovering the Samaniego [10]signature decomposition.

ii)The Bridge system lifetime can be set as $T = (T_1 \vee T_2) \wedge (T_1 \vee T_3 \vee T_5) \wedge (T_2 \vee T_3 \vee T_4) \wedge (T_4 \vee T_5)$. where T_1, T_2, T_3, T_4, T_5 are independent and identically distributed lifetimes.

The Samaniego [9] system signatures are:

$\alpha_1 = P(T = T_{(1)}) = 0$, $\alpha_2 = P(T = T_{(2)}) = \frac{1}{5}$, $\alpha_3 = P(T = T_{(3)}) = \frac{3}{5}$, $\alpha_4 = P(T = T_{(4)}) = \frac{1}{5}$ and $\alpha_5 = P(T = T_{(5)}) = 0$ and the signature system distribution lifetime decomposition is

$$P(T \leq t) = \frac{1}{5}P(T_{(2)} \leq t) + \frac{3}{5}P(T_{(3)} \leq t) + \frac{1}{5}P(T_{(4)} \leq t).$$

As $\frac{P(T=T_{(1)})}{P(T \geq T_{(1)})} = 0$, $\frac{P(T=T_{(2)})}{P(T \geq T_{(2)})} = \frac{1}{5}$, $\frac{P(T=T_{(3)})}{P(T \geq T_{(3)})} = \frac{3}{4}$, $\frac{P(T=T_{(4)})}{P(T \geq T_{(4)})} = 1$ and $\frac{P(T=T_{(5)})}{P(T \geq T_{(5)})} = 1$ and follows that the

the dynamical signature are:

$\beta_1 = 0, \beta_2 = \frac{1}{5}, \beta_3 = \frac{11}{20}, \beta_4 = \frac{1}{4}, \beta_5 = 0$ and the dynamic signature system distribution lifetime decomposition is

$$P(T \leq t|\Im_t) = \frac{1}{5}1_{\{T_{(2)} \leq t\}} + \frac{11}{20}1_{\{T_{(3)} \leq t\}} + \frac{1}{4}1_{\{T_{(4)} \leq t\}}.$$

Now, in the set $\{T_{(2)} \leq t < T_{(3)}\} \cap \{T > t\}$ the system signature actualizes to $\beta_3 = \frac{3}{4}$ and $\beta_4 = 1 - \frac{3}{4} = \frac{1}{4}$.

the decomposition system signature actualizes to

$$P(T \leq t + x|\{T_{(2)} \leq t < T_{(3)}\} \cap \{T > t\}) =$$

$\frac{3}{4}P(T_{(3)} \le t+x|\{T_{(2)} \le t < T_{(3)}\}\cap\{T > t\})+\frac{1}{4}P(T_{(4)} \le t+x|\{T_{(2)} \le t < T_{(3)}\}\cap\{T > t\})$.

Also

$$P(T \le t) = \frac{1}{5}P(T_{(2)} \le t) + \frac{4}{5}[\frac{3}{4}P(T_{(3)} \le t) + \frac{1}{4}P(T_{(4)} \le t)]$$

in the set $\{T_{(4)} \le t\}$, recovering the Samaniego [10] signature decomposition.

iii) If T_1, T_2, T_3, T_4 are independent and identically distributed component's lifetimes of the system with lifetime $T = T_1 \vee (T_2 \wedge T_3 \wedge T_4)$, then $\alpha_1 = P(T = T_{(1)}) = 0$, $\alpha_2 = P(T = T_{(2)}) = \frac{1}{2}$ and $\alpha_3 = P(T = T_{(3)}) = \frac{1}{4}$ and $\alpha_4 = P(T = T_{(4)}) = \frac{1}{4}$ and the signature system distribution lifetime decomposition is

$$P(T \le t) = \frac{1}{2}P(T_{(2)} \le t) + \frac{1}{4}P(T_{(3)} \le t) + \frac{1}{4}P(T_{(4)} \le t).$$

However $\frac{P(T=T_{(1)})}{P(T\ge T_{(1)})} = 0$, $\frac{P(T=T_{(2)})}{P(T\ge T_{(2)})} = \frac{1}{2}$, $\frac{P(T=T_{(3)})}{P(T\ge T_{(3)})} = \frac{1}{2}$ and $\frac{P(T=T_{(4)})}{P(T\ge T_{(4)})} = 1$.
Follows that the dynamical signature are: $\beta_1 = 0$, $\beta_2 = \frac{1}{2}$, $\beta_3 = 0$, and $\beta_4 = \frac{1}{2}$ the ·namic signature system distribution lifetime decomposition is

$$P(T \le t|\Im_t) = \frac{1}{2}1_{\{T_{(2)}\le t\}} + \frac{1}{2}1_{\{T_{(4)}\le t\}}.$$

In the set $\{T_{(2)} \le t < T_{(3)}\} \cap \{T > t\}$ we have $\beta_3 = \frac{1}{2}$ and $\beta_4 = 1 - \frac{1}{2} = \frac{1}{2}$. the dynamical system signature actualizes to

$$P(T \le t+x|\{T_{(2)} \le t < T_{(3)}\} \cap \{T > t\})) = \frac{1}{2}P(T_{(2)} \le t+x) + \frac{1}{2}.$$

$$[\frac{1}{2}P(T_{(3)} \le t+x|\{T_{(2)} \le t < T_{(3)}\}\cap\{T > t\})+\frac{1}{2}P(T_{(4)} \le t+x|\{T_{(2)} \le t < T_{(3)}\}\cap\{T > t\})]$$

$$P(T \le t) = \frac{1}{2}P(T_{(2)} \le t) + \frac{1}{2}[\frac{1}{2}P(T_{(3)} \le t) + \frac{1}{2}P(T_{(4)} \le t)],$$

recovering the Samaniego signature decomposition as in [10].

Remarks 2.1.7: As the dynamic signatures actualizes itself on time and in the set $\{T_{(n)} < t\}$, $\alpha_i = \beta_i$, $\forall i$. we can rewrite either Theorem 1.3 [6] and Theorem 1.5 [11] in the cited papers in an unified Theorem:

Theorem 2.1.8 Consider two mixed systems based on n original components with i.i.d. lifetimes having the common continuous distribution F. The first system having lifetime S_1, signature vector α_1 and dynamic signature vector β_1. The second one having lifetime S_2, signature vector α_2 and dynamic signature vector. Then

a) if

$$\alpha_1 \leq_{st} \alpha_2 \Longrightarrow (S_1|\Im_t \leq_{st} (S_2|\Im_t);$$

b) if

$$\alpha_1 \leq_{hr} \alpha_2 \Longrightarrow (S_1|\Im_t) \leq_{hr} (S_2|\Im_t);$$

c) if F is absolutely continuous and if .

$$\beta_1 \leq_{lr} \beta_2 \Longrightarrow (S_1|\Im_t) \leq_{lr} (S_2|\Im_t);$$

Proof Firstly we always have $\Sigma_{j=i}^{n}\beta_j = 1, \forall i, 1 \leq i \leq n$ in the way that both the vectors β_1 and β_2 are not significant for stochastically comparing systems lifetimes with respect to stochastic ordering and hazard rate ordering. However, as the stochastic comparisons must holds for all time $t \geq 0$, the constants β_i actualizes itself to α_i in the set $\{T_{(i-1)} \leq t < T_{(i)}\} \cap \{T > t\}$ and $\beta_i = \alpha_i \;\; \forall i$ in $t > T_{(n)}$, either the vectors α_1 and α_2 are relevant for those comparisons and are the sufficient conditions.

As the atoms in $\Im_t$ is of the kind $\{T_{(i)} \leq t < T_{(i+1)}\}$, $i = 1, 2, ..., n$ with $T_{(0)} = 0$ and $T_{(n+1)} = \infty$, the proof of parts a) and b) follows from Theorems 1.3 of [6] and Theorem 1.5. from [11].

To prove part c) we have to consider the likelihood ratio ordering between the vectors β_1 and β_2 and the prove follows as in Theorem 1.5. [11].

Remarks 2.1.9: Samaniego et al. [11] analyses the NBU dynamic version of aging in the signature context and defines the class of New Better Than Used Distribution conditioned to the event $\{T > t\} \cap \{T_{(i)} \leq t < T_{(i+1)}\}$.

Definition 2.1.10 a) Consider a mixed system based on n components with i.i.d lifetimes $T_1, ..., T_n$ with absolutely continuous distribution F. Let T be the systems lifetime and let $E_i = \{T_{(i)} \leq t < T_{(i+1)}\}$. For fixed $i \in \{0, 1, ..., n-1\}$, T is conditionally NBU, given i failed components (denoted by i-NBU) if for all $t > 0$ either

i) $P(\{T > t\} \cap E_i) = 0$ or

ii) $P(\{T > t\} \cap E_i) > 0$ and

$$P(T > x) \geq P(T > t + x|\{T > t\} \cap E_i), \quad \forall x > 0.$$

b) An n-component system is said to be $UNBU$ if it is i-NBU for $i \in \{0, 1, ..., n-1\}$.

Samaniego et al. [11] observes that if the system is uniformly NBU then it is NBU but the reverse does not hold, and then give sufficient conditions for a system based on n components with i.i.d. lifetimes to be $UNBU$:

Theorem 2.1.11 Let $s_n(n)$ be the signature, and T the lifetime, of a mixed system based on n components whose lifetimes are i.i.d. with common continuous distribution F. Assume that F is NBU and that

$$\mathbf{s}_n(n) \geq_{st} \mathbf{s}_n(n-i), \quad i = 1, 2, ..., n-1.$$

Then the system is $UNBU$.

We report to the work by Arjas [1] of conditioned $MNBU|\Im_t$. To that we remember the concept of Upper Set: A Borel set $U \subset R^n$ is called *upper* if for any $\mathbf{x}, \mathbf{y} \in R^n$, $\mathbf{x} \in U$ and $\mathbf{x} \leq \mathbf{y}$ together imply that $\mathbf{y} \in U$. ($\mathbf{x} \leq \mathbf{y}$ means that $x_i \leq y_i, 1 \leq i \leq n$).

The Arjas [1] definition is:

Definition 2.1.12 a) We say that $\mathbf{T} = (T_1, ..., T_n)$ is multivariate new better than used relative to $\Im_t$, and abbreviate this by $MNBU|\Im_t$, if

$$P(\theta_t \mathbf{T} \in U|\Im_t) \leq P(\theta_t \mathbf{T} \in U|\Im_0)$$

for all $t \geq 0$ and all open upper set U. In the special case $n = 1$

b) T is $NBU|\Im_t$, if for all $t \geq 0$ and $s \in R$

$$P(\theta_t T > s|\Im_t) \leq P(\theta_t T > s|\Im_0)$$

As a lifetime T is NBU if,, and only if, it is $NBU|\sigma\{1_{\{T>s\}}, 0 \leq s \leq t\}$, this definition is an extension of the classical concept and it turns out that this distribution class have most of what could be called desirable properties of any extension of the conventional NBU-class. In particular if we consider

$$\Im_t = \sigma\{1_{\{T_{(i)}>s\}}, X_i = j, 1 \leq j \leq n, j \in E, 0 < s \leq t\},$$

satisfying the Dellacherie conditions of right continuity and completeness we have:

Theorem 2.1.13 i) Suppose that $\mathbf{T}$ is $MNBU|\Im_t$. If $\mathbf{T}_0 = (T_i, i \in I_0)$ is any subvector of $\mathbf{T}$ and

$$\Re_t = \sigma\{1_{\{T_{(i)}>s\}}, X_i = j, i \in I_0, j \in E, 0 < s \leq t\},$$

then $\mathbf{T}_0$ is $MNBU|\Re_t$. In particular each T_i is NBU.

ii) Suppose that $\mathbf{T}$ is $MNBU|\Im_t$. If $(\Re_t)_{t\geq 0}$ is another family of σ-algebras such that $\Re_t \subset \Im_t$ for all $t \geq 0$ and $\Re_0 = \Im_0$ then $\mathbf{T}$ is $MNBU|\Re_t$. In particular let T be the lifetime of a coherent system and $\Re_t = \sigma\{1_{\{T>s\}}, 0 \leq s \leq t\}$, then T is NBU.

iii) Suppose that $T_1, ..., T_n$ are independent and NBU. Then $\mathbf{T}$ is $MNBU|\Im_t$.

Follows that Theorem 2.1.11, in [11], also holds in the general context of dynamic signatures. Others concepts of distributions classes relative to $\Im_t$ can be analyzed, such as $MIFR|\Im_t$,(see Arjas [1]).

2.2. The mathematical details.

The natural tool to consider the increasing information on time through a family of sub-σalgebras is the martingale theory and, in our case, the point process martingale theory.

The simple marked point $N_{(i),j}(t) = 1_{\{T_{(i)} \le t, X_i = j\}}$ is an $\Im_t$-submartingale and from the Doob-Meyer decomposition we know that there exists a unique $\Im_t$-predictable process $(A_{(i),j}(t))_{t\ge 0}$, called the $\Im_t$-compensator of $N_{(i),j}(t)$, with $A_{(i),j}(0) = 0$ and such that $N_{(i),j}(t) - A_{(i),j}(t)$ is an $\Im_t$-martingale. $A_{(i),j}(t)$ is absolutely continuous by the totally inaccessibility of $T_i, 1 \le i \le n$.

The compensator process is expressed in terms of the conditional probability, given the available information and generalize the classical notion of hazards. Intuitively, this corresponds to producing wether the failure is going to occur now, on the basis of all observations available up to, but not including, the present.

As $N_{(i),j}(t)$ can only count on the time interval $(T_{(i-1)}, T_{(i)}]$, the corresponding compensator differential $dA_{(i),j}(t)$ must vanish outside this interval. To count the $i-th$ failure we let $N_{(i)}(t) = \Sigma_{j\ge 1} N_{(i),j}(t)$ with $\Im_t$-compensator process $A_{(i)}(t) = \Sigma_{j\ge 1} A_{(i),j}(t)$.

Follows that the $\Im_t$-compensator of $N(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} N_{(i),j}(t)$ is

$$A(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{(i),j}(t) 1_{\{T_{(i-1)} < t \le T_{(i)}\}},$$

which can be written, without loss of generality (see Bremaud [4]) as

$$A(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{(i),j}(t) 1_{\{T_{(i)} \le t < T_{(i+1)}\}}.$$

In this former notation $(A(t))_{t\ge 0}$ is an $\Im_t$-predictable process and therefore unique.

The $\Im_t$-stopping times T_i are rarely of directly concerning in reliability theory. One is more interested in the system lifetime

$$T = \min_{1\le j\le k} \max_{i\in K_j} T_i,$$

where $K_j, 1 \le j \le k$ are minimal cut sets, that is, a minimal set of components whose joint failure causes the system fail.

Conveniently, we can define the critical level $Y_{(i),j}$, as the first time from which onwards the failure of component j lead to system failure at $\{T = T_{(i)}, X_i = j\}$. We consider the $\Im_t$-compensator process $(A_\Phi(t))_{t\ge 0}$ of the point process $N_\Phi(t) = 1_{\{T\le t\}}$, of the system lifetime T, such that $N_\Phi(t) - A_\Phi(t)$ is an zero mean $\Im_t$-martingale with $P(T \le t) = E[N_\Phi(t)] = E[A_\Phi(t)]$.

Theorem 2.2.1 Under the above notation, in the set $\{T > t\}$, the $\Im_t$-compensator of $N_\Phi(t) = 1_{\{T \leq t\}}$, is

$$A_\Phi(t) = \Sigma_{i=1}^n \Sigma_{j=1}^n [A_{(i),j}(t) - A_{(i),j}(Y_{(i),j})]^+ 1_{\{T_{(i)} \leq t < T_{(i+1)}\}},$$

where $a^+ = \max\{a, 0\}$.
Proof

As $A_{(i),j}(s)$ is the $\Im_s$-compensator of $N_{(i),j}(s)$ we have that

$$E[\int_0^\infty C_{(i),j}(s) dN_{(i),j}(s)] = E[\int_0^\infty C_{(i),j}(s) dA_{(i),j}(s)]$$

holds true for all non-negative $\Im_t$-predictable process $(C_{(i),j}(t))_{t \geq 0}$. Follows that, for any $\Im_t$-predictable process $(C(t))_{t \geq 0}$, we can define the $\Im_t$-predictable process $C_{(i),j}(t) = C(t) 1_{\{Y_{(i),j} < t \leq t \wedge T\}}$ and therefore

$$E[\int_0^\infty C(s) 1_{\{Y_{(i),j} < s \leq t \wedge T\}} dN_{(i),j}(s)] =$$

$$E[\int_0^\infty C(s) 1_{\{Y_{(i),j} < s \leq t \wedge T\}} dA_{(i),j}(s)].$$

Also we note that

$$\{T \in ds\} = \cup_{T_{(i)} > Y_{(i),j}} \{T_{(i)} \in ds, X_i = j\}$$

in the way that

$$N_\Phi(t) = 1_{\{T \leq t\}} = \int_0^\infty \sum_{i=1}^n \sum_{j=1}^n 1_{\{Y_{(i),j} < s \leq t \wedge T\}} dN_{(i),j}(s).$$

Follows that $E[\int_0^\infty C(s) dN_\Phi(s)] =$

$$E[\int_0^\infty C(s) \sum_{i=1}^n \sum_{j=1}^n 1_{\{Y_{(i),j} < s \leq T\}} dA_{(i),j}(s) 1_{\{T_{(i)} \leq t < T_{(i+1)}\}}] = E[\int_0^\infty C(s) dA_\Phi(s)].$$

At this point we can prove our main result.

The approach is as the following: in addition to $(\Im_t)_{t \geq 0}$, at each time t, the observer knows if the events $\{T \leq t\}$ have either occurred or not and if they have, he knows exactly the value of T.

The mathematical formulation of such an additional observation is given by a family of sub σ-algebras of $\Im$, denoted by $(\Re_t)_{t \geq 0}$, where

$$\Re_t = \sigma\{1_{\{T > s\}}, 0 < s \leq t\},$$

satisfies the Dellacherie conditions of right continuity and completeness .

Note that, the series parallel representation of T is

$$T = \Phi(\mathbf{T}) = \min_{1 \leq j \leq k} \max_{i \in K_j} T_i,$$

where $K_j, 1 \leq j \leq k$ are the minimal cut sets. Follows that

$$\{T > s\} = \bigcap_{1 \leq j \leq k} \bigcup_{i \in K_j} \{T_i > s\}$$

and therefore $\Re_t \subset \Im_t$ for all $t \geq 0$. Actually and abusively, we continue to use $\Im_t$ for $\Im_t \bigvee \Re_t$ representing the complete information level.

Theorem 2.2.2 Let T be the lifetime of a coherent system of order n, with component lifetimes $T_1, ..., T_n$ which are totally inaccessible $\Im_t$-stopping time . Then, under the above notation and at complete information level, we have

$$P(T \leq t | \Im_t) = \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} \frac{P(T = T_{(i)} | \Im_t)}{P(T \geq T_{(i)}} 1_{\{T_{(i)} \leq t < T_{(i+1)}\}}$$

with $T_{(n+1)} = \infty$.

Proof From the Projection Theorem,(Bremaud [4]), follows that the $\Re_t$-compensator is given by $E[A_\Phi(t)|\Re_t]$. As $A_\Phi(t)$ is calculated in the set $\{T > t\}$ we have $E[dA_{(i),j}(s)|\Re_s] = \frac{dA_{(i),j}(s)}{P(T \geq s)}$.

Now, we have

$$P(T \leq t | \Im_t) = \Sigma_{i=1}^{n} \Sigma_{j=1}^{n} E[\int_0^\infty \frac{1_{\{Y_{(i),j} < s \leq T \wedge t\}}}{P(T \geq s)} dA_{(i),j}(s)] 1_{\{T_{(i)} \leq t < T_{(i+1)}\}} =$$

$$\Sigma_{i=1}^{n} \Sigma_{j=1}^{n} E[\int_0^\infty \frac{1_{\{Y_{(i),j} < s \leq T \wedge t\}}}{P(T \geq s)} dN_{(i),j}(s)] 1_{\{T_{(i)} \leq t < T_{(i+1)}\}} =$$

$$\Sigma_{i=1}^{n} \Sigma_{j=1}^{n} \frac{P(T = T_{(i)} | \Im_t)}{P(T \geq T_{(i)}} 1_{\{T_{(i)} \leq t < T_{(i+1)}\}}.$$

Remarks 2.2.3 i) The above results proves the Remark 2.1.1 of Subsection 2.1, in the case of independent and identically distributed lifetimes in which

$$P(T \leq t | \Im_t) = \Sigma_{i=1}^{n} \frac{P(T = T_{(i)})}{P(T \geq T_{(i)})} 1_{\{T_{(i)} \leq t < T_{(i+1)}\}}.$$

ii) Navarro, et al. [7] consider the mixture representation of residual lifetimes of used systems. In its conclusion asked about the general case of dependent components which remain an interesting open question. Clearly, it is not seemingly true to think the general case of dependent components in the signatures context. However, as Navarro et all. (1988) asked, it is plausible to analyse the case of dependent and identically distributed lifetimes (any way, its holds true for exchangeable distribution). In this case we have

$$P(T \leq t|\Im_t) = \Sigma_{i=1}^{n} \frac{P(T = T_{(i)}|\Im_t)}{P(T \geq T_{(i)})} 1_{\{T_{(i)} \leq t < T_{(i+1)}\}}.$$

and the terms $\frac{P(T=T_{(i)}|\Im_t)}{P(T \geq T_{(i)})}$ depends on the distributions lifetimes, as we see in the sequel:

We consider the system lifetime $T = T_1 \wedge (T_2 \vee T_3)$ with three components lifetimes T_1, T_2, T_3 with the three dimensional standard exponential distribution of Marshall and Olkin is used with the the survival probability $P(T_1 > t_1, T_2 > t_2, T_3 > t_3)$ given by

$$\exp\{-[t_1 + t_2 + t_3 + \max\{t_1, t_2\} + \max\{t_1, t_3\} + \max\{t_2, t_3\} + \max\{t_1, t_2, t_3\}]\}.$$

An interpretation of this distribution is as follows: Suppose independent sources of shocks are present in the environment. A shock from source i destroys component $i, 1 \leq i \leq 3$ and it occurs at a random time $U_i, 1 \leq i \leq 3$ respectively. A shock from source i, j destroys either the components i and j, simultaneously at a random time $U_{ij}, 1 \leq i \leq 3, 2 \leq j \leq 3, i < j$. Finally, a shock from source $1, 2, 3$ destroys the three components simultaneously at the random time U_{123}. The lifetimes $U_1, U_2, U_3, U_{12}, U_{13}, U_{23}$ and U_{123} are independent and identically distributed with standard exponential distribution.

The components lifetimes are define

$$T_1 = \min\{U_1, U_{12}, U_{13}, U_{123}\};$$

$$T_1 = \min\{U_2, U_{12}, U_{23}, U_{123}\};$$

$$T_1 = \min\{U_3, U_{13}, U_{23}, U_{123}\},$$

and

$$P(T_1 > t) = P(T_2 > t) = P(T_3 > t) = \exp\{-4t\}.$$

Then the lifetimes are dependent but identically distributed. However

$$P(T = T_{(1)}|T_{(1)} > t) = \frac{P(t < T_1 \leq \min\{T_2, T_3\})}{P(T_{(1)} > t)} =$$

$$\frac{P(t < U_1 \leq \min\{U_2, U_3, U_{12}, U_{13}, U_{23}\})}{\exp[-7t]} = \frac{\exp[-7t]}{7\exp[-7t]} = \frac{1}{7},$$

and $P(T = T_{(1)}|T_{(1)} > t)$ is independent of $F(t)$. As the Marshall and Olkin is an exchangeable distribution we have an expected result.

Otherwise, if we consider the following Type I Gumbel exponential lifetime multivariate distribution with survival function

$$P(T_1 > t_1, T_2 > t_2, T_3 > t_3) = \exp\{-[t_1 + t_2 + t_3 + t_2 t_3 + t_1 t_2 t_3]\}.$$

As $P(T_2 > t_2, T_3 > t_3) = \exp\{-[t_2 + t_3 + t_2t_3]\}$ the components are dependents. Also, $P(T_1 > t) = P(T_2 > t) = P(T_2 > t) = \exp\{-[t]\}$ and the components are identically distributed. . Furthermore the component lifetime T_1 is independent of T_2 and T_3. Follows that

$$P(T = T_{(1)}|T_{(1)} > t) = \frac{P(t < T_1 \leq \min\{T_2, T_3\})}{P(T_{(1)} > t)} =$$

$$\frac{1}{\exp -[3t + t^2 + t^3]} \int_t^\infty P(\min\{T_2, T_3\} > s) \exp\{-[s]\} ds =$$

$$\frac{1}{\exp -[3t + t^2 + t^3]} \int_t^\infty \exp\{-[2\frac{3}{2}s + s^2]\} ds =$$

$$\frac{\exp[\frac{9}{4}]\sqrt{\Pi}}{\exp -[3t + t^2 + t^3]} \sqrt{2} \frac{1}{\sqrt{2\Pi}} \int_t^\infty \exp\{-\frac{1}{2} \frac{[s - (-\frac{3}{2})]^2}{\frac{1}{2}}\} ds,$$

which depends in the particular time t and of the particular distribution F.

Therefore in this case we see that the quantities $P(T = T_{(1)}|T_{(1)} > t)$ depends in the particular component lifetime distribution. Therefore, in working with system signatures in the dependent and identically distributed components lifetimes (other than the exchangeable distribution) we does not have the nice properties that we have in the context of independent and identically distributed (i.i.d.) absolutely continuous component lifetimes, in which case they are distribution free measures of system quality, depending solely on the design characteristics of the system and independent of the behavior of the systems components .

References

[1] Arjas, E. (1981). A stochastic process approach to multivariate reliability system. Notions based on conditional stochastic order. Mathematical of Operations Research 6, 263-276.

[2] Aven, T. and Jensen, U. (1999). *Stochastic Models in Reliability.* Springer Verlag, New York.

[3] Barlow and Proschan,F. (1981). *Statistical Theory of Reliability and Life Testing: Probability models.* Hold, Reinhart and Wiston , Inc. Silver Spring, MD.

[4] Bremaud ,P. (1981). *Point Processes and Queues: Martingale Dynamics.*Springer-Verlag, New York.

[5] P.J. Boland and Samaniego, F.(2004). *The signatue of a coherent system and its applications in reliability.* Mathematical reliability: An expository perspectiva. R. Soyer, T. Mazzuchi, and N.D. Singpurwalla (Editors), Kluwer Publishers, Boston. 1 - 29.

[6] Kochar, S., Mukherjee, H., Samaniego, F.(1999). The signature of a coherent system and its application to comparisons among systems. Naval Research Logistic. 46, 507 - 523.

[7] Navarro, J., Balakrishnan, N. and Samaniego,F.J.(2008). Mixture representation of residual lifetimes of used systems. Journal of Applied Probability. 45, 1097 - 1112.

[8] Samaniego, F. (1985). On closure of the IFR class under formation of coherent systems. IEEE Transactions in Reliability. R-34, 69-72.

[9] Samaniego,F.J. (2006). *On comparison of engineered systems of different sizes,* in Proceedings of the 12th Annual Army Conference on Applied Statistics. Aberdeen Proving Ground, Army Research Laboratory.

[10] Samaniego,F.J. (2007). *System signatures and their applications in engineering reliability.* International Series in Operation Research and Management Science, Vol 110, Springer, New York.

[11] Samaniego,F.J., Balakrishnan, N. and Navarro, J., (2009). Dynamic signatures and their use in comparing the reliability of a new and used systems. Naval Research Logistic. 56, 577 - 596.

[12] Shaked, M., Suarez-Llorens, A. (2003). On the comparison of reliability experiments based on the convolution order. Journal of American Statistical Association. 98, 693 - 702.

[13] Shaked, M., and Shanthikumar, J.G.(2007). *Stochastic Orders.*Springer, New York.

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