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by

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A GENERALIZED COCHRANE-ORCUTT-TYPE ESTIMATOR FOR TIME SERIES REGRESSION MODELS

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Abstract

Given the regression model $y_t = x_t'\beta + u_t$, $t=1,\dots,T$, where $x_t = (x_{1t},\dots,x_{kt})'$ and $\beta = (\beta_1,\dots,\beta_k)'$, we shall consider a two stage estimation procedure for β, from observations on $\{y_t, x_t\}$, under certain assumptions on $\{x_t\}$ and $\{u_t\}$, the latter considered a stationary sequence. The method is basically a version of the Cochrane-Orcutt procedure, where the order of the auto-regression fitted to the residuals of first stage is obtained via some information criterium (AIC, BIC,etc.). We shall prove the consistency, asymptotic normality and asymptotic efficiency of the proposed estimator.

Key words: Autoregression; AIC; BIC; Cochrane-Orcutt estimator; Regression; Stationary series.

1. Introduction

In this work we will be interested in the model

$$y_t = x_t'\beta + u_t \ , \ t = 1,\ldots,T \ , \tag{1.1}$$

where $x_t = (x_{1t},\ldots,x_{kt})'$ and $\beta = (\beta_1,\ldots,\beta_k)'$. The aim is to estimate β from observations $\{y_t,x_t\}$, $t=1,\ldots,T$, assuming that $\{x_t\}$ is an observable deterministic sequence and $\{u_t\}$ is a non-observable stationary random sequence. The model (1.1) can be written in the form

$$y = X\,\beta + u \ , \tag{1.2}$$

with $y = (y_1,\ldots,y_T)'$, $u = (u_1,\ldots,u_T)'$ and $X = (x_1,\ldots,x_T)$ a $T\times k$ matrix.

It is known that if $\{u_t\}$ is not a white noise sequence, then the ordinary least squares (OLS) estimate of β, given by

$$\hat{\beta}_{OLS} = (X'X)^{-1}\,X'y \ , \tag{1.3}$$

it is not, in general, optimal (BLUE). Optimal procedures will involve the replacement of $X'X$ and $X'y$ in (1.3) by $X'V^{-1}X$ and $X'V^{-1}y$, respectively, where V is the $T\times T$ covariance matrix on u. If V is known we have the usual generalized least squares (GLS) of β. See Harvey (1981) for details.

Since V is generally unknown, GLS estimators will involve two stages:

(A) Estimation of the process $\{u_t\}$.

(B) Use of the results of (A) to estimate β.

To implement stage (A) it will be necessaire to obtain a consistent estimator for β, generally through OLS, and then to obtain the first stage regression residuals $\hat{u}_t$, given by

$$\hat{u}_t = y_t - x'_t \hat{\beta}_{OLS} \tag{1.4}$$

Some alternative ways to implement stages (A) and (B) are:

(A) Parametric method (AR and ARMA modelling); non-parametric method (spectral modelling).

(B) Brute force method; Generalized Cochrane-Orcutt method; Hannan method.

Thus, in stage (A) we can fit an AR (autoregressive) model or an ARMA (autoregressive, moving average) model to the residuals , or we can estimate the spectral density of the residual process. Given that we have used AR or ARMA modelling in stage (A), we can proceed to stage (B) by using the brute force method or the generalized Cochrane-Orcutt procedure. In the former, we obtain a consistent estimator $\hat{V}$ for the matrix V and form the GLS estimator

$$\hat{\beta}_{GLS} = (X'V^{-1}X)^{-1} X'\hat{V}^{-1}y \ . \tag{1.5}$$

In the latter case, we transform the data y_t and x_t using the same model proposed for the residuals and then obtain GLS estimates regressing the transformed y_t on the transformed x_t. See Cochrane-Orcutt (1949), Theil (1971) and Harvey (1981) for details.

If we use any of the methods of estimation in stage (A) we can proceed to stage (B) and obtain (1.5) in the frequency domain. This is accomplished replacing the x_t and y_t by their Fourier transforms and V by a diagonal matrix whose elements are spectral estimates of u_t in different frequencies. See Hannan (1963) and Harvey (1978).

Another possibility is to build an ARMA model for u_t and then to proceed by maximum likelihood. See Pierce (1971). In this case, the order (p,q) of the ARMA model is assumed to be fixed.

The purpose of this paper is to propose an estimating procedure, which is a generalization of the Cochrane-Orcutt method, where the order of the autoregression fitted to the residuals (1.4) is not fixed, but determined according to some criterion. Under certain assumptions on the sequencies x_t and u_t, we will derive the consistency, asymptotic normality and asymptotic efficiency of the second-stage estimator.

2. Assumptions

In this section we collect the main assumptions which will be used in what follows.

We assume that the sequence $\{u_t\}$ is generated by an autoregressive model with infinite order,

$$\sum_{j=0}^{\infty} a(j)\, u_{t-j} = \varepsilon_t \,, \tag{2.1}$$

where $a(0) = 1$, $a(1)$, $a(2)$, ... are real numbers and:

(A1) ε_t is a sequence of independent, normal random variables, with zero means and variance $\sigma_\varepsilon^2 > 0$;

(A2) $\sum_{j=0}^{\infty} j^{1/2} |a(j)| < \infty$;

(A3) $A(z) = \sum_{j=0}^{\infty} a(j)\, z^j \neq 0$, $|z| \leq 1$.

Conditions (A2) and (A3) imply that $\{u_t\}$ is stationary and invertible, with bounded non null spectral density function.

For the sequence $\{x_t\}$ we will assume that it is deterministic, depending on T, satisfying:

(B1) $\lim_{T\to\infty} d_j^2(T) = \infty$, $d_j^2(T) = \sum_{t=1}^{T} x_{j,t}^2$, $j=1,\ldots,k$.

(B2) $\lim_{T\to\infty} \dfrac{x_{j,T}^2}{d_j^2(T)} = 0$, $j=1,\ldots,k$.

(B3) $\lim_{T\to\infty} \dfrac{\sum_{t=1}^{T-|\ell|} x_{j,t}\, x_{i,t+|\ell|}}{(d_j^2(T))^{1/2}(d_i^2(T))^{1/2}} = r_{i,j}(\ell)$, $\ell=0,\pm1,\ldots$

Condition (B1) is necessary to warrant the consistency of the estimators that will be proposed. If the conditions (B1) - (B3) are satisfied, then the k×k matrix $R(\ell) = \left[r_{i,j}(\ell)\right]$ is non-negative definite (n.n.d) and hence can be written as

$$R(\ell) = \int_{-\pi}^{\pi} e^{i\ell\lambda} F_x'(d\lambda) , \qquad (2.2)$$

where $F_x(\lambda_2) - F_x(\lambda_1)$ is a n.n.d matrix, for all λ_1 , λ_2 such that $-\pi \leq \lambda_1 < \lambda_2 \leq \pi$.

In the scalar case, k=1, we shall ommit the indices in (B1)-(B3); we shall write, for example x_T in place of $x_{j,T}$ and $d^2(T)$ in place of $d_j^2(T)$. In the next sections we restric attention to the scalar case, for the ease of notation and exposition. The extensions to the general case k>1 can be obtained with no further difficulties.

3. The Ordinary Least Squares Estimators

Consider (1.1) with k=1, and assume that $\{u_t\}$ follows (2.1). We establish the consistency and asymptotic normality of $\hat{\beta}_{OLS}$.

Theorem 3.1. If conditions (A1)-(A3) hold for $\{u_t\}$, if $\{x_t\}$ satisfies (B3) and

$$\text{(B'1)} \quad d^2(T) = \sum_{t=1}^{T} x_t^2 = O(T^\delta) \text{ , for some } \delta > 0 \text{ ,}$$

then the estimator $\hat{\beta}_{OLS}$ given by (1.3) is consistent.

Proof. We have that $\mathrm{Var}(\hat{\beta}_{OLS}) = (x'x)^{-1} x'Vx(x'x)^{-1}$, where $V = [\gamma(i-j)]$, $\gamma(i-j) = E(u_{t-i}u_{t-j})$, $i,j=0,\ldots,T-1$.

Since $x'Vx = \sum_{t=1}^{T} \sum_{s=1}^{T} x_t x_s \gamma(t-s) = \sum_{k=-T+1}^{T-1} \gamma(k) \sum_{t=1}^{T-|k|} x_t x_{t+|k|}$,

it follows that $x'Vx = O(T^{\delta})$ and hence $\mathrm{Var}(\hat{\beta}_{OLS}) = O(T^{-\delta})$, since (A1)-(A3) imply $\sum_k |\gamma(k)| < \infty$ (Shibata, 1980). □

Theorem 3.2. Under the conditions (A1)-(A3) for $\{u_t\}$ and (B1)-(B3) for $\{x_t\}$, we have that

$$\left(\sum_{t=1}^{T} x_t^2\right)(\hat{\beta}_{OLS} - \beta) \xrightarrow{L} N(0,W) ,$$

where $W = \sum_{\ell=-\infty}^{\infty} r(\ell)\cdot\gamma(\ell)$, $r(\ell)$ given in (B3).

Proof. We have that

$$\left(\sum_{t=1}^{T} x_t^2\right)^{1/2} (\hat{\beta}_{OLS} - \beta) = \left(\sum_{t=1}^{T} x_t^2\right)^{-1/2} \sum_{t=1}^{T} u_t x_t$$

By (A1)-(A3), we can write

$$u_t = \sum_{j=0}^{\infty} b(j)\varepsilon_{t-j} , \tag{3.1}$$

where the b(j) satisfy

$$\sum_{j=0}^{\infty} j^{1/2} |b(j)| < \infty , \tag{3.2}$$

(see Hannan and Kavalieris, 1986). The conditions of Theorem 6.3.4 of Fuller (1976, p.251) are satisfied and the result follows. □

4. A Two-Stage Generalized Cochrane-Orcutt Estimator

Asymptotically eficient estimators of the parameters of regression models with correlated erros have been suggested in the literature. In geral, when model (1.1) is considered, it is assumed that $\{u_t\}$ is generated by an AR(p) or ARMA(p,q) model, where the orders p or (p,q) are fixed. A more realistic assumption is that $\{u_t\}$ is generated by an infinite order autoregressive model. Our aim is to present an estimation procedure under such assumption on $\{u_t\}$ and derive some properties of the estimators. The relevant references here are Berk (1974), An, Chen and Hannan (1982), Hannan and Kavalieris (1983, 1984a, 1984b, 1986) and Shibata (1980).

The suggested estimation procedure follows the stages:

(a) Compute the OLS estimator of β, $\hat{\beta}_{OLS}$.

(b) Compute the residuals of the OLS regression,

$$\hat{u}_t = y_t - \hat{\beta}_{OLS}\, x_t \;, \; t=1,\ldots,T. \tag{4.1}$$

and fit an autoregression of order h to $\hat{u}_t$. The order h is chosen such that

$$\log \hat{\sigma}_h^2 + h \cdot C(T)/T \tag{4.2}$$

is minimized, for $h \leq H(T)$, where $H(T)$ increases with T and $C(T)/T \to 0$. If $C(T) = 2$, we are led to the AIC criterion (Akaike, 1973), if $C(T) = \log T$, we have the BIC criterion (Akaike, 1977) and if $C(T) = c \log \log T$, $c > 2$, we have HQ criterion (Hannan and Quinn, 1979).

The estimates of the coefficients of the autoregression fitted to $\hat{u}_t$ will be denoted by $\hat{a}_h(j)$, $j=1,\ldots,h$, and are Yule-Walker estimates and can be obtained through the Durbin-Levinson recursion (Durbin,.1960; Morettin,1984). In (4.2), $\hat{\sigma}_h^2$ is the estimated residual variance for a fitted model of order h.

(c) Form the sequences

$$\hat{\eta}_t = \sum_{j=0}^{h} \hat{a}_h(j) y_{t-j} \ , \ t=h+1,\ldots,T \tag{4.3}$$

$$\hat{\Psi}_t = \sum_{j=0}^{h} \hat{a}_h(j) x_{t-j} \ , \ t=h+1,\ldots,T \ , \tag{4.4}$$

where $\hat{a}_h(0) = 1$.

(d) The final estimator of β, $\hat{\beta}$, is given by the regression of $\hat{\eta}_t$ on $\hat{\Psi}_t$, for $t = h+1,\ldots,T$, namely

$$\hat{\beta} = \frac{\sum_{t=h+1}^{T} \hat{\eta}_t \hat{\Psi}_t}{\sum_{t=h+1}^{T} \hat{\Psi}_t^2} \ . \tag{4.5}$$

We call $\hat{\beta}$ a <u>generalized Cochrane-Orcutt estimator in two stages</u> (GCO2). The rationale for such a procedure is that model (1.1) is being replaced by

$$\hat{\eta}_t = \beta \hat{\Psi}_t + \hat{\varepsilon}_{t,h} \ , \tag{4.6}$$

where

$$\hat{\varepsilon}_{t,h} = \sum_{j=0}^{h} \hat{a}_h(j) u_{t-j} \tag{4.7}$$

We can see $\hat{\varepsilon}_{t,h}$ as a substitute for ε_t , when we approximate (2.1) by (4.7). With $h = h_T$, $\hat{\varepsilon}_{t,h}$ gets closer to ε_t, as h increases (Hannan, 1986, Theorem 2.2) and (4.6) gets closer to a model with uncorrelated errors.

The consistency of $\hat{\beta}$ is proved in section 7 and the asymptotic normality and efficiency of $\hat{\beta}$ are proved in section 8. In section 6 we present some results necessaire to prove these properties and in section 5 we introduce some definitions an notation that will be used in the sequel.

5. Notation and Definition

Let $\gamma(t) = E(u_s u_{s+t})$ the autocovariance function of u_t and $\Gamma(h)$ the h×h matrix with elements $\gamma(j-k)$, j,k=1,...,h.

Define

$$c(t) = \begin{cases} \frac{1}{T} \sum_{s=1}^{T-t} u_s u_{s+t} , & 0 \le t \le T-1 \\ 0 & , t \ge T \end{cases} \tag{5.1}$$

$$c(-t) = c(t) \tag{5.2}$$

and let $\hat{c}(t)$ the function obtained from $c(t)$, replacing u_t by $\hat{u}_t$ in (5.1) and (5.2).

Let $\tilde{\Gamma}(h)$ and $\hat{\Gamma}(h)$ the matrices with elements $c(j-k)$ and $\hat{c}(j-k)$, $j,k = 1,\dots,h$, respectively and let the vectors

$$a_h = \left[a_h(1),\dots,a_h(h)\right]' , \quad \tilde{a}_h = \left[\tilde{a}_h(1),\dots,\tilde{a}_h(h)\right]' ,$$

$$\hat{a}_h = \left[\hat{a}_h(1),\dots,\hat{a}_h(h)\right]'$$

be specified, respectively, by

$$a_h' \, \Gamma(h) = -\left[\gamma(1),\dots,\gamma(h)\right] , \tag{5.3}$$

$$\tilde{a}_h' \, \tilde{\Gamma}(h) = -\left[c(1),\dots,c(h)\right], \tag{5.4}$$

$$\hat{a}_h' \, \hat{\Gamma}(h) = -\left[\hat{c}(1),\dots,\hat{c}(h)\right]. \tag{5.5}$$

Define η_t, ψ_t and $\varepsilon_{t,h}$ in an analogous fashion to $\hat{\eta}_t$, $\hat{\psi}_t$ and $\hat{\varepsilon}_{t,h}$, given in (4.3), (4.4) and (4.7), respectively, when we replace $\hat{a}_h(j)$ by $a_h(j)$.

Denote by $\|x\|_\infty$ the uniform norm of x, that is, $\|x\|_\infty = \max|x_j|$ and for any matrix A, $\|A\|_\infty = \max\limits_i \sum\limits_j |a_{ij}|$.

If $\{y_t\}$, $t=1,2,\dots$ is a sequence of random variables and $\{g_t\}$, $t=1,2,\dots$ is a sequence of positive real numbers, we shall write $y_t = O_p(g_t)$ if for any $\varepsilon > 0$, there is a positive real number M_ε such that $P\{|y_t| \geq M_\varepsilon g_t\} \leq \varepsilon$, for all t. We shall write $y_t = o_p(g_t)$ if for any $\varepsilon > 0$, $\lim\limits_{t\to\infty} P\{|y_t| > \varepsilon g_t\} = 0$. We write

$y_t = O(g_t)$ if $\limsup_{t\to\infty} |y_t|/g_t < \infty$, almost surely, and $y_t = o(g_t)$ if "$< \infty$" is replaced by "$= 0$".

6. Some Preliminary Results

The purpose here is to establish the convergence (in probability) of the estimators of the autoregressive coefficients. For this, we prove initially the convergence of the sample covariances of the residuals $\hat{u}_t$ to their true values.

Lemma 6.1. The Gaussian sequence $\{u_t\}$ is regular if and only if it can be written as

$$u_t = \sum_{j=0}^{\infty} c_j \, e_{t-j} \, ,$$

where $\sum_{j=0}^{\infty} |c_j|^2 < \infty$ and the $\{e_t\}$ are independent and normaly distributed.

For a proof, see Ibragimov and Linnik (1971, Cap. 17).

Theorem 6.1. Consider model (1.1), with $\{u_t\}$ given by (2.1) and satisfying (A1)-(A3) and $\{x_t\}$ satisfying (B1)-(B3). Then,

$$\sup_{1 \leq t < \infty} |\hat{c}(t) - \gamma(t)| = O_p\{(\log T/T)^{1/2}\}$$

Proof. For $0 \leq t \leq T-1$ we have

$$\hat{c}(t) - c(t) = \frac{1}{T}\left\{\sum_{t=1}^{T-t} \hat{u}_s \hat{u}_{s+t} - \sum_{s=1}^{T-t} u_s u_{s+t}\right\} =$$

$$= \frac{1}{T}\left\{\sum_{s=1}^{T-t} (y_s - \hat{\beta}_{OLS} x_s)(y_{s+t} - \hat{\beta}_{OLS} x_{s+t}) - \sum_{s=1}^{T-t} u_s u_{s+t}\right\} =$$

$$= \frac{1}{T} (\beta - \hat{\beta}_{OLS})^2 \sum_{s=1}^{T} x_s^2 \frac{\sum_{s=1}^{T-t} x_s x_{s+t}}{\sum_{s=1}^{T} x_s^2} +$$

$$+ (\beta - \hat{\beta}_{OLS}) \left(\sum_{s=1}^{T} x_s^2\right)^{1/2} \left[\frac{\sum_{s=1}^{T-t} x_s u_{s+t}}{\left(\sum_{s=1}^{T} x_s^2\right)^{1/2}} + \frac{\sum_{s=1}^{T-t} x_{s+t} u_s}{\left(\sum_{s=1}^{T} x_s^2\right)^{1/2}}\right]$$

By Theorem 3.2, $\left(\sum_{s=1}^{T} x_s^2\right)^{1/2} (\hat{\beta}_{OLS} - \beta) = O_p(1)$, and by Theorem 6.3.4 of Fuller (1976), $\left(\sum_{s=1}^{T} x_s^2\right)^{1/2} \sum_{s=1}^{T-t} x_s u_{s+t} = O_p(1)$, $t = 0,1,\ldots,T-1$. Also, by (B3), $\sum_{s=1}^{T-t} x_s x_{s+t} \Big/ \sum_{s=1}^{T} x_s^2 = O(1)$, $t=0,1,\ldots,T-1$, and hence

$$\sup_{0 \le t < \infty} |\hat{c}(t) - c(t)| = O_p(T^{-1}). \tag{6.2}$$

Conditions (A1)-(A3) imply that the assumptions of Lemma 6.1 are satisfied, hence $\{u_t\}$ is regular and consequently ergodic. By Theorem 3 of An, Chen and Hannan (1982),

$$\sup_{0\le t<\infty} |c(t)-\gamma(t)| = O_p\{(\log T/T)^{1/2}\} \tag{6.3}$$

From (6.2) and (6.3) we obtain (6.1). □

Lemma 6.2 below is due to Hannan and Kavalieris(1984a) and Lemma 6.3 is due to Berk (1974).

<u>Lemma 6.2</u>. Let $\{u_t\}$ generated by (2.1), satisfying (A2) and (A3) and assume that ε_t satisfies $E(\varepsilon_t\varepsilon_s) = \delta_{st}\sigma^2$, $E\{\varepsilon_t|F_{t-1}\} = 0$, $E(\varepsilon_t^4) < \infty$ and $E\{\varepsilon_t^2|F_{-\infty}\} = \sigma^2$, when F_t is the σ-algebra generated by ε_s, $s \le t$. If $H(T) = T$, then

$$\max_{1\le j\le H(T)} \left[T^{-1} \sum_{t=j+1}^{T} \varepsilon_t\, u_{t-j}\right] = 0\,\{(\log T/T)^{1/2}\}. \tag{6.4}$$

<u>Lemma 6.3</u>. Let $\{u_t\}$ satisfying (2.1) with $\sum_{j=0}^{\infty} |a(j)| < \infty$. Define $A_h(z) = \sum_{j=0}^{h} a_h(j)z^j$, $a_h(0) = 1$ and $a_h(j)$, $j=1,\dots,h$ specified by (5.3). Then $\lim_{h\to\infty} A_h(z) = A(z)$, $|z| \le 1$.

<u>Theorem 6.2</u>. Let $\{y_t\}$ generated by (1.1) with $\{u_t\}$ given by (2.1) and satisfying (A1)-(A3), $\{x_t\}$ satisfying (B1)-(B3). Then, for $H(T) = o\{(T/\log T)^{1/2}\}$, we have

$$\max_{1\le j\le h} |\hat{a}_h(j) - a_h(j)| = O_p\{(\log T/T)^{1/2}\}, \tag{6.5}$$

uniformly for $h \le H(T)$.

<u>Proof</u>. We have that

$$\sum_{j=1}^{h} \{\hat{a}_h(j) - a_h(j)\} \hat{c}(j-k) = - \sum_{j=0}^{h} a(j) \hat{c}(j-k) +$$

$$+ \sum_{j=1}^{h} a(j) \hat{c}(j-k) - \sum_{j=1}^{h} a(j) \gamma(j-k) + \sum_{j=0}^{h} a(j) \gamma(j-k) +$$

$$+ \sum_{j=1}^{h} a_h(j) \gamma(j-k) - \sum_{j=1}^{h} a_h(j) \hat{c}(j-k) =$$

$$= - \sum_{j=0}^{h} a(j) \hat{c}(j-k) + \sum_{j=1}^{h} \{a(j) - a_h(j)\}\{\hat{c}(j-k) - \gamma(j-k)\}+$$

$$+ \sum_{j=0}^{h} a(j) \gamma(j-k) \tag{6.6}$$

By Theorem 6.1, the second term of the right-handed side (RHS) in (6.6) is

$$O_p \{(\log T/T)^{1/2}\} \sum_{j=1}^{h} \{a(j) - a_h(j)\} \; .$$

and by Lemma 6.3, $\sum_{j=1}^{h} \{a(j) - a_h(j)\} = o(1)$, uniformly in h, the above mentioned second term is $o_p\{(\log T/T)^{1/2}\}$, uniformly in h.

Since $\lim_{t\to\infty} t^{1/2}\gamma(t) = o(1)$, using (A2) we have

$$\left| \sum_{j=T+1}^{\infty} a(j)\gamma(j-k) \right| = O(T^{-1/2}) \tag{6.7}$$

and consequently for the last term of RHS of (6.6) we have

$$\sum_{j=0}^{h} a(j) \gamma(j-k) = - \sum_{j=h+1}^{\infty} a(j)\gamma(j-k) = - \sum_{j=h+1}^{T} a(j)\{\gamma(j-k)$$

$$- \hat{c}(j-k)\} - \sum_{j=h+1}^{T} a(j)\hat{c}(j-k) + O(T^{-1/2}) = \sum_{j=h+1}^{T} a(j)\hat{c}(j-k) +$$

$$+ O_p\{(\log T/T)^{1/2}\} \sum_{j=k+1}^{T} a(j) + O(T^{-1/2}) =$$

$$= - \sum_{j=h+1}^{T} a(j)\hat{c}(j-k) + o_p\{(\log T/T)^{1/2}\}.$$

Therefore (6.6) becomes

$$\sum_{j=1}^{h}\{\hat{a}_h(j) - a_h(j)\}\hat{c}(j-k) = - \sum_{j=0}^{h} a(j)\hat{c}(j-k) + o_p\{(\log T/T)^{1/2}\}. \tag{6.8}$$

Using (6.2), the first term of the RHS of (6.8) is

$$- \sum_{j=0}^{T} a(j)c(j-k) + O_p(T^{-1}) \tag{6.9}$$

and, as proved by Hannan and Kavalieris(1986), the first term of (6.9) is

$$- \frac{1}{T} \sum_{j=0}^{T} \sum_{t=k+1}^{T} u_{t-j} u_{t-k},$$

hence

$$\sum_{j=0}^{T} a(j)c(j-k) = \frac{1}{T} \sum_{t=k+1}^{T} \left[\sum_{j=0}^{\infty} a(j)u_{t-j} - \sum_{j=T+1}^{\infty} a(j)u_{t-j} \right] u_{t-k} =$$

$$= \frac{1}{T} \sum_{t=k+1}^{T} \varepsilon_t u_{t-k} + o(T^{-1/2}). \tag{6.10}$$

By substitution of (6.10) in (6.8) we obtain

$$\sum_{j=1}^{h} \{\hat{a}_h(j) - a_h(j)\}\hat{c}(j-k) = -\frac{1}{T}\sum_{t=k+1}^{T} \varepsilon_t u_{t-k} + o_p\{(\log T/T)^{1/2}\},$$

and by Lemma 6.2,

$$\max_{1\le k\le h} \sum_{j=1}^{k} \{\hat{a}(j)-a_h(j)\}\hat{c}(j-k) = O_p\{(\log T/T)^{1/2}\}, \qquad (6.11)$$

uniformly in $h \le H(T)$, $H(T) = o\{(T/\log T)^{1/2}\}$.

On the other hand,

$$\max_{1\le j\le h} |\hat{a}_h(j)-a_h(j)| = \|\hat{a}_h-a_h\|_\infty = \|\hat{\Gamma}(h)^{-1}\hat{\Gamma}(h)\left[\hat{a}_h-a_h\right]\|_\infty$$

$$\le \|\hat{\Gamma}(h)^{-1}\|_\infty \|\hat{\Gamma}(h)\left[\hat{a}_h-a_h\right]\|_\infty , \qquad (6.12)$$

and hence using (6.11), (6.12) and the fact that $\|\hat{\Gamma}(h)\|_\infty^{-1}$ is uniformly bounded (Hannan and Kavalieris, 1984a), we obtain the desired result. □

7. The Consistency of The Estimator

Let $\hat{\beta}$ the GCO2 estimator defined by (4.5) and let

$$\hat{\beta}^* = \frac{\sum_{t=h+1}^{T} \eta_t \psi_t}{\sum_{t=h+1}^{T} \psi_t^2} \qquad (7.1)$$

Theorem 7.1. Under the conditions of Theorem 6.2,

$$\left(\sum_{t=1}^{T} x_t^2\right)^{1/2} (\hat{\beta}^* - \beta) = o_p(1),$$

uniformly in $h \leq H(T)$.

Proof. $\hat{\eta}_t = \eta_t + \sum_{j=1}^{h} \left[\hat{a}_h(j) - a_h(j)\right] y_{t-j}$,

$$\hat{\psi}_t = \psi_t + \sum_{j=1}^{h} \left[\hat{a}_h(j) - a_h(j)\right] x_{t-j} ,$$

$$\hat{\varepsilon}_{t,h} = \varepsilon_{t,h} + \sum_{j=1}^{h} \left[\hat{a}_h(j) - a_h(j)\right] u_{t-j} .$$

Hence,

$$\left(\sum_{t=1}^{T} x_t^2\right)^{-1/2} \sum_{t=h+1}^{T} \hat{\psi}_t \hat{\varepsilon}_{t,h} = \left(\sum_{t=1}^{T} x_t^2\right)^{-1/2} \left\{ \sum_{t=h+1}^{T} \psi_t \varepsilon_{t,h} + \right.$$

$$+ \sum_{t=h+1}^{T} \psi_t \sum_{j=1}^{h} \left[\hat{a}_h(j) - a_h(j)\right] u_{t-j} + \sum_{t=h+1}^{T} \varepsilon_{t,h} \sum_{j=1}^{h} \left[\hat{a}_h(j) - \right.$$

$$\left. - a_h(j)\right] x_{t-j} + \sum_{t=h+1}^{T} \sum_{j=1}^{h} \sum_{i=1}^{h} \left[\hat{a}_h(j) - a_h(j)\right] \left[\hat{a}_h(i) - \right.$$

$$\left. \left. - a_h(i)\right] x_{t-j} u_{t-i} \right\} . \tag{7.2}$$

The second term of RHS of (7.2) is

$$\sum_{j=1}^{h} \left[\hat{a}_h(j)-a_h(j)\right]\left\{\left(\sum_{t=1}^{T} x_t^2\right)^{-1/2}\left[\sum_{t=h+1}^{T} x_t\, u_{t-j} + a_h(1)\right.\right.$$

$$\left.\left.\sum_{t=h+1}^{T} x_{t-1}\, u_{t-j} + \ldots + a_h(h) \sum_{t=h+1}^{T} x_{t-h}\, u_{t-j}\right]\right\} . \qquad (7.3)$$

From Theorem 6.3.4 of Fuller (1976) and Theorem 6.2 and from the fact that $h \le H(T) = o\{(T/\log T)^{1/2}\}$, (7.3) is $o_p(1)$, uniformly in $h \le H(T)$. Similarly, the third and fourth term of the RHS of (7.2) are $o_p(1)$. Hence (7.2) becomes

$$\left(\sum_{t=1}^{T} x_t^2\right)^{-1/2} \sum_{t=h+1}^{T} \hat{\psi}_t\, \hat{\varepsilon}_{t,h} = \left(\sum_{t=1}^{T} x_t^2\right)^{-1/2} \sum_{t=h+1}^{T} \psi_t \varepsilon_{t,h} + o_p(1), \qquad (7.4)$$

uniformly in $h \le H(T)$. In an analogous way, it can be shown that

$$\left(\sum_{t=1}^{T} x_t^2\right)^{-1} \sum_{t=h+1}^{T} \hat{\psi}_t^2 = \left(\sum_{t=1}^{T} x_t^2\right)^{-1} \sum_{t=h+1}^{T} \psi_t^2 + o_p(1), \qquad (7.5)$$

uniformly in $h \le H(T)$. From (7.4) and (7.5) it follows that

$$\left(\sum_{t=1}^{T} x_t^2\right)^{1/2} \frac{\sum_{t=h+1}^{T} \hat{\psi}_t\, \hat{\varepsilon}_{t,h}}{\sum_{t=h+1}^{T} \hat{\psi}_t^2} = \left(\sum_{t=1}^{T} x_t^2\right)^{1/2} \frac{\sum_{t=h+1}^{T} \psi_t\, \varepsilon_{t,h}}{\sum_{t=h+1}^{T} \psi_t^2} +$$

$$+\, o_p(1) , \qquad (7.6)$$

and the theorem follows. □

Theorem 7.2. Under the conditions of Theorem 6.2, $\hat{\beta}^* \to \beta$ in probability, as $T \to \infty$, for all $h \le H(T)$

Proof. It is easy to verify that $\{\psi_t\}$ satisfies the assumptions (B1)-(B3) and that

$$\sum_{t=h+1}^{T} \psi_t^2 \bigg/ \sum_{t=1}^{T} x_t^2 = O(1). \tag{7.7}$$

Since $\hat{\beta}^* - \beta = \sum_{t=h+1}^{T} \psi_t \varepsilon_{t,h} \bigg/ \sum_{t=h+1}^{T} \psi_t^2$, then

$$\left(\sum_{t=h+1}^{T} \psi_t^2\right)^{1/2} (\hat{\beta}^* - \beta) = \left(\sum_{t=h+1}^{T} \psi_t^2\right)^{-1/2} \left[\sum_{t=h+1}^{T} \psi_t u_t + \right.$$

$$\left. + \sum_{j=1}^{h} a_h(j) \sum_{t=h+1}^{T} \psi_t u_{t-j}\right].$$

Since $\left(\sum_{t=h+1}^{T} \psi_t^2\right)^{1/2} \sum_{t=h+1}^{T} \psi_t u_{t-j} = O_p(1)$, $j=0,1,\ldots,h$, $h \le H(T)$, then it follows that $\left(\sum_{t=h+1}^{T} \psi_t^2\right)^{1/2} (\hat{\beta}^* - \beta) = O_p(1)$, $h \le H(T)$. □

Theorem 7.3. Under the conditions of Theorem 6.2, $\hat{\beta} \to \beta$ in probability, as $T \to \infty$, uniformly in $h \le H(T)$.

Proof. From Theorems 7.1, 7.2 and (7.7), we have

$$\left(\sum_{t=1}^{T} x_t^2\right)^{1/2} (\hat{\beta} - \beta) = \left(\sum_{t=1}^{T} x_t^2\right)^{1/2} (\hat{\beta} - \beta^*) +$$

$$+ \left(\sum_{t=1}^{T} x_t^2\right)^{1/2} (\tilde{\beta}^* - \beta) = O_p(1),\ h \leq H(T) \text{ and the result}$$

follows. □

8. The Asymptotic Normality and Efficiency of $\hat{\beta}$

The asymptotic normality of $\hat{\beta}$ will be proved based on a central limit theorem established by Hannan (1973). The theorem gives the limit distribution of the statistic

$$\sum_{t=1}^{T} x_t u_t \Bigg/ \left(\sum_{t=1}^{T} x_t^2\right)^{1/2} \tag{8.1}$$

in four distinct situations, each derived under certain assumptions on $\{u_t\}$ and $\{x_t\}$.

Theorem 8.1. Under the conditions of Theorem 6.2, and assuming that $\int_{-\pi}^{\pi} f_h(\lambda)\, F_x(d\lambda) > 0$ a.s., where

$$f_h(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi} \left| \sum_{k=0}^{\infty} \sum_{j=0}^{h} a_h(j) b(k-j) e^{ik\lambda} \right|^2 ,$$

and $F_x(\lambda)$ was defined in (2.2) and the $b(j)$ were defined in (3.1), the asymptotic distribution of $\left(\sum_{t=h+1}^{T} \psi_t^2\right)^{1/2} (\tilde{\beta}^* - \beta)$ is normal, with zero mean and variance given by

$$\int_{-\pi}^{\pi} 2\pi f_h(\lambda) F_\psi(d\lambda), \quad h \leq H(T) , \tag{8.2}$$

where $F_\psi(\lambda)$ is the function defined by

$$\rho(\ell) = \lim_{T\to\infty} \frac{\sum_{t=h+1}^{T-|\ell|} \psi_t \psi_{t+|\ell|}}{\sum_{t=h+1}^{T} \psi_t^2} = \int_{-\pi}^{\pi} e^{i\ell\lambda} F_\psi(d\lambda). \tag{8.3}$$

Proof. We know that

$$\left(\sum_{t=h+1}^{T} \psi_t^2\right)^{1/2} (\hat{\beta}^* - \beta) = \sum_{t=h+1}^{T} \psi_t \varepsilon_{t,h} \Bigg/ \left(\sum_{t=k+1}^{T} \psi_t^2\right)^{1/2} ,$$

where $\varepsilon_{t,h} = \sum_{j=0}^{h} a_h(j) u_{t-j}$, $a_h(0) = 1$. From (3.1), it follows that

$$\varepsilon_{t,h} = \sum_{j=0}^{h} a_h(j) \sum_{i=0}^{\infty} b(i)\varepsilon_{t-i-j} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_h(j) b(i) \varepsilon_{t-i-j}$$

$$= \sum_{k=0}^{\infty} c_h(k)\varepsilon_{t-k} , \tag{8.6}$$

where $c_h(k) = \sum_{j=0}^{h} a_h(j) b(k-j)$ and

$$\sum_{k=0}^{\infty} |c_h(k)| < \infty , \quad h \leq H(T) , \tag{8.7}$$

using condition (A2). The representation (8.6), with (8.7) holding, warrants that $\{\varepsilon_{t,h}\}$ is regular, for every $h \leq H(T)$. Since $\{\psi_t\}$ satisfies (B1)-(B3), the conditions of part (i) of Theorem 1 of Hannan (1973) are satisfied and the theorem is proved. □

<u>Theorem 8.2</u>. Under the conditions of Theorem 8.1, the asymptotic distribution of $\left(\sum_{t=1}^{T} x_t^2\right)(\hat{\beta} - \beta)$ is normal, with mean zero and variance given by

$$V_h = \int_{-\pi}^{\pi} 2\pi f_h(\lambda) F^*(d\lambda) \quad , \quad h \leq H(T) \quad , \tag{8.8}$$

where $F^*(\lambda)$ is such that

$$v(\ell) = \lim_{T\to\infty} \frac{\sum_{t=1}^{T} x_t^2 \sum_{t=h+1}^{T-|\ell|} \psi_t \psi_{t+|\ell|}}{\left(\sum_{t=h+1}^{T} \psi_t^2\right)^2} = \int_{-\pi}^{\pi} e^{i\ell\lambda} F^*(d\lambda) . \tag{8.9}$$

<u>Proof</u>. Immediate consequence of Theorems 7.1 and 8.2. □

We now show that the asymptotic variance V_h of $\hat{\beta}$, for $h \to \infty$, coincides with the asymptotic variance of the generalized least squares estimators, establishing the asymptotic efficiency of $\hat{\beta}$.

<u>Theorem 8.3</u>. Under the conditions of Theorem 8.1,

$$\lim_{h\to\infty} V_h = 2\pi \left[\int_{-\pi}^{\pi} \frac{1}{f_u(\lambda)} F_x\,(d\lambda) \right]^{-1}, \tag{8.10}$$

where $f_u(\lambda)$ is the spectral density of $\{u_t\}$.

Proof. We have that

$$f_h(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi} \left| \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} a_h(j) b(k-j) e^{ik\lambda} \right|^2 =$$

$$= \frac{\sigma_\varepsilon^2}{2\pi} \left| \sum_{k=0}^{\infty} b(k-j)\ e^{(k-j)\lambda} \sum_{j=0}^{h} a_h(j) e^{ij\lambda} \right|^2 .$$

Since $\lim_{h\to\infty} A_h(e^{i\lambda}) = A(e^{i\lambda})$, $A^{-1}(e^{i\lambda}) = B(e^{i\lambda})$, then $\lim_{h\to\infty} f_h(\lambda) = \sigma_\varepsilon^2/2\pi$. On the other hand,

$$\lim_{h\to\infty} \lim_{T\to\infty} \sum_{t=h+1}^{T} \psi_t^2 \Bigg/ \sum_{t=1}^{T} x_t^2 = \lim_{h\to\infty} \sum_{j=0}^{h} \sum_{k=0}^{h} a_h(j) a_h(k) r(j-k)$$

$$= \lim_{h\to\infty} \int_{-\pi}^{\pi} A_h(e^{i\lambda})\ A_h^*(e^{i\lambda})\ F_x(d\lambda)\ , \tag{8.11}$$

using (2.2), $A^*(\cdot)$ being the complex conjugate of $A(\cdot)$.

Recalling that

$$f_u(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi} \left[A(e^{i\lambda})\ A^*(e^{i\lambda}) \right]^{-1}$$

it follows that the limit of RHS of (8.11) becomes $\frac{\sigma_\varepsilon^2}{2\pi}\int_{-\pi}^{\pi} f_u^{-1}(\lambda)\, F_x(d\lambda)$. From this and (8.9), we obtain

$$\lim_{h\to\infty} v(0) = \lim_{h\to\infty} \int_{-\pi}^{\pi} F^*(d\lambda) = \frac{2\pi}{\sigma_\varepsilon^2}\left[\int_{-\pi}^{\pi} \frac{1}{f_u(\lambda)}\, F_x(d\lambda)\right]^{-1},$$

hence (8.10) holds. □

9. Further Comments

In the case of the general model (1.1), the COG2 estimator is obtained as in section 4, replacing $\hat{\psi}_t$ by

$$\hat{\psi}_{i,t} = \sum_{j=h+1}^{T} \hat{a}_h(j)\, x_{i,t-j}\ , \ i=1,\dots,k, \tag{9.1}$$

and if $\hat{\beta} = (\hat{\beta}_1,\dots,\hat{\beta}_k)'$, each $\hat{\beta}_i$ is obtained as in (4.5). It can be shown that $\hat{\beta}$ is consistent and the asymptotic distribution of $D_T(\hat{\beta} - \beta)$ is normal, where $D_T = \mathrm{diag}\left[\left(\sum_{t=1}^{T} x_{i,t}^2\right)^{1/2},\dots,\left(\sum_{t=1}^{T} x_{k,t}^2\right)^{1/2}\right]$

Grenander and Rosenblatt (1957) gave conditions under which the asymptotic variance of the OLS estimator, given by Theorem 3.2 is equal to (8.10).

We can show after some algebra that

$$\hat{\beta} \simeq \frac{\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{I_{yx}(\omega)}{I_{xx}(\omega)}\left\{I_{xx}(\omega)\left|\sum_{j=0}^{h} \hat{a}_h(j)e^{-ij\omega}\right|\right\}^2 d\omega}{\frac{1}{2\pi}\int_{-\pi}^{\pi} I_{xx}(\omega)\left|\sum_{j=1}^{h} \hat{a}_h(j)e^{-ij\omega}\right|^2 d\omega}, \tag{9.2}$$

where $I_{xx}(\omega)$ is the periodogram of x_t and $I_{yx}(\omega)$ is the cross-periodogram of y_t and x_t , $t=1,\ldots,T$. Since $I_{yx}(\omega)/I_{xx}(\omega)$ is an estimate of β, we can see (9.2) as a weighted average of estimates of β, one for each frequency. The weighting is given by

$$g(\omega) = I_{xx}(\omega) \left| \sum_{j=0}^{h} \hat{a}_h(j)\, e^{-ij\omega} \right|^2 , \tag{9.3}$$

which is an estimate of $f_x(\omega)/f_u(\omega)$, where $f_x(\omega)$ is the spectrum of x_t and $f_u(\omega)$ is the spectrum of u_t, given by

$$f_u(\omega) = \frac{\sigma_\varepsilon^2}{2\pi} \left| \sum_{j=0}^{\infty} a(j)\, e^{-ij\omega} \right|^{-2} .$$

If we note that the optimal weighting is $I_{xx}(\omega) \left| \sum_{j=0}^{\infty} a(j) e^{-ij\omega} \right|^2$, we conclude that:

(a) If $f_x(\omega)$ is concentrated in a narrow band of frequencies then we do not have to get a good estimate of $f_u(\omega)$. It follows that h can increase slowly with T, or that C(T) increases fast with T.

(b) If $f_x(\omega)$ is very variable, we need a good estimate of $f_x(\omega)$ and, hence, good estimates of a(j). In this case C(T) must increase slowly with T or even be a constant.

Since the Yule-Walker estimates are known to present appreciable bias in some instances (see Shaman and Stine, 1988, and references therein) , possibly better estimates can be obtained using least squares or maximum likelihood estimates in stage (b) of section 4.

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