A methodology for modeling variable axial laminates: a case study of a uniformly loaded circular plate

Maísa M. A. D. Maciel[a]*, José Humberto S. Almeida Jr.[b], Marcelo L. Ribeiro[a], Volnei Tita[a]

[a]Department of Aeronautical Engineering, São Carlos School of Engineering, University of São Paulo, Av. João Dagnone, 1100, 13573-120 São Carlos, SP.
maisa.madm@usp.br , malribei@usp.br , voltita@sc.usp.br.

[b]Department of Mechanical Engineering, Aalto University, Espoo, Finland
humberto.almeida@aalto.fi

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Abstract. *Variable axial (VA) composites are increasingly being used as an alternative to alleviate stress concentrations and distribute more uniformly the load throughout the design domain. However, studies dealing with damage initiation and propagation on VA composites are still scarce. To tackle this problem, a multi-scale approach is proposed here. First, a transversely isotropic representative volume element (RVE) with circular fibers embedded in a square bulk polymeric matrix is developed and the effective coefficients of the constitutive tensor are calculated. Then, the effective coefficients are used as input in a macro-scale model of a uniformly loaded circular plate to predict the mechanical response of isotropic, unidirectional, and VA configurations.*

1. INTRODUCTION

Continuous carbon fiber reinforced plastics (CFRP) are extensively used in lightweight structures due to their high specific strength and stiffness properties [1]. CFRP laminates are usually manufactured with unidirectional layers at different orientations to achieve a quasi-isotropic behavior [1]. In certain load cases and applications, such conventional laminates do not fully exploit the anisotropy of such materials. Allowing fibers to varying within the plane of the laminate increases the ability to tailor laminates for non-uniform stress states in a continuous manner [2]. Also, curved fiber paths present tailoring possibilities that can lead to an alternation of load paths to result in favorable stress distribution within the laminate, thus improving bending and buckling properties [2]. The first works on this field suggested the placement of fibers along principal stress directions to alleviate stress concentrations around discontinuities [3].

Gürdal and Olmedo, (1993) introduced a fiber path definition and formulated a closed-form numerical solution for rectangular plates. Their initial work was very important in the definition of the in-point variation of stiffness and Poisson's ratio [4]. Waldhart et al (1996) included the rotational angle to increase the buckling performance due to the stiffness variation [5]. Recent works have reported in the literature the benefits of the variable axis composites(VA) over traditional straight fiber paths [6–10], which is due to the advancement in manufacturing techniques that enables the production of VA with great reliability. Manufacturing of VA laminates may be achieved through automated fiber placement (AFP), continuous tow shearing (CTS), and tailored fiber placement (TFP) technologies [11].

As VA configurations are being increasingly used in aircraft structures, mainly considering the potentialities for optimization of the fiber orientation in terms of buckling and vibration response [8,11]. However, there are some challenges to be overcome, such as the prediction of damage and failure in VA composites. Therefore, an investigation of damage and failure of VA composite materials is the first important step to define a consistent material model to predict the behavior of structures made of this type of structure. Thus, this work proposes a methodology to predict the mechanical response of VA structures.

2. PROPOSED METHODOLOGY

2.1. Micromechanical model

For that, firstly, a representative volume element (RVE) is generated via Python script and analyzed numerically via finite element (FE) analysis in the Abaqus package. In this case, the RVE is a 1 mm3 cube, and the fiber is a central cylinder. The fiber volume fraction (Vf) used is 62%. Also, a thin layer is added at the boundary between the fiber and the matrix to represent the interphase. Based on Tita et al., 2015. The interface volume fraction (I_f) must be determined based on a relation of interphase thickness and the fiber radius, which leads to an I_f of 0.12% [12].

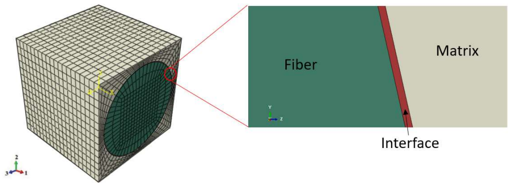


Figure 1 – RVE used in this work with three phases in detail.

Each element (fiber, matrix, and interface) is considered as being isotropic and homogenous. For this work, the interface properties were considered equal to the matrix as in to represent resin pockets. The constitutive equation of the composites studied can be written in the matrix form as Eq. (1) For a transversally isotropic composite laminate, where σ_{ij} and ε_{kl} are the stress and infinitesimal strain tensors, where the contracted Voigt notation is used.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{33} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{Bmatrix} \tag{1}$$

The homogenization approach for composite materials relies on finding a dependence between the average variables of the material model, which may represent coherent physical behavior [13]. Based on the theorem of average, the mechanical properties of the RVE are taken from the average properties of the composite laminate and using the FE analysis the average stress tensor can be calculated following the Eq.(2) [14].

$$\overline{T_{ij}} = \frac{1}{|V|} \sum_{n=1}^{nel} T_{ij}^{(n)} V^{(n)} \tag{2}$$

where nel is the number of elements of the RVE, $Tij^{(n)}$ and $V^{(n)}$ are the evaluated tensor and volume of each element. It is assumed that the average mechanical properties of an RVE are equal to the average properties of the composite laminate.

2.2. Variable axial fiber path definitions

Gürdal and Olmedo (1993) [4], in-plane analysis of rectangular panels made of balanced symmetric angle-ply laminates had a variety of fiber angles $[\pm\theta(x)]_s$ along the axes x. The linear variation along *x* axis centered in a panel of length a resulted in Eq. (3).

$$\theta(x) = \frac{2(T_1 - T_0)}{a}|x| + T_0 \quad (3)$$

Where T_0 is the fiber orientation angle at panel center, $x = 0$, and T_1 is the fiber orientation at the edge of the panel, $x = \pm a/2$.

Waldhart et al (1996) [5], presented a generalized equation of fiber path which allowed the direction of the fiber to be rotated with respect to the coordinate direction x. starting from an arbitrary reference point A with a fiber orientation angle ϕ from coordinate axis x the fiber orientation is assumed to reach a value T_1 at a distance *d* from the reference point. With the linear variation of the fiber orientation angle between A and B, the equation for the fiber orientation angle along this reference path takes the form of Eq. (4).

$$\theta(x') = \phi + (T_1 - T_0)\frac{|x'|}{d} + T_0 \quad (4)$$

The representation of a single curvilinear layer may be specified by $\phi\langle T_0|T_1\rangle$.

3. CASE STUDY

The problem tackled in this work is a uniformly loaded circular plate with clamped edges, as shown in Fig. 2. Initially, a homogenization procedure to obtain the coefficients is performed and then a macro model that represents the circular plate is built up with shell elements (S4R). The plates have a radius of 0.125 m and a thickness of 0.014 m. A pressure of 2.5 MPa is applied.

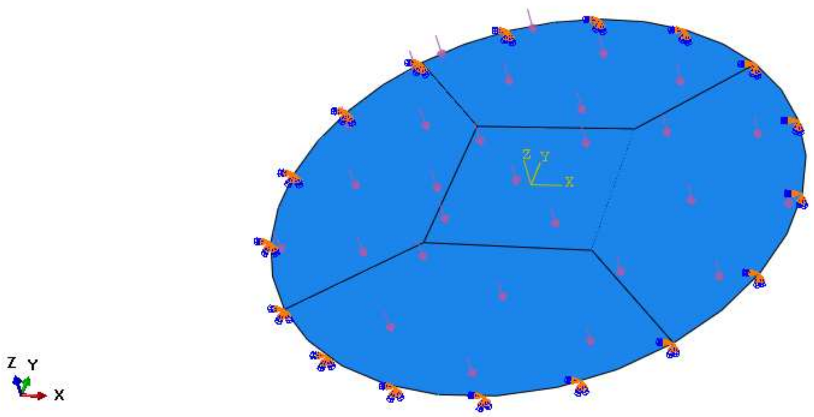
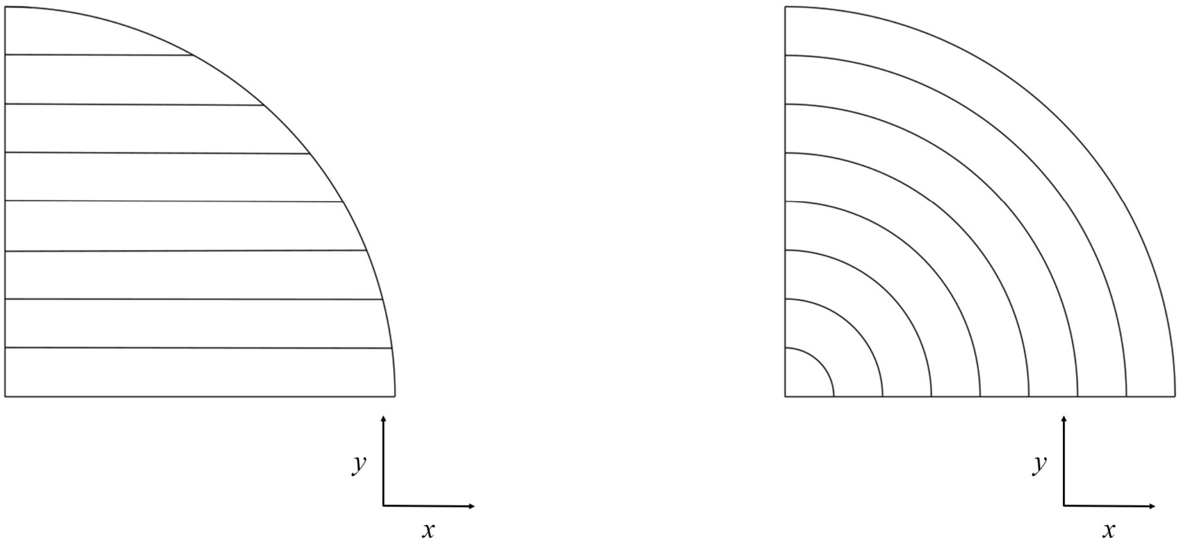


Figure 2– Loads and boundary conditions of the plate.

The VA plate is designed following the fiber path proposed by Gurdäl & Olmedo (1993) [4], which was described in item 1.2.

There is two fiber paths used in this case study and both are illustrated in Fig.3.



(a) (b)

Figure 3– Illustration of the first quadrant of the simulated circular plate for two different fiber paths (a) Unidirectional (b) Variable axial.

3.1. Material properties

The material used was a Hexcel M9/M10 prepreg, which was hand laminated and autoclave cured by 120 °C, with pressure between 0,3 and 5 bar for 60 min [15]. More detail can be obtained in the thesis of Tita (2003) [15]. Table 1 shows the properties used for the micromechanical properties.

Table 1 – Properties used for the micromechanical model used for homogenization

Properties	Values
Fiber modulus	220 GPa
Fiber Poisson coefficient	0.17
Matrix modulus	3.2 GPa
Matrix Poisson coefficient	0.3

3.2. Implementation via UMAT (user material subroutine)

The UMAT subroutine is used to calculate the stress tensor. At this stage of this work, the procedures performed by the UMAT is the identification of the material model variables, the calculation of the Jacobian matrix, and the prediction of the stress according to the fiber direction.

4. PRELIMINARY RESULTS

Following the methodology described by the homogenization procedure, all six effective coefficients are calculated and presented in Table 1. In Table 2, a comparison of the effective coefficients obtained by homogenization and the coefficients calculate in Tita(2003) [15] is presented.

Table 2– Effective coefficients for the material. Units are in GPa.

	C_{11}^{eff}	C_{12}^{eff}	C_{33}^{eff}	C_{13}^{eff}	C_{44}^{eff}	C_{66}^{eff}
Perfect interface	138.404	5.088	20.324	5.088	4.363	6.494
First step degraded interface	137.813	0.966	3.248	0.966	1.086	1.119
Tita (2003)	128.132	3.331	10.089	-	-	5.400

As observed in Table 1, there is a difference between the calculated values by Tita [14] when compared to the ones herein calculated. This difference is due to the homogenization process, which considers the properties of the constituents (fiber and matrix) as well as the interface. Also, Tita [14] calculated these values based on experimental observations, whereas the values here are theoretical.

For this case study, a circular plate with clamped edges is uniformly loaded. Table 3 shows the maximum stress in direction 1 and the maximum deflection at the center of the plate.

Table 3– Effective coefficients for the material.

Material	**Analysis**		**Relative error**
	Abaqus	UMAT	
Isotropic	**S_{11} Max**		
	97.24 MPa	83.26 MPa	14%
	U_3		
	-2.899e-04 m	-2.353e-04 m	18%
Unidirectional	S_{11} Max		
	97.24 MPa	163.30 MPA	40%
	U_3		
	-2.899e-04 m	-6.202e-04 m	53%
VA	**S_{11} Max**		
	--	237.00 MPa	-
	U_3		
	--	-4.781e-04 m	-

The error for the isotropic material properties is quite small, being 14% and 18% in terms of S_{11} and U_3, respectively. The error for the UD material is higher for the same properties, 40% and 45% for S_{11} and U_3, respectively. However, the stress fields are quite similar, as shown in Fig. 4.

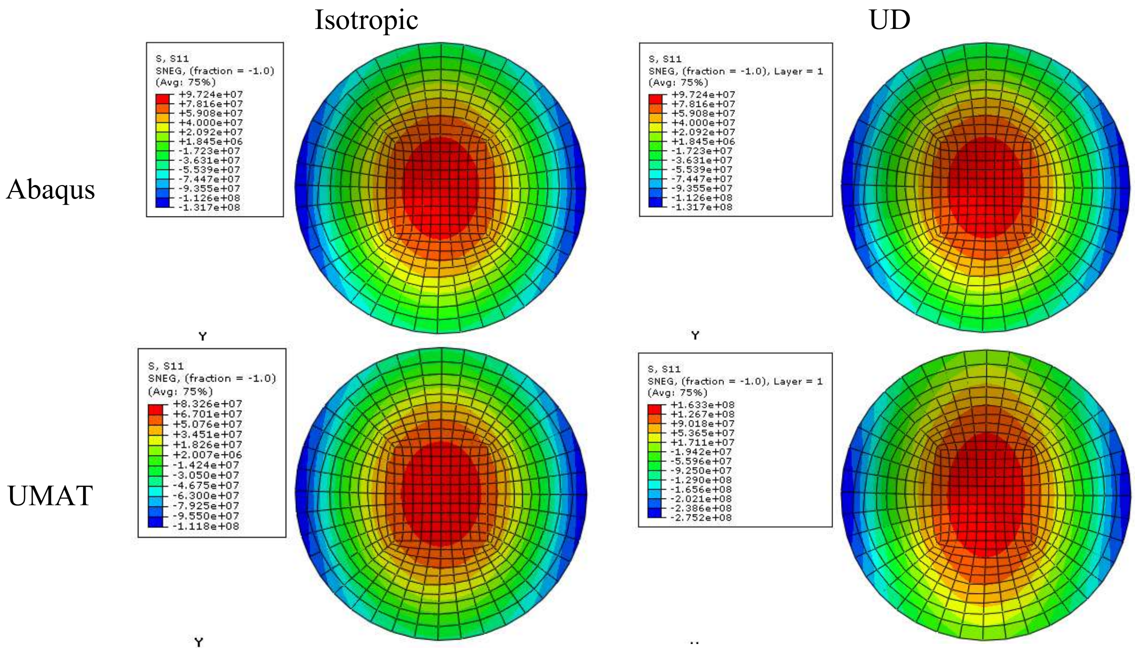


Figure 4– Stress fields S_{11} for all cases.

5. CONCLUSION

Firstly, the effective coefficients for the constitutive matrix were successfully obtained for the composites in this study. Then, and UMAT subroutine was developed for the isotropic case to compare with the analytical solution. Preliminary results shown here are promising, which more comprehensive results will be presented in the final version of this work.

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