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Local Influence in Measurement Error Regression Models with Null Intercept

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SUMMARY. In this paper we discuss the application of local influence in measurement error regression models with null intercepts and dependent populations. We derive the appropriate matrices for assessing the local influence for different perturbation schemes. A real data is used as an illustration of the usefulness of the application.

KEY WORDS: Influence diagnostic; Likelihood displacement; Lack of fit; Measurement error models; pretest/posttest data.

1 Introduction

The main object of this paper is to discuss the application of the local influence method (Cook, 1986) in the measurement error regression models with null intercept and dependent populations. A univariate version of the model was originally considered in Chan and Mak (1979). Wong (1995) considered some structures of dependency between the covariates of two samples. We are going to apply the model described in Section 2 to the data from a pretest/posttest study presented in Singer and Andrade (1997). In that study, designed to compare two types of toothbrushes with respect to the efficacy in removing dental plaque, 26 preschoolers were evaluated with respect to a dental plaque index before and after toothbrushing either with a conventional or with an experimental (hugger) toothbrush. The reason for considering null intercepts is that null pretest dental plaque indices imply null expected posttest values. As the same individuals were evaluated under two different experimental conditions (toothbrushes), we need a model which takes into account the possible within subjects correlation structure. We are going to discuss the model in detail in the next section. Influence diagnostic is an important step in the analysis of a data set, as it provides us indication of bad model fitting or of influential observations. Usually the analysis is based on case-weight perturbation scheme where the weights are either 0 or 1 so that the case is either deleted or retained and then the individual impact of cases are assessed in the estimation process, see Cook (1986) for a wide reference. Cook (1986) proposed a very important method for assessing the local influence of minor perturbations of a statistical model. Since then many works has been written with respect to the local influence, but little work has been found in the literature for the measurement error regression models. Lee and Zhao (1996) employed local influence approach in generalized linear measurement error models and Abdullah (1995) compared several methods for detecting influential observations in a functional measurement error models. Recently, Kim (2000) applied the local influence method in the structural measurement error models. We are going to illustrate the application using a real data from pretest/posttest study described earlier in this section. The appropriate matrices necessary to construct the influence graphs are given in closed form expressions. Section 2 presents the model. Section 3 reviews the concept of the local influence and the application to the model defined in Section 2. Finally, in Section 4 we present the illustrative application.

2 Measurement regression error model with null intercept

In this section we are going to describe the model. The basic model is given by

$$Y_{ij} = \beta_i x_{ij} + e_{ij}, \tag{1}$$

$$X_{ij} = x_{ij} + u_{ij}, \tag{2}$$

where Y_{ij} and X_{ij} respectively denote the observed values of the response and explanatory variables for population i and subject j, $(i = 1,\ldots,p,\ j = 1,\ldots,n)$, x_{ij}, correspond to the true values of the latter, β_i, $i = 1,\ldots,p$ stand for the (unknown) slopes and

$$\begin{pmatrix} e_{ij} \\ u_{ij} \\ x_{ij} \end{pmatrix} \sim N_3 \left[\begin{pmatrix} 0 \\ 0 \\ \mu \end{pmatrix}, \begin{pmatrix} \lambda_i \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma_x^2 \end{pmatrix} \right], \tag{3}$$

are independently distributed, $i = 1,\ldots,p$, $j = 1,\ldots,n$. An extra term was included in the model to allow for a possible within subjects correlation structure, leading to

$$x_{ij} = \mu + a_j \tag{4}$$

$i = 1,\ldots,p$, $j = 1,\ldots,n$ with the a_j independently distributed as $N(0,\sigma_x^2)$ and independent of the u_{ij}. Considering the case of two dependent populations and under the model specified by equations (1), (2), (3) and (4), the vector of observations $(X_{1j}, Y_{1j}, X_{2j}, Y_{2j})^T$ is distributed according to a four-dimensional normal distribution with mean vector $(\mu, \beta_1\mu, \mu, \beta_2\mu)^T$ and covariance matrix

$$
\mathbf{V} = \begin{bmatrix} \sigma_x^2+\sigma^2 & \beta_1\sigma_x^2 & \sigma_x^2 & \beta_2\sigma_x^2 \\ \beta_1\sigma_x^2 & \beta_1^2\sigma_x^2+\lambda_1\sigma^2 & \beta_1\sigma_x^2 & \beta_1\beta_2\sigma_x^2 \\ \sigma_x^2 & \beta_1\sigma_x^2 & \sigma_x^2+\sigma^2 & \beta_2\sigma_x^2 \\ \beta_2\sigma_x^2 & \beta_1\beta_2\sigma_x^2 & \beta_2\sigma_x^2 & \beta_2^2\sigma_x^2+\lambda_2\sigma^2 \end{bmatrix}. \tag{5}
$$

By using general properties of the multivariate normal distribution it follows that the log-likelihood function for $\theta = (\beta_1, \beta_2, \mu, \sigma_x^2, \sigma^2, \lambda_1, \lambda_2)^T$ can be written as

$$
\begin{aligned}
L(\theta) = const - \frac{n}{2}\log[\sigma^6\Delta] + \frac{1}{\sigma^2\Delta}\Bigg\{ \sigma_x^2 \Bigg[\lambda_1\lambda_2 \sum_{j=1}^{n} X_{1j}X_{2j} + \beta_1\beta_2 \sum_{j=1}^{n} Y_{1j}Y_{2j} + \\
\beta_1\lambda_2 \left(\sum_{j=1}^{n} X_{1j}Y_{1j} + \sum_{j=1}^{n} X_{2j}Y_{1j} \right) + \beta_2\lambda_1 \left(\sum_{j=1}^{n} X_{1j}Y_{2j} + \sum_{j=1}^{n} X_{2j}Y_{2j} \right) \Bigg] + \\
\sigma^2\mu \left[\lambda_1\lambda_2 \left(\sum_{j=1}^{n} X_{1j} + \sum_{j=1}^{n} X_{2j} \right) + \beta_1\lambda_2 \sum_{j=1}^{n} Y_{1j} + \beta_2\lambda_1 \sum_{j=1}^{n} Y_{2j} \right] - \\
\frac{1}{2}\Bigg\{ n\sigma^2\mu^2\Gamma + \left(\Delta - \lambda_1\lambda_2\sigma_x^2\right) \left(\sum_{j=1}^{n} X_{1j}^2 + \sum_{j=1}^{n} X_{2j}^2 \right) - \\
\left[\sigma^2\lambda_2 + \left(\beta_2^2 + 2\lambda_2\right)\sigma_x^2\right] \sum_{j=1}^{n} Y_{1j}^2 + \left[\sigma^2\lambda_1 + \left(\beta_1^2 + 2\lambda_1\right)\sigma_x^2\right] \sum_{j=1}^{n} Y_{2j}^2 \Bigg\}\Bigg\},
\end{aligned} \tag{6}
$$

where $\Gamma = \beta_1^2\lambda_2 + \beta_2^2\lambda_1 + 2\lambda_1\lambda_2$ and $\Delta = \Gamma\ \sigma_x^2 + \sigma^2\lambda_1\lambda_2$. Maximum Likelihood estimates for the vector of parameters θ has to be obtained by iterative procedures. One such procedure is the EM algorithm, which are described in the Appendix A.

3 Local influence diagnostics

Let $L(\theta)$ denote the log-likelihood function given in (6), where $\theta = (\beta_1,\ \beta_2,\ \mu,\ \sigma_x^2,\ \sigma^2,\ \lambda_1,\ \lambda_2)^T$. The perturbation is introduced in the model by the vector ω, $q \times 1$, where $\omega \in \Omega \subseteq \mathbf{R}^q$, Ω an open subset. Denoting $L(\theta/\omega)$ the log-likelihood of the perturbed model, we are going to assume that there is a vector $\omega_0 \in \Omega$ such that $L(\theta) = L(\theta/\omega_0)$, $\forall\ \theta$. To assess the influence of the perturbation on the maximum likelihood estimates of θ, we may consider the likelihood displacement

$$LD(\omega) = 2\left[L\left(\hat{\theta}\right) - L\left(\hat{\theta}_\omega\right)\right], \tag{7}$$

where $\hat{\theta}$ and $\hat{\theta}_w$ denotes the maximum likelihood estimator under the postulated model and under the perturbed model $L(\theta/\omega)$, respectiverly. The idea of local influence (Cook, 1986) is concerned in characterizing the behavior of $LD(\omega)$ at ω_0, which can be summarized by the normal curvatures at $LD(\omega_0)$ as $LD(\omega)$ is a nonnegative function with a global minimum at ω_0. Cook (1986) showed that the normal curvature at $LD(\omega_0)$ in the unit direction $\mathbf{d}$, $C_{\mathbf{d}}$, can be expressed as

$$C_{\mathbf{d}} = 2 \mid \mathbf{d}^T \ddot{F} \mathbf{d} \mid, \tag{8}$$

where, $\ddot{F}$=$\Delta^T \left(\ddot{L}\right)^{-1} \Delta$, $\Delta_{ij} = \partial^2 L(\theta/\omega) / \partial\theta_i \partial\omega_j$ evaluated at $\theta = \hat{\theta}$ and $\omega = \omega_0$, $i = 1, \ldots, p$, $j = 1, \ldots, q$ and $-\ddot{L}$ is the observed information matrix for the postulated model. "Large" values of $C_{\mathbf{d}}$ indicates sensitivity to the perturbation introduced in the direction $\mathbf{d}$. Notice that the direction $\mathbf{d}_{max}$ corresponding to the largest curvature $C_{\mathbf{d}_{max}}$ is the eigenvector of the largest eigenvalue ($C_{\mathbf{d}_{max}}$) of the matrix $\ddot{F}$. The index plot of $\mathbf{d}_{max}$ may reveal those observations that under small perturbations exert notable influence on $LD(\omega)$.

There are situations in which our interest is on a subset θ_1 of $\theta = (\theta_1^T, \theta_2^T)^T$. In our case the interest is in $\theta_1 = (\beta_1, \beta_2)^T$. In this case, the likelihood displacement can be defined as

$$LD_s(\omega) = 2\left[L\left(\hat{\theta}\right) - L\left(\hat{\theta}_{1\omega},\ g\left(\hat{\theta}_{1\omega}\right)\right)\right],$$

where g is the function which maximazes $L(\theta_1, \theta_2)$ for each fixed θ_1 and $\hat{\theta}_{1\omega}$ is determined by the partition $\hat{\theta}_\omega^T = \left(\hat{\theta}_{1\omega}^T, \hat{\theta}_{2\omega}^T\right)^T$. The normal curvature on the direction **d** is given by (Cook, 1986)

$$C_{\mathbf{d}}(\theta_1) = 2 \mid \mathbf{d}^T \Delta^T \left(\ddot{L}^{-1} - B_{22}\right) \Delta \mathbf{d} \mid,$$

where, $\| \mathbf{d} \| = 1$, $B_{22} = \begin{pmatrix} 0 & 0 \\ 0 & L_{22}^{-1} \end{pmatrix}$ and L_{22} is determined by the partition $\ddot{L} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}$. After some algebraic manipulations we obtained the observed information matrix $\ddot{L}$ under the model defined by (1), (2), (3) and (4), which is given in Appendix B. We are going to define some perturbations under the model defined by (1), (2), (3) and (4) and obtain the matrix Δ. The maximum likelihood estimator of the model parameters are going to be denoted by $\hat{\theta} = \left(\hat{\beta}_1, \hat{\beta}_2, \hat{\mu}, \hat{\sigma_x^2}, \hat{\sigma^2}, \hat{\lambda}_1, \hat{\lambda}_2\right)$.

3.1 Perturbation of case weights

Let us denote by $\mathbf{Z}_j$, the vector of observed values, i.e., $\mathbf{Z}_j = (X_{1j}, Y_{1j}, X_{2j}, Y_{2j})^T$, $j = 1, \ldots, n$, then $\mathbf{Z}_j \sim N_4(\mathbf{m}, \Sigma)$, with $\mathbf{m} = (\mu, \beta_1\mu, \mu, \beta_2\mu)^T$ and Σ as given in (5). Taking the log-likelihood function of the perturbed model as

$$L\left((X_{1j}, Y_{1j}, X_{2j}, Y_{2j}), \theta/\omega\right) = \omega_j L\left((X_{1j}, Y_{1j}, X_{2j}, Y_{2j}), \theta\right), \text{or}$$

$$L(\theta/\omega) = -2Log(2\pi \mid \Sigma \mid^{\frac{1}{2}}) \sum_{j=1}^{n} \omega_j - \frac{1}{2} \sum_{j=1}^{n} \omega_j (\mathbf{Z}_j - \mathbf{m})^T \Sigma^{-1} (\mathbf{Z}_j - \mathbf{m}),$$

we may obtain after algebraic manipulations the matrix Δ, given by

$$\Delta = \frac{-1}{\Delta_1}\left[\mathbf{1}_n^T A_1,\ \mathbf{1}_n^T A_2,\ \mathbf{1}_n^T A_3,\ 1/2\ \mathbf{1}_n^T A_4,\ 1/(2\hat{\sigma}^2)\mathbf{1}_n^T A_5,\ 1/2\ \mathbf{1}_n^T A_6,\ 1/2\ \mathbf{1}_n^T A_7\right],$$

where $A_i = diag\{a_{i1}, \ldots, a_{in}\}, i = 1, \ldots, 7$, with

$$a_{1j} = \hat{\beta}_1 \hat{\lambda}_2 \hat{\sigma}_x^2 - F_j R_{2j}/(\hat{\sigma}^2 \Delta_1), \quad a_{2j} = \hat{\beta}_2 \hat{\lambda}_1 \hat{\sigma}_x^2 - F_j R_{1j}/(\hat{\sigma}^2 \Delta_1),$$

$a_{3j} = G_j$, $a_{4j} = \Phi + \hat{\lambda}_1\hat{\lambda}_2 - G_j^2\Delta_1^{-1}$,

$a_{5j} = 3\Delta_1 + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 + M_j - \left(\Delta_1 + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\right)(\hat{\sigma}^2 M_j + \hat{\sigma}_x^2 N_j)/(\hat{\sigma}^2\Delta_1)$,

$a_{6j} = P_2 - R_{2j}^2/(\hat{\sigma}^2\Delta_1)$, $a_{7j} = P_1 - R_{1j}^2/(\hat{\sigma}^2\Delta_1)$,

$F_j, G_j, M_j, N_j, R_{1j}, R_{2j}, P_1, P_2, \Delta_1$ and Φ as given in the Appendix B, $j = 1, \ldots, n$. The vector ω_0 is given by $\omega_0 = \mathbf{1}_n$.

3.2 Perturbation of the response variables

Lets introduce the following perturbation in the response variables

$$Y_{1j} \rightarrow Y_{1j} + S_{Y1}\omega_{1j}$$

$$Y_{2j} \rightarrow Y_{2j} + S_{Y2}\omega_{2j}$$

i.e., we are going to introduce an additive perturbation ω_{ij} multiplied by a scaling factor S_{Yi} in the response variable Y_{ij}, $i = 1, 2$, $j = 1, \ldots, n$. In this case, $\omega_0 = (0, \ldots, 0)^T$ and we can use, for example, the inverse of the sample deviance of the $\mathbf{Y}_i$, as the scaling factor. The perturbed log likelihood function is given by

$$L(\theta/\omega) = -2n log(2\pi) - \frac{n}{2} log \mid \Sigma \mid - \frac{1}{2}\sum_{j=1}^{n}\left(\mathbf{Z}_j^* - \mathbf{m}\right)^T \Sigma^{-1}\left(\mathbf{Z}_j^* - \mathbf{m}\right),$$

with $\mathbf{Z}_j^* = (X_{1j},\ Y_{1j} + S_{Y1}w_{1j},\ X_{2j},\ Y_{2j} + S_{Y2}w_{2j})^T$, $\mathbf{m} = (\mu, \beta_1\mu, \mu, \beta_2\mu)^T$ and Σ as given in (5). In this case, the matrix Δ can be expressed as

$$\Delta = \frac{1}{\Delta_1}\left[1_{2n}^T B_1,\ 1_{2n}^T B_2,\ 1_{2n}^T B_3,\ 1_{2n}^T B_4,\ 1_{2n}^T B_5,\ 1_{2n}^T B_6,\ 1_{2n}^T B_7\right]^T,$$

where $B_i = diag\left\{b_{i11}, \ldots, b_{i1n}, b_{i21}, \ldots, b_{i2n}\right\}, i = 1, \ldots, 7$, with

$b_{11j} = S_{Y1}\hat{\sigma}^{-2}\left[Q_{2j} + 2\hat{\beta}_1\hat{\lambda}_2\hat{\sigma}_x^2\Delta_1^{-1}\left(P_2 Y_{1j} - \hat{\beta}_1 Q_{2j}\right)\right]$,

$b_{12j} = -\hat{\beta}_2\hat{\sigma}_x^2 S_{Y2}/(\hat{\sigma}^2\Delta_1)\left[2\hat{\beta}_1\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2 Y_{2j} + \left(\hat{\beta}_1^2\hat{\lambda}_2\hat{\sigma}_x^2 - \hat{\lambda}_1 P_2\right)Y_{1j}\right]$,

$b_{21j} = -\hat{\beta}_1\hat{\sigma}_x^2 S_{Y1}/(\hat{\sigma}^2\Delta_1)\left[2\hat{\beta}_1\hat{\beta}_2\hat{\lambda}_2\hat{\sigma}_x^2 Y_{1j} + \left(\hat{\beta}_2^2\hat{\lambda}_1\hat{\sigma}_x^2 - \hat{\lambda}_2 P_1\right)Y_{2j}\right]$,

$$b_{22j} = S_{Y2}\hat{\sigma}^{-2}\left[Q_{1j} + 2\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2\Delta_1^{-1}\left(P_1Y_{2j} - \hat{\beta}_2Q_{1j}\right)\right],$$

$$b_{31j} = \hat{\beta}_1\hat{\lambda}_2S_{Y1},\ b_{32j} = \hat{\beta}_2\hat{\lambda}_1S_{Y2},\ b_{41j} = -\hat{\beta}_1\hat{\lambda}_2S_{Y1}G_j\Delta_1^{-1},\ b_{42j} = -\hat{\beta}_2\hat{\lambda}_1S_{Y2}G_j\Delta_1^{-1},$$

$$b_{51j} = -S_{Y1}\hat{\sigma}^{-2}\left[\hat{\lambda}_2\left(Y_{1j} - \hat{\beta}_1\hat{\mu}\right) + \left(\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 + \Delta_1\right)\left(\hat{\beta}_1\hat{\beta}_2\hat{\sigma}_x^2Y_{2j} - P_2Y_{1j}\right)/(\hat{\sigma}^2\Delta_1)\right],$$

$$b_{52j} = -S_{Y2}\hat{\sigma}^{-2}\left[\hat{\lambda}_1\left(Y_{2j} - \hat{\beta}_2\hat{\mu}\right) + \left(\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 + \Delta_1\right)\left(\hat{\beta}_1\hat{\beta}_2\hat{\sigma}_x^2Y_{1j} - P_1Y_{2j}\right)/(\hat{\sigma}^2\Delta_1)\right],$$

$$b_{61j} = -S_{Y1}P_2/(\hat{\sigma}^2\Delta_1)\left(\hat{\beta}_1Q_{2j} - P_2Y_{1j}\right),\ b_{62j} = -\hat{\beta}_1\hat{\beta}_2\hat{\sigma}_x^2S_{Y2}/(\hat{\sigma}^2\Delta_1)\left(P_2Y_{1j} - \hat{\beta}_1Q_{2j}\right)$$

$$b_{71j} = -\hat{\beta}_1\hat{\beta}_2\hat{\sigma}_x^2S_{Y1}/(\hat{\sigma}^2\Delta_1)\left(P_1Y_{2j} - \hat{\beta}_2Q_{1j}\right),\ b_{72j} = -S_{Y2}P_1/(\hat{\sigma}^2\Delta_1)\left(\hat{\beta}_2Q_{1j} - P_1Y_{2j}\right)$$

$G_j, Q_{1j}, Q_{2j}, P_1, P_2$ and Δ_1 as given in Appendix B, $j = 1, \ldots, n$.

3.3 Perturbation of the explanatory variables

As we have done in the response variables, we are going to introduce the following perturbation in the explanatory variables

$$X_{1j} \to X_{1j} + S_{X1}\omega_{1j},$$

$$X_{2j} \to X_{2j} + S_{X2}\omega_{2j},$$

Again, we can consider the inverse of the sample deviance of the explanatory variables as the scale factor S_{X1} and S_{X2}, for example. The vector $\omega_0 = (0, \ldots, 0)^T$ and the perturbed log likelihood function is given by

$$L(\theta/\omega) = -2nlog(2\pi) - \frac{n}{2}log \mid \Sigma \mid -\frac{1}{2}\sum_{j=1}^{n}\left(\mathbf{Z}_j^{**} - \mathbf{m}\right)^T\Sigma^{-1}\left(\mathbf{Z}_j^{**} - \mathbf{m}\right),$$

with $\mathbf{Z}_j^{**} = [X_{1j} + S_{X1}w_{1j},\ Y_{1j},\ X_{2j} + S_{X2}w_{2j},\ Y_{2j}]^T$, $\mathbf{m} = (\mu, \beta_1\mu, \mu, \beta_2\mu)^T$ and Σ as given in (5). The matrix Δ may be expressed as

$$\Delta = \frac{1}{\Delta_1}\left[1_{2n}^TC_1,\ 1_{2n}^TC_2,\ 1_{2n}^TC_3,\ 1_{2n}^TC_4,\ 1_{2n}^TC_5,\ 1_{2n}^TC_6,\ 1_{2n}^TC_7\right]^T,$$

where $C_i = diag\left\{c_{i11}, \ldots, c_{i1n}, c_{i21}, \ldots, c_{i2n}\right\}, i = 1, \ldots, 7$, with

$$c_{11j} = S_{X1}e_{1j},\ c_{12j} = S_{X2}e_{1j},\ e_{1j} = -\hat{\lambda}_2\hat{\sigma}_x^2/(\hat{\sigma}^2\Delta_1)\left(\hat{\beta}_1T_{1j} - \hat{\lambda}_1R_{2j}\right),$$

$$c_{21j} = S_{X1}e_{2j},\ c_{22j} = S_{X2}e_{2j}\ ,\quad e_{2j} = -\hat{\lambda}_1\hat{\sigma}_x^2/(\hat{\sigma}^2\Delta_1)\left(\hat{\beta}_2 T_{2j} - \hat{\lambda}_2 R_{1j}\right),$$

$$c_{31j} = S_{X1}\hat{\lambda}_1\hat{\lambda}_2\ ,\ c_{32j} = S_{X2}\hat{\lambda}_1\hat{\lambda}_2\ ,\ c_{41j} = -S_{X1}\hat{\lambda}_1\hat{\lambda}_2 G_j\Delta_1^{-1}\ ,\ c_{42j} = -S_{X2}\hat{\lambda}_1\hat{\lambda}_2 G_j\Delta_1^{-1},$$

$$c_{51j} = -S_{X1}\hat{\sigma}^{-2}\left\{\hat{\lambda}_1\hat{\lambda}_2\left(X_{1j}-\hat{\mu}\right) - \left(\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 + \Delta_1\right)/(\hat{\sigma}^2\Delta_1)\left[\left(\Phi\hat{\sigma}_x^2 + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\right)X_{1j} - \hat{\lambda}_1\hat{\lambda}_2\left(X_{2j}\hat{\sigma}_x^2 + \hat{\mu}\hat{\sigma}^2\right) - \hat{\sigma}_x^2 L_j\right]\right\},$$

$$c_{52j} = -S_{X2}\hat{\sigma}^{-2}\left\{\hat{\lambda}_1\hat{\lambda}_2\left(X_{2j}-\hat{\mu}\right) - \left(\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 + \Delta_1\right)/(\hat{\sigma}^2\Delta_1)\left[\left(\Phi\hat{\sigma}_x^2 + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\right)X_{2j} - \hat{\lambda}_1\hat{\lambda}_2\left(X_{1j}\hat{\sigma}_x^2 + \hat{\mu}\hat{\sigma}^2\right) - \hat{\sigma}_x^2 L_j\right]\right\},$$

$$c_{61j} = -\hat{\beta}_1\hat{\lambda}_2\hat{\sigma}_x^2 R_{2j}S_{X1}/(\hat{\sigma}^2\Delta_1),\ c_{62j} = -\hat{\beta}_1\hat{\lambda}_2\hat{\sigma}_x^2 R_{2j}S_{X2}/(\hat{\sigma}^2\Delta_1),$$

$$c_{71j} = -\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2 R_{1j}S_{X1}/(\hat{\sigma}^2\Delta_1),\ c_{72j} = -\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2 R_{1j}S_{X2}/(\hat{\sigma}^2\Delta_1),$$

$L_j, G_j, T_{1j}, T_{2j}, R_{1j}, R_{2j}, \Delta_1$ and Φ, as given in Appendix B, $j = 1, \ldots, n$.

3.4 Perturbation on the variance of the measurement errors

Under the model specified by the equations (1), (2), (3) and (4), the variances of the measurement errors were considered the same for all individuous. We are going to perturb the specified model assuming that $\sigma_j^2 = \sigma^2/\omega_j$, so that

$$\mathbf{Z}_j = (X_{1j},\ Y_{1j},\ X_{2j},\ Y_{2j})^T \sim N_4(\mathbf{m},\ \Sigma_j),$$

$j = 1, \ldots, n$, with $\mathbf{m} = (\mu,\ \beta_1\mu,\ \mu,\ \beta_2\mu)^T$ and

$$\Sigma_j = \begin{bmatrix} \sigma_x^2 + \sigma^2/\omega_j & \beta_1\sigma_x^2 & \sigma_x^2 & \beta_2\sigma_x^2 \\ \beta_1\sigma_x^2 & \beta_1^2\sigma_x^2 + \lambda_1\sigma^2/\omega_j & \beta_1\sigma_x^2 & \beta_1\beta_2\sigma_x^2 \\ \sigma_x^2 & \beta_1\sigma_x^2 & \sigma_x^2 + \sigma^2/\omega_j & \beta_2\sigma_x^2 \\ \beta_2\sigma_x^2 & \beta_1\beta_2\sigma_x^2 & \beta_2\sigma_x^2 & \beta_2^2\sigma_x^2 + \lambda_2\sigma^2/\omega_j \end{bmatrix},$$

i.e., a heteroscedastic model. The log likelihood function is given by

$$L(\theta/\omega) = -2n log(2\pi) - \frac{1}{2}\sum_{j=1}^{n} log \mid \Sigma_j \mid - \frac{1}{2}\sum_{j=1}^{n}(\mathbf{Z}_j - \mathbf{m})^T\,\Sigma_j^{-1}(\mathbf{Z}_j - \mathbf{m}),$$

and $\omega_0 = \mathbf{1}_n$. The matrix Δ may be expressed as

$$\Delta = \frac{1}{\Delta_1^2}\left[\mathbf{1}_n^T D_1,\ \mathbf{1}_n^T D_2,\ -\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\mathbf{1}_n^T D_3,\ \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\mathbf{1}_n^T D_4,\ 1/2\mathbf{1}_n^T D_5,\ 1/2\mathbf{1}_n^T D_6,\ 1/2\mathbf{1}_n^T D_7\right]^T,$$

where $D_i = diag\{d_{i1}, \ldots, d_{in}\}, i = 1, \ldots, 7$, with

$$d_{1j} = -\hat{\lambda}_1\hat{\lambda}_2^2\hat{\beta}_1\hat{\sigma}^2\hat{\sigma}_x^2 + \hat{\sigma}^{-2}\left\{\hat{\lambda}_2\left\{\hat{\lambda}_1\hat{\lambda}_2\hat{\mu}\hat{\sigma}^4\left(Y_{1j} - \hat{\beta}_1\hat{\mu}\right) + \hat{\beta}_1\hat{\sigma}_x^2\left[\hat{\sigma}_x^2 N_j + 2\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\Delta_1^{-1}\times\right.\right.\right.$$
$$\left.\left.(\hat{\sigma}^2 M_j + \hat{\sigma}_x^2 N_j)\right]\right\} - \hat{\sigma}_x^2\left(\Delta_1 + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\right)\left[\hat{\beta}_1\hat{\lambda}_2\left(X_{1j}^2 + X_{2j}^2\right) - \hat{\lambda}_2\left(X_{1j}Y_{1j} + X_{2j}Y_{1j}\right) + \right.$$
$$\left.\left. Y_{2j}\left(\hat{\beta}_1 Y_{2j} - \hat{\beta}_2 Y_{1j}\right)\right]\right\},$$

$$d_{2j} = -\hat{\lambda}_1^2\hat{\lambda}_2\hat{\beta}_2\hat{\sigma}^2\hat{\sigma}_x^2 + \hat{\sigma}^{-2}\left\{\hat{\lambda}_1\left\{\hat{\lambda}_1\hat{\lambda}_2\hat{\mu}\hat{\sigma}^4\left(Y_{2j} - \hat{\beta}_2\hat{\mu}\right) + \hat{\beta}_2\hat{\sigma}_x^2\left[\hat{\sigma}_x^2 N_j + 2\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\Delta_1^{-1}\times\right.\right.\right.$$
$$\left.\left.(\hat{\sigma}^2 M_j + \hat{\sigma}_x^2 N_j)\right]\right\} - \hat{\sigma}_x^2\left(\Delta_1 + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\right)\left[\hat{\beta}_2\hat{\lambda}_1\left(X_{1j}^2 + X_{2j}^2\right) - \hat{\lambda}_1\left(X_{1j}Y_{2j} + X_{2j}Y_{2j}\right) + \right.$$
$$\left.\left. Y_{1j}\left(\hat{\beta}_2 Y_{1j} - \hat{\beta}_1 Y_{2j}\right)\right]\right\},$$

$$d_{3j} = G_j, \qquad d_{4j} = -\left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)/2 + G_j^2\Delta_1^{-1},$$

$$d_{5j} = \hat{\lambda}_1\hat{\lambda}_2\left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)\hat{\sigma}_x^2 + \hat{\sigma}^{-2}\left\{\hat{\sigma}_x^2\hat{\sigma}^{-2}\left(\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 + \Delta_1\right)N_j + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\Delta_1^{-1}\left[\hat{\lambda}_1\hat{\lambda}_2(\hat{\sigma}^2 M_j + \right.\right.$$
$$\left.\left. 2\hat{\sigma}_x^2 N_j) - \left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)\hat{\sigma}_x^2 M_j\right]\right\},$$

$$d_{6j} = \hat{\lambda}_2^2\hat{\beta}_1^2\hat{\sigma}^2\hat{\sigma}_x^2 - R_{2j}\hat{\sigma}^{-2}\left\{\hat{\beta}_1\hat{\sigma}_x^2\left[\hat{\lambda}_2\left(X_{1j} + X_{2j}\right) + \hat{\beta}_2 Y_{2j}\right] - \left(\hat{\beta}_2^2 + 2\hat{\lambda}_2\right)\hat{\sigma}_x^2 Y_{1j} - \right.$$
$$\left.\hat{\lambda}_2\hat{\sigma}^2\Delta_1^{-1}\left(\hat{\beta}_1\hat{\sigma}_x^2 G_j + \hat{\lambda}_1 R_{2j}\right)\right\},$$

$$d_{7j} = \hat{\lambda}_1^2\hat{\beta}_2^2\hat{\sigma}^2\hat{\sigma}_x^2 - R_{1j}\hat{\sigma}^{-2}\left\{\hat{\beta}_2\hat{\sigma}_x^2\left[\hat{\lambda}_1\left(X_{1j} + X_{2j}\right) + \hat{\beta}_1 Y_{1j}\right] - \left(\hat{\beta}_1^2 + 2\hat{\lambda}_1\right)\hat{\sigma}_x^2 Y_{2j} - \right.$$
$$\left.\hat{\lambda}_1\hat{\sigma}^2\Delta_1^{-1}\left(\hat{\beta}_2\hat{\sigma}_x^2 G_j + \hat{\lambda}_2 R_{1j}\right)\right\},$$

$N_j, M_j, G_j, R_{1j}, R_{2j}, \Delta_1$ and Φ as given in Appendix B, $j = 1, \ldots, n$.

4 Application

Considering the pretest/posttest data presented in Singer and Andrade (1997), we applied the perturbations described in the last section. The Figures 1 and 2 correspond to the index plots of $\mathbf{d}_{max}$ to assess the influence of the perturbation ω on the maximum likelihood estimator of the full parameter vector θ and on the subset $\theta = (\beta_1, \beta_2)^T$, respectiverly. The plots (a), (b), (c) and (d), refers to the application of perturbation schemes described in Section 3, namely perturbation on the variance of the measurement errors, case weights, response variables and explanatory variables, respectiverly. In the plots (c) and (d) the index from 1 to 26 refers to the data corresponding to the experimental toothbrush, while the index from 27 to 52 refers to the data corresponding to the conventional toothbrush.

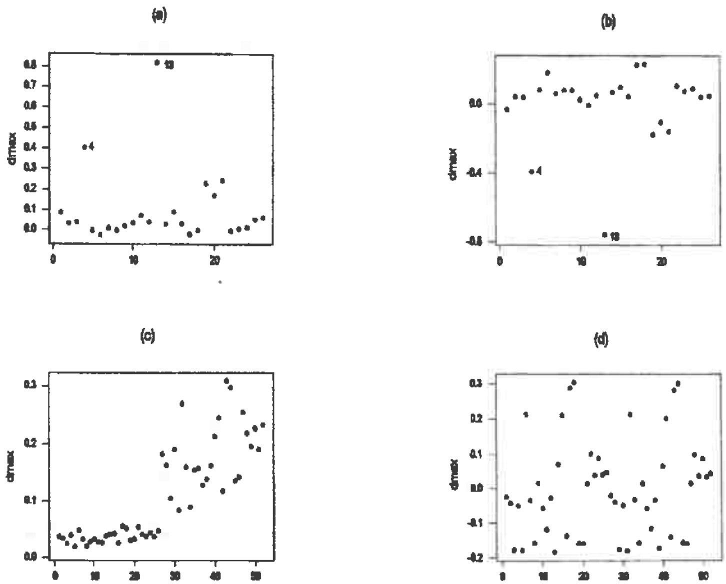

Figure 1: Influence Graphics: (a) Perturbations on scale, (b) Case weights, (c) Response variables and (d) Explanatory variables.

Considering Figure 1, we observe that in the plots (a) and (b) the observations 4 and 13 stands out. These observations corresponds to the ones which had the one of the greatest dental plaque index after toothbrushing and which had one of the least reduction of plaque index after toothbrushing with the use of the experimental toothbrush. The values of $| C_{\mathbf{d}_{max}} |$ were 3,437 and 2,857, respectively. To assess the influence of these observations on the maximum likelihood estimative (mle) of the parameters, we have obtained the mle of the parameters (see Appendix A) with the full data and without the observation 13, as well as without the observations 13 and 4. These values can be seen in Table 1.

Table 1: Maximum likelihood estimates and asymptotic standard deviates

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\mu}$	$\hat{\sigma}_x^2$	$\hat{\sigma}^2$	$\hat{\lambda}_1$	$\hat{\lambda}_2$
	0,147 (0,025)	0,454 (0,045)	1,759 (0,172)	0,540 (0,200)	0,481 (0,040)	0,102 (0,122)	0,267 (0,123)
without obs. 13	0,135 (0,020)	0,464 (0,042)	1,760 (0,175)	0,594 (0,208)	0,367 (0,097)	0,091 (0,117)	0,310 (0,147)
without obs. 13 and 4	0,124 (0,017)	0,463 (0,044)	1,765 (0,181)	0,610 (0,216)	0,370 (0,100)	0,059 (0,025)	0,330 (0,157)

Notice that in these cases the greatest differences were on the estimatives of the variances of the measurement error. Considering plots (c) and (d), which refers to the perturbation of the response variables and explanatory variables the corresponding values of $| C_{\mathbf{d}_{max}} |$ were 1,306 and 1,358. Analysing the plot (c) we can cleary see that the data refering to the conventional toothbrush stands out compared to the data refering to the experimental toothbrush, which means that the data obtained after the use of the conventional toothbrush has greater effect with small local changes in the parameters estimates. Considering plot (d), which refers to the perturbation of the explanatory variables, none of the observations stands out. In this case, both samples (with the use of the experimental and conventional toothbrushes) have the same distribution for the explanatory variables, while the response variables have different distributions for each sample. The estimated asymptotic variance of the response variable with the use of the experimental toothbrush is given by 0,06 while this value for the convencional one is given by 0,24.

Considering Figure 2, the values of $| C_{\mathbf{d}_{max}} |$ for the plots (a), (b), (c) and (d)

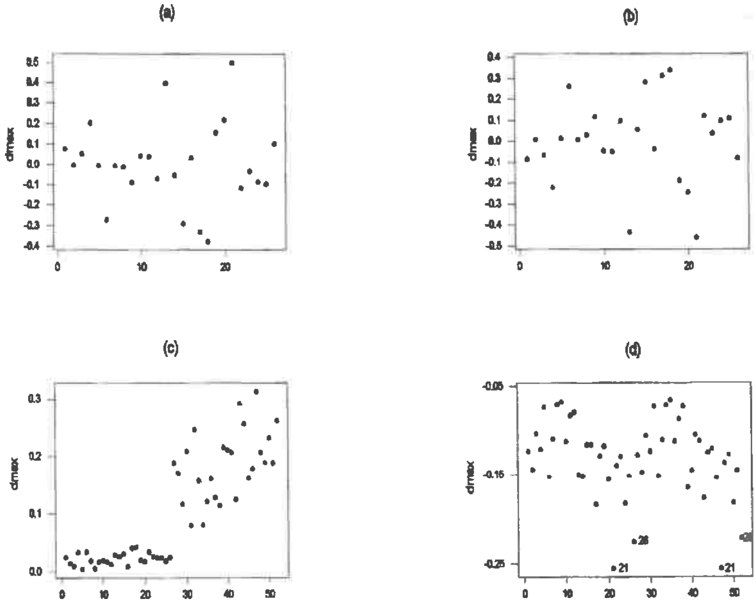

Figure 2: Influence Graphics: (a) Perturbations on scale, (b) Case weights, (c) Response variables and (d) Explanatory variables.

were 1,754; 1,509; 0,354 and 0,717, respectiverly. Notice that in this case none of the observations stands out in the plots (a) and (b), what is in accordance, for instance, with the results obtained previously when the observations 13 and 4 were taken out and we had observed that the estimated values of the β_1 and β_2 were not much affected. Analysing plots (c) and (d), we observe that plot (c) has the same pattern as the plot (c) of the Figure 1, which gives us the same conclusions. In the plot (d) the observations 21 and 26 stands out, but since the value of $\mid C_{\mathbf{d}_{max}} \mid$ refering to this perturbation is "small" we would expect that these observations would not have great influence on the estimates of the parameters of interest β_1 and β_2 as can be confirmed in Table 2.

Table 2: Maximum likelihood estimates and asymptotic standard deviates

	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\mu}$	$\hat{\sigma}_x^2$	$\hat{\sigma}^2$	$\hat{\lambda}_1$	$\hat{\lambda}_2$
without obs. 21 and 26	0,145 (0,028)	0,499 (0,050)	1,616 (0,171)	0,480 (0,186)	0,463 (0,122)	0,111 (0,045)	0,252 (0,125)

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Appendix A: EM Algorithm

Considering the model defined by (1), (2), (3) and (4), the log likelihood function is given by (6). However there are no explicit solutions for the likelihood equations. In that way, we are going to implement the EM algorithm. Let us define by $\mathbf{T_j} = (x_j, \mathbf{Z_j}^T)^T$, with $\mathbf{Z_j} = (X_{1j}, Y_{1j}, X_{2j}, Y_{2j})^T$, where x_j is the true unobserved value of the explanatory variables, X_{ij} (Y_{ij}) is the observed explanatory (response) variable for the population i, $i = 1, 2$, and individual j, $j = 1, \ldots, n$. Then $T_j \sim N_5(\mathbf{m}, V)$, with $\mathbf{m} = (\mu, \mu, \beta_1\mu, \mu, \beta_2\mu)^T$ and

$$\mathbf{V} = \begin{bmatrix} \sigma_x^2 & \sigma_x^2 & \beta_1\sigma_x^2 & \sigma_x^2 & \beta_2\sigma_x^2 \\ \sigma_x^2 & \sigma_x^2 + \sigma^2 & \beta_1\sigma_x^2 & \sigma_x^2 & \beta_2\sigma_x^2 \\ \beta_1\sigma_x^2 & \beta_1\sigma_x^2 & \beta_1^2\sigma_x^2 + \lambda_1\sigma^2 & \beta_1\sigma_x^2 & \beta_1\beta_2\sigma_x^2 \\ \sigma_x^2 & \sigma_x^2 & \beta_1\sigma_x^2 & \sigma_x^2 + \sigma^2 & \beta_2\sigma_x^2 \\ \beta_2\sigma_x^2 & \beta_2\sigma_x^2 & \beta_1\beta_2\sigma_x^2 & \beta_2\sigma_x^2 & \beta_2^2\sigma_x^2 + \lambda_2\sigma^2 \end{bmatrix}.$$

By using general properties of the multivariate normal distribution it follows that the complete log-likelihood function is given by

$$L(\theta/\mathbf{Z}_1,\ldots,\mathbf{Z}_n) = const - \frac{n}{2}\log[\lambda_1\lambda_2\sigma^8\sigma_x^2] - \frac{1}{2}\left[\sum_{j=1}^{n}\frac{(x_j-\mu)^2}{\sigma_x^2} + \sum_{j=1}^{n}\frac{(X_{1j}-x_j)^2}{\sigma^2} + \sum_{j=1}^{n}\frac{(X_{2j}-x_j)^2}{\sigma^2} + \sum_{j=1}^{n}\frac{(Y_{1j}-\beta_1 x_j)^2}{\lambda_1\sigma^2} + \sum_{j=1}^{n}\frac{(Y_{2j}-\beta_2 x_j)^2}{\lambda_2\sigma^2}\right], \quad (9)$$

with $\theta = (\beta_1, \beta_2, \mu, \sigma_x^2, \sigma^2, \lambda_1, \lambda_2)^T$.

E Step

Here the expectation of the complete data log likelihood function given in (9) is obtained, given the observed data $\mathbf{Z} = (\mathbf{Z}_1, \ldots, \mathbf{Z}_n)^T$ and the current estimated parameters. To implement this step, it suffices to obtain the conditional expectation of the sufficient statistics over the distribution of $\mathbf{T} = (\mathbf{T}_1, \ldots, \mathbf{T}_n)^T$, given $\mathbf{Z}$ and θ, as the log likelihood function is from regular exponential family (Dempster et al., 1977). As the sufficient statistics depend on x_j, only through x_j and x_j^2, the E step is defined by the equations

$$\begin{aligned}\hat{x}_j &= E(x_j/\mathbf{Z},\theta)\\ &= \mu + \frac{\sigma_x^2\left[\beta_1\lambda_2 Y_{1j} + \beta_2\lambda_1 Y_{2j} - \beta_1^2\lambda_2\mu - \beta_2^2\lambda_1\mu + \lambda_1\lambda_2(X_{1j}+X_{2j}) - 2\mu\lambda_1\lambda_2\right]}{\lambda_1\lambda_2\sigma^2 + \sigma_x^2\left(\beta_1^2\lambda_2 + \beta_2^2\lambda_1 + 2\lambda_1\lambda_2\right)}\end{aligned}$$

and

$$\begin{aligned}\hat{x}_j^2 &= E(x_j^2/\mathbf{Z},\theta)\\ &= \frac{\lambda_1\lambda_2\sigma^2\sigma_x^2}{\lambda_1\lambda_2\sigma^2 + \sigma_x^2\left(\beta_1^2\lambda_2 + \beta_2^2\lambda_1 + 2\lambda_1\lambda_2\right)} + (\hat{x}_j)^2,\end{aligned}$$

$j = 1, \ldots, n$.

M Step

The step M is implemented by maximizing the complete data log likelihood function given in (9). Deriving equation (9) with respect to the parameters and equating to zero, we obtain after algebraic manipulations

$$\hat{\mu} = \frac{u_x}{n},\ \hat{\beta}_1 = \frac{u_{xY_1}}{u_{xx}},\ \hat{\beta}_2 = \frac{u_{xY_2}}{u_{xx}},\ \hat{\sigma}^2 = \frac{1}{n}\left[\frac{u_{X_1X_1} + u_{X_2X_2}}{2} + u_{xx} - u_{xX_1} - u_{xX_2}\right],$$

$$\hat{\sigma}_x^2 = \frac{u_{xx}}{n} - \left(\frac{u_x}{n}\right)^2,\ \hat{\lambda}_1 = \frac{u_{Y_1Y_1} - (u_{xY_1})^2/u_{xx}}{\frac{u_{X_1X_1}+u_{X_2X_2}}{2} + u_{xx} - u_{xX_1} - u_{xX_2}},$$

$$\hat{\lambda}_2 = \frac{u_{Y_2Y_2} - (u_{xY_2})^2/u_{xx}}{\frac{u_{X_1X_1}+u_{X_2X_2}}{2} + u_{xx} - u_{xX_1} - u_{xX_2}},$$

with,

$u_{X_1X_1} = \sum_{j=1}^n X_{1j}^2$, $u_{X_2X_2} = \sum_{j=1}^n X_{2j}^2$, $u_{Y_1Y_1} = \sum_{j=1}^n Y_{1j}^2$, $u_{Y_2Y_2} = \sum_{j=1}^n Y_{2j}^2$, $u_x = \sum_{j=1}^n \hat{x}_j$, $u_{xx} = \sum_{j=1}^n \hat{x}_j^2$, $u_{xX_1} = \sum_{j=1}^n \hat{x}_j X_{1j}$, $u_{xX_2} = \sum_{j=1}^n \hat{x}_j X_{2j}$, $u_{xY_1} = \sum_{j=1}^n \hat{x}_j Y_{1j}$ and $u_{xY_2} = \sum_{j=1}^n \hat{x}_j Y_{2j}$, $j = 1, \ldots, n$.

The EM algorithm cycles between equations given in the E step and the equations given in M step until convergence. Observe that no additional iterative procedure is requered to solve the M step within each cycle of the algorithm, making the above procedure extremely simple to implement and computationally inexpensive. In a general setting, each cycle of the EM algorithm increases the observed likelihood function given in (6) (Dempster et.al., 1977).

Considering the methods of moment estimators (MME) as initial values, we have obtained the following estimates of the parameters.

Table 3:Maximum likelihood estimative (MLE) of the parameters under the model defined by equations (1), (2), (3) and (4), via EM algorithmith for the data presented in Singer and Andrade (1997).

	Parameter						
	β_1	β_2	μ	σ_x^2	σ^2	λ_1	λ_2
MME	0.156	0.436	1.769	0.408	0.562	0.074	0.363
MLE	0.147	0.454	1.758	0.539	0.482	0.102	0.267

Appendix B: Observed Information matrix

$$\ddot{L} = \frac{\partial^2 L(\theta)}{\partial\theta_l \partial\theta_s}\Big|_{\theta=\hat{\theta}} = \{l_{ls}\}, \quad l, s = 1, \ldots, 7 \text{ e } \theta = \left(\beta_1,\ \beta_2,\ \mu,\ \sigma_x^2,\ \sigma^2,\ \lambda_1,\ \lambda_2\right)$$

where

$$\begin{aligned}
l_{11} &= -\Delta_1^{-2}\Big\{n\hat{\lambda}_2\hat{\sigma}_x^2\left(P_2\hat{\lambda}_1 - \hat{\beta}_1^2\hat{\lambda}_2\hat{\sigma}_x^2\right) + \hat{\sigma}^{-2}\textstyle\sum_{j=1}^n\Big[Q_{2j}F_j - \hat{\lambda}_2\hat{\sigma}_x^2R_{2j}\,(Y_{1j} - \\
&\quad 4\hat{\beta}_1T_{1j}\Delta_1^{-1})\Big]\Big\}, \\
l_{12} &= \hat{\sigma}_x^2\Delta_1^{-2}\Big\{2n\hat{\beta}_1\hat{\beta}_2\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}_x^2 - \hat{\sigma}^{-2}\textstyle\sum_{j=1}^n\Big[\left(\hat{\beta}_1Y_{2j} - 2\hat{\beta}_2Y_{1j}\right)F_j - \hat{\lambda}_1R_{2j}\,(Y_{2j} - \\
&\quad 4\hat{\beta}_2F_j\Delta_1^{-1})\Big]\Big\}, \\
l_{13} &= -\hat{\lambda}_2\Delta_1^{-2}\textstyle\sum_{j=1}^n\left(\hat{\beta}_1T_{1j} - \hat{\lambda}_1R_{2j}\right), \\
l_{14} &= -\hat{\lambda}_2\Delta_1^{-2}\Big[n\hat{\beta}_1\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 - \Delta_1^{-1}\textstyle\sum_{j=1}^n G_j\left(\hat{\beta}_1T_{1j} - \hat{\lambda}_1R_{2j}\right)\Big], \\
l_{15} &= \Delta_1^{-2}\Big\{n\hat{\beta}_1\hat{\lambda}_1\hat{\lambda}_2^2\hat{\sigma}_x^2 + \hat{\sigma}^{-2}\textstyle\sum_{j=1}^n\Big[\hat{\mu}\hat{\lambda}_1\hat{\lambda}_2R_{2j} + F_j\left(\hat{\lambda}_2\left(Y_{1j} - \hat{\beta}_1\hat{\mu}\right) - \right. \\
&\quad \left. R_{2j}\left(\Delta_1 + 2\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\right)/(\hat{\sigma}^2\Delta_1)\right)\Big]\Big\}, \\
l_{16} &= \Delta_1^{-2}\Big[n\hat{\beta}_1\hat{\lambda}_2\hat{\sigma}_x^2P_2 + \hat{\sigma}^{-2}\textstyle\sum_{j=1}^n R_{2j}\left(Q_{2j} - 2P_2T_{1j}/\Delta_1^{-1}\right)\Big], \\
l_{17} &= -\hat{\beta}_2\hat{\sigma}_x^2\Delta_1^{-2}\Big\{n\hat{\beta}_1\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2 + 1/(\hat{\sigma}^2\Delta_1)\textstyle\sum_{j=1}^n R_{1j}\Big[\hat{\lambda}_1R_{2j} - \hat{\beta}_1\left(\hat{\beta}_1\hat{\lambda}_2\hat{\sigma}_x^2Y_{1j} + \right. \\
&\quad \left.\hat{\lambda}_1Q_{2j}\right)\Big]\Big\}, \\
l_{22} &= -\Delta_1^{-2}\Big\{n\hat{\lambda}_1\hat{\sigma}_x^2\left(P_1\hat{\lambda}_2 - \hat{\beta}_2^2\hat{\lambda}_1\hat{\sigma}_x^2\right) + \hat{\sigma}^{-2}\textstyle\sum_{j=1}^n\Big[Q_{1j}F_j - \hat{\lambda}_1\hat{\sigma}_x^2R_{1j}\,(Y_{2j} - \\
&\quad 4\hat{\beta}_2T_{2j}\Delta_1^{-1})\Big]\Big\}, \\
l_{23} &= -\hat{\lambda}_1\Delta_1^{-2}\textstyle\sum_{j=1}^n\left(\hat{\beta}_2T_{2j} - \hat{\lambda}_2R_{1j}\right), \\
l_{24} &= -\hat{\lambda}_1\Delta_1^{-2}\Big[n\hat{\beta}_2\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 - \Delta_1^{-1}\textstyle\sum_{j=1}^n G_j\left(\hat{\beta}_2T_{2j} - \hat{\lambda}_2R_{1j}\right)\Big], \\
l_{25} &= \Delta_1^{-2}\Big\{n\hat{\beta}_2\hat{\lambda}_1^2\hat{\lambda}_2\hat{\sigma}_x^2 + \hat{\sigma}^{-2}\textstyle\sum_{j=1}^n\Big[\hat{\mu}\hat{\lambda}_1\hat{\lambda}_2R_{1j} + F_j\left(\hat{\lambda}_1\left(Y_{2j} - \hat{\beta}_2\hat{\mu}\right) - \right. \\
&\quad \left. R_{1j}\left(\Delta + 2\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2\right)/(\hat{\sigma}^2\Delta_1)\right)\Big]\Big\},
\end{aligned}$$

$$l_{26} = -\hat{\beta}_1\hat{\sigma}_x^2\Delta_1^{-2}\left\{n\hat{\beta}_1\hat{\beta}_2\hat{\lambda}_2\hat{\sigma}_x^2 + 1/(\hat{\sigma}^2\Delta_1)\sum_{j=1}^n R_{2j}\left[\hat{\lambda}_2 R_{1j} - \hat{\beta}_2\left(\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2 Y_{2j} + \hat{\lambda}_2 Q_{1j}\right)\right]\right\},$$

$$l_{27} = \Delta_1^{-2}\left[n\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2 P_1 + \hat{\sigma}^{-2}\sum_{j=1}^n R_{1j}\left(Q_{1j} - 2P_1T_{2j}\Delta_1^{-1}\right)\right],$$

$$l_{33} = -n\Delta_1^{-1}\left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right), \quad l_{34} = \Delta_1^{-2}\left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)\sum_{j=1}^n G_j,$$

$$l_{35} = \hat{\lambda}_1\hat{\lambda}_2\Delta_1^{-2}\sum_{j=1}^n G_j, l_{36} = -\hat{\beta}_1\hat{\lambda}_2\Delta_1^{-2}\sum_{j=1}^n R_{2j},$$

$$l_{37} = -\hat{\beta}_2\hat{\lambda}_1\Delta_1^{-2}\sum_{j=1}^n R_{1j},$$

$$l_{44} = \left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)\Delta_1^{-2}\left[n\left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)/2 - \Delta_1^{-1}\sum_{j=1}^n G_j^2\right],$$

$$l_{45} = \hat{\lambda}_1\hat{\lambda}_2\Delta_1^{-2}\left[n\left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)/2 - \Delta_1^{-1}\sum_{j=1}^n G_j^2\right],$$

$$l_{46} = \hat{\beta}_1\hat{\lambda}_2\Delta_1^{-2}\left[n\hat{\beta}_1\hat{\lambda}_2\hat{\sigma}^2/2 + \Delta_1^{-1}\sum_{j=1}^n G_jR_{2j}\right],$$

$$l_{47} = \hat{\beta}_2\hat{\lambda}_1\Delta_1^{-2}\left[n\hat{\beta}_2\hat{\lambda}_1\hat{\sigma}^2/2 + \Delta_1^{-1}\sum_{j=1}^n G_jR_{1j}\right],$$

$$l_{55} = 1/(\hat{\sigma}^2\Delta_1)\left\{n\left(3\Delta_1^2 + \hat{\lambda}_1^2\hat{\lambda}_2^2\hat{\sigma}^4\right)/(2\hat{\sigma}^2\Delta_1) - \sum_{j=1}^n\left[\hat{\lambda}_1^2\hat{\lambda}_2^2\Delta_1^{-2}\left(\hat{\sigma}^2M_j + \hat{\sigma}_x^2N_j\right) + \hat{\sigma}_x^2N_j\left(\hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2 + \Delta_1\right)/(\hat{\sigma}^4\Delta_1)\right]\right\},$$

$$l_{56} = -1/(2\Delta_1^2)\left\{n\hat{\beta}_1^2\hat{\lambda}_2^2\hat{\sigma}_x^2 + R_{2j}/(\hat{\sigma}^4\Delta_1)\sum_{j=1}^n\left[\hat{\lambda}_2\hat{\sigma}^2\left(\hat{\lambda}_1R_{2j} + \hat{\beta}_1\hat{\sigma}_x^2G_j\right) + \Delta_1\hat{\sigma}_x^2\left[\left(\hat{\beta}_2^2 + 2\hat{\lambda}_2\right)Y_{1j} - \hat{\beta}_1\left[\hat{\beta}_2Y_{2j} + \hat{\lambda}_2\left(X_{1j} + X_{2j}\right)\right]\right]\right]\right\},$$

$$l_{57} = -1/(2\Delta_1^2)\left\{n\hat{\beta}_2^2\hat{\lambda}_1^2\hat{\sigma}_x^2 + R_{1j}/(\hat{\sigma}^4\Delta_1)\sum_{j=1}^n\left[\hat{\lambda}_1\hat{\sigma}^2\left(\hat{\lambda}_2R_{1j} + \hat{\beta}_2\hat{\sigma}_x^2G_j\right) + \Delta_1\hat{\sigma}_x^2\left[\left(\hat{\beta}_1^2 + 2\hat{\lambda}_1\right)Y_{2j} - \hat{\beta}_2\left[\hat{\beta}_1Y_{1j} + \hat{\lambda}_1\left(X_{1j} + X_{2j}\right)\right]\right]\right]\right\},$$

$$l_{66} = P_2\Delta_1^{-2}\left(nP_2/2 - \sum_{j=1}^n R_{2j}^2/(\hat{\sigma}^2\Delta_1)\right),$$

$$l_{67} = \hat{\beta}_1\hat{\beta}_2\hat{\sigma}_x^2\Delta_1^{-2}\left(n\hat{\beta}_1\hat{\beta}_2\hat{\sigma}_x^2/2 + 1/(\hat{\sigma}^2\Delta_1)\sum_{j=1}^n R_{1j}R_{2j}\right),$$

$$l_{77} = P_1\Delta_1^{-2}\left(nP_1/2 - \sum_{j=1}^n R_{1j}^2/(\hat{\sigma}^2\Delta_1)\right),$$

with

$\Phi = \hat{\beta}_1^2\hat{\lambda}_2 + \hat{\beta}_2^2\hat{\lambda}_1 + \hat{\lambda}_1\hat{\lambda}_2$, $\Delta_1 = \left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right)\hat{\sigma}_x^2 + \hat{\lambda}_1\hat{\lambda}_2\hat{\sigma}^2$, $W_j = \hat{\mu}\hat{\sigma}^2 + \hat{\sigma}_x^2\left(X_{1j} + X_{2j}\right)$,

$P_1 = \left(\hat{\beta}_1^2 + 2\hat{\lambda}_1\right)\hat{\sigma}_x^2 + \hat{\lambda}_1\hat{\sigma}^2$, $P_2 = \left(\hat{\beta}_2^2 + 2\hat{\lambda}_2\right)\hat{\sigma}_x^2 + \hat{\lambda}_2\hat{\sigma}^2$, $Q_{1j} = \hat{\beta}_1\hat{\sigma}_x^2Y_{1j} + \hat{\lambda}_1W_j$,

$Q_{2j} = \hat{\beta}_2\hat{\sigma}_x^2Y_{2j} + \hat{\lambda}_2W_j$, $R_{1j} = P_1Y_{2j} - \hat{\beta}_2Q_{1j}$, $R_{2j} = P_2Y_{1j} - \hat{\beta}_1Q_{2j}$,

$T_{1j} = \hat{\beta}_1\hat{\lambda}_2\hat{\sigma}_x^2Y_{1j} + \hat{\lambda}_1Q_{2j}$, $T_{2j} = \hat{\beta}_2\hat{\lambda}_1\hat{\sigma}_x^2Y_{2j} + \hat{\lambda}_2Q_{1j}$, $L_j = \hat{\beta}_1\hat{\lambda}_2Y_{1j} + \hat{\beta}_2\hat{\lambda}_1Y_{2j}$,

$F_j = \hat{\lambda}_1\hat{\lambda}_2W_j + \hat{\sigma}_x^2L_j$, $G_j = \hat{\mu}\left(\Phi + \hat{\lambda}_1\hat{\lambda}_2\right) - \hat{\lambda}_1\hat{\lambda}_2\left(X_{1j} + X_{2j}\right) - L_j$,

$N_j = \Phi\left(X_{1j}^2 + X_{2j}^2\right) + \left(\hat{\beta}_1^2 + 2\hat{\lambda}_1\right)Y_{2j}^2 + \left(\hat{\beta}_2^2 + 2\hat{\lambda}_2\right)Y_{1j}^2 - 2\left[\hat{\beta}_1\hat{\lambda}_2\left(X_{1j}Y_{1j} + X_{2j}Y_{1j}\right)\right.$

$$+\hat{\beta}_2\hat{\lambda}_1\left(X_{1j}Y_{2j}+X_{2j}Y_{2j}\right)+\hat{\beta}_1\hat{\beta}_2Y_{1j}Y_{2j}+\hat{\lambda}_1\hat{\lambda}_2X_{1j}X_{2j}\Big],$$

$$M_j=\hat{\mu}G_j-\left(\hat{\lambda}_1\hat{\lambda}_2\left(X_{1j}+X_{2j}\right)+L_j\right)\hat{\mu}+\hat{\lambda}_1\left(\hat{\lambda}_2X_{1j}^2+Y_{2j}^2\right)+\hat{\lambda}_2\left(\hat{\lambda}_1X_{2j}^2+Y_{1j}^2\right),$$

$j=1,\ldots,n.$

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