RT-MAT 2004-26

ALGEBRAS WHICH ARE STANDARDLY STRATIFIED IN ALL ORDERS

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Outubro 2004

Esta é uma publicação preliminar ("preprint").

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October 14, 2004

Abstract

The aim of this work is to characterize the algebras which are standardly stratified with respect to any order of simple. We show that such algebras are exactly the algebras with idempotent ideals projective, we also conclude as a corollary a characterization for hereditary algebras, which was given originally by Dlab and Ringel [D].

Preliminaries

In this paper all algebras are finite dimensional K-algebras, basic and indecomposable, where K is an algebraically closed field. Using a fundamental Theorem of Gabriel all such algebras Λ are, up to isomorphism, of the form $\Lambda = \frac{KQ}{I}$ where Q is a finite quiver and I an admissible ideal.
Let $v_1, ...v_n$ be the vertices of Q in a fixed order and $S_1, ..., S_n$ the corresponding order of the simples modules, P_i the projective cover of S_i. Define the standard module Δ_i as a maximal quotient of P_i with composition factors $S_j, j \leq i$ ([R]).
Given a module A and a set of modules B define the trace of B in A, denoted by $\tau_B(A)$, as the sum of all images of morphisms of modules of B in A, that is $\tau_B(A) = \sum Im : B \longrightarrow A$.
An alternative definition of the standard modules is the following $\Delta_i = \frac{P_i}{U_i}$, where U_i is the sum of all images of morphisms $P_j \to P_i$ for $j > i$, that is $U_i = \tau_{\coprod_{j>i} P_j}(P_i)$, is the trace of all posterior projective modules P_i in it.

Another description is $\Delta_i = \frac{\Lambda e_i}{\Lambda \varepsilon_{i+1} \Lambda e_i}$, where $\varepsilon_k = e_k + ... + e_n$ for $1 \leq k \leq n$ and $\varepsilon_{n+1} = 0$.

Given $\Delta = \{\Delta_1, ..., \Delta_n\}$, consider $F(\Delta)$, the full subcategory of $mod\Lambda$, formed by the modules $M \in mod\Lambda$ such that M has a filtration with factors in Δ, that is there is a filtration $0 = M_0 \subset M_1 \subset ... \subset M_t = M$ with $\frac{M_i}{M_{i-1}} \simeq \Delta_k$, I. E. The factors are isomorphic to some module in Δ.

The algebra Λ is called standardly stratified if $\Lambda \in F(\Delta)$. If also, the endomorphisms ring of each standard module is simple Λ is called quasi-hereditary (see for example [R] and [X]).

An algebra Λ with idempotent ideals projective [P], denote by **iip**, is an algebra where all idempotent ideal of Λ are projective Λ-module.This means that all ideals I such that $I^2 = I$ are projective modules.

One characterization of **iip** algebras which is useful for us was given by M. I. Platzeck, in the Proposition 1.2 of [P], is the following: Λ is **iip** if and only if $\tau_{\widehat{P_i}}(\Lambda)$ is projective for all i, where $\widehat{P_i} = \underset{j \neq i}{\oplus} P_j$.
The main result of this note is to show that these algebras are exactly the algebras which are standardly stratified with respect to every order.

1 Preparatory Lemmas

In this section, we give several lemmas that will permit a better comprehension of our main results, some of these lemmas are known results, they have important by themselves, for sake of completeness we decide to state and give their proofs in this section.

Lemma 1 *If Λ is standardly stratified in the order $e_1, ..., e_n$ then $\frac{\Lambda}{\Lambda \varepsilon_i \Lambda}$ is standardly stratified in the order $e_1, ..., e_{i-1}$, where $\varepsilon_i = e_i + ... + e_n$.*

Proof.

Denote by $B = \frac{\Lambda}{\Lambda \varepsilon_i \Lambda}$ and $\Delta_k(\Lambda)$ and $\Delta_k(B)$ the $\Delta-$ modules of Λ and B, respectively.

Firstly observe that $\Delta_j(\Lambda), j = 1, ..., n$ are B- modules, because $\Delta_j(\Lambda) = \frac{\Lambda e_j}{\Lambda \varepsilon_i \Lambda e_j}$ and as $j < i$ this is a Λ- module annihilated by $\Lambda \varepsilon_i \Lambda$, in other words it is a B-module.

All modules $\Delta_j(\Lambda)$ have Top S_j and $\Delta_r(\Lambda)$ has composition factors only in the set $\{S_k, k < r\}$.

We show that these $\Delta_j(\Lambda)$ are exactly the $\Delta_j(B)$.

Observe that $\Delta_r(\Lambda)$ has Top S_r and it has all composition factors in the set $\{S_k, k < r\}$, moreover it is a B-module then $\Delta_r(\Lambda)$ is a quotient of $P_r(B)$ with composition factors in the set $\{S_k, k < r\}$, and as $\Delta_r(B)$ it is the maximal quotient of $P_r(B)$ with this property, therefore we have an epimorphism from $\Delta_r(B)$ to $\Delta_r(\Lambda)$.

We also have an epimorphism from $P_r(\Lambda)$ to $P_r(B)$ and another from $P_r(B)$ to $\Delta_r(B)$, thus we have an epimorphism from $P_r(\Lambda)$ to $\Delta_r(B)$, thus

$\Delta_r(B)$ is quotient of $P_r(\Lambda)$ that has Top S_r with all composition factors in the set $\{S_k, k < r\}$ and as $\Delta_r(\Lambda)$ is the maximal quotient of $P_r(\Lambda)$ with this property, we have an epimorphism of $\Delta_r(\Lambda)$ to $\Delta_r(B)$.

Therefore $\Delta_r(\Lambda) \simeq \Delta_r(B)$.

Let us show now that $P_r(B) \in F_B(\Delta), \forall r$.

Recall that $P_r(B) = \frac{P_r(\Lambda)}{(\Lambda\varepsilon_i\Lambda)P_r(\Lambda)}$ and that if M is a Λ -module $\frac{M}{(\Lambda\varepsilon_i\Lambda)M}$ is a B - module.

Now because Λ is standardly stratified, we have $P_r(\Lambda) \in F_\Lambda(\Delta)$, so that we have a filtration

$Q_r^s \subset ... \subset Q_r^2 \subset Q_r^1 \subset P_r(\Lambda)$ with factors $\Delta_k(\Lambda), k < r$.

Passing to factors have

$\frac{Q_r^s}{(\Lambda\varepsilon_i\Lambda)Q_r^s} \subset ... \subset \frac{Q_r^2}{(\Lambda\varepsilon_i\Lambda)Q_r^2} \subset \frac{Q_r^1}{(\Lambda\varepsilon_i\Lambda)Q_r^1} \subset P_r(B) = \frac{P_r(\Lambda)}{(\Lambda\varepsilon_i\Lambda)P_r(\Lambda)}$

Then $P_r(B) \in F_B(\Delta)$. ∎

Lemma 2 *If Λ is an algebra with idempotent ideals projective, then $\tau_P(Q)$ is projective for P indecomposable projective and Q projective.*

Proof.

$\tau_P(Q)$ is a summand of $\tau_P(\Lambda^s)$, if we prove that $\tau_P(\Lambda)$ is projective, we will prove our claim, also this is clear, because $P = \Lambda e$ with e idempotent, thus $\tau_P(\Lambda) = \Lambda e\Lambda$, that is an idempotent ideal and therefore projective. ∎

Moreover if $P = P_1^{n_1} \amalg ... \amalg P_t^{n_t}$, with $P_i \not\cong P_j$ indecomposable then $\tau_P(\Lambda) = \tau_{P_1\amalg...\amalg P_t}(\Lambda)$

Lemma 3 *If Λ is an algebra with idempotent ideals projective, then $\tau_{P_1\amalg...\amalg P_t}(\Lambda)$ is projective where the P_i are non isomorphic, indecomposable, projective modules.*

Proof.

Clearly $P_1\amalg...\amalg P_t = \Lambda e_1 + ... + \Lambda e_t = \Lambda(e_1 + ... + e_t)$, then $\tau_{P_1\amalg...\amalg P_t}(\Lambda) = \Lambda(e_1 + ... + e_t)\Lambda$, that is an idempotent ideal and therefore projective. ∎

2 The main result

We are now in conditions to prove the main result of this note.

Theorem 4 *The algebra Λ is standardly stratified in all orders if and only if it is an algebra with idempotent ideals projective.*

Proof. We are going to prove first that if Λ is standardly stratified in all orders then is an algebra with idempotent ideals projective.

Let Λ be an algebra which is standardly stratified in all orders. We can select one order such that $l(P_i) \leqslant l(P_{i+1}), \forall i$

We claim that in this case $P_k = \Delta_k$ for all k.

We have by definition that $P_n = \Delta_n$, we also have an exact sequence $0 \to \tau_{P_n}(P_{n-1}) \to P_{n-1} \to \Delta_{n-1} \to 0$.
If $\tau_{P_n}(P_{n-1}) \neq 0 \Rightarrow \tau_{P_n}(P_{n-1}) \simeq P_n^{k_n}$, since Λ is standardly stratified, this cannot happens because of our hypothesis on the length of the projective modules, therefore $\tau_{P_n}(P_{n-1}) = 0$. We continue by induction.

Let us assume that $\Delta_n, \Delta_{n-1}, ..., \Delta_{n-j+1}$ are projective, if $\tau_{\coprod_{r>n-j} P_r}(P_{n-j}) \neq 0 \Rightarrow \tau_{\coprod_{r>n-j} P_r}(P_{n-j}) \simeq \coprod_{r>n-j} P_r^{k_r}$ but this can not be because the hypothesis over the length, then $P_j = \Delta_j$.

Thus we have an order where $P_k = \Delta_k$ and therefore $F(\Delta) = Proj$

Observe also that with the order defined above $Hom(P_j, P_i) = Hom(\Delta_j, \Delta_i) = 0$ for $j > i$. An algebra Λ with this property is usually called quasi triangular.

In this algebra the vertex v_n is a source if we forget the loops in it. That is there is no arrow starting as v_n and endind at another vertex.

Let be $\Lambda = \frac{KQ}{I}$, then consider $\overline{Q}$ the quiver obtained from Q by elimination of v_n and $\overline{I}$ the ideal generated by the relations of I that remain after eliminate the arrows that start at v_n.

Then $\frac{\Lambda}{\Lambda e_n \Lambda} = \frac{K\overline{Q}}{\overline{I}}$ is also standardly stratified in all orders, this is consequence of Lemma 1, and the fact that $Hom(P(n), P(j)) = 0$ if $n \neq j$.

We write $\Lambda = 1\Lambda 1 = (e_n + \widehat{e_n})\Lambda(e_n + \widehat{e_n}) = (e_n \Lambda e_n) + (e_n \Lambda \widehat{e_n}) + (\widehat{e_n} \Lambda e_n) + (\widehat{e_n} \Lambda \widehat{e_n})$.

$L = (e_n \Lambda e_n)$ is a local algebra therefore it is standardly stratified.

$(e_n \Lambda \widehat{e_n}) = 0 = Hom\left(P_n, \widehat{P_n}\right)$.

$M = (\widehat{e_n} \Lambda e_n) = \tau_{\widehat{P_n}}(P_n)$

$U = (\widehat{e_n} \Lambda \widehat{e_n}) = \frac{\Lambda}{\Lambda e_n \Lambda} = \frac{K\overline{Q}}{\overline{I}}$

Thus

$$\Lambda \simeq \begin{pmatrix} L & 0 \\ M & U \end{pmatrix}$$

We can describe the projective modules of the algebra Λ,as triples in the following way.

There is the projective Q_n=$(P_n, M \otimes P_n, id)$, where P_n is the projective of the local algebra L, the other projectives can be viewed in the form and $Q_i = (0, P_i, 0)$ for $i \in \{1, ..., n-1\}$ where Q_i are the projective modules of U.

By a result of Marcos, Merklen, Saenz in "Standardly Stratified Split and Lower Triangular Algebras" [MMS] Proposition 16 we have that $M = (\widehat{e_n} \Lambda e_n) = \tau_{\widehat{P_n}}(P_n) \in F_{(\widehat{e_n} \Lambda \widehat{e_n})}(\Delta)$.

Because Λ is standardly stratified in all orders, we can choose one order so that e_n is the first.

We show, by induction in the number of simples, that Λ is **iip**

It is clear that a local algebra is **iip**.

Now we see that Λ is an algebra with idempotents ideals projective, for this is enough to show, using a result of Platzeck, [P] that $\tau_{\widehat{P_i}}(\Lambda)$ is projective for all i.

We also have that U is **iip**, by induction hypothesis because it has less simple, non isomorphic modules, than Λ.

So $\tau_{\widehat{Q_i}}(\Lambda) = \tau_{\widehat{Q_i}}(Q_n \oplus Q_1 \oplus ... \oplus Q_{n-1}) = \tau_{\widehat{Q_i}}(Q_n) \oplus \tau_{\widehat{Q_i}}\left(\coprod_{j\neq n} Q_j\right)$

If $i \neq n, \tau_{\widehat{Q_i}}(Q_n) = Q_n$, which is projective because Q_n is summand of $\widehat{Q_i}$, hence $i \neq n$, $\tau_{\widehat{Q_i}}\left(\coprod_{j\neq n} Q_j\right) \simeq \tau_{\widehat{P_i}}(U)$ that is projective since U **iip**, then $\tau_{\widehat{Q_i}}(\Lambda)$ is projective.

If $i = n$, $\tau_{\widehat{Q_i}}(Q_n) = (0, M, 0)$, which is isomorphic to $\tau_{\widehat{P_i}}(P_n) = M$ and $\tau_{\widehat{Q_i}}\left(\coprod_{j\neq n} Q_j\right) \simeq \tau_{\widehat{P_i}}(U) = U$, because $\widehat{Q_n} \simeq \coprod_{j\neq n} P_n = U$, clearly $\tau_{\widehat{Q_i}}\left(\coprod_{j\neq n} Q_j\right)$ is projective.

It remains only to see that $(0, M, 0)$ is projective or equivalently that M is projective.

To see that M is projective, since $U = (\widehat{e_n}\Lambda\widehat{e_n}) = \frac{\Lambda}{\Lambda e_n \Lambda} = \frac{K\overline{Q}}{\overline{I}}$ is standardly stratified in all orders, then we can choose an order such that $P_k = \Delta_k$, and therefore $F_{(\widehat{e_n}\Lambda\widehat{e_n})}(\Delta) = Proj$ and as $M = (\widehat{e_n}\Lambda e_n) = \tau_{\widehat{P_n}}(P_n) \in F_{(\widehat{e_n}\Lambda\widehat{e_n})}(\Delta)$, therefore M is projective.

We will prove now the other implication.
That is: If Λ is an algebra with idempotent ideals projective then Λ is standardly stratified in all orders.

Let Λ be an algebra with idempotent ideals projective and $e_1, ..., e_n$ an order of idempotents, we will show that Λ is standardly stratified in this order.

For this we show that $P_k \in F(\Delta), \forall k$.

First $P_n = \Delta_n \in F(\Delta)$.

Suppose that $P_n, P_{n-1}, ..., P_{n-k+1} \in F(\Delta)$ and we prove that it follows that $P_{n-k} \in F(\Delta)$.

If $P_{n-k} = \Delta_{n-k}$, it is clear $P_{n-k} \in F(\Delta)$, if not we have a sequence $0 \to \tau_{\coprod_{r>n-k} P_r}(P_{n-k}) \to P_{n-k} \to \Delta_{n-k} \to 0$, and because $\tau_{\coprod_{r>n-k} P_r}(P_{n-k})$ is projective, we get as consequence of Lemmas 2 e 3, $\tau_{\coprod_{r>n-k} P_r}(P_{n-k}) = \coprod_{r>n-k} P_r^{s_r} \in F(\Delta)$, and this shows the result. ∎

3 Some applications

In this section we get various consequences of the main theorem4 proved in the previous section.

Corollary 5 *If Λ is standardly stratified in all orders then there exists a order such that $F(\Delta) = Proj$.*

Proof.

If Λ is standardly stratified in all orders, as in the proof of Theorem4, we can choose one order such that $l(P_i) \leqslant l(P_{i+1}), \forall i$, and in that case all $P_k = \Delta_k$ and therefore $F(\Delta) = Proj$. ■

The next corollary is linked with the modules of finite projective dimension.

Corollary 6 *If Λ is standardly stratified in all orders then there is an order such that $F(\Delta) = P^{<\infty}$, where $P^{<\infty}$ is the subcategory of modules of finite projective dimension.*

Proof.

If Λ is standardly stratified in all orders then Λ is an algebra with idempotent ideals projective and in this case we can take a order such that $Hom(P_j, P_i) = 0$ for $j < i$, thus $\Delta_i = \frac{P_i}{\tau_{\overline{P}_i}(P_i)}$ [P] and $F(\Delta) = P^{<\infty}$. ■

Using the previous results we can obtain a characterization of hereditary algebras that generalize one result of Dlab and Ringel which appears in [D].

Corollary 7 *The following conditions are equivalents:*

1. *$gl \dim \Lambda < \infty$ and Λ is standardly stratified in all orders.*
2. *Q does not have oriented cycles and Λ is quasi hereditary in all orders.*
3. *Λ is quasi hereditary in all orders.*
4. *Λ is hereditary.*

Proof.

$2 \Rightarrow 1$ Clear.

$1 \Rightarrow 2$

If Λ is standardly stratified in all orders then by the previous Corollary there exists one order such that $F(\Delta) = P^{<\infty}$ and since $gl \dim \Lambda < \infty$ it follows that $F(\Delta) = P^{<\infty} = mod\Lambda \Rightarrow \Delta_i = S_i \Rightarrow Q$ does not have oriented cycles.

Because Λ is standardly stratified in all orders and $gl \dim \Lambda < \infty$ then it is quasi hereditary in all orders.

$2 \Rightarrow 3$ Evident.

$3 \Rightarrow 1$

It is clear that if Λ is quasi hereditary in all orders then Λ is standardly stratified in all orders and we have $gl\dim\Lambda < \infty$.

$1 \Rightarrow 4$

If Λ is standardly stratified in all orders then by the previous Corollary there exists an order such that $F(\Delta) = P^{<\infty}$ and by $gl\dim\Lambda < \infty$ then $F(\Delta) = P^{<\infty} = mod\Lambda$.

If follows from the work [CMMP] of Coelho, Marcos, Merklen, Platzeck that if Λ is an algebra with idempotent ideals projective then $fd\Lambda \leq 1$, so $gl\dim\Lambda = fd\Lambda \leq 1$, therefore Λ is hereditary.

$4 \Rightarrow 1$

Let be now Λ an hereditary algebra, it is clear that $gl\dim\Lambda < \infty$, if $e_1, ..., e_n$ is a order of idempotents, show that Λ is standardly stratified in this order.

For this prove that $P_k \in F(\Delta), \forall k$.

First $P_n = \Delta_n \in F(\Delta)$.

Suppose that $P_n, P_{n-1}, ..., P_{n-k+1} \in F(\Delta)$ and prove that $P_{n-k} \in F(\Delta)$.

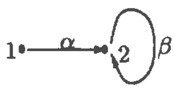
If $P_{n-k} = \Delta_{n-k}$ is clear, if not we have a sequence $0 \to \tau_{\underset{r>n-k}{\amalg} P_r}(P_{n-k}) \to P_{n-k} \to \Delta_{n-k} \to 0$, and $\tau_{\underset{r>n-k}{\amalg} P_r}(P_{n-k})$ is projective, because it is a submodule of P_{n-k}, then, $\tau_{\underset{r>n-k}{\amalg} P_r}(P_{n-k}) = \underset{r>n-k}{\amalg} P_r^{s_r} \in F(\Delta)$. ■

4 Remarks and Examples

Remark 8 *If A is a local algebra then$F(\Delta) = Proj = P^{<\infty}$.*

Remark 9 *It can exist, as the next examples show, orders, not necessarily distinct such that $F(\Delta) = Proj$ and $F(\Delta) = P^{<\infty}$, and A not necessarily with idempotent ideals projective.*

1. Let be $A = \frac{KQ}{I}$ where Q is the quiver



and I is the ideal generated by the relations $\beta\alpha = 0$ and $\beta^2 = 0$, in the order $1, 2$ this algebra is not standardly stratified because $P_1 \notin F(\Delta)$, although in the order $2, 1$, $F(\Delta) = Proj$, thus A is not **iip**, but if we analyze the modules of finite projective dimension, they are the projective modules, because if a module has finite projective dimension then it has even dimension as K-space and the only indecomposable modules with even dimension are the projective modules $P^{<\infty}$.

2. Even in the case that exists orders distinct such that $F(\Delta) = Proj$ and $F(\Delta) = P^{<\infty}$, A is not necessarily **iip**, as shown in the following example:

   Let be $A = \frac{KQ}{I}$ where Q is the quiver $\overset{n}{\bullet} \overset{\alpha_{n-1}}{\longrightarrow} \overset{n-1}{\bullet} \ldots\ldots \overset{2}{\bullet} \overset{\alpha_1}{\longrightarrow} \overset{1}{\bullet}$, and I is the ideal generated by the relations $\alpha_{i+1}\alpha_i$, for $i = 1, ..., n-2$, in this case in the order $1, 2, ..., n$, $F(\Delta) = Proj$ and in the order $n, n-1, ..., 2, 1$, $F(\Delta) = P^{<\infty}$, but this algebra is not standardly stratified in all orders because it is of finite global dimension and if it was standardly stratified, it would be hereditary by the Corollary. We also can show directly that this algebra is not standardly stratified in the order $n, 1, 2, ..., n-1$.

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