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The nonwandering set of flows on a
Reeb foliation

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THE NONWANDERING SET OF FLOWS ON A REEB FOLIATION

1 - INTRODUCTION

Let $\chi(S^3)$ be the space of the C^r vector fields, $r \geq 4$, tangent to a Reeb foliation σ on the sphere S^3 [1] with the usual C^r topology.

A periodic orbit α (in particular a singular point) of $X \in \chi(S^3)$ is called *hyperbolic* if it is hyperbolic in the usual sense for the restriction $X|_{F_p}$ where F_p denotes the leaf of σ containing $p \in \alpha$. When α is a singular point or is the boundary of a disc contained in its leaf, the qualitative behaviour of small perturbations of the vector field X is given by the Kupka-Smale theorem. Let α_1, α_2 be generators of the fundamental group of the compact leaf T^2 and $H(\alpha_i)$ the holonomy given by

$$\begin{cases} g_1(y) = y, & y \leq 0 \\ g_1(y) < y, & y > 0 \end{cases} \quad \text{and} \quad \begin{cases} g_2(y) > y, & y < 0 \\ g_2(y) = y, & y \geq 0. \end{cases}$$

Suppose $\alpha_X \subset T^2$ a hyperbolic periodic orbit attractor for $X|_{T^2}$. When α_X is homotopic to α_1 there exists an open neighborhood V of α_X in S^3 such that the periodic orbits of small perturbations $Y \in \chi(S^3)$ of the vector field X with points in V have the same type of α_X , are contained in V and define a cylinder with boundary $\alpha_Y \subset T^2$ which is the ω -limit of all the points $p \in V$. If α_X is homotopic to α_2 the periodic orbits of Y which are completely contained in V define a cylinder with boundary $\alpha_Y \subset T^2$ which is an attractor except for α_Y which also admits a 2-dimensional

invariant and unstable manifold. When α_X is homotopic to $\alpha_1^m \alpha_2^n$, $mn \neq 0$, α_Y is the only periodic orbit completely contained in V . A hyperbolic periodic orbit $\alpha_X \subset T^2$ which is an attractor for $X|_{T^2}$ is called a *saddle type orbit* if it is homotopic to $\alpha_1^m \alpha_j^n$, $m \leq 0$ with $n < 0$ if $m = 0$.

A *saddle connection* is a regular trajectory $\gamma \subset T^2$ whose α and ω limit sets are saddle, saddle node or saddle type orbit and is not interior to the 2-dimensional invariant manifold of the saddle node.

The periodic orbits which are quasi-generic [3] in their own leaves and the saddle connections, with $\text{trace}DX|_{F_p(p)} \neq 0$ in the loop at p [3], are called *quasi generic elements* if they are generic respect the local parameter of leaves. The structure of orbits locally in the quasigeneric orbits and in the connections between singular points are given in [3].

In [4] was considered a residual set G_2 in $X(S^3)$ of vector fields such that only the saddle connections between singular points are generic respect the local parameter of the leaves. There was given also an open proper subset of $X(S^3)$ of vector fields without saddle type orbit where the vector fields without Ω -explosion are dense.

In this work we are allowing for the saddle type orbits and all the types of saddle connections. We define a dense set G_3 in $X(S^3)$ of vector fields such that all the saddle connections are quasigeneric elements and conclude the density of the vector fields without Ω -explosion in the entire space $X(S^3)$.

Now we state the main results of this work whose proofs will be outlined in the subsequent parts.

Theorem: There exists G_3 , a dense set of vector fields $X \in \chi(S^3)$ such that:

- a) $X|_{T^2}$ is Morse-Smale
- b) the periodic orbits are hyperbolic or quasi-generic
- c) the saddle connections are quasi-generic
- d) each leaf of σ contains at most one quasi-generic element

Corollary: There exists in $\chi(S^3)$ a dense set of vector fields without Ω -explosion.

2. PROOF OF THEOREM

Consider $\chi(S^3) \supset G_1 \supset G_2(T) \supset G_2(T;S)$ each open and dense in the preceding, already described in [4] :

G_1 denotes a set of vector fields $X \in \chi(S^3)$ whose singular points are hyperbolic but for a finite number of quasi-generic points, $X|_{T^2}$ is a Morse-Smale vector field and each leaf of σ contains at most one quasi generic singular point .

$G_2(T)$ denotes a set of vector fields $X \in G_1$ whose periodic orbits with period $\leq T$ are hyperbolic or quasi-generic and each leaf contains at most one quasi-generic periodic orbit with period τ , $0 \leq \tau \leq T$.

$G_2(T, S)$ denotes a set of vector fields $X \in G_2(T)$ without saddle connection between saddle node and saddle type orbit, whose saddle connection between singular points with length $\leq S$ are quasi-generic and lie at most one in each leaf with the additional condition that the quasi-generic periodic orbits of such a leaf have period $> T$.

Take $X \in G_1$ and $\alpha_i (i=1, 2, \dots, n)$ its saddle type orbits. Consider open neighborhoods V^i of α_i and \mathcal{V} of X in G_1 such that $\bar{V}^i \cap \bar{V}^j = \emptyset$, $Y \in \mathcal{V}$ admits, for each i , only one saddle type orbit $\alpha_i(Y) \subset V^i$, of the same type of α_i , that is the only orbit completely contained in \bar{V}^i except for the local cylinder of hyperbolic orbits at $\alpha_i(Y)$. We fix the neighborhoods V and \mathcal{V} and suppose $\alpha_i(Y)$ to be the only periodic orbits of $Y|_{T_2}$ at V .

Let $\gamma \subset V$ be a trajectory of $Y \in \mathcal{V}$, $t_1 = \inf \{t \in \mathbb{R} : \forall s \geq t, \gamma(s) \in V\}$ and $t_2 = \sup \{t \in \mathbb{R} : \forall s \leq t, \gamma(s) \in V\}$. The length of $\gamma(t)$ with $-\infty \leq t_2 < t < t_1 \leq \infty$ will be denoted $length \gamma$. For $\gamma \subset V$, $length \gamma$ is zero.

Fixed X, V and \mathcal{V} we can suppose the existence of a number $0 < m < 1$ and an open $A \subset S^3$, $\bar{A} \cap \bar{V} = \emptyset$ containing the singular points of any $Y \in \mathcal{V}$ such that the saddle node separatrices, the orbits on the central manifold of saddle type orbit and the saddle separatrices not completely contained in A have $length > m$.

Given $T, S, R > 0$ we denote by $\mathcal{V}(X, T, S, R)$ the set of all the vector fields $Y \in \mathcal{V} \cap G_2(T, S)$ whose connections between saddle and saddle type orbit with $length \leq R$ are quasi-generic and lie at most one in each leaf with the additional condition that the quasi-generic saddle connections between critical points and the quasi-generic periodic orbits of such a leaf have $length > S$ and $period > T$ respectively.

Proposition 1: $\mathcal{V}(X, T, S, R)$ is open and dense in \mathcal{V} when $1 \leq R < S$

Lemma 1: Take $X \in G_1$ and $p \notin T^2$

- a) $\omega(p)$ is a cycle when $\omega(p) \subset F_p$
- b) $\omega(p)$ is a periodic orbit or contains a saddle type orbit when $\omega(p) \cap T^2 \neq \emptyset$
- c) $\alpha(p)$ is a saddle type orbit γ when $\omega(p)$ is a saddle type orbit and $\alpha(p) \supset \gamma$

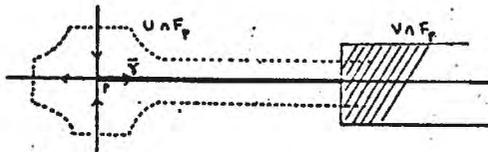
Lemma 2: Take $Y \in \mathcal{V}$, $p \notin T^2$ and γ a non trivial periodic orbit such that $\gamma \subset \omega(p)$. Then, for each $R > 0$, there exist neighborhoods $V_p \subset S^3 - T^2$ of p and $v_p \subset \mathcal{V}$ of Y such that any regular orbit of $Z \in v_p$, but eventually for those in central manifold of saddle type orbit, at V_p have $length > R$.

Lemma 3: If $Y \in \mathcal{V}(X, T, S, R)$ and $0 < R \leq S$ then Y admits a finite number of connections between saddle

and saddle type periodic orbit with $length \leq R$.

Proof: If there were a infinite number of such connections γ_n between saddles p_n , with $p_n \rightarrow p$, and a saddle type periodic orbit α we would have a chain of saddle connections with lengths $\leq R \leq S$ in the leaf F_p . \square

Take $Y \in \mathfrak{d}(X, T, S, R)$ and $\gamma = \gamma(Y)$ a connection between a saddle $p \in T^2$ and a saddle type orbit α with $\omega(\gamma) = \alpha$ and $length \gamma \leq R$. Denote $\bar{\gamma} = \{\gamma(t) : -\infty < t \leq t_1\}$ (see definition of $length \gamma$). Consider open neighborhoods U of $\bar{\gamma}$ and \mathfrak{U} of Y in $\mathfrak{d} \cap G_2(T, S)$ such that $Z \in \mathfrak{U}$ admits only one connection $\gamma(Z)$ between saddle and saddle type orbit, with $\overline{\gamma(Z)} \subset U$, which is quasi generic. Furthermore, the $length o_Z(x)$, with $x \in U - \gamma(Z)$ and $o_Z(x) \cap V \neq \emptyset$, is greater than R . For $Z \in \mathfrak{U}$ we also suppose that in the saturated set of U by the foliation the periodic orbits with period $\leq T$ are hyperbolic and the saddle connections between singular points have $length > S$.



Proof of proposition 1: For $Y \in \mathcal{V}(X, T, S, R)$ consider, like above, convenient neighborhoods U^i and \mathcal{U}^i determined by the connections between saddle and saddle type orbit with length $\leq R$. The continuity of arc length, the lemmas above and the local behaviour in the saddle, saddle-node and saddle connections between singular points with length $\leq R < S$ [2], [3] guarantee the existence, for each $x \in K = S^3 - V - U$ of open neighborhoods U_x of x and \mathcal{U}_x of Y in $\mathcal{V} \cap G_2(T, S)$ such that the connections between saddle and saddle type orbit, of $Z \in \mathcal{U}_x$, by U_x have $length > R$. The openness follows from the compactness of K . The density will follow like proposition II-2.3 of [3]. Let $\alpha_i(Y)$ be the saddle type orbits of $Y \in \mathcal{V} \cap G_2(T, S)$ and D_i the fundamental domain of the Poincaré map associated to $\alpha_i(Y)$ with $D_i \cap T^2 = \emptyset$. Consider for $\ell > 0$ the compact $K_\ell = \{x \in U \overline{D}_i : o_Y(x) \text{ is saddle-separatrix with } length \leq \ell\}$. For $x \in K_\ell, o_Y(x) = \gamma, \omega(x) = \alpha_i(Y), \alpha(x) = p$, consider the above neighborhoods U, \mathcal{U} such that for $Z \in \mathcal{U}$ the connections between saddle $q \in U$ and $\alpha_i(Z)$ by $\overline{U} \cap \overline{V}$ have $length > \frac{2}{3} length \gamma + m$ (m in the beginning of this section) and the connections between saddle and $\alpha_j(Z)$ by $\overline{U} \cap \overline{V}$ have $length > \max(S, (\frac{4}{3})^k m)$ where k is a natural number > 1 such that $(\frac{4}{3})^{k-1} m \leq R < (\frac{4}{3})^k m$. Beginning with $\ell = \frac{4}{3} m$, cover the compact $\overline{D}_i \cap K_\ell$ with a finite number of open U_s^i such that $(\bigcup_{i \neq j} \overline{U}_s^i) \cap (\bigcup_{i \neq j} \overline{U}_s^j) = \emptyset$. For each i we perturb the vector

field in these neighborhoods without change in $S^3 - U \cup U_S^i$ to have a finite number of connections between $\alpha_i(Y)$ and saddle point with $length \leq \ell = \frac{4}{3} m$, which are quasi-generic. By a small change we obtain a vector field Z with, at most, one such connection in each leaf where the saddle connections between critical points have $length > S$ and the periodic orbits with period $\tau, 0 \leq \tau \leq T$ are hyperbolic. For each connection γ_i between saddle and saddle type orbit $\alpha_i(Z)$ with $length \leq \frac{4}{3} m$ consider the neighborhoods U^i such that $\bar{U}^i \cap \bar{U}^j = \emptyset$, the connections between saddle and α_j ($i \neq j$) by $\bar{U}^i \cap \bar{V}$ have $length > \max(S, (\frac{4}{3})^k m)$ and between saddle $q \notin U^i$ and α_i , have $length > S + \frac{2}{3} m > (\frac{4}{3})^2 m$ since the connections between saddles and between saddle and saddle node in F_{γ_i} have $length > S$. Repeat the initial process for $\ell = (\frac{4}{3})^2 m$ perturbing the connections between saddle and saddle type orbit of $length \tau, \frac{4}{3} m < \tau \leq (\frac{4}{3})^2 m$ without breaking the initial perturbation; thus successively until $\ell = (\frac{4}{3})^k m$. \square

Given $L > 0$ we call $\mathcal{V}(X, T, S, R, L)$ the set of the vector fields $Y \in \mathcal{V}(X, T, S, R)$ whose connections between two saddle type orbits with $length \leq L$ are quasi generic and lie at most one in each leaf with the additional condition that the quasi-generic periodic orbits, the saddle connections between singular points and between saddle and saddle type orbit of such a leaf have period $> T$, $length > S$ and $> R$ respectively.

Lemma 4: Let $Y \in \mathfrak{V}(X, T, S, R, L)$ and $0 < L \leq R \leq S$. Then Y has a finite number of connections between saddle type orbits with *length* $\leq L$.

Proposition 2: $\mathfrak{V}(X, T, S, R, L)$ is open and dense in $\mathfrak{V}(X, T, S, R)$ for $1 \leq L \leq R < S$.

Proof: Take $Y \in \mathfrak{V}(X, T, S, R)$ and keep notation of the proof of proposition 1. Consider the compact $F = \{x \in U \bar{D}_i : \alpha_Y(x) \text{ is a connection between saddle type orbits with } \textit{length} \leq L\}$. Call $\alpha_j(Y)$ the saddle type orbits which are attractor sets on T^2 . For each $x \in K \cap \bar{D}_j$ consider convenient open neighborhoods $U^j \subset S^3 - V$ of a segment of $\alpha_Y(x)$ and U^j of Y in $\mathfrak{V}(X, T, S, R)$ such that if $\alpha(x) = \alpha_j(Y)$ and $\omega(x) = \alpha_{k_j}(Y)$, then, any trajectory by \bar{U}^j not connecting $\alpha_j(Z)$ and $\alpha_{k_j}(Z)$, has *length* $> L$. From the compacity of $F \cap \bar{D}_i$ we have a finite number of such neighborhoods U_k^j with $(U_{\bar{k}}^j)_{i \neq j} \cap (U_{\bar{k}}^i) = \emptyset$. Begin changing the vector field in $U_{\bar{k}}^1$ such that any connection between $\alpha_1(Y)$ and saddle type orbit with *length* $\leq L$ is quasi generic. Repeating the process for $j = 2, 3, \dots$ we have a vector field whose saddle type orbits admit only a finite number of such connections. For a small change we have a vector field in $\mathfrak{V}(X, T, S, R, L)$. \square

The theorem follows by considering

$$G_3 = \bigcup_{X \in G_1} \left(\bigcap_{m, n \in \mathbb{N}} \mathfrak{V}(X, m, n+2, n+1, n) \right)$$

3. ON THE NONWANDERING SET

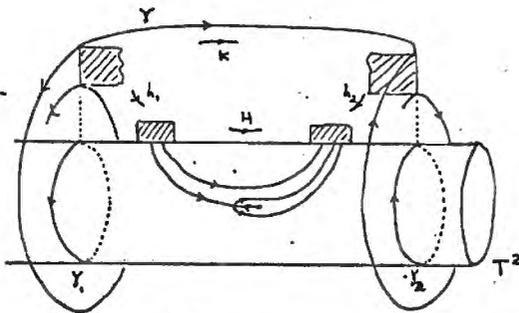
Let γ be a quasi-generic connection between two saddle type periodic orbits γ_1, γ_2 of $X \in G_1$ with $\omega(\gamma) = \gamma_1, \alpha(\gamma) = \gamma_2$. Call P_1, P_2 the Poincaré maps of $X, (-X)$ associated to γ_1 and γ_2 respectively. In coordinates (see [5]) we have $P_1(x, y, X) = (\lambda_1 x, g_X(y))$, $P_2(x, y, X) = (\lambda_2 x, g_X(y))$ where $g_X \in C^\infty$, $g_X(0) = g_X^{(n)}(0) = 0$ for $n > 1$, $g_X'(0) = 1$. Suppose $\lambda_1 > \lambda_2 > 1$ then $g_X(y) < y$ for $y > 0$.

For $x > 0$ small enough and $y \in]g_X(1), 1]$ define $h_1(x, y, X) = (\lambda_1 x^{n_1(x, y, X)}, g_X^{n_1(x, y, X)}(y))$ where $0 < a \leq \lambda_1 x < \lambda_1 a$

$h_2(x, y, X) = (\lambda_2 x^{n_2(x, y, X)}, g_X^{n_2(x, y, X)}(y))$ where

$0 < a \leq \lambda_2 x < \lambda_2 a$

Suppose $a = 1$.



The next results stated for X are obviously extended for a neighborhood of X in G_1 .

Lemma 5: Under the above conditions, $\pi_2 h_1(x, y, X) >$
 $> \pi_2 h_2(x, y, X)$ for $x > 0$ small enough and
 $y \in]g_X(1), 1]$ where $\pi_2(x, y) = y$.

Proof: Given a natural $N > 0$ we have $n_2(x, y, X) - n_1(x, y, X) > N$
 for small $x > 0$ and $y \in]g_X(1), 1]$ since
 $\frac{\ln 1/x}{\ln \lambda_i} \leq n_i(x, y, X) < \frac{\ln \lambda_i/x}{\ln \lambda_i}$, $i = 1, 2$. \square

Suppose $p \in T^2$, $\alpha(p) = \gamma_1$, $\omega(p) = \gamma_2$. On certain open subsets of $\{(x, y) : x > 0, y \in]g_X(1), 1]\}$, we can define Hh_1, h_2K where H and K are given by the flows of X and $(-X)$ respectively. With the notation of lemma 5 we have

$$-1 < n_2(x, y, X) - n_2(K(x, y), X) - \frac{\ln K_1(x, y) / x}{\ln \lambda_2} < 1$$

where $K_1(x, y) = \pi_1 K(x, y)$. From $\frac{\partial K_1}{\partial x}(0, y) > a > 0$ for any $y \in]g_X(1), 1]$ we deduce the existence of a natural number m such that $n_2(K(x, y), X) = n_2(x, y, X) + i$, $-m \leq i \leq m$
 for small x . From $\pi_2 H h_1(x, y, X) = g_X^{n_1(x, y, X) + k}(y)$, $k \geq 0$
 and lemma 5 we have $\pi_2 h_2(K(x, y), X) = g_X^{n_2(K(x, y), X)} <$
 $< \pi_2 H h_1(x, y, X)$ for x small enough. Then

Lemma 6: Under the hypothesis above, γ and the canonical component of $X|_{T^2}$ containing p are contained in $\hat{\Omega}(X)$.

Lemma 7: Under the same hypothesis, $\gamma \subset \overline{\text{Per } X}$

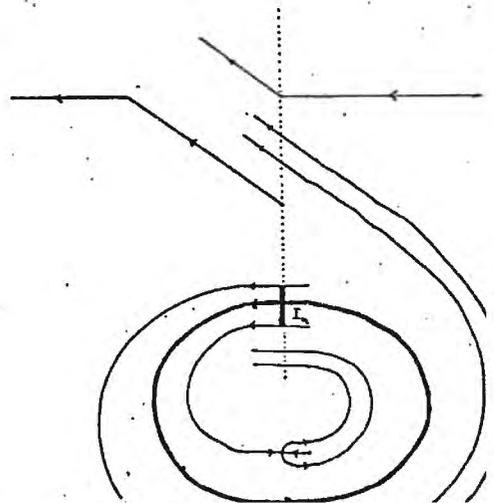
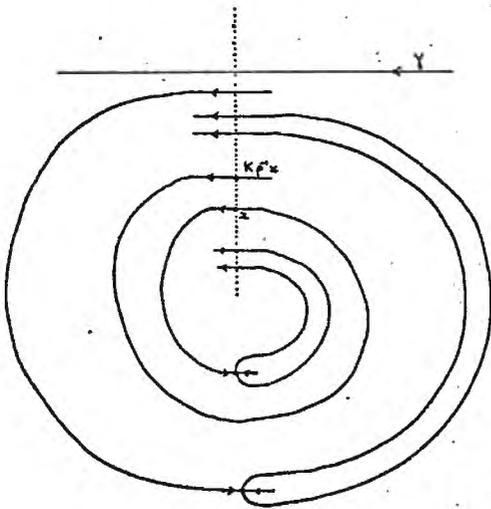
Proof: For $\rho(x,y) = \pi_1 h_2^{-1} H h_1(x,y)$ and x_1, x_2 small enough with $\pi_2 h_1(x_1,y) = \pi_2 h_1(x_2,y)$ we have

$$\left| \frac{\rho(x_1,y) - \rho(x_2,y)}{x_1 - x_2} \right| \geq M \left(\frac{\lambda_1}{\lambda_2} \right)^{n_1(x_1,y)}, \quad M > 0 \text{ constant}$$

and then,

$$|K\rho^{-1}(x_1,y) - K\rho^{-1}(x_2,y)| \leq c |x_1 - x_2|, \quad 0 < c < 1$$

Thus, for $y_n \rightarrow y$ we deduce the existence of intervals I_n where $K\rho^{-1}$ is a contraction, hence, the existence of a hyperbolic fixed point in I_n



F_y

REFERENCES:

- [1] G. Reeb, Sur certains propriétés topologiques des variétés feuilletées, Actualités Sci. Ind. 1183, Herman, Paris (1952).
- [2] M.M. Peixoto, Structural Stability in two dimensional manifolds, Topology 1 (1962).
- [3] J. Sotomayor, Generic one parameter families of vector fields on two-dimensional manifolds, I.H.E.S.. Publ. Math. 43 (1974).
- [4] E.M. Sallum, Vector fields tangent to a Reeb foliation on S^3 , J. Diff. Equation, 34 (1979).
- [5] L.A.F. de Oliveira, Um caso de estabilidade local não detectável por jatos, tese de mestrado, IME-USP, (1978).

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