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PARTICLES AND FIELDS

Neutrino Magnetic Moment and the Muon $g-2$ Tension

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Abstract

It is suggested that the precession of the boosted neutrino magnetic moments in the external magnetic field may explain the tension between the muon anomalous g-factor measured in the Muon $g-2$ experiments and the theoretical value predicted by the Standard Model. As the neutrinos and positron resulting from the muon decay are entangled prior to the positron detection, the interaction of the neutrino dipoles with the magnetic field gives origin to an uncertainty in the time modulation of the positron angular distribution. The translated uncertainty in the muon g-factor, $\delta a_\mu \approx (1.96 \pm 0.68)$ ppm, presents 1σ agreement with the anomaly measured at Fermilab, $\delta a_\mu \approx (2.14 \pm 0.50)$ ppm. A dependence of the anomaly on the positron energy is also predicted and can be experimentally verified. A fitting of the Brookhaven binned data suggests such a dependence, with $\chi_r^2 = 0.86$, better than with an energy-independent anomaly, which gives $\chi_r^2 = 0.78$.

Keywords Relativistic entanglement · Neutrino magnetic moment · Muon g-factor · Muon decay

1 Introduction

The discrepancy between the theoretical value of the muon magnetic anomaly [1] and that recently observed in the Fermilab Muon $g-2$ experiment [2, 3] (and earlier at Brookhaven [4]) has raised claims about possible new physics beyond the standard model of elementary particles. There is still, however, some space to explore existing alternatives in the context of known physics, as for example new theoretical computations of the hadronic contributions from lattice QCD [5]. In this note we explore the effect of a non-zero neutrino magnetic dipole on the distribution of positrons in these experiments, which would affect the precision of the muon magnetic moment determination. That Dirac neutrinos have magnetic moments is a result of the minimal extension of the standard model with massive neutrinos [6]. As we shall see, they precess in the magnetic field with a frequency that, corrected by the corresponding Lorentz boosts, precisely agrees with the anomaly measured in the muon Larmor frequency. Although there is no physical interaction (in the classical sense) between the neutrinos and positron resulting from the muon decay, they are entangled prior to the positron detection, and tracing over the neutrino helicity states leaves an uncertainty in the positron time modulation that could explain the tension. If this is the case, it would mean not only a verification of the theoretical prediction for the muon g-factor but also for the neutrino magnetic moment, as well as an interesting evidence of quantum entanglement and non-locality [7].

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2 A Curious Correlation

In the Muon $g-2$ experiment [2] the muon magnetic dipole precesses around a uniform magnetic field, the muon decays into two neutrinos and a positron, and the resulting time modulation in the direction of positron emission is used to determine the muon $g-2$ value. We will try to show that the precession of the neutrinos magnetic dipoles affects the measurement significantly, resulting in a correction to the muon magnetic moment given by

$$\delta\mu_\mu \gamma_\mu = 2\mu_\nu \gamma_\nu, \tag{1}$$

where μ_ν is the neutrino magnetic moment and the Lorentz γ-factors are due to the muon and neutrinos boosts in the magnetic field. We would then have, in natural units,

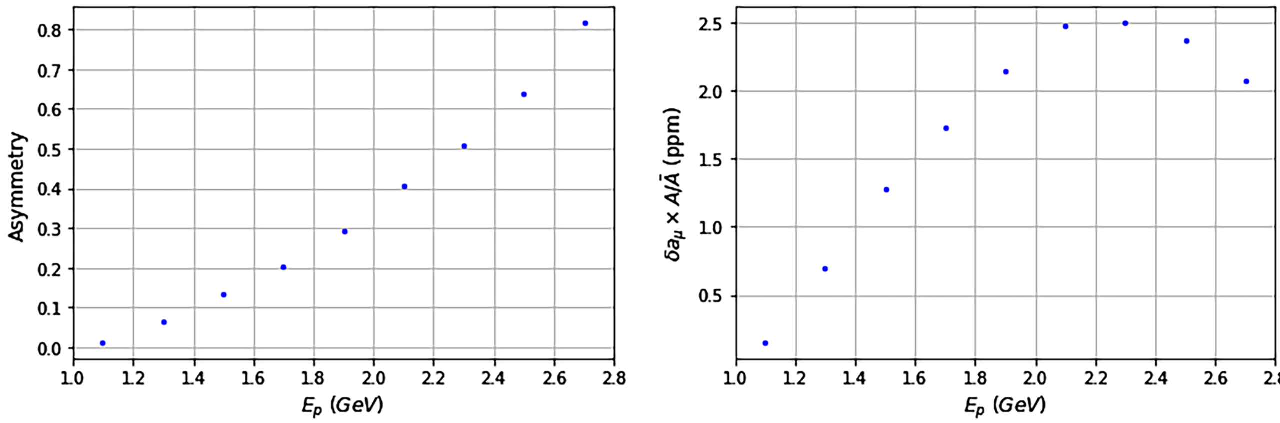

Fig. 1 Positron asymmetry [8] (left panel) and the asymmetry weighted $g-2$ anomaly (right panel) as functions of the positron energy

$$\left(\frac{\delta g}{2}\right)\left(\frac{e}{2m_\mu}\right)\gamma_\mu = 2\mu_\nu\gamma_\nu, \tag{2}$$

where δg is the corresponding correction to the muon g-factor, or yet

$$\delta a_\mu\,\mu_B\left(\frac{m_e}{m_\mu}\right)\gamma_\mu = 2\mu_\nu\gamma_\nu. \tag{3}$$

In these expressions, $a_\mu = (g-2)/2$ is the muon magnetic anomaly, e is the elementary charge, μ_B is the Bohr magneton, and m_μ and m_e are respectively the muon and electron masses.

In the minimal extension of the standard model that accommodates massive Dirac neutrinos by including right-handed singlets, their magnetic moment is [6]

$$\mu_\nu \approx \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} \approx 3.2\times 10^{-19}\mu_B\left(\frac{m_\nu}{1\mathrm{eV}}\right), \tag{4}$$

where m_ν is the neutrino mass and G_F is the Fermi's constant. On the other hand, if the produced neutrinos carry in the laboratory frame a fraction f of the muon energy, we also have

$$f m_\mu\gamma_\mu = 2m_\nu\gamma_\nu. \tag{5}$$

In this way, we obtain

$$\delta a_\mu \approx 3.2\times 10^{-19}\left(\frac{m_\mu}{m_e}\right)\left(\frac{m_\mu}{1\mathrm{eV}}\right)f. \tag{6}$$

Note that δa_μ does not depend on the neutrino mass, but the more energetic the neutrinos, the stronger the effect. Using $m_\mu \approx 106$ MeV and $m_e \approx 0.5$ MeV we have

$$\delta a_\mu \approx (7.2f)\times 10^{-9}, \tag{7}$$

that is, the average discrepancy found in the experiments relative to the standard model prediction, provided that $f \approx 0.35$. It means that the positron energy in the lab frame is $E_p \approx 0.65\,E_\mu$. The experiment uses muons with $E_\mu \approx 3.1$ GeV, which leads to $E_p \approx 2$ GeV. This is indeed the central value for the energy of detected positrons. The measured 1σ interval $\delta a_\mu \approx (2.51 \pm 0.59)\times 10^{-9}$ [2] corresponds to 1.7 GeV $< E_p <$ 2.3 GeV, in agreement with the observed interval of highest probability (see Fig. 15 of [3]). This central value for the positron energy is in fact that expected theoretically. The positron spectrum in the muon rest frame is given by Eq. (6) of [4]. The preferred direction of emission is aligned to the muon polarisation, along which the spectrum is reduced to

$$\frac{dP}{dy d\Omega} = \frac{2y^2}{\pi}(1-y), \tag{8}$$

where $y = E_p^*/E_{\max}$ is the normalised positron energy, with $E_{\max} \approx m_\mu/2$. It has a maximum for $y \approx 0.67$.

Going further, we can try to predict the measured value of the anomaly by averaging Eq. (7) over the positron energy. As the $g-2$ measurement rests on the asymmetry in the positron emission, which depends on its energy, the average must be weighted with the asymmetry. In Fig. 1 we show the asymmetry A (left panel) and the weighted anomaly normalised with the average asymmetry (right panel) as functions of E_P. Because the asymmetry increases monotonically with the positron energy while the anomaly Eq. (7) decreases monotonically, the resulting curve presents a maximum, near the central value found above. The average anomaly in the energy interval[1] 1 GeV $< E_P < 2.7$ GeV is $\delta a_\mu \approx (1.96 \pm 0.68)$ ppm, in 1σ agreement with the observed value $\delta a_\mu \approx (2.14 \pm 0.50)$ ppm.

[1] For $E_P < 1$ GeV the asymmetry becomes negative, the error bars are too large, and for this reason the experiment is biased to mostly catch positrons with energy above this limit. The highest sensitivity per unit energy interval is reached for energies around 2.6 GeV.

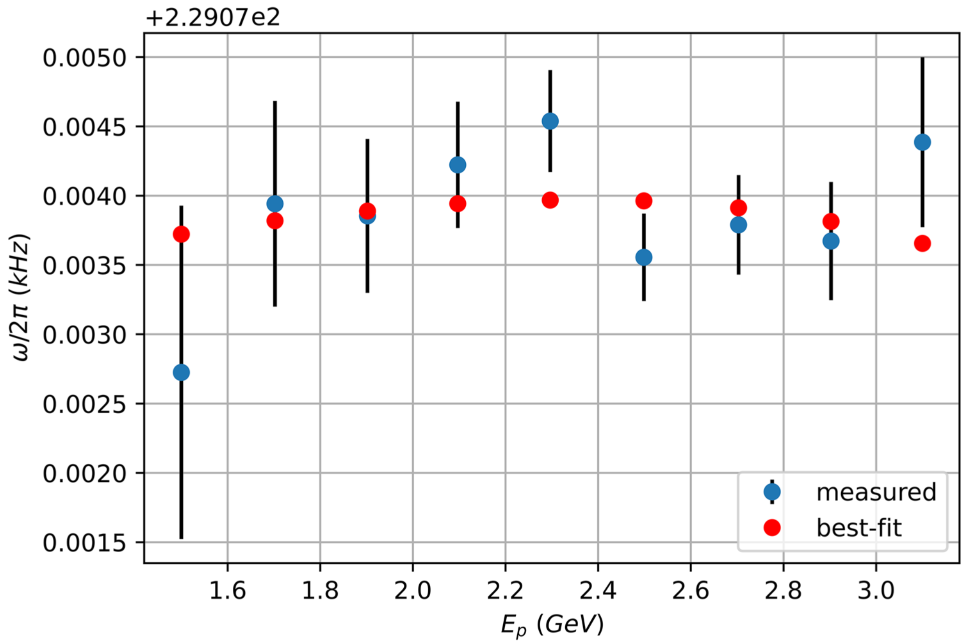

Fig. 2 Best fit of the measured frequency ω_a as a function of the positron energy [4]. The red dots correspond to the anomaly Eq. (7) weighted with the positron asymmetry

Besides correctly predicting the measured average anomaly, Eq. (7) also presents a dependence on the positron energy that can, in principle, be tested.[2] We can find binned data in Fig. 38 of [4], which can be fitted with our curve of the weighted anomaly (right panel of Fig. 1). In order to include all the data points, ranging from 1.5 GeV to 3.1 GeV, we have used the asymmetry given in [4], which does not differ too much from that shown in the left panel of Fig. 1. The result can be seen in Fig. 2. The red dots correspond to our predicted anomaly, and the blue dots to the points measured at Brookhaven. The relation between the experimental anomaly δa_μ and the measured frequency ω_a is given by [4]

$$\omega_a \approx \omega_{a0}\left(1 + \frac{\delta a_\mu}{a_\mu}\right), \tag{9}$$

where $a_\mu \approx 1.17 \times 10^{-3}$ and ω_{a0} is the frequency for $\delta a_\mu = 0$, left as a free parameter. The best fit gives $\omega_{a0}/2\pi = 229.07361 \pm 0.00014$ kHz. The reduced χ^2 is $6.9/8 \approx 0.86$, a little better than that obtained with a constant ω_a, i.e. $6.2/8 \approx 0.78$ [4].

3 Entanglement in the Muon Decay?

In order to understand the origin of correction Eq. (1) we need to remember the basic principle behind the experiments, i.e. the correlation between the muon magnetic moment and the direction of the positron emission. Its derivation can be found, e.g., in [9], from where we have extracted Fig. 3. Although that derivation involves a summation over the helicities, the figure shows that the correlation also includes the neutrinos spins and, as positron and neutrinos form a quantum system entangled at the muon decay, the precession of the latter in the external magnetic field is correlated with an additional time modulation in the positron probability angular distribution. This additional modulation can be derived from first principles, with no need to refer to experimental details. As in any entanglement measurement, it does not result from an interaction between the different parts of the quantum system, but from the uncertainty principle applied to the whole system. Like in other similar contexts (e.g. the Bohm-Aharanov effect), the wave function presents unitary evolution and does not collapse in the magnetic field. Nevertheless, the interaction between the latter and the two neutrino dipoles leads to an uncertainty in the energy of the system given by

$$\delta E = 2\mu_\nu B\gamma_\nu, \tag{10}$$

where the Lorentz factor comes from the neutrinos boost in the magnetic field. This uncertainty is related to our ignorance about the neutrinos polarisation. The interaction is

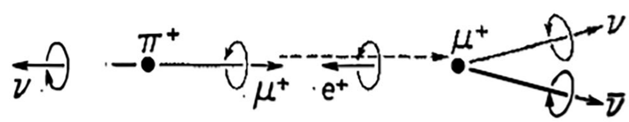

Fig. 3 Correlation between the muon spin and the direction of positron emission [9]

[2] We are thankful to colleagues from Muon $g-2$ Collaboration for calling our attention to this possibility.

maximised for neutrinos moving orthogonally to the magnetic field, when it is enhanced by the neutrino's boost. In the neutrinos frame[3], each dipole acquires an energy $-\gamma_\nu \mathbf{B} \cdot \boldsymbol{\mu}_\nu$ and the maximum uncertainty Eq. (10) corresponds to the dipoles alignment with the field.

At the same time, the helicity states are not eigenstates of the Hamiltonian. The neutrino helicities oscillate with a frequency of maximum value δE, and tracing over them leaves an uncertainty in the time modulation of the positron distribution. For each individual neutrino the energy × time indeterminacy relation is saturated, i.e. $\delta E_\nu \delta t_\nu = \hbar/2$, where $\delta E_\nu = \mu_\nu B \gamma_\nu$ and $\delta t_\nu = 1/\delta E$ is the minimum period of neutrinos precession. For the whole system, the indeterminacy relation gives the period of the modulation anomaly, $\delta t = 1/(2\delta E)$.

Entanglement can be made explicit if we formally write the time evolution of the system wave function as

$$\Psi(t|s,p) = e^{-i\hat{H}_s t}\left[e^{-i\hat{H}_0 t}\,\Psi(0|s,p)\right], \tag{11}$$

where s, p generically represent the particles spins and momenta, $\hat{H}_0$ is the Hamiltonian operator for the muon decay, and $\hat{H}_s$ is the Hamiltonian for the dipoles interaction with the magnetic field. The final state is measured at a time t after the muon decay. It depends on the initial and final spins and momenta in a non-trivial way dictated by general conservation laws [10, 11]. Finding the final distribution of positrons involves to trace over the neutrinos helicities,

$$|\Psi(t|p)|^2 = \sum_{s(t)} |\Psi(t|s,p)|^2. \tag{12}$$

In the presence of a magnetic field the helicities oscillate, expansion Eq. (12) is in a basis of non-stationary states, and the measured probability depends on the flying time between the muon decay and the positron detection, which leaves an uncertainty in the modulation frequency of the positron distribution. Nevertheless, this is in fact a formal reasoning, because we cannot really follow the time evolution of interacting relativistic quantum systems, i.e. the explicit time dependence of the probability amplitude Eq. (12) through the varying helicities. Only the initial and final states are given and, for this reason, a postulate regarding the entanglement of the involved particles is necessary.

The corresponding correction in the muon Larmor frequency involves a further Lorentz factor owing to the muon boost in the laboratory frame. Identifying the muon proper frequency correction $2\delta\mu_\mu B\gamma_\mu$ with $1/\delta t$, we obtain

$$\delta E = \delta\mu_\mu B \gamma_\mu, \tag{13}$$

[3] Note that, for relativistic muons, the neutrinos are emitted nearly in the same direction in the lab frame, with approximately equal γ factors.

which, by using Eq. (10), finally leads to Eq. (1). Its precise agreement with the measured anomaly suggests that the $g-2$ experiment verifies not only the muon theoretical g-factor but also the neutrino magnetic moment, at the same time that it evidences quantum entanglement and non-locality in the muon decay.

Here, a remark is in order on the choice of reference frames when describing relativistic entanglement [12, 13]. The reader should note that Eq. (10) corresponds to the neutrinos frequency in their rest frame, while Eq. (13) represents (half) the uncertainty in the muon frequency at the muon rest frame. In fact, as the periods of clocks in relative motion depend on the frame, the only meaningful statement is the correlation between the proper frequencies, measured by observers comoving with the entangled particles. That is what we seem to find in the $g-2$ experiments: *Eq. (1) expresses the equality between the neutrinos maximum proper frequency and (half) the corresponding uncertainty in the muon proper frequency*.

A further argument may also help to elucidate the correlation above. Before the muon decay, the minimum error in the muon Larmor period is measured at the muon frame and is given by the inverse of $\delta\mu_\mu\gamma_\mu B$. After the decay, the minimum error caused by the neutrinos precession in the modulation period is measured by an observer comoving with the neutrinos, being given by the inverse of $2\mu_\nu\gamma_\nu B$. As the system presents unitary evolution until the positron detection, no information is lost and the minimal uncertainty is the same before and after the decay, which leads again to Eq. (1).

Our estimation of the modulation uncertainty is quite general, with the neutrino dipoles pointing to any direction. If we assume at the end of the day that neutrino masses are incorporated to the standard model by the inclusion of right-handed singlets, their polarisation is longitudinal and, as they are emitted horizontally, Eq. (10) is in fact a definite frequency, not only a superior limit. Therefore, the corresponding uncertainty in the muon frequency also becomes a precise correction. It has a positive value, because neutrinos and muon precess in the same direction.

4 Concluding Remarks

The entanglement between neutrinos and positrons in the context of $g-2$ experiments certainly deserves further studies and a better understanding, specially in view of the lack of a definitive description of relativistic entanglement [12]. The correlation found above, between frequencies measured by comoving observers, is not only suggested by Lorentz invariance, but also by the quantum mechanical interpretation of entanglement as a non-local action between particles that were once in causal contact. As this "action at a distance” is maintained along the time, the correlation between comoving frames is natural.

If such an effect is indeed present, there are some interesting by-products. First, the neutrino magnetic moment is given by Eq. (4) only in the case of Dirac neutrinos, whereas Majorana neutrinos present only transition magnetic moments but no diagonal dipoles. On the other hand, Eq. (4) is derived within a minimal extension of the Standard Model that includes massive neutrinos [6]. In this respect, the $g-2$ experiments would be confirming these two hypotheses. Furthermore, quantum entanglement is usually manifest in correlated destructive measurements. It has already appeared in experiments with neutrino oscillations [14] and neutral kaons decay [15]. The present case is an example where a non-collapsing interaction of part of a system, namely between the neutrinos and the external magnetic field, affects a measurement performed on the other entangled component. This may be confirmed or ruled out when binned data from the Fermilab experiment, possibly more accurate than those used in Fig. 2, are available.

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Declarations

Conflict of Interest The authors declare no competing interests.

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