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Charged exotic charmonium states

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In this review, we present and discuss all the experimental information about the charged exotic charmonium states, which have been observed over the last five years. We try to understand their properties such as masses and decay widths with QCD sum rules. We describe this method, show the results and compare them with the experimental data and with other theoretical approaches.

Keywords: Exotic states; non-perturbative methods.

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1. Introduction

Since its first observation in 2003 by the Belle Collaboration,[1] the $X(3872)$ has attracted the interest of all the hadronic community. It is the most well studied state among the new charmonium states and has been confirmed by five collaborations: CDF,[2] D0,[3] BaBar,[4] LHCb[5] and CMS.[6] There is little doubt in the community that the $X(3872)$ structure is more complex than just a $c\bar{c}$ state. Besides the $X(3872)$, the other recently observed charmonium states that clearly have a more complex structure than $c\bar{c}$ are the charged states. Up to now, there are some experimental evidences for seven charged states, which are shown in Table 1.

The first charged charmonium state, the $Z^+(4430)$, was observed by the Belle Collaboration in 2008, produced in $B^+ \to K(\psi'\pi^+)$.[7] However, the BaBar Collaboration[8] searched for the $Z^-(4430)$ signature in four decay modes and concluded that there is no significant evidence for a signal peak in any of these processes. Very recently, the Belle Collaboration has confirmed the $Z^+(4430)$ observation and has determined the preferred assignment of the quantum numbers to be $J^P = 1^{+1}$.[9] Curiously, there are no reports of a Z^+ signal in the $J/\psi\pi^+$ decay channel.

Table 1. Charged exotic charmonium states.

State (mass)	Experiment (year)	J^P	Decay mode	Ref.
$Z^+(4430)$	BELLE (2008)	1^+	$B^+ \to K\psi'\pi^+$	7
$Z_1^+(4050)$	BELLE (2008)	?	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	10
$Z_2^+(4250)$	BELLE (2008)	?	$\bar{B}^0 \to K^-\pi^+\chi_{c1}$	10
$Z_c^+(3900)$	BESIII (2013)	1^+	$Y(4260) \to (J/\psi\pi^+)\pi^-$	12
$Z_c^+(4025)$	BESIII (2013)	?	$e^+e^- \to (D^*\bar{D}^*)^\pm\pi^\mp$	15
$Z_c^+(4020)$	BESIII (2013)	?	$e^+e^- \to (\pi^+h_c)\pi^-$	16
$Z_c^+(3885)$	BESIII (2013)	?	$e^+e^- \to (D\bar{D}^*)^\pm\pi^\mp$	17

The $Z^+(4430)$ observation motivated further studies of other $\bar{B}^0$ decays. The Belle Collaboration has reported the observation of two resonance-like structures, called $Z_1^+(4050)$ and $Z_2^+(4250)$, in the exclusive process $\bar{B}^0 \to K^-\pi^+\chi_{c1}$, in the $\pi^+\chi_{c1}$ mass distribution.[10] Once again the BaBar Collaboration did not confirm these observations.[11]

After these non-confirmations, it was with great excitement that the hadron community heard about the observation of the $Z_c^+(3900)$. The $Z_c^+(3900)$ was first observed by the BESIII Collaboration in the $(\pi^\pm J/\psi)$ mass spectrum of the $Y(4260) \to J/\psi\pi^+\pi^-$ decay channel.[12] This structure, was also observed at the same time by the Belle Collaboration[13] and was confirmed by the authors of Ref. 14 using CLEO-c data.

Soon after the $Z_c^+(3900)$ observation, the BESIII related the observation of other three charges states: $Z_c^+(4025)$,[15] $Z_c^+(4020)$[16] and $Z_c^+(3885)$.[17] Up to now, it is not clear if the states $Z_c^+(3900)$–$Z_c^+(3885)$ and the states $Z_c^+(4025)$–$Z_c^+(4020)$ are the same states seen in different decay channels, or if they are independent states.

All these charged states cannot be $c\bar{c}$ states and they are natural candidates for molecular or tetraquark states. These exotic states are allowed by the strong interactions, both at the fundamental level and at the effective level, and their absence in the experimentally measured spectrum has always been a mystery. The theoretical tools to address these questions are lattice QCD, chiral perturbation theory, QCD sum rules (QCDSR), effective lagrangian approaches and quark models. For more details, we refer the reader to the more comprehensive Ref. 18 and to the more recent and also more specific Ref. 19 review articles.

In this rapidly evolving field, periodic accounts of the status of theory and experiment are needed. There are already several reviews of the recent charmonium spectroscopy. The present one is focused on the charged states and on the QCDSR approach to them. In the following sections, we discuss some of these new charmonium states using the QCDSR approach.

2. QCD Sum Rules

The method of the QCDSR is a powerful tool to evaluate the masses and decay widths of hadrons based on first principles. It was first introduced by Shifman,

Vainshtein and Zakharov[20] to the study of mesons, and was latter extended to baryons by Ioffe[21] and Chung *et al.*[22,23] Since then, the QCDSR technique has been applied to study numerous hadronic properties with various flavor content and has been discussed in many papers[24–28] emphasizing different aspects of the method. The method is based on identities between two- or three-point correlation functions, which connect hadronic observables with QCD fundamental parameters, such as quark masses, the strong coupling constant, and quantities which characterize the QCD vacuum, i.e. the condensates. The correlation function is of a dual nature: it represents a quark–antiquark fluctuation for short distances (or large momentum) and can be treated in perturbative QCD, while at large distances (or small momentum) it can be related to hadronic observables. The sum rule calculations are based on the assumption that in some range of momentum, both descriptions are equivalent. One, thus, proceeds by calculating the correlation function for both cases and by eventually equating them to obtain information on the properties of the hadrons.

In principle, QCDSR allow first-principle calculations. In practice, however, in order to extract results, it is necessary to make expansions, truncations, and other approximations that may reduce the power of the formalism and introduce large errors. However, if one can find ways to control these errors, the method can provide important informations about the structure of the hadrons.

2.1. *Hadron masses*

The QCDSR calculations of the mass of a hadronic state are based on the correlator of two hadronic currents. A generic two-point correlation function is given by

$$\Pi(q) \equiv i \int d^4x \, e^{iq\cdot x} \langle 0|T[j(x)j^\dagger(0)]|0\rangle \,, \tag{1}$$

where $j(x)$ is a current with the quantum numbers of the hadron we want to study. In the QCDSR approach, the correlation function is evaluated in two different ways: at the quark level in terms of quark and gluon fields and at the hadronic level introducing hadron characteristics such as the mass and the coupling of the hadronic state to the current $j(x)$.

The hadronic side, or phenomenological side of the sum rule is evaluated by writing a dispersion relation to the correlator in Eq. (1):

$$\Pi^{\rm phen}(q^2) = -\int ds \frac{\rho(s)}{q^2 - s + i\epsilon} + \cdots \,, \tag{2}$$

where ρ is the spectral density given by the absorptive part of the correlator and the dots represent subtraction terms.

Since the current j ($j^\dagger$) is an operator that annihilates (creates) all hadronic states that have the same quantum numbers as j, $\Pi(q)$ contains information about all these hadronic states, including the low mass hadron of interest. In order for the QCDSR technique to be useful, one must parametrize $\rho(s)$ with a small number of

parameters. In general, one parametrizes the spectral density as a single sharp pole representing the lowest resonance of mass m, plus a smooth continuum representing higher mass states:

$$\rho(s) = \lambda^2 \delta(s - m^2) + \rho_{\rm cont}(s)\,, \tag{3}$$

where λ gives the coupling of the current with the low mass hadron, H: $\langle 0|j|H\rangle = \lambda$. With this ansatz, the phenomenological side of the sum rule becomes:

$$\Pi^{\rm phen}(q^2) = -\frac{\lambda^2}{q^2 - m^2} - \int_{s_{\rm min}}^{\infty} ds \frac{\rho_{\rm cont}(s)}{q^2 - s + i\epsilon} + \cdots\,. \tag{4}$$

In the QCD side, or OPE side, the correlation function is evaluated by using the Wilson's operator product expansion (OPE):[29]

$$\Pi^{\rm OPE}(q) = \sum_n C_n(Q^2)\hat{O}_n\,, \tag{5}$$

where the set $\{\hat{O}_n\}$ includes all local gauge invariant operators expressible in terms of the gluon fields and the fields of light quarks, which are represented in the form of vacuum condensates. The lowest dimension condensates are the quark condensate of dimension three: $\hat{O}_3 = \langle\bar{q}q\rangle$, and the gluon condensate of dimension four: $\hat{O}_4 = \langle g^2 G^2\rangle$. The lowest-dimension operator with $n = 0$ is the unit operator associated with the perturbative contribution.

For non-exotic mesons, i.e. normal quark–antiquark states, such as ρ and J/ψ, the contributions of condensates with dimensions higher than four are suppressed by large powers of $1/Q^2$. Therefore, the expansion in Eq. (5) can be safely truncated after dimension four condensates, even at intermediate values of Q^2 (~ 1 GeV^2). However, for molecular or tetraquark states, higher dimension condensates like the dimension five mixed-condensate: $\hat{O}_5 = \langle\bar{q}g\sigma\cdot Gq\rangle$, the dimension six four-quark condensate: $\hat{O}_6 = \langle\bar{q}q\bar{q}q\rangle$ and even the dimension eight quark condensate times the mixed-condensate: $\hat{O}_8 = \langle\bar{q}q\bar{q}g\sigma\cdot Gq\rangle$, can play an important role. The three-gluon condensate of dimension-six: $\hat{O}_6 = \langle g^3 G^3\rangle$ can be safely neglected, since it is suppressed by the loop factor $1/16\pi^2$.

The precise evaluation of the $D = 6$, $\hat{O}_6$, and $D = 8$, $\hat{O}_8$, condensates require a involved analysis including a nontrivial choice of factorization scheme.[30] Therefore, in our calculations, we assume that their vacuum saturation values are given by:

$$\langle\bar{q}q\bar{q}q\rangle = \langle\bar{q}q\rangle^2\,, \qquad \langle\bar{q}q\bar{q}g\sigma\cdot Gq\rangle = \langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle\,. \tag{6}$$

The OPE side can also be written in terms of a dispersion relation as:

$$\Pi^{\rm OPE}(q^2) = -\int_{s_{\rm min}}^{\infty} ds \frac{\rho^{\rm OPE}(s)}{q^2 - s + i\epsilon} + \cdots\,, \tag{7}$$

where

$$\rho^{\rm OPE}(s) = \frac{1}{\pi}\,{\rm Im}[\Pi^{\rm OPE}(s)]\,. \tag{8}$$

To keep the number of parameters as small as possible, in general, in the QCDSR approach, one assumes that the continuum contribution to the spectral density, $\rho_{\rm cont}(s)$ in Eq. (4), vanishes below a certain continuum threshold, s_0. Above this threshold, one uses the ansatz

$$\rho_{\rm cont}(s) = \rho^{\rm OPE}(s)\Theta(s - s_0)\,. \tag{9}$$

Using Eq. (9) in Eq. (4), we get

$$\Pi^{\rm phen}(q^2) = -\frac{\lambda^2}{q^2 - m^2} - \int_{s_0}^{\infty} ds \frac{\rho^{\rm OPE}(s)}{q^2 - s + i\epsilon} + \cdots\,. \tag{10}$$

To improve the matching of the two descriptions of the correlator, one applies the Borel transformation. The Borel transformation removes the subtraction terms in the dispersion relation, and exponentially suppresses the contribution from excited resonances and continuum states in the phenomenological side. In the OPE side, the Borel transformation suppresses the contribution from higher dimension condensates by a factorial term.

After performing a Borel transform on both sides of the sum rule, and transferring the continuum contribution to the OPE side, the sum rule can be written as

$$\lambda^2 e^{-m^2/M^2} = \int_{s_{\rm min}}^{s_0} ds\, e^{-s/M^2} \rho^{\rm OPE}(s)\,. \tag{11}$$

A good sum rule is obtained in the case that one can find a range of M^2 called Borel window, in which the two sides have a good overlap and information on the lowest resonance can be extracted. To determine the allowed Borel window, one analyses the OPE convergence and the pole contribution: the minimum value of the Borel mass is fixed by considering the convergence of the OPE, and the maximum value of the Borel mass is determined by imposing the condition that the pole contribution must be bigger than the continuum contribution.

The mass of the hadronic state, m, can be obtained by taking the derivative of Eq. (11) with respect to $1/M^2$, and dividing the result by Eq. (11):

$$m^2 = \frac{\int_{s_{\rm min}}^{s_0} ds\, e^{-s/M^2} s\rho^{\rm OPE}(s)}{\int_{s_{\rm min}}^{s_0} ds\, e^{-s/M^2} \rho^{\rm OPE}(s)}\,. \tag{12}$$

Using the formalism described above, we can compute the masses of the new states. A compilation of results of the states discussed here is shown in Table 2. These numbers will be discussed in detail in the next sections.

2.2. *Hadron decay widths*

The QCDSR calculations for the coupling constant in a hadronic vertex are based on the correlator of three hadronic currents. A generic three-point correlation function associated with a vertex of three mesons M_1, M_2 and M_3 is given by

$$\Gamma(p, p', q) = \int d^4x\, d^4y\, e^{ip'\cdot x}\, e^{-iq\cdot y} \langle 0|T\{j_3(x) j_2^\dagger(y) j_1^\dagger(0)\}|0\rangle\,, \tag{13}$$

Table 2. Masses obtained with QCDSR.

State	J^{PC}	Current	Mass	Ref.
$X(3872)$	1^{++}	Tetraquark	(3.92 ± 0.13) GeV	42
$X(3872)$	1^{++}	$D\bar{D}^*$ Molecule	(3.87 ± 0.07) GeV	43
$Z^+(4430)$	0^-	$D^*\bar{D}_1$ Molecule	(4.40 ± 0.10) GeV	53
$Z^+(4430)$	0^-	Tetraquark	(4.52 ± 0.09) GeV	54
$Z^+(4430)$	1^-	Tetraquark	(4.84 ± 0.14) GeV	54
$Z_1^+(4020)$	0^+	$D^*\bar{D}^*$ Molecule	(4.15 ± 0.12) GeV	59
$Z_2^+(4250)$	1^-	$D_1\bar{D}$ Molecule	(4.19 ± 0.22) GeV	59
$Z_c^+(3930)$	1^+	Tetraquark	(3.92 ± 0.13) GeV	64
$Z_c^+(4025)$	1^+	$D^*\bar{D}^*$ Molecule	(3.950 ± 0.105) GeV	69
$Z_c^+(4025)$	2^+	$D^*\bar{D}^*$ Molecule	(3.946 ± 0.104) GeV	69

where $q = p' - p$ and the current j_i represents states with the quantum numbers of the meson i. As in the case of the two-point correlation function, the function in Eq. (13) is evaluated in two ways. In the OPE side, we consider that the currents are composed by quarks and we use the Wilson's OPE to evaluate the correlation function. In the phenomenological side, we insert, in Eq. (13), intermediate states for the mesons M_1, M_2 and M_3. We then write the correlation function in terms of the coupling of these mesons with the corresponding currents, and in terms of the form factor, $g_{M_1M_2M_3}(q^2)$, in the hadronic vertex, which is defined by the generalization of the on-mass-shell matrix element, $\langle M_3M_2|M_1\rangle$, for an off-shell M_2 meson:

$$\langle M_3(p')M_2(q)|M_1(p)\rangle = g_{M_1M_2M_3}(q^2) f_{M_1,p} f_{M_2,p'} f_{M_3,q}\,, \tag{14}$$

which can be extracted from the effective Lagrangian that describes the coupling between these three mesons. In Eq. (14), the functions $f_{M_i,k}$ are obtained from the Lagrangian and are related with the quantum numbers of the meson M_i. After evaluating both sides separately, we equate one description with the other and we can extract the form factor from the sum rule.

The coupling constant is defined as the value of the form factor at the meson pole: $Q^2 = -m_2^2$, where m_2 is the mass of the meson M_2 that was off-shell. Very often, in order to determine the coupling constant, we have to extrapolate the QCDSR results to a Q^2 region where the sum rules are no longer valid (since the QCDSR results are valid in the deep Euclidian region). To do that, in general, we parametrize the QCDSR results through a analytical form, like a monopole or an exponential function. For more details we refer the reader to Ref. 31.

3. $X(3872)$

The $X(3872)$ was first observed by Belle Collaboration in 2003 in the decay $B^+ \to X(3872)K^+ \to J/\psi\pi^+\pi^-K^+$,[1] and has been confirmed by other five

collaborations.[2–6] The current world average mass is $m_X = (3871.68 \pm 0.17)$ MeV and its total width is less than 1.2 MeV.[32] The LHCb Collaboration determined $J^{\rm PC} = 1^{++}$ quantum numbers with more than 8σ significance.[33]

3.1. *Mass*

Calculations using constituent quark models give masses for possible charmonium states, with $J^{\rm PC} = 1^{++}$ quantum numbers, which are much bigger than the observed $X(3872)$ mass: $2\ ^3P_1(3990)$ and $3\ ^3P_1(4290)$.[34] These results, together with the coincidence between the X mass and the $D^{*0}D^0$ threshold: $M(D^{*0}D^0) = (3871.81 \pm 0.36)$ MeV,[35] inspired the proposal that the $X(3872)$ could be a molecular $(D^{*0}\bar{D}^0 + \bar{D}^{*0}D^0)$ bound state with small binding energy.[36–40]

Other interesting possible interpretation of the $X(3872)$, first proposed by Maiani *et al.*,[41] is that it could be a tetraquark state resulting from the binding of a diquark and an antidiquark.

The first QCDSR calculation of the mass of the $X(3872)$ considered as a $J^{\rm PC} = 1^{++}$ tetraquark state was done in Ref. 42. Following this calculation, a $J^{\rm PC} = 1^{++}$, $D^*\bar{D}$ molecular current was considered in Ref. 43. The corresponding interpolating currents used in these calculations are:

$$j_\mu^{\rm di} = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(q_a^T C\gamma_5 c_b)(\bar{q}_d\gamma_\mu C\bar{c}_e^T) + (q_a^T C\gamma_\mu c_b)(\bar{q}_d\gamma_5 C\bar{c}_e^T)]\,, \tag{15}$$

for a tetraquark current, and

$$j_\mu^{\rm mol} = \frac{1}{\sqrt{2}}[(\bar{q}_a\gamma_5 c_a)(\bar{c}_b\gamma_\mu q_b) - (\bar{q}_a\gamma_\mu c_a)(\bar{c}_b\gamma_5 q_b)]\,, \tag{16}$$

for a molecular $D\bar{D}^*$ current. In Eqs. (15) and (16), q denotes a u or d quark.

In the OPE side, the calculations were done at leading order in α_s and contributions of condensates up to dimension eight were included. In both cases, it was possible to find a Borel window where the pole contribution is bigger than the continuum contribution and with a reasonable OPE convergence.

The mass obtained in Ref. 42, considering the allowed Borel window and the uncertainties in the parameters, was $m_X = (3.92 \pm 0.13)$ GeV whereas the result for the mass obtained in Ref. 43 was $m_X = (3.87 \pm 0.07)$ GeV, as shown in Table 2.

We see that, in both cases, a good agreement with the experimental mass was obtained. Up to now, there are many QCDSR calculations[28] of the mass of the $X(3872)$ considering different currents and in all cases, good agreement with the experimental mass is found. Even with a mixed charmonium-molecular current, the value obtained for the mass does not change significantly.[44] These calculations only confirm the result presented in Ref. 45 that shows that the calculation of the mass of a given state, in the QCDSR approach, is very insensitive to the choice of the current. However, this may not be the case for the decay width.[45]

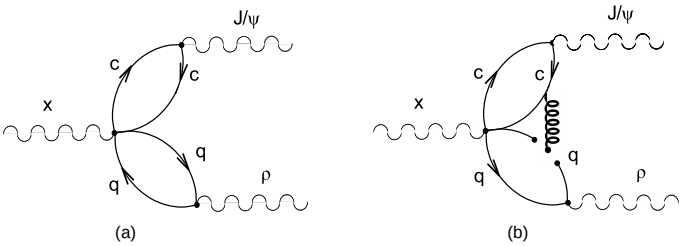

Fig. 1. Generic decay diagrams of the $X(3872) \to J/\psi\rho$ decay.

3.2. *Decay width*

The first QCDSR calculation of the width of the $X(3872)$ was done in Ref. 46. In particular, in Ref. 46, the $X(3872)$ was considered as a tetraquark state described by the current in Eq. (15) and a very large decay width was obtained: $\Gamma(X \to J/\psi\rho \to J/\psi\pi^+\pi^-) = (50 \pm 15)$ MeV. A similar width was obtained in Ref. 44 with a molecular current such as the one in Eq. (16). Indeed, large partial decay widths are expected when the coupling constant is obtained from QCDSR, in the case of multiquark states, when the initial state contains the same number of valence quarks as the number of valence quarks in the final state. An example is the case of the light scalars σ and κ studied in Ref. 47, whose widths are of the order of 400 MeV. In the case of the $X \to J/\psi\rho$ decay, the generic decay diagram in terms of quarks has two "petals", one associated with the J/ψ and the other with the ρ. Among the possible diagrams, there are two distinct subsets. Diagrams with no gluon exchange between the petals, as the one shown in Fig. 1(a), and therefore, no color exchange between the two final mesons in the decay. If there is no color exchange, the final state containing two color singlets was already present in the initial state. In this case, the tetraquark had a component similar to a $J/\psi - \rho$ molecule. The other subset of diagrams is the one where there is a gluon exchange between the petals, as the one shown in Fig. 1(b). This type of diagram represents the case where the X is a genuine four-quark state with a complicated color structure. These diagrams are called color-connected (CC). Considering only the CC diagrams in the calculation, the decay width obtained in Ref. 46 was:

$$\Gamma_{\rm CC}(X \to J/\psi\,\rho \to J\psi\pi^+\pi^-) = (0.7 \pm 0.2)\ {\rm MeV}\,, \tag{17}$$

in a very good agreement with the experimental upper limit.

This procedure may appear somewhat unjustified. However, if the initial state has a nontrivial color structure only CC diagrams should contribute to the calculation. Unfortunately, although the initial tetraquark current has a nontrivial color structure, it can be rewritten as a sum of molecular type currents with trivial color configuration through a Fierz transformation. This is the reason why the diagrams without gluon exchange between the two "petals" survive in the QCDSR calculation. Therefore, the approach of considering only CC diagrams can be considered as a form of simulating a real tetraquark state with nontrivial color structure.

Other possible approach to reduce the large width is to consider the $X(3872)$ as a mixture between a $c\bar{c}$ current and a molecular current, as done in Ref. 44:

$$J_\mu(x) = \sin(\alpha) j_\mu^{\rm mol}(x) + \cos(\alpha) j_\mu^2(x)\,, \tag{18}$$

with $j_\mu^{\rm mol}(x)$ given in Eq. (16) and

$$j_\mu^2(x) = \frac{1}{6\sqrt{2}}\langle\bar{q}q\rangle[\bar{c}_a(x)\gamma_\mu\gamma_5 c_a(x)]\,. \tag{19}$$

The necessity of mixing a $c\bar{c}$ component with the $D^0\bar{D}^{*0}$ molecule was already pointed out in some papers.[48–51] In particular, in Ref. 52, a simulation of the production of a bound $D^0\bar{D}^{*0}$ state with binding energy as small as 0.25 MeV, obtained a cross-section of about two orders of magnitude smaller than the prompt production cross-section of the $X(3872)$ observed by the CDF Collaboration. The authors of Ref. 52 concluded that S-wave resonant scattering is unlikely to allow the formation of a loosely bound $D^0\bar{D}^{*0}$ molecule in high energy hadron collision.

As discussed above, there is no problem in reproducing the experimental mass of the $X(3872)$, using the current in Eq. (18), for a wide range of the mixture angle α. However, the value of the $XJ/\psi\rho$ coupling constant and, therefore, the value of the $X \to J/\psi\ (n\pi)$ decay width, is strongly dependent on this angle. It was shown in Ref. 44 that for a mixing angle $\alpha = 9^0 \pm 4^0$, it is possible to describe the experimental mass of the $X(3872)$ with a decay width $\Gamma(X \to J/\psi(n\pi)) = (9.3 \pm 6.9)$ MeV, which is compatible with the experimental upper limit. Therefore, in a QCDSR calculation, the $X(3872)$ can be well described basically by a $c\bar{c}$ current with a small, but fundamental, admixture of molecular ($D\bar{D}^*$) or tetraquark ($[cq][\bar{c}\bar{q}]$) currents.

4. $Z^+(4430)$

This resonance was found by Belle Collaboration in the channel $B^+ \to K\psi'\pi^+$ and it was the first charged charmonium state observed, with mass $M = (4433^{+15+19}_{-12-13})$ MeV and width $\Gamma = (109^{+86+74}_{-43-56} \pm 18 \pm 30)$ MeV.[7] Curiously, there is no signal of this resonance in the $J/\psi\pi^+$ channel. Since the minimal quark content of this state is $c\bar{c}u\bar{d}$, this can only be achieved in a multiquark configuration.

The BaBar Collaboration searched the $Z^-(4430)$ in the four decay modes $B^+ \to K^0_S\psi'\pi^-$, $B^+ \to K^0_S J/\psi\pi^-$, $B^+ \to K^+\psi'\pi^0$ and $B^+ \to K^+ J/\psi\pi^0$. No significant evidence of a signal peak was found in any of the processes investigated.[8]

Since the $Z^+(4430)$ mass is close to the D^*D_1 threshold, it was suggested that it could be an S-wave threshold effect or a D^*D_1 molecular state. Considering the $Z^+(4430)$ as a weakly bound S-wave D^*D_1 molecular state, its quantum numbers may be $J^P = 0^-, 1^-, 2^-$. The 2^- assignment is probably suppressed in the $B^+ \to Z^+K$ decay because of the small phase space. Other possible interpretations are a tetraquark state, a cusp in the D^*D_1 channel, a baryonium state, a radially excited $c\bar{s}$ state and a hadrocharmonium state.[28]

There are QCDSR calculations for the $Z^+(4430)$ assuming that the state could have $J^P = 0^-$ or $J^P = 1^-$ quantum numbers.[53,54] In the first case, the obtained

masses were $m_{\rm mol}(0^-) = (4.40 \pm 0.10)$ GeV for a $D^* D_1$ molecular current[53] and $m_{\rm di}(0^-) = (4.52 \pm 0.09)$ GeV for a diquark-antidiquark current.[54] In the second assignment and for a diquark–antidiquark current, the obtained mass was $m_{\rm di}(1^-) = (4.84 \pm 0.14)$ GeV.[54] These numbers are displayed in Table 2.

From these results, the preliminary conclusion, at the time, was that the assignment $J^P = 1^-$ was disfavored and that the configuration $J^P = 0^-$, in both molecular and tetraquark states, would lead to a mass which is in agreement with the data. However, a recent reanalysis of the Belle data revealed that the favored quantum numbers are $J^P = 1^+$.[9] It is important to mention that soon after the $Z^+(4430)$ was first observed, Maiani *et al.* have suggested that the $Z^+(4430)$ could be the first radial excitation of a charged partner of the $X(3872)$, and therefore, would have $J^P = 1^+$ quantum numbers.[55] The existence of a charged partner of the $X(3872)$ was first proposed in Ref. 41.

Clearly, in view of the recent experimental reanalysis, if the $Z^+(4430)$ really exists, it could be a $\psi' \pi^+$ resonance or a tetraquark excitation, which invalidates a QCDSR calculation.

5. $Z_1^+(4050)$ and $Z_2^+(4250)$

After the observation of the $Z^+(4430)$ other $\bar{B}^0 \to K^- \pi^+ (c\bar{c})$ decays were carefully investigated. Two resonance-like structures, called $Z_1^+(4050)$ and $Z_2^+(4250)$, were observed by the Belle Collaboration in the exclusive process $\bar{B}^0 \to K^- \pi^+ \chi_{c1}$, in the $\pi^+ \chi_{c1}$ mass distribution.[10] The significance of each of the $\pi^+ \chi_{c1}$ structures exceeds 5σ and, since they were observed in the $\pi^+ \chi_{c1}$ channel, they must have the quantum numbers $I^G = 1^-$. Also in this case, the BaBar Collaboration did not confirm these observations.[11] When fitted with two Breit–Wigner resonance amplitudes, the resonance parameters are $m_1 = (4051 \pm 14^{+20}_{-41})$ MeV, $\Gamma_1 = (82^{+21+47}_{-17-22})$ MeV, $m_2 = (4248^{+44+180}_{-29-35})$ MeV and $\Gamma_2 = (177^{+54+316}_{-39-61})$ MeV.

Since the masses of the $Z_1^+(4050)$ and $Z_2^+(4250)$ are close to the $D^* \bar{D}^*(4020)$ and $D_1 \bar{D}(4085)$ thresholds, it is natural to interpret these states as molecular states or threshold effects. However, calculations using meson exchange models do not agree with each other. In Ref. 56, a strong attraction in the $D^* \bar{D}^*$ with $J^P = 0^+$ was found, while in Ref. 57 the interpretation of $Z_1^+(4050)$ as a $D^* \bar{D}^*$ molecule was not favored. In any case, it is very difficult to understand a bound molecular state whose mass is above the $D^* \bar{D}^*$ threshold. In Ref. 58, the interpretation of $Z_2^+(4250)$ as a $D_1 \bar{D}$ or $D_0 \bar{D}^*$ molecule was disfavored.

Soon after the observation of these states, QCDSR were used[59] to study the $D^* \bar{D}^*$ and $D_1 \bar{D}$ molecular states with $I^G J^P = 1^- 0^+$ and $1^- 1^-$ respectively. The currents used in both cases were of the type of Eq. (16). As shown in Table 2, for the $D^* \bar{D}^*$ system, the obtained mass was $m_{D^* D^*} = (4.15 \pm 0.12)$ GeV. Since the central value of the mass is around 130 MeV above the $D^* D^*(4020)$ threshold, we can conclude that there are repulsive interactions between the two D^* mesons. Therefore, it is not clear whether this structure is a resonance or not. For the $D_1 \bar{D}$

system, the obtained mass was $m_{D_1D} = (4.19 \pm 0.22)$ GeV. Here, in contrast to the previous case, the central value is around 100 MeV below the $D_1D(4285)$ threshold, and, considering the errors, consistent with the mass of the $Z_2^+(4250)$ resonance structure. Therefore, in this case, there seems to be an attractive interaction between the mesons D_1 and D and the molecular interpretation of this state seems more justified.

QCDSR estimate always contain some uncertainties. In the study of the masses of the charged Z states, part of the theoretical uncertainty comes from the width of the state. In most cases, the width is neglected. In the present case, when the width is included in the phenomenological side of the sum rule, the mass of the corresponding state increases.[60] It becomes then possible to obtain a mass $m_{D_1D} = 4.25$ GeV with a width $40 \leq \Gamma \leq 60$ MeV. Following the same trend, the mass of the $D^*\bar{D}^*$ molecule will be far from the $Z_1^+(4050)$ mass. In view of these facts, the authors of Ref. 60 concluded that it is possible to describe the $Z_2^+(4250)$ as a $D_1\bar{D}$ molecular state with $I^GJ^P = 1^-1^-$ quantum numbers. They also concluded that the $D^*\bar{D}^*$ state is probably a virtual state that is not related with the $Z_1^+(4050)$ resonance-like structure. Since the D^*D^* threshold (4020) is so close to the $Z_1^+(4050)$ mass and the $\eta_c''(3^1S_0)$ mass is predicted to be around 4050 MeV, the $Z_1^+(4050)$ is probably only a threshold effect.

6. $Z_c^+(3900)$

After the non-confirmed observations of $Z^+(4430)$, $Z_1^+(4050)$ and $Z_2^+(4250)$, only seen by Belle, the BESIII and Belle Collaborations reported the observation of a charged charmonium-like structure in the $M(\pi^\pm J/\psi)$ mass spectrum of the $Y(4260) \to J/\psi\pi^+\pi^-$ decay channel.[12,13] The existence of this structure, called $Z_c(3900)$, was promptly confirmed by the the authors of Ref. 14 using CLEO-c data.

In most of the theoretical calculations, it is relatively easy to reproduce the masses of the states. In the case of the $Z_c(3900)$, assuming SU(2) symmetry, the mass obtained in QCDSR for the Z_c is exactly the same one obtained for the $X(3872)$. As discussed in Sec. 3.2, it is, however, much more difficult to reproduce their measured decay widths. The $Z_c(3900)$ decay width represents a challenge to theorists. While its mass is very close to the $X(3872)$ mass, which may be considered its isosinglet partner, it has a much larger decay width. Indeed, while the $Z_c(3900)$ decay width is in the range 40–50 MeV, the $X(3872)$ width is smaller than 1.2 MeV.

This difference can be attributed to the fact that the $X(3872)$ may contain a significant $|c\bar{c}\rangle$ component,[44] which is absent in the $Z_c(3990)$. As pointed out in Ref. 61, this would also explain why the Z_c has not been observed in B decays.

According to the experimental observations, the $Z_c(3900)$ decays into $J/\psi\pi^+$ with a relatively large decay width. This is unexpected for a $D^* - \bar{D}$ molecular state, in which the distance between the D^* and the $\bar{D}$ is large. This decay must involve

Table 3. Coupling constants and decay widths in different channels.

Vertex	Coupling constant (GeV)	Decay width (MeV)
$Z_c^+(3900)J/\psi\pi^+$	3.89 ± 0.56	29.1 ± 8.2
$Z_c^+(3900)\eta_c\rho^+$	4.85 ± 0.81	27.5 ± 8.5
$Z_c^+(3900)D^+\bar{D}^{*0}$	2.5 ± 0.3	3.2 ± 0.7
$Z_c^+(3900)\bar{D}^0 D^{*+}$	2.5 ± 0.3	3.2 ± 0.7

the exchange of a charmed meson, which is a short range process and hence unlikely to occur in large systems. In Ref. 62 it was shown that, in order to reproduce the measured width, the effective radius must be $\langle r_{\rm eff}\rangle \simeq 0.4$ fm. This size scale is small and pushes the molecular picture to its limit of validity. In another paper,[63] the new state was treated as a charged $D^* - \bar{D}$ molecule and the authors explored its electromagnetic structure, arriving at the conclusion that its charge radius is of the order of $\langle r^2\rangle \simeq 0.11$ fm^2. Taking this radius as a measure of the spatial size of the state, we conclude that it is more compact than a J/ψ, for which $\langle r^2\rangle \simeq 0.16$ fm^2. In Ref. 64, the combined results of Refs. 62 and 63 were taken as an indication that the Z_c is a compact object, which may be better understood as a quark cluster, such as a tetraquark. Moreover, the $Z_c(3900)$ was interpreted as the isospin 1 partner of the $X(3278)$, as the charged state predicted in Ref. 41. Therefore, the quantum numbers for the neutral state in the isospin multiplet were assumed to be $I^G(J^{\rm PC}) = 1^+(1^{+-})$. The interpolating field for $Z_c^+(3900)$ used in Ref. 64 is given by Eq. (15) with the plus signal changed by a minus signal. The three-point QCDSR were used to evaluate the coupling constants in the vertices $Z_c^+(3900)J/\psi\pi^+$, $Z_c^+(3900)\eta_c\rho^+$, $Z_c^+(3900)D^+\bar{D}^{*0}$ and $Z_c^+(3900)\bar{D}^0 D^{*+}$. In all cases only color-connected diagrams were considered, since the $Z_c(3900)$ is expected to be a genuine tetraquark state with a nontrivial color structure. The obtained couplings, with the respective decay widths, are given in Table 3. A total width of $\Gamma = (63.0 \pm 18.1)$ MeV was found for the $Z_c(3900)$, in good agreement with the two experimental values: $\Gamma = (46 \pm 22)$ MeV from BESIII,[12] and $\Gamma = (63 \pm 35)$ MeV from BELLE.[13]

From the results in Table 3, it is possible to evaluate the ratio

$$\frac{\Gamma(Z_c(3900) \to D\bar{D}^*)}{\Gamma(Z_c(3900) \to \pi J/\psi)} = 0.22 \pm 0.12\,. \tag{20}$$

The QCDSR analysis performed in Ref. 65 also supports the identification of $X(3872)$ and $Z_c^+(3900)$ as the $J^{\rm PC} = 1^{++}$ and $J^{\rm PC} = 1^{+-}$ diquark–antidiquark type tetraquark states, respectively.

7. $Z_c^+(4025)$, $Z_c^+(4020)$ and $Z_c^+(3885)$: Are They Real?

Very recently, the BESIII Collaboration reported the observation of other three charges states: $Z_c^+(4025)$,[15] $Z_c^+(4020)$[16] and $Z_c^+(3885)$.[17]

In the BESIII set-up a reaction $e^+e^- \to (D^*\bar{D}^*)^\pm\pi^\mp$ was performed at $\sqrt{s} = 4.26$ GeV and a peak was seen in the $(D^*\bar{D}^*)^\pm$ invariant mass distribution just

about 10 MeV above the threshold. The peak was identified as a new particle, the $Z_c^+(4025)$.[15] The authors assume in the paper that the $(D^*\bar{D}^*)^\pm$ pair is created in a S-wave and then the $Z_c^+(4025)$ must have $J^P = 1^+$ to match, together with the pion, the quantum numbers $J^P = 1^-$ of the virtual photon from the e^+e^- pair. However, they also state that the experiment does not exclude other spin-parity assignments. Since the $(D^*\bar{D}^*)^\pm$ has charge, the isospin must be $I = 1$.

In parallel with the experimental works many theoretical papers were devoted to understand these new states. In Ref. 66, Heavy Quark Spin Symmetry (HQSS) was used to make predictions for states containing one D or D^* and one $\bar{D}$ or $\bar{D}^*$. Assuming the $X(3872)$ to be $D\bar{D}^*$ molecule, the authors found a series of new hadronic molecules, including the $Z_c^+(3900)$ and the $Z_c^+(4025)$. They would correspond to bound states (with uncertainties of about 50 MeV in the binding) of $D\bar{D}^*$ and $D^*\bar{D}^*$ respectively, with quantum numbers $I(J^P) = 1(1^+)$. Remarkably, even with uncertainties, these states always appear in the bound region. In Refs. 67 and 68, using QCDSR and assuming a structure of $D^*\bar{D}^*$, the authors obtained a possible $I(J^P) = 1(1^+)$ state compatible with the $Z_c^+(4025)$ albeit with around 250 MeV uncertainty in the energy. Recently,[69] a study of the $D^*\bar{D}^*$ system has also been done within QCDSR, projecting the correlation function on spin-parity 0^+, 1^+ and 2^+. In the three cases, a state with mass 3950 ± 100 MeV was found. The central value of the mass of these states is more in line with the results of Refs. 70 and 71, although with the error bar, they could as well be related to a resonance. In Ref. 72, the new Z_c states were investigated from a different perspective and, using pion exchange, a $D^*\bar{D}^*$ state with $I(J^P) = 1(1^+)$ compatible with the $Z_c(4025)$ was obtained. One should note that the input used in this latter work is quite different from the one in Ref. 66, since in HQSS, the pion exchange is subdominant. Finally, in Ref. 73, using a tetraquark structure and QCDSR, a state with $I(J^P) = 1(2^+)$ compatible with $Z_c(4025)$ was obtained, once again with a large error in the energy of 190 MeV. In a different analysis, in Ref. 74, a pion and the $D^*\bar{D}^*$ state are produced from the $X(4260)$ and the $D^*\bar{D}^*$ state is left to interact, while the pion remains a spectator (initial single-pion emission mechanism). Although it is not mentioned, whether the $D^*\bar{D}^*$ interaction produces a resonance with certain quantum numbers, the authors show that the mechanism can produce some enhancement in the $D^*\bar{D}^*$ invariant mass distribution just above threshold.

Bumps close to the threshold of a pair of particles should be treated with caution. Sometimes they are identified as new particles, but they can also be a reflection of a resonance below threshold. In a similar reaction, $e^+e^- \to J/\psi(D\bar{D})$, the Belle Collaboration reported[75] a bump close to the threshold in the $(D\bar{D})$ invariant mass distribution, which was tentatively interpreted as a new resonance. However, in Ref. 76, it was shown that the bump was better interpreted in terms of a $(D\bar{D})$ molecular state, below the $(D\bar{D})$ threshold (the so called $X(3700)$). Similarly, in Ref. 77, the $\phi\omega$ threshold peak measured[78] in the $J/\psi \to \gamma\phi\omega$ reaction was better interpreted as a signal of the $f_0(1710)$ resonance, below the $\phi\omega$ threshold, which couples strongly to $\phi\omega$. Further examples of this phenomenon may be found in

Ref. 79. In that paper, the theory of $D^*\bar{D}^*$ interactions is reviewed and it is pointed out that a $(D^*\bar{D}^*)$ state with a mass above the threshold is very difficult to support. In particular, in Ref. 71, it was found that there is only one bound state of $(D^*\bar{D}^*)$ in $I^G = 1^-$, with quantum numbers $J^{\rm PC} = 2^{++}$ with a mass around 3990 MeV and a width of about 100 MeV. Both mass and width are compatible with the reanalysis of data carried out in Ref. 79. Therefore, we can conclude that such $J^P = 2^+$ $D^*\bar{D}^*$ bound state provides a natural explanation for the state observed in Ref. 15.

An argument against the existence of a new resonance above the threshold is the fact that if the state were a $J^P = 1^+$ produced in S-wave, as assumed in the experimental work, it would easily decay into $J/\psi\pi$ exchanging a D meson in the t-channel. This is also the decay channel of the $Z_c(3900)$, which would then have the same quantum numbers as the state claimed in Ref. 15. However, while a peak is clearly seen in the $J/\psi\pi$ invariant mass distribution, in the case of the $Z_c(3900)$, no trace of a peak is seen around 4025 MeV, in spite of using the same reaction and the same e^+e^- energy.

Less than a month after the observation of the $Z_c^+(4025)$, the BESIII Collaboration reported the observation of the $Z_c^+(4020)$, a structure observed in the $h_c\pi^\pm$ mass spectrum.[16] The difference between the parameters of this structure and the $Z_c^+(4025)$, observed in the $D^*\bar{D}^*$ final state, is within 1.5σ and it is not clear whether they are the same state or not. The authors do not find a significant signal for $Z_c^+(3900) \to h_c\pi^\pm$.

Since the $Z_c^+(4025)$ and the $Z_c^+(4020)$ have almost the same mass and their quantum numbers were not yet accurately determined, we might think that they are, in fact, the same particle. Looking only at the most natural quantum numbers of the final states, the S-wave $D^*\bar{D}^*$ states have the quantum numbers $J^P = 0^+$, 1^+ and 2^+, while the S-wave $h_c\pi^\pm$ states have the quantum numbers $J^P = 1^-$. Therefore the $Z_c^+(4025)$ and $Z_c^+(4020)$ would be different particles. However, it is also possible to have a P-wave $h_c\pi^\pm$ system with quantum numbers $J^P = 0^+$ 1^+ and 2^+. In this case, the $Z_c^+(4025)$ and the $Z_c^+(4020)$ could be the same particle.

In the analysis presented in Ref. 80, the author concluded that QCDSR do not support the picture of $Z_c^+(4025)$ and $Z_c^+(4020)$ as diquark–antidiquark vector tetraquark states with $J^P = 1^-$. A short time later, in Ref. 81, the author concluded that, for these two states (treated as a single state), the QCDSR analysis supports the assignments $J^P = 1^+$ and $J^P = 2^+$ in a diquark–antidiquark configuration.

Shortly after the observation of the $Z_c^+(4020)$, the same collaboration reported the measurement of the $Z_c^+(3885)$, a charged structure observed in the $(D\bar{D}^*)\pm$ invariant mass distribution.[17] The mass and width of this structure are 2σ and 1σ, respectively, below those of the $Z_c^+(3900)$. The angular distribution of the $\pi Z_c(3885)$ system favors the $J^P = 1^+$ assignment and disfavors $J^P = 1^-$ or $J^P = 0^-$. Regarding the fact that this state could be the $Z_c(3900)$, seen in a different decay channel, the only comment from the experimental side is that if the $Z_c^+(3900)$ and $Z_c^+(3885)$ are the same state, then the ratio

$$\frac{\Gamma(Z_c(3885) \to D\bar{D}^*)}{\Gamma(Z_c(3900) \to \pi J/\psi)} = 6.2 \pm 1.1 \pm 2.7\,, \tag{21}$$

is determined.[17]

Comparing the results in Eqs. (20) and (21) we can conclude that, if the ratio in Eq. (21) is confirmed, the states $Z_c^+(3900)$ and $Z_c^+(3885)$ are not the same state.

Here again, as in the case of the $Z_c^+(4025)$ discussed above, it is possible that the $Z_c^+(3885)$) is not a real state but a manifestation of a resonance with a mass below the $(D\bar{D}^*)$ threshold. This point remains to be clarified.

8. Towards a New Spectroscopy

The proliferation of new charmonium states motivates attempts to group them into families. One possible way to organize some of the charmonium and bottomonium new states was suggested in Ref. 82 and it is summarized in Fig. 2. In this figure, we present the charm and bottom spectra in the mass region of interest. On the left (right), we show the charm (bottom) states with their mass differences in MeV. The comparison between the two left lines with the two lines on the right emphasizes the similarity between the spectra. In the bottom of the second column, we have the newly found $Z_c^+(3900)$.

The existence of a charged partner of the $X(3872)$ was first proposed in Ref. 41. A few years later,[55] the same group proposed that the $Z^+(4430)$, observed by BELLE,[7] would be the first radial excitation of the charged partner of the $X(3872)$. This suggestion was supported by the fact that the mass difference corresponding to a radial excitation in the charmonium sector is given by $M_{\Psi(2S)} - M_{\Psi(1S)} \sim$

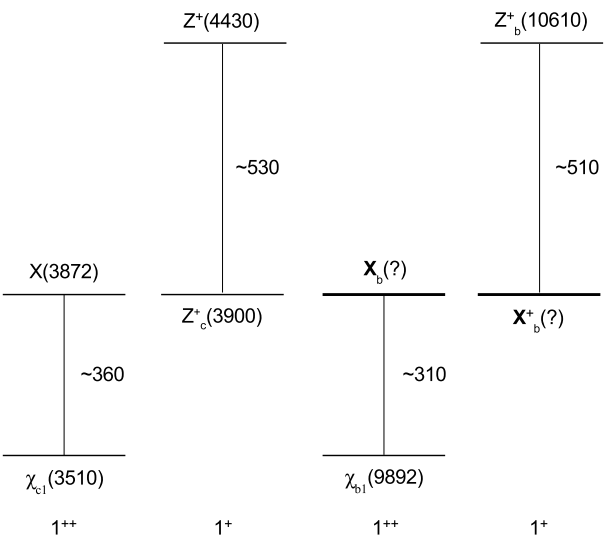

Fig. 2. Charm and bottom energy levels.

590 MeV. This number is close to the mass difference $M_{Z^+(4430)} - M_{X^+(3872)} \sim$ 560 MeV. A similar connection between $Z^+(4430)$ and $Z_c^+(3900)$ was found in the hadro-charmonium approach,[83] where the former is essentially a Ψ' embedded in light mesonic matter and the latter a J/ψ also embedded in light mesonic matter. In Ref. 82, this reasoning was extended to the bottom sector and it was conjectured that the $Z_b^+(10610)$, observed by the BELLE Collaboration in Ref. 84, might be a radial excitation of an yet unmeasured X_b^+, predicted in Ref. 42. The observation of $Z_c^+(3990)$ gives support to this conjecture and should motivate new experimental searches of this bottom charged state and its neutral partner, the only missing states in the diagram.

9. Conclusion

The most important message from the experimental program carried out at Belle and BESIII is that definitely there is something really new happening in the charmonium spectroscopy. This started in 2003 with the measurement of the $X(3872)$, which has a very robust experimental signature and has been measured by many different groups. The $X(3872)$ is electrically neutral and hence its multiquark nature was not clear from the beginning. Five years later, in 2008, the observation of $Z^+(4430)$, $Z_1^+(4050)$ and $Z_2^+(4250)$ would have been the proof of the existence of multiquark configurations in the charmonium sector. However, the nonconfirmation of these measurements rendered this claim weak. Another five years later, in 2013, the confirmation of the observation of the $Z^+(4430)$ together with the measurements of the $Z_c^+(3900)$ (which was measured by BESIII and confirmed by other groups) and also of the $Z_c^+(4025)$, $Z_c^+(4025)$ and $Z_c^+(3885)$, reinforced our belief that we are observing multiquark states. What has to be done next? From the experimental side, it is necessary to determine unambiguously, the quantum numbers of all these states and eliminate the suspicion that they are mere threshold effects and not real particles. As suggested in Ref. 79, this can be done by performing an energy scan in the e^+e^- reactions. Moreover, a more refined analysis will allow us to determine whether all these states are really different. From the theoretical side, its necessary to focus on the calculation of the decay widths in all the different approaches, since, as we have discussed, the masses are easily obtained by different methods and they are not sufficient to discriminate between different theoretical models. If our present picture of these states survives all these tests and improvements, we will have found multiquark states. This is in itself very interesting! Whether meson molecules, tetraquarks or hadrocharmonium, these are novel objects which will induce a small revolution in our understanding of hadrons.

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