Holographic phase transition and quasinormal modes in Lovelock gravity

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In this work we aim at discussing the effects of the higher order curvature terms on the Lovelock AdS black holes quasinormal spectrum and, in the context of gauge/gravity correspondence, their consequences for the formation of holographic superconductors. We also explore the hydrodynamic limit of the $U(1)$ gauge field perturbations in d dimensions.

I. INTRODUCTION

The AdS/CFT relation, discovered in the framework of string theory [1,2], has surpassed its natural cradle to spread into the realm of condensed matter by means of the holographic construction [3–5]. In such a case, it does not matter much what is the type of the black hole in the AdS space as a physical object, but rather what is the CFT theory described in the process. Indeed, the CFT theory is the backbone of the construction and classical perturbations of the gravity set up may lead, changing the black hole, or generally speaking changing the AdS set up, to valuable information about the CFT counterpart. We move to new condensed matter systems, thus to new physics.

Recently, a series of models have been considered, with various degrees of success to obtain models concerning condensed matter systems, see [6,7] for a partial and incomplete list. Several different physical situations have been touched, such as superconductivity, for perturbations of Reissner-Nordstrom solutions [3] and superfluidity [7], as well as when dealing with time dependent solutions [8], density waves [9]. Applications in high energy physics have been particularly important [10,11]. Higher order corrections to the gravity counterpart have been often used, but a general discussion is still missing [12–15].

Here, we are going to discuss details of the higher order corrections to gravity and their consequences for the holographic field theory. In particular, we consider Lagrangians whose field equations are at most of second order [16] which, in the case of generalizations of gravity lead us to the Lovelock Lagrangians [17]. The paper is organized as follows. Section II provides a review of the d-dimensional Lovelock gravity and the black hole solutions considered in this work. In Sec. III we obtain the quasinormal spectrum of d-dimensional charged Lovelock black holes due to a scalar probe field. In Secs. IV and V we explore in the probe limit the formation of holographic

superconductors in the presence of higher order corrections to the curvature. Also we compute the real time R-current correlators due to electromagnetic perturbations due to electromagnetic field. In Sec. VI we discuss the results and some final comments are given.

II. THE LOVELOCK GRAVITY

String theory brought the idea that higher dimensional curvature terms in the gravity action are basically mandatory to cope with quantum corrections at the Planck scale [18]. On the other hand, field equations with higher time derivatives are unstable. Such a result, originally relying upon Ostrogradsky [19] long ago on very general grounds has been rederived in simple terms [20]. Nevertheless, there are theories with complex dynamics involving higher order terms in the Lagrangian but whose equations of motion are at most second order in time [16,17,21]. We discuss here the cases of Lovelock gravity as discussed in [22], where, in several space-time dimensions we have Einstein gravity corrected by higher order terms but with second order differential equations for the fundamental metric fields. The solutions of the field equations we are considering are those of Refs. [22] with nonvanishing charge, that is, the gravity sector in d dimensions is described by the action

$$S = -\frac{1}{16\pi G} \int d^d x \sum_{p=0}^{k} \mathcal{L}^{(p)}, \tag{1}$$

where k is an integer strictly smaller than $\frac{d+1}{2}$ labeling inequivalent theories, the individual Lagrange densities are

$$\mathcal{L}^{(p)} = \frac{l^{p-2k}}{d-2k} \left(\frac{k}{p}\right) \epsilon_{\mu_1 \cdot \mu_d} \epsilon^{a_1 \dots a_d} \mathcal{R}^{\mu_1 \mu_2}_{a_1 a_2} \dots \mathcal{R}^{\mu_{2p-1} \mu_{2p}}_{a_{2p-1} a_{2p}} e^{\mu_{2p+1}}_{a_{2p+1}} \dots e^{\mu_d}_{a_d},$$

with l denoting the d-dimensional AdS radius related to the cosmological constant Λ by

$$\Lambda = -\frac{(d-1)(d-2)}{2l^2}$$

and the curvature is

$$\mathcal{R}_{ab}^{\mu\nu} = R_{ab}^{\mu\nu} + \frac{1}{l^2} e_a^\mu e_b^\nu,$$

where e_a^μ is the vielbein.

It is an established result that the equations of motion are second order in the time derivatives. The Einstein equations have been solved [22] and a series of AdS black hole solutions emerge from these actions, the most important result used in the present work. Solutions are labeled by the space-time dimension d, the integer k defined above.

In order to consider charged solutions, the gravity action (1) has to be supplemented by the Maxwell action

$$S_M = -\frac{1}{4\epsilon} \int d^d x \sqrt{-g} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}, \qquad (2)$$

where ϵ is related to the vacuum permeability ϵ_0.

Solutions to the Einstein-Maxwell system are labeled by the space-time dimension d, the integer k defined above and the charge Q. From [22,23], those solutions read

$$ds^2 = -\left(\eta + \frac{r^2}{l^2} - g_k(r)\right) dt^2 + \frac{dr^2}{\eta + \frac{r^2}{l^2} - g_k(r)} + r^2 d\Sigma_{d-2}^2, \qquad (3)$$

where $\eta = -1, 0, 1$ defines the topology. For $\eta = 1$, $d\Sigma_{d-2}^2$ is the angular measure on the sphere, $\eta = 0$ implies a flat black hole where $d\Sigma_{d-2}^2$ is the flat metric and $\eta = -1$ corresponds to the hyperbolic metric. The metric depends on the charge Q of the black hole by means of the expression

$$g_k = \left(\frac{2\hat{G}M + \delta_{d-2k,1}}{r^{d-2k-1}} - \frac{\epsilon\hat{G}}{d-3} \frac{Q^2}{r^{2(d-k-2)}}\right)^{1/k}, \qquad (4)$$

where $\hat{G}$ is the gravitational constant, $\delta_{d-2k,1}$ is the standard Kronecker delta and M is the black hole mass. For a generic value of k, the black holes described by the line element (3) have two horizons (r_-, r_+) located at the zeros of g_{tt}, satisfying $r_- < r_+$. The black hole family describe by (3) include the d-dimensional Reissner-Nordström AdS-black holes for $k = 1$ and the charged AdS-Gauss-Bonnet black holes for $k = 2$ and $d > 5$. For a discussion on the causal structure of Gauss-Bonnet gravity, see [24]. Also, the line element (3) is asymptotically AdS for all values of k and d.

III. LOVELOCK SCALAR QUASI NORMAL MODES

In this section we are going to explore the quasinormal spectrum of charged AdS-Lovelock black holes considering a probe scalar field evolving at such geometry.

The black hole quasinormal modes (QNM) of asymptotically AdS black holes is obtained by considering probe fields dynamics supplemented by ingoing boundary conditions at the event horizon and Dirichlet boundary conditions at spatial infinity [25,26]. In the context of AdS/CFT correspondence the decay of QNM is interpreted as the return to equilibrium of a thermal state in the quantum field theory at finite temperature living at the AdS boundary [27]. For a recent review on the subject see [28,29] and the references therein. In particular, scalar fluctuations on the bulk geometry are related to the poles of thermal retarded Green function [30] and the electromagnetic perturbations are associated to the poles of retarded Green functions of R-symmetry currents at the boundary.

The next procedure is standard but new. We consider the scalar perturbations of the above system. Scalar perturbations are easily obtained. We rewrite the metric in a form that we use in the numerical analysis below, that is,

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma_{d-2}^2, \qquad (5)$$

with

$$f(r) = \eta + \frac{r^2}{l^2} - \left[\frac{M}{r^{d-2k-1}} - \frac{Q^2}{r^{2(d-k-2)}}\right]^{\frac{1}{k}}. \qquad (6)$$

Depending on the curvature (thus on η), the angular part of (5) changes accordingly. We can rewrite the parameters in terms of the inner horizon r_- and the event horizon r_+ as

$$M = \frac{1}{r_+^{d-3} - r_-^{d-3}} \left[r_+^{2d-4-2k} \left(\eta + \frac{r_+^2}{l^2}\right)^k - r_-^{2d-4-2k} \left(\eta + \frac{r_-^2}{l^2}\right)^k\right],$$

$$Q^2 = \frac{1}{r_-^{3-d} - r_+^{3-d}} \left[r_+^{d-1-2k} \left(\eta + \frac{r_+^2}{l^2}\right)^k - r_-^{d-1-2k} \left(\eta + \frac{r_-^2}{l^2}\right)^k\right]. \qquad (7)$$

In this paper, we study the planar black hole case, namely $\eta = 0$, and without loss the generality, we set $l = 1$. Now, it is standard to compute the scalar modes. Because it is an anti-de Sitter spacetime, we should use Horowitz-Hubeny method [27] to calculate the scalar quasinormal modes of this black hole. According to this method, we set $v = t + \int \frac{dr}{f(r)}$, so the metric is rewritten as

$$ds^2 = -f(r) dv^2 + 2 dv dr + r^2 d\Sigma_{d-2}^2, \qquad (8)$$

and then the scalar equation is given by

$$f(r)\phi'' + (f' - 2i\omega)\phi' - V(r)\phi = 0, \qquad (9)$$

where $V = \frac{(d-2)f'}{2r} + (d-4)(d-2)\frac{f}{4r^2} + (d-2)\frac{L^2}{r^2}$. The transformation $z = \frac{1}{r}$ is introduced, so that the region of
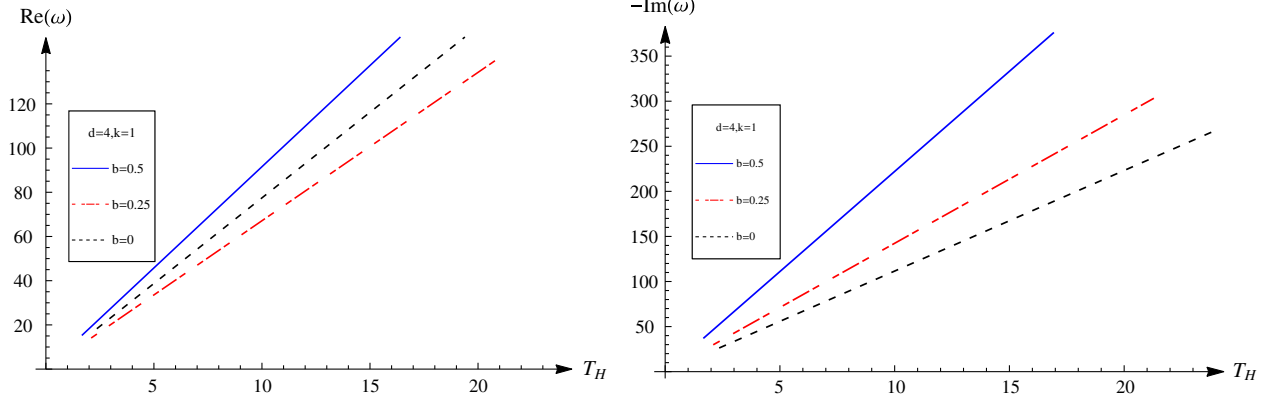
FIG. 1 (color online). Real (left) and imaginary (right) quasinormal modes behavior in terms of the Hawking temperature for $d = 4, k = 1$.

variable becomes $0 \leq z \leq h$ ($h = \frac{1}{r_+}$). The boundary condition at event horizon require $\phi(r_+) = 1$ but ϕ should vanish at infinity. Therefore, the scalar field equation is given by

$$s(z)\frac{d^2\phi}{dz^2} + \frac{t(z)}{z-h}\frac{d\phi}{dz} + \frac{u(z)\phi}{(z-h)^2} = 0, \quad (10)$$

where

$$s(z) = -\frac{z^4 f}{z-h},$$

$$t(z) = -z^2\left(z^{-2}\frac{df}{dz} + 2zf + 2i\omega\right),$$

$$u(z) = (z-h)V. \quad (11)$$

we can expand $s(z) = \sum s_i(x-h)^i$, $t(z) = \sum t_i(x-h)^i$, and $u(z) = \sum u_i(x-h)^i$ and $\phi(z) = \sum a_i(x-h)^i$ at event horizon $z = h$. Considering the boundary condition at horizon, we have $a_0 = 1$, and substitute $s, u, t,$ and ϕ into Eq. (11), we obtain the recursion relation

$$a_n = -\frac{1}{P_n}\sum_{i=0}^{n-1}[i(i-1)s_{n-i} + it_{n-i} + u_{n-i}]a_i, \quad (12)$$

where $P_n = n(n-1)s_0 + nt_0$, so all the a_i can be obtained. Finally, according to another boundary condition $\phi(0) = 0$, we always can get the value of ω from equation $\sum_i a_i = 0$. It is very convenient to use *Wolfram Mathematica* to realize the above process, so we use this software to calculate the quasinormal modes of this black hole. We find a sequence of quasinormal frequencies as function of the temperature of the black hole. It is a tedious but straightforward procedure. Nevertheless, we find some interesting results.

The first noteworthy result with angular quantum number $l = 0$ is the fact that spaces with higher values of k are stiffer, namely have larger values for the imaginary part of the frequency, as shown in Figs. 1–4. Also, we clearly see that the real and imaginary part of frequencies scales linearly with the Hawking temperature, which is expected of AdS black holes [27]. We have the results for various values of the temperature as given in Table I, where b is the value of black hole charge Q.
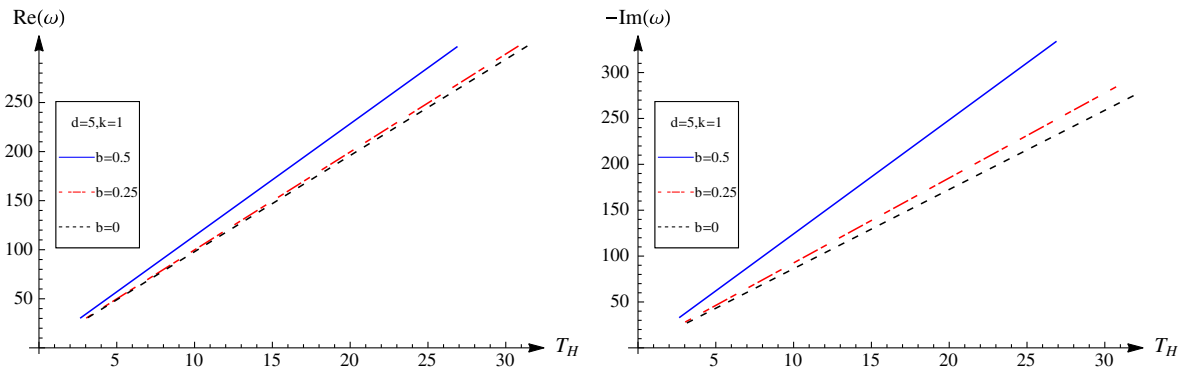

FIG. 2 (color online). Real (left) and imaginary (right) quasinormal modes behavior in terms of the Hawking temperature for $d = 5, k = 1$.
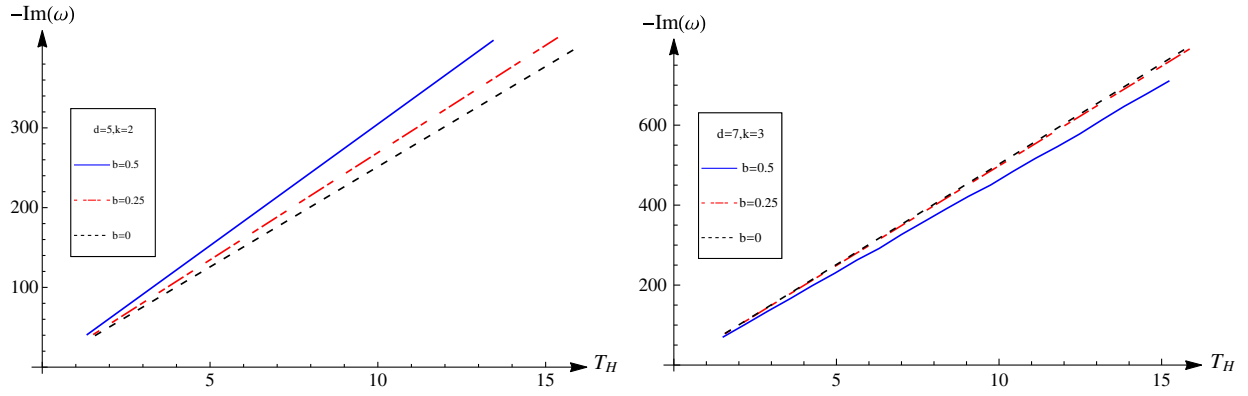
FIG. 3 (color online).   Purely damped quasinormal modes behavior in terms of the Hawking temperature for $d = 5, k = 2$ and $d = 7, k = 3$.

As we see in Fig. 3, the scalar quasinormal frequencies for a five and seven-dimensional Lovelock black hole, with $k = 2$ and $k = 3$, respectively, are purely damped, namely there is no oscillatory phase for the perturbation. Such a result seems to be a general feature of Lovelock theories with $d = 2k + 1$ since the gravity theory reduces to Chern-Simons gravity in these cases. The purely damped frequencies are not new in literature. The same result has been found in the behavior of scalar quasinormal modes of the three dimensional Lifshitz black hole [31], whose gravity theory is the new massive gravity (NMG). The corresponding action, as in the Lovelock case, contains

higher order corrections in the curvature. The dynamics of probe scalar fields in higher dimensional Lifshitz black holes $(d = 5, …, 10)$ do not show an oscillating phase either [32]. Thus, at least in case of Lovelock gravity with $d = 2k + 1$ and Lifshitz black holes, the purely damped modes are related to the higher curvature terms.

Also, we observe that the effect of adding charge to the black hole is to increase the quasinormal frequency value. There is also a quite important increase in the imaginary part of the frequencies for $k = 2$, when the models seems to get stiffer. This effect is less pronounced for the six-dimensional case, see Fig. 4.
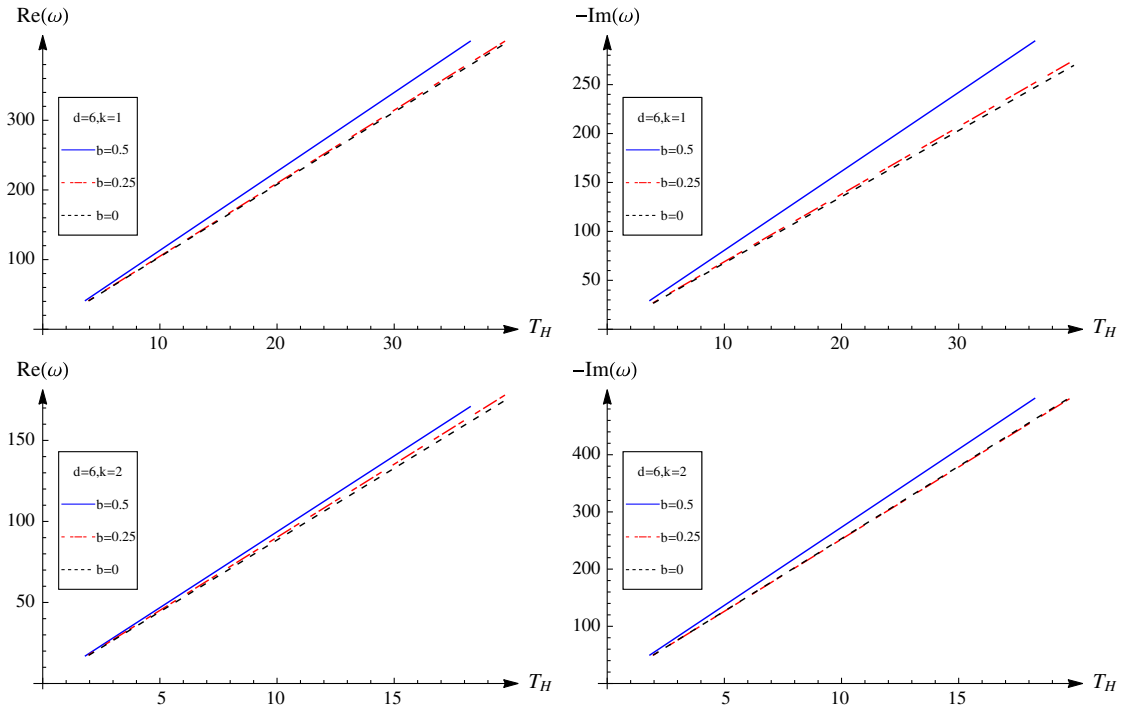

FIG. 4 (color online).   Effect on the real part (left) and imaginary part (right) of quasinormal frequencies by adding charge to the six-dimensional case for $k = 1, k = 2$.

TABLE I.   Various values of quasinormal frequencies, where T is the Hawking temperature.

b	$d = 4$	$d = 5, k = 1$	$d = 5, k = 2$	$d = 6, k = 1$	$d = 6, k = 2$
0	$(7.747 - 11.158i)T$	$(9.800 - 8.620i)T$	$(-25.133i)T$	$(10.394 - 6.769i)T$	$(8.854 - 25.320i)T$
0.25	$(6.712 - 14.240i)T$	$(9.981 - 9.257i)T$	$(-26.899i)T$	$(10.483 - 6.902i)T$	$(9.021 - 25.235i)T$
0.5	$(9.156 - 22.211i)T$	$(11.416 - 12.4206i)T$	$(-30.480i)T$	$(11.33 - 8.069i)T$	$(9.361 - 27.322i)T$

IV. THE PHASE TRANSITION AND CONDUCTIVITY

According to the AdS/CFT dictionary, the scalar perturbation ψ corresponds, at the AdS border, to the order parameter of the conformal field theory. The gauge field perturbation gives rise to the border source and to the current. We can thus analyze whether we can have a superconducting phase and compute the conductivity. As it turns out, we have the conductivity as a function of the frequency, what is a physically relevant object to study the properties of the conformal field theory at the border (or else, of the condensed matter system at the border). For later purposes it is going to be useful to write the above function in terms of the event horizon radius r_+ and the Cauchy horizon $r_c \equiv \gamma r_+$. From this point on we shall work on flat topology, $\eta = 0$ and $l = 1$. We have

$$M = \frac{\gamma^{2d-4} - 1}{b^{d-3} - 1} r_+^{d-1}, \qquad Q^2 = \frac{\gamma^{2d-4} - \gamma^{d-3}}{\gamma^{d-3} - 1} r_+^{2d-4}, \qquad (13)$$

where $\gamma = \frac{r_-}{r_+}$. In terms of the new parameters we have

$$f(r) = r^2 - r_+^{\frac{d-1}{k}} r^{2(1+\frac{2-d}{k})} \left[\frac{1 - \gamma^{2d-4}}{1 - \gamma^{d-3}} r^{d-3} - \gamma^{d-3} \frac{1 - \gamma^{d-1}}{1 - \gamma^{d-3}} \right]^{\frac{1}{k}}, \qquad (14)$$

and the Hawking temperature is given in terms of the local gravity at the black hole event horizon,

$$T_c = \frac{d - 1 - \gamma^{d-3}[2d - 4 - \gamma^{d-1}(d - 3)]}{4k(1 - \gamma^{d-3})\pi} r_+. \qquad (15)$$

In order to obtain the phase transition, we consider the Lovelock gravity action (1) coupled to a classical charged scalar field ψ and the electromagnetic gauge field A_μ, whose action is

$$S_c = \int d^d x \sqrt{-g} \left[-\frac{1}{4\epsilon} F_{\mu\nu} F^{\mu\nu} - |\nabla \psi - iqA\psi|^2 - m^2 |\psi|^2 \right], \qquad (16)$$

where ∇ is the covariant derivative, q and m are the scalar field charge and mass, respectively.

We consider the equation of motion of the matter and gauge fields, in such a way that scalars are functions of the radial variable in order to define an order parameter at the border. The scalar potential corresponding to the gauge field ($\phi \equiv \mathcal{A}_0$) is, consequently, a function of the radial variable. The vectorial components of the gauge field are functions of t and r. Without loss of generality we consider only $\mathcal{A}_x$, whose time dependence is harmonic, that is, $\mathcal{A}_x(\vec{x}, t) = \mathcal{A}_x(\vec{x})e^{-i\omega t}$.

We are going to consider the equations of motion of the electric potential $A_0 = \Phi(r)$, of the scalar field $\Psi(r)$ and the x-component of the vector potential, $\mathcal{A}_x(r)$. Moreover, it is useful to change variables from r to $z = \frac{1}{r}$. Also, foreseeing the asymptotic behavior of the fields, we redefine them as $\phi(z) = \Phi(1/z)$, $\psi(z) = \Psi(1/z)z^{-\lambda_f}$, $A_x(z) = \mathcal{A}_x(1/z)$. We in this paper use the shooting method to calculate numerically the holographic superconductor and the conductivity. The Maxwell-Klein Gordon equations in the probe limit for $k = 1$ and $d = 5$ read

$$\psi_{5,1}'' + \frac{3(\gamma^4 + b^2)z^6 - (\gamma^4 + b^2 + 1)z^4 - 3}{(z^3 - z)[(\gamma^4 + \gamma^2)z^4 - z^2 - 1]} \psi_{5,1}'$$
$$+ \frac{m^2(1 - z^2)[(\gamma^4 + \gamma^2)z^4 - z^2 - 1] + z^2\phi_{5,1}}{(z^3 - z)^2[(\gamma^4 + \gamma^2)z^4 - z^2 - 1]^2} \psi_{5,1} = 0,$$

$$\phi_{5,1}'' - \frac{\psi_{5,1}'}{z} - \frac{2\psi_{5,1}^2 \phi_{5,1}}{z^2(z^2 - 1)[(\gamma^4 + \gamma^2)z^4 - z^2 - 1]} = 0,$$

$$A_{x,5,1}'' + \frac{5(\gamma^4 + \gamma^2)z^6 - 3(\gamma^4 + \gamma^2 + 1)z^4 - 1}{(z^3 - z)[(\gamma^4 + \gamma^2)z^4 - z^2 - 1]} A_{x,5,1}'$$
$$+ \frac{z^2\omega^2 - 2(z^2 - 1)[(\gamma^4 + \gamma^2)z^4 - z^2 - 1]\psi_{5,1}^2}{(z^3 - z)^2[(\gamma^4 + \gamma^2)z^4 - z^2 - 1]^2} A_{x,5,1} = 0.$$

By the shooting method, we choose the value of the fields near the horizon, solve the differential equations to the spatial infinity and compare with the boundary condition. For solving the differential equation we choose the functions as power series of $z - 1$. We thus obtain the value of the function at the boundary and compare with the boundary condition. We subsequently consider the cases $d = 5, k = 2, d = 6, k = 1$, and $d = 6, k = 2$, whose equations of motion are given in the Appendix.

A. Numerical analysis

Let us first concentrate on the 5-dimensional case, where the function $f(r)$ defining the metric is given by
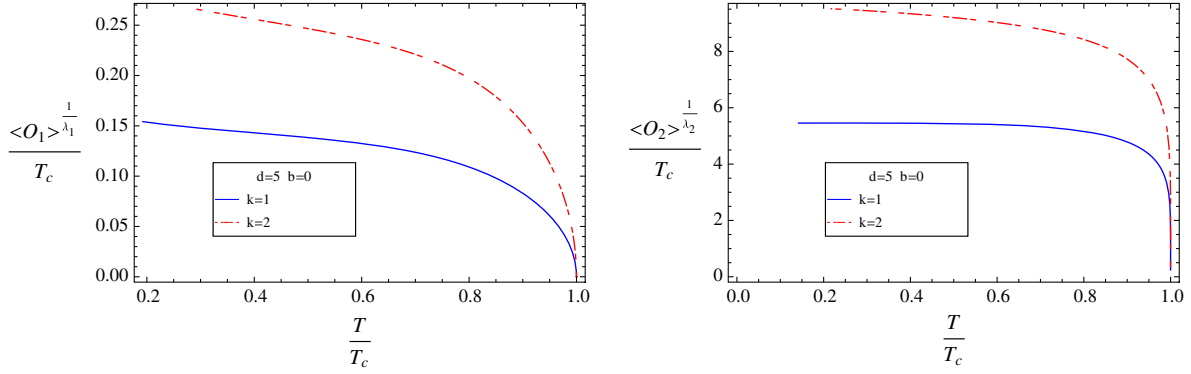
FIG. 5 (color online).   Condensation of operators $\langle\mathcal{O}_1\rangle$ and $\langle\mathcal{O}_1\rangle$ for the five-dimensional uncharged case ($b = 0$).

$$f(r) = \eta + \frac{r^2}{R^2} - \frac{r_e^{\frac{4}{k}-2} r^{2-\frac{6}{k}}}{(b^2-1)^{\frac{1}{k}}b^2}\left[b^6(r^2 - r_e^2)\left(\frac{b^2 r_e^2}{R^2} + \eta\right)^k\right.$$
$$\left. - b^{2k}(r^2 - r_e^2 b^2)\left(\frac{r_e^2}{R^2} + \eta\right)^k\right]^{\frac{1}{k}}. \tag{17}$$

Here, $\eta = +1, 0, -1$ depending on whether the solution has positive, zero or negative curvature. The parameter b is a measure of the charge of the black hole, R is the inverse of the cosmological constant and r_e the event horizon.

We search for static solutions for the electric potential and for the scalar field seeking at the order parameter at the border. Moreover, we look for a vector potential at a given frequency (as above) in order to test the Ohm's law. The fields obey the coupled differential equations

$$\psi''(r) + \left(\frac{f'(r)}{f(r)} + \frac{d-2}{r}\right)\psi' + \left[\frac{\varphi^2(r)}{f^2(r)} - \frac{m^2}{f(r)}\right]\psi(r) = 0, \tag{18}$$

$$\phi'' + \frac{d-2}{r}\phi' - \frac{2q^2\psi^2(r)}{f(r)}\phi(r) = 0, \tag{19}$$

$$A_x''(r) + \left(\frac{f'(r)}{f(r)} + \frac{d-4}{r}\right)A_x'(r) + \left(\frac{\omega^2}{f^2(r)} - \frac{2\psi^2(r)}{f(r)}\right)$$
$$\times A_x(r) = 0. \tag{20}$$

B. Results for phase transition

According to the usual AdS/CFT dictionary, when we approach the AdS boundary the expansion of the perturbations near the boundary leads to CFT fields with well-defined physical interpretation [2,3]. For the scalar field, in particular, we have the expansion

$$\psi(r) = \psi^{(1)}\frac{1}{r} + \psi^{(2)}\frac{1}{r^2} + \text{higher order in}\left(\frac{1}{r}\right). \tag{21}$$

The expansion coefficients $\langle\mathcal{O}_i\rangle = \sqrt{2}\psi^{(i)}$ are, according to the above mentioned dictionary, order parameters of the boundary theory, as long as we choose appropriated boundary conditions, that is, if $\psi^{(1)} = 0$ we define $\langle\mathcal{O}_2\rangle$ and for $\psi^{(2)} = 0$ we define $\langle\mathcal{O}_1\rangle$. The computation of either field uses the shooting method found by [4].

We considered various choice of parameters. Generally speaking, the order parameter $\langle\mathcal{O}_2\rangle$ is larger than $\langle\mathcal{O}_1\rangle$, and the one corresponding to $k = 2$ larger than the one corresponding to $k = 1$, see Figs. 5 and 6. This result about k means that the nonlinearity enhances the order parameter, but strangely enough the conductivity goes the other way
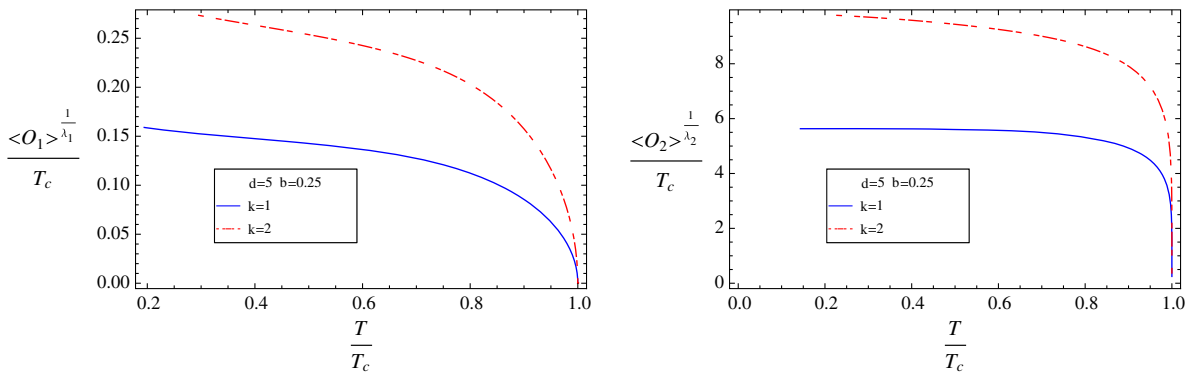
FIG. 6 (color online).   Condensation of operators $\langle\mathcal{O}_1\rangle$ and $\langle\mathcal{O}_1\rangle$ for the five-dimensional charged case ($b = 0.25$).
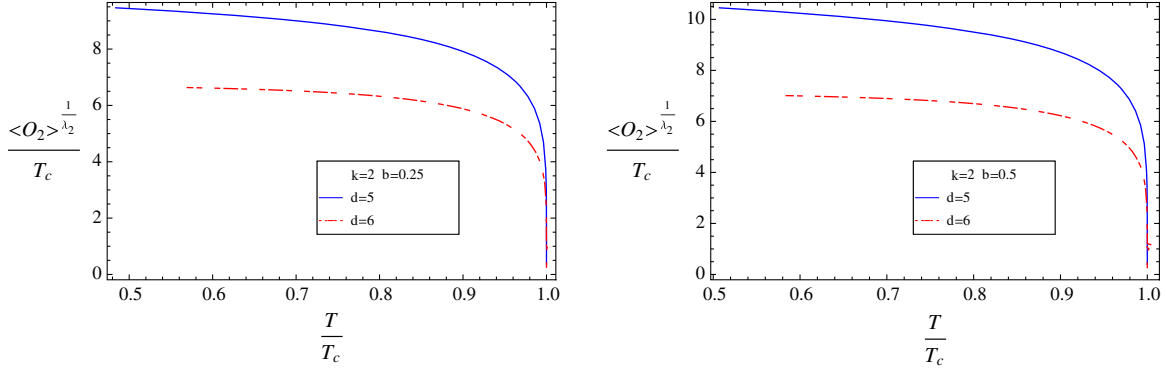
FIG. 7 (color online).   Condensation dependence on the dimensionality and the values of k.

(see next subsection), namely the conductivity (both real and imaginary part) are smaller for $k = 2$. Thus, order does not mean, in this case, better conductivity properties.

The effect of dimensionality upon the phase transition is to lower the value of the condensate as the number of spatial dimensions increase. Such an effect is present in both condensates $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$, see Fig. 7 for an example.

C. Results for conductivity

Now, we are going to compute the conductivity for each boundary operator $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$ following the standard AdS/CFT recipe [3]. Solving numerically Eq. (20), imposing ingoing wave boundary conditions at the black hole event horizon and considering the asymptotic behavior of A_x for large r, we have that the leading term is the current $\langle J_\mu \rangle$ and the subleading one the dual source $A_x^{(0)}$, both defined at the AdS border.

Having these two quantities, we compute the conductivity $\sigma(\omega)$ through the Ohm's law

$$\sigma(\omega) = -\frac{i\langle J_\mu \rangle}{\omega A_x}. \tag{22}$$

We present in Figs. 8–11 the real end imaginary part of conductivity $\sigma(\omega)$ of $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$ for five dimensional black hole in charged and uncharged cases. The conductivity

phenomena is qualitatively very similar in both cases $k = 1$ and $k = 2$ for the two operators, but the $k = 2$ corrections to the curvature seems to lower the conductivity comparing to the $k = 1$ case.

V. R-CURRENT CORRELATORS AND HYDRODYNAMICAL QUASINORMAL MODES

In this section we are going to apply the AdS/CFT correspondence [2,33] in order to compute the real time R-current correlators, which can be expressed in terms of the boundary value of the gauge invariant quantities such as the electric field at the spatial infinity. As one knows, the electromagnetic fluctuations, in the AdS/CFT context, give rise to the correlators associated to the R-symmetry at the boundary field theory.

Following the procedure outlined in [33,34], the imposition of Dirichlet boundary conditions on the gauge invariant variables lead to the poles of the field theory correlation functions and, according to [35], the quasinormal frequency spectra of the asymptotic AdS black hole considered. Moreover, a consequence of applying the approach [35] is that the electromagnetic quasinormal spectra presents a set of modes which behaves like a diffusion wave in the long wavelength and low frequency limit, such a limit is called hydrodynamic limit of
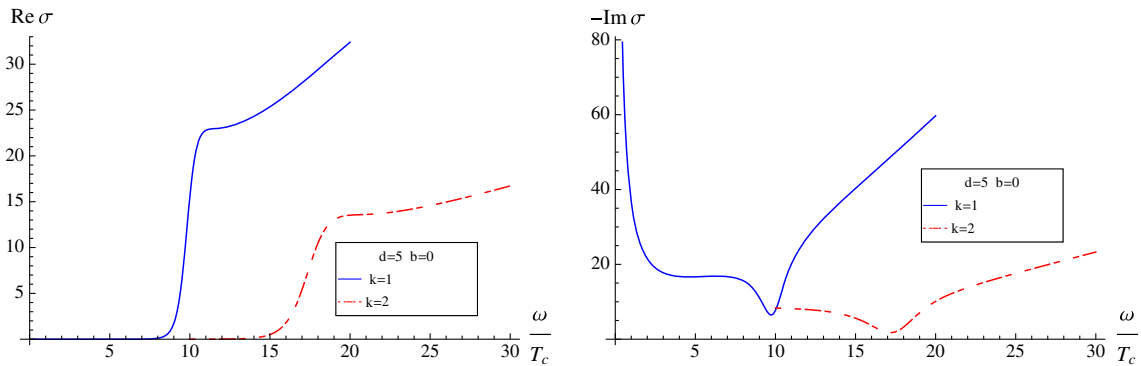
FIG. 8 (color online).   Real (left) and imaginary (right) parts of $\langle \mathcal{O}_1 \rangle$ conductivity for zero black hole charge in five dimensions and varying k.
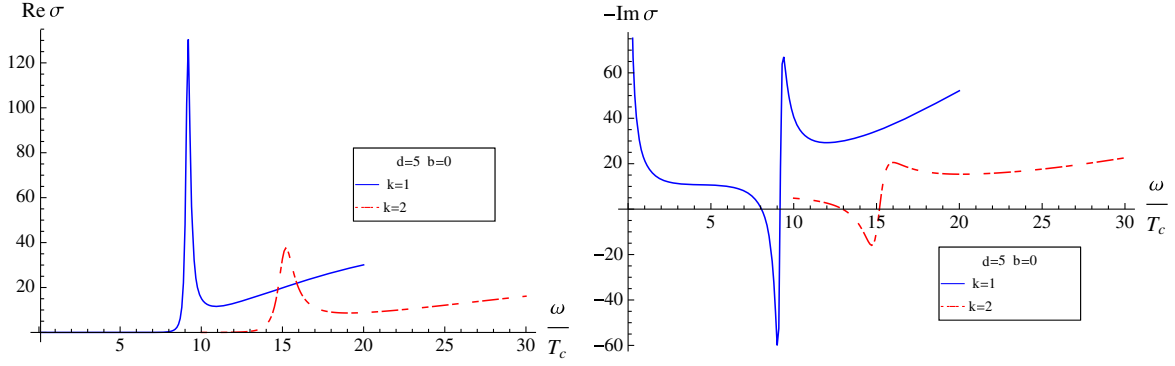
FIG. 9 (color online). Real (left) and imaginary (right) parts of $\langle \mathcal{O}_2 \rangle$ conductivity for zero black hole charge in five dimensions and varying k.
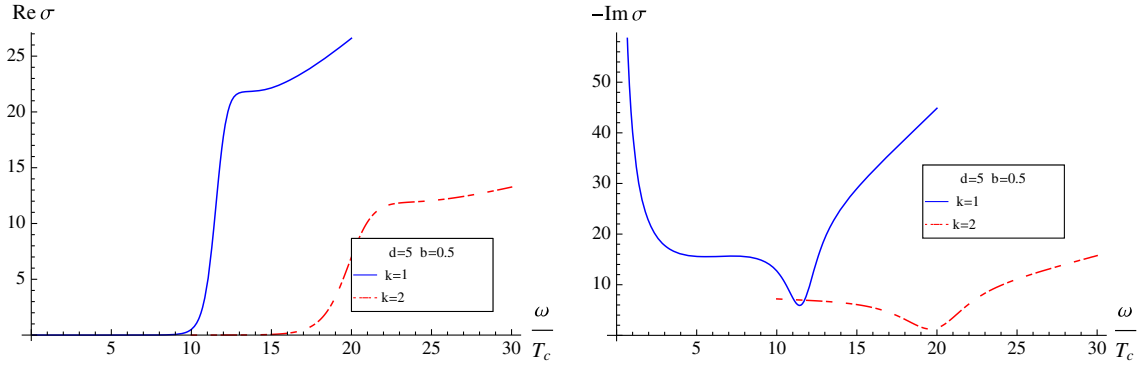

FIG. 10 (color online). Real (left) and imaginary (right) parts of $\langle \mathcal{O}_1 \rangle$ conductivity for charged black hole in five dimensions and varying k.
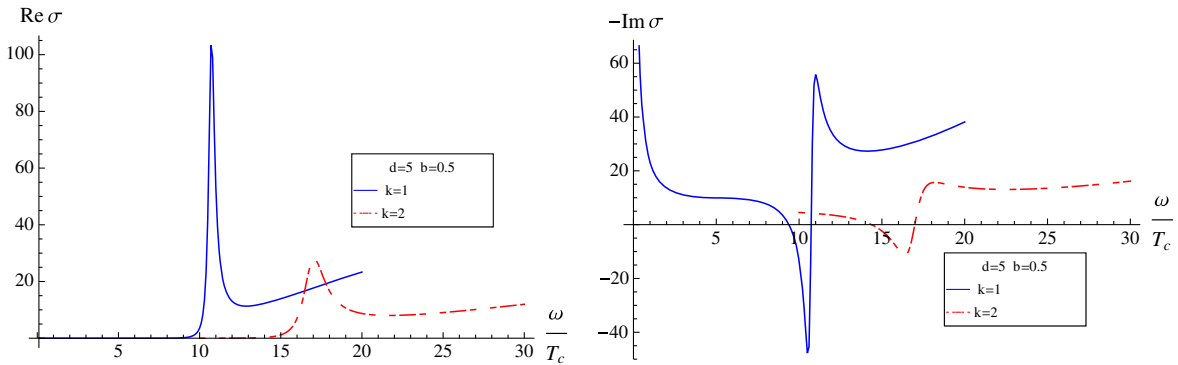

FIG. 11 (color online). Real (left) and imaginary (right) parts of $\langle \mathcal{O}_2 \rangle$ conductivity for charged black hole in five dimensions and varying k.

perturbations. The two main results of the section is the explicit form of correlators in the field theory defined at the boundary of Lovelock black holes and the frequency of diffusion quasinormal modes for dimension $d \geq 4$.

A. Correlators due to electromagnetic field

We are going to consider as our bulk geometry the uncharged d-dimensional planar Lovelock black hole, represented by the following line element

$$ds^2 = \frac{r_0^2}{l^2 u^{2/(d-3)}} \left[-g(u)dt^2 + \sum_i^{d-3} dx_i^2 \right] + \frac{r_0^2}{u^{2/(d-3)}} d\phi^2$$

$$+ \frac{l^2}{(d-3)^2 u^2 g(u)} du^2. \tag{23}$$

The function $g(u)$ is the horizon function given by

$$g(u) = 1 - u^\gamma, \qquad \gamma = \frac{(d-1)}{k(d-3)}.$$

The event horizon is located at $r = r_0$ or $u = 1$, the radial coordinate $r \in [r_0, +\infty]$ is mapped to $u \in [1, 0]$ through $u = r_0/r$. In order to have a black hole with planar topology, at least one of the extra dimensions has to be compact [23], so in the above metric $\phi \in [0, 2\pi]$, and the remaining directions have the domain $x_i \in [-\infty, +\infty]$, where $i = 1 \ldots d - 3$.

In AdS/CFT context, the electromagnetic field evolving in the region near the AdS boundary couples to the holographic field theory current-current two-point correlation function. Therefore, we have to compute the perturbations on the electromagnetic potential A_μ, whose equations governing its dynamics are the Maxwell equations,

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0, \qquad (24)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the metric components which enter in Maxwell equation are those given by (23). Using the isometries of black hole spacetime, we can decompose the gauge field A_μ in Fourier as following

$$A_\mu(t, x_i, \phi, u) = \frac{1}{(2\pi)^{d-1}} \int (dw)(dm)(dq_i)^{d-3}$$
$$\times e^{-i\omega t + im\phi + iq_i x^i} A_\mu(\omega, m, q_i, u). \quad (25)$$

It is possible, without loss of generality, choose a $(d-1)$- dimensional wave vector $\vec{p} = (-\omega, Q_a)$ (with $a = 1, \ldots, d-2$), where $Q_a = (m, q_i) = (0, q, 0)$, so, as initial configuration, have the gauge field A_μ propagating in one of the planar directions $x_i = (x, \vec{0})$. Such choice allows us to consider the perturbations on the gauge field as two orthogonal sets [36] [34], the odd perturbations A_ϕ and the even perturbations A_t, A_x, A_u.

Our gauge choice is the radial gauge where $A_r = A_u = 0$, and the fundamental gauge invariant variables for the two classes of perturbations are the transverse component of electric field E_ϕ for the odd perturbations and the component E_x for the even perturbations. The equations governing the dynamics are obtained from the Maxwell equations (24) written on the spacetime (23):

$$E_\phi'' + \frac{g(u)'}{g(u)} E_\phi' + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 g(u)}{(d-3)^2 g(u)^2 u^{\frac{2d-4}{d-3}}} E_\phi = 0, \quad (26)$$

$$E_x'' + \frac{g(u)' \mathfrak{w}^2}{g(u)[\mathfrak{w}^2 - \mathfrak{q}^2 g(u)]} E_x' + + \frac{\mathfrak{w}^2 - \mathfrak{q}^2 g(u)}{(d-3)^2 g(u)^2 u^{\frac{2d-4}{d-3}}} E_x = 0, \quad (27)$$

where the primes refers to derivatives with respect to u direction. For convenience, we have normalized the quantities $\mathfrak{w}$ and $\mathfrak{q}$ in terms of black hole Hawking temperature

$$T = \frac{(d-1)}{4\pi l^2 k} r_0,$$

namely,

$$\mathfrak{w} = \frac{(d-1)}{4\pi k} \frac{\omega}{T}, \qquad \mathfrak{q} = \frac{(d-1)}{4\pi k} \frac{q}{T}.$$

Following the AdS/CFT recipe [30], the current-current two point correlators are given by the field E_μ ($\mu = \phi, x$) near the AdS boundary, which in our case, is obtained through the solution of Eqs. (26) and (27) at $u \approx 0$:

$$E_\phi = a_\phi(\mathfrak{w}, \mathfrak{q}) + b_\phi(\mathfrak{w}, \mathfrak{q})u, \qquad (28)$$

$$E_x = a_x(\mathfrak{w}, \mathfrak{q}) + b_x(\mathfrak{w}, \mathfrak{q})u, \qquad (29)$$

furthermore, close to the event horizon $E_\mu = g(u)^{\pm \frac{ik}{d-1}\mathfrak{w}}$, where the positive exponent corresponds to outgoing waves and the negative exponent to ingoing waves at the event horizon, also the choice of sign means the electric field at AdS boundary is taken as classical source of retarded (negative) or advanced (positive) current-current two point correlators. The ingoing waves at the event horizon are physically motivated boundary conditions for a classical black hole, thus, we are going to adopt the negative exponent for the electrical field E_μ meaning that we are considering the retarded correlators of the holographic field theory.

The next step is to consider the electromagnetic action at the AdS boundary ($u \approx 0$) with the results (28) and (29) we have

$$S = \frac{(d-3)r_0^{(d-3)}}{2\eta^2 l^{d-3}} \int \frac{d\omega dq}{(2\pi)^2} \left[\frac{g(u)}{\mathfrak{q}^2 g(u) - \mathfrak{w}^2} \right.$$
$$\left. \times E_x(u, -\vec{p})E_x'(u, \vec{p}) - \frac{g(u)}{\mathfrak{w}^2} E_\phi(u, -\vec{p})E_\phi'(u, \vec{p}) \right]. \quad (30)$$

where η^2 is the normalization of the action, from [37] one finds

$$\frac{1}{\eta^2} = \frac{(d-1)\Gamma[\frac{d}{2}]}{2^{(d-1)}\pi^{\frac{d}{2}}\Gamma[d]} (N_c^2 - 1),$$

with N_c representing the number of D-branes. Also, we can rewrite the electric field at the AdS boundary in terms of the gauge field in the same region $A_\mu^0(\vec{p}) = A_\mu(u \to 0, \vec{p})$ and applying the Lorentzian prescription [30],

$$C_{\mu\nu}(\omega, \vec{p}) = \frac{2\delta^2 S}{\delta A_\mu^0(\vec{p})\delta A_\nu^0(-\vec{p})}, \qquad (31)$$

one finds the current-current correlators

$$C_{tt}(\omega, q) = \frac{(d-3)r_0^{d-3}}{\eta^2 l^{d-3}} \frac{b_x(\mathfrak{w}, \mathfrak{q})}{a_x(\mathfrak{w}, \mathfrak{q})} \frac{\mathfrak{q}^2}{\mathfrak{w}^2 - \mathfrak{q}^2}, \qquad (32)$$

$$C_{xx}(\omega, q) = \frac{(d-3)r_0^{d-3}}{\eta^2 l^{d-3}} \frac{b_x(\mathfrak{w}, \mathfrak{q})}{a_x(\mathfrak{w}, \mathfrak{q})} \frac{\mathfrak{w}^2}{\mathfrak{w}^2 - \mathfrak{q}^2}, \qquad (33)$$

$$C_{\phi\phi}(\omega, q) = \frac{(d-3)r_0^{d-3}}{\eta^2 l^{d-3}} \frac{b_\phi(\mathfrak{w}, \mathfrak{q})}{a_\phi(\mathfrak{w}, \mathfrak{q})}, \qquad (34)$$

$$C_{tx}(\omega, q) = \frac{(d-3)r_0^{d-3}}{\eta^2 l^{d-3}} \frac{b_x(\mathfrak{w}, \mathfrak{q})}{a_x(\mathfrak{w}, \mathfrak{q})} \frac{\mathfrak{w}\mathfrak{q}}{\mathfrak{w}^2 - \mathfrak{q}^2}. \qquad (35)$$

Using the components of $C_{\mu\nu}$ it is possible to express the transversal $\Pi^T(\omega, q)$ and longitudinal $\Pi^L(\omega, q)$ self-energies of the $(d-1)$ holographic thermal field theory

$$\Pi^T(\omega, q) = \frac{(d-3)r_0^{d-3}}{\eta^2 l^{d-3}} \frac{b_\phi(\mathfrak{w}, \mathfrak{q})}{a_\phi(\mathfrak{w}, \mathfrak{q})}, \qquad (36)$$

$$\Pi^L(\omega, q) = \frac{(d-3)r_0^{d-3}}{\eta^2 l^{d-3}} \frac{b_x(\mathfrak{w}, \mathfrak{q})}{a_x(\mathfrak{w}, \mathfrak{q})}. \qquad (37)$$

Therefore, the electromagnetic correlation functions are fully determined by the relations $b_\phi(\mathfrak{w}, \mathfrak{q})/a_\phi(\mathfrak{w}, \mathfrak{q})$ and $b_x(\mathfrak{w}, \mathfrak{q})/a_x(\mathfrak{w}, \mathfrak{q})$ and the poles of the correlators are the same as the zeros of $a_\phi(\mathfrak{w}, \mathfrak{q})$ and $a_x(\mathfrak{w}, \mathfrak{q})$ [35]. To find the poles, we impose Dirichlet boundary conditions on the electric field at AdS boundary and ingoing wave conditions at the black hole event horizon.

B. Diffusion quasinormal modes

To determine the self-energies found in the preceding computation, we have to solve the differential equations for E_x and E_ϕ. Analytical solutions are unknown, unless in the so-called *hydrodynamical limit* of the perturbations. Such a limit is achieved by considering a set of perturbations with small frequencies and small wave numbers,

$$\mathfrak{w} \ll 1, \qquad \mathfrak{q} \ll 1.$$

From the point of view of the thermal field theory, at least one of the electromagnetic quasinormal frequencies has to behave as a diffusion mode in the hydrodynamical limit. So, if we impose Dirichlet and ingoing-wave boundary conditions to the differential equations (26) and (27), we found that there is not a transversal diffusion mode, namely, does not exist a value of ω that is compatible with $E_\phi = 0$ at AdS boundary. Such a result is independent on the dimensionality of the bulk and the flavor of the Lovelock theory, in other words, independent on d and k. However, we found that for the longitudinal mode, there is a hydrodynamical mode given by

$$\mathfrak{w} = -i\frac{\mathfrak{q}^2}{(d-3)} \Rightarrow \omega = -i\frac{(d-1)}{4\pi(d-3)kT}q^2, \qquad (38)$$

whose diffusion coefficient can be read off

$$D = \frac{(d-1)}{4\pi(d-3)kT}. \qquad (39)$$

This is the main result of the section. We found that the diffusion coefficient depends crucially on the flavor of Lovelock gravity. As we increase the corrections to the curvature in Lovelock Lagrangian the diffusion coefficient tend to zero, so the charge diffusion in longitudinal direction in thermal field theory is diminished in gravity duals with corrections to the curvature.

VI. CONCLUDING REMARKS

In this work we have studied the effects of higher order corrections to the gravity upon the scalar and hydrodynamical quasinormal modes spectrum, the condensation of holographic operators and their conductivity.

Regarding to the scalar quasinormal modes, we found that the corrections to the curvature diminish the quasinormal modes oscillating phase, it is similar to the dynamics of a perturbation in a very dense material medium. We see from Fig. 3 the case where the real part of the frequencies are zero, so these modes are purely damped. Moreover, we found in the hydrodynamical limit a purely damped diffusive quasinormal mode $\omega = -i(d-1)q^2/4\pi(d-3)kT$, which depends strongly on the k parameter.

We obtained explicitly the phase transition giving the condensation of operators $\langle \mathcal{O}_1 \rangle$ and $\langle \mathcal{O}_2 \rangle$. The influence of curvature corrections of the Lovelock gravity is to increase the value of the condensate, in both charged and uncharged cases. Also, we compute the conductivity, where we found that the considered gravity bulk diminish the real part and imaginary part of $\sigma(\omega)$ as we add more corrections to curvature.

As an extension of this work, it would be interesting to consider charged fermions fields evolving on the gravity bulk given by the family of Lovelock black holes in order to investigate if purely damped quasinormal frequencies are allowed in this case. Another problem which will be address in a future work is the question of gravitational stability of Lovelock black holes and the computation of holographic stress-energy tensor of field theory on the spacetime AdS boundary.

ACKNOWLEDGMENTS

APPENDIX EQUATIONS OF MOTION FOR $(d = 5, k = 2)$, $(d = 6, k = 1)$ AND $(d = 6, k = 2)$

The general equations of motion of the scalar and gauge fields are

$$\psi_{5,2}'' + \frac{((b^4 + b^2 + 1)z^2 - 3\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)})}{z\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)} + (b^4 + b^2)z^5 - (b^4 + b^2 + 1)z^3}\psi_{5,2}'$$

$$+ \frac{(m^2(z^2\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)} - 1) + z^2\varphi_{5,2}^2)}{z^2(z^2\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)} - 1)^2}\psi_{5,2} = 0,$$

$$\varphi_{5,2}'' - \frac{\varphi_{5,2}'}{z} + \frac{2\varphi_{5,2}\psi_{5,2}^2}{z^2(z^2\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)} - 1)} = 0,$$

$$A_{x,5,2}'' + \left[\frac{(\frac{(b^4 + b^2)z}{\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)}} - \frac{2}{z^3})}{\frac{1}{z^2} - \sqrt{1 - b^2(b^2 + 1)(z^2 - 1)}} + \frac{1}{z}\right]A_{x,5,2}' + \frac{2(z^2\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)} - 1)\psi_{5,2}^2 + \omega^2 z^2}{z^2(z^2\sqrt{1 - b^2(b^2 + 1)(z^2 - 1)} - 1)^2}A_{x,5,2} = 0.$$

$$\psi_{6,1}'' + \frac{-2(b^2 + b + 1) - 3(b + 1)(b^2 + 1)(b^4 + 1)z^5 + 6b^3(b^4 + b^3 + b^2 + b + 1)z^8}{z(b^2 - (b + 1)(b^2 + 1)(b^4 + 1)z^5 + b^3(b^4 + b^3 + b^2 + b + 1)z^8 + b + 1)} - \frac{2}{z}\psi_{6,1}'$$

$$+ \frac{\varphi_{6,1}^2 - m^2[\frac{1}{z^2} - z^6(\frac{b^8 - 1}{z^3(b^3 - 1)} + \frac{b^3 - b^8}{b^3 - 1})]}{z^4[\frac{1}{z^2} - z^6(\frac{b^8 - 1}{z^3(b^3 - 1)} + \frac{b^3 - b^8}{b^3 - 1})]^2}\psi_{6,1} = 0,$$

$$\varphi_{6,1}'' - \frac{2}{z}\varphi_{6,1}' - \frac{2(b^3 - 1)\varphi_{6,1}\psi_{6,1}^2}{(b^3 - 1)z^2 - (b^8 - 1)z^7 + (b^3 - b^8)z^{10}} = 0,$$

$$A_{x,6,1}'' + \frac{\frac{3(1 - b^8)z^2}{b^3 - 1} + \frac{6b^3(b^4 + b^3 + b^2 + b + 1)z^5}{b^2 + b + 1} - \frac{2}{z^3}}{\frac{1}{z^2} - z^6(\frac{b^8 - 1}{z^3(b^3 - 1)} + \frac{b^3 - b^8}{b^3 - 1})}A_{x,6,1}' + \frac{\omega^2 - 2[\frac{1}{z^2} - z^6(\frac{b^8 - 1}{z^3(b^3 - 1)} + \frac{b^3 - b^8}{b^3 - 1})]\psi(z)^2}{z^4[\frac{1}{z^2} - z^6(\frac{b^8 - 1}{z^3(b^3 - 1)} + \frac{b^3 - b^8}{b^3 - 1})]^2}A_{x,6,1} = 0.$$

$$\psi_{6,2}'' - \left(\frac{\frac{b^8 - 4(b^5 - 1)b^3 z^3 - 1}{2\sqrt{b^3 - 1}\sqrt{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}} + \frac{2}{z^3}}{\frac{1}{z^2} - \sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}}} + \frac{2}{z}\right)\psi_{6,2}' \frac{m^2 z^2\sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}} - m^2 + z^2\varphi_{6,2}^2}{z^2(z^2\sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}} - 1)^2}\psi_{6,2} = 0,$$

$$\varphi_{6,2}'' - \frac{2}{z}\varphi_{6,2}' + \frac{2\varphi_{6,2}\psi_{6,2}^2}{z^4\sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}} - z^2} = 0,$$

$$A_{x,6,2}'' + \frac{-\frac{b^8 - 4(b^5 - 1)b^3 z^3 - 1}{2(b^3 - 1)\sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}}} - \frac{2}{z^3}}{\frac{1}{z^2} - \sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}}}A_{x,6,2}' \frac{2z^2\sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}}\psi_{6,2}^2 + \omega^2 z^2 - 2\psi_{6,2}^2}{z^2(z^2\sqrt{\frac{z(b^8 - (b^5 - 1)b^3 z^3 - 1)}{b^3 - 1}} - 1)^2}A_{x,6,2} = 0 \tag{A1}$$

We get the results for different values of b (representing the charge) k and the dimension d.

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