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ENTIRE FUNCTIONS ON BANACH SPACES WITH U PROPERTY

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ABSTRACT. Let E and F be complex Banach spaces. We show that if E has U property and contains no subspace isomorphic to l_1, then every holomorphic functions from E into F which is bounded on weakly compact sets of E, is bounded on bounded sets. In particular, this is true in the Banach spaces that are M-ideal in their bi-dual

1. Introduction

Let E and F be complex Banach spaces. We denote by $C_w(E,F)$ and $C_{wu}(E,F)$ the spaces of functions from E into F that are weakly continuous and weakly uniformly continuous respectively, on bounded subsets of E. Valdivia showed in [16] that $C_{wu}(E,F) = C_w(E,F)$ if and only if the space E is reflexive. Afterwards Aron, Hervés and Valdivia show in [1] that for every Banach space E and every polynomial $P \in C_w(E,F)$ we have always that $P \in C_{wu}(E,F)$ and raised the following question: Does is every holomorphic $f \in C_w(E,F)$, weakly uniformly continuous on bounded sets?. This problem today is know as l_1-problem ([5]) because in [1] it also is shown that a positive answer for l_1 implies a positive answer in the general case.

Obviously the problem l_1 has an affirmative answer when E is a reflexive space, however, no positive answer is well known for a Banach space arbitrary and for a long time the only example of a non-reflexive Banach space that answered affirmatively to the problem was c_0, the Banach space of sequences tending to zero, under the sup norm. This example was given by Dineen in [6].

By adapting Dineen's techniques we generalize the result obtained for c_0 and show that the l_1−problem has an positive answer when E is a Banach space with the U property and contains no subspace isomorphic to l_1. Examples of these spaces are the spaces with shirinking and uncondicional basis e more generally Banach spaces that are M-ideal in their bidual.

2. Notation and basic definitions

The letters E, F always denote complex Banach spaces and E' denotes the dual topological of E. Also $S(E) = \{x \in E : \|x\| = 1\}$ denotes the unity sphere of E.

If $f : E \to F$ is a function and $A \subset E$ we denote by

$$\|f\|_A := \sup_{x \in A} \|f(x)\|.$$

In this article, we denote by $P(^nE,F)$ the space of all n- homogeneous from E into F. For each polynomial $P_n \in P(^nE,F)$ we denote by $\overset{\vee}{P}_n$the n-linear symmetric form associate, that is $P(x) = \overset{\vee}{P}_n(x,...,x)$ for every $x \in E$. We also denote by $H(E,F)$ the space of all holomorphic functions from E into F. When $F = \mathbb{C}$ we denote $P(^nE) := P(^nE,\mathbb{C})$ and $H(E) := H(E,\mathbb{C})$. We refer to [10] or [5] for the properties of polynomials and holomorphic functions in infinite dimensional spaces .

Let $(y_i) \subset E$ be a sequence in E and consider a formal series $y = \sum_{i=1}^{\infty} y_i$. For m, n integers positive numbers with $0 = m < n$, we define $q^n(y) := \sum_{i=1}^{n} y_i$ and $q_m^n(y) := \sum_{m+1}^{n} y_i$. Let (P_n) be a sequence

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of n-homogeneous polynomials $n \geq 1$, and a sequence $m_1 < m_2 < m_3 < \ldots$ of integers positive. Then by Leibniz formula [10, thm 1.8]

$$\begin{aligned} \|P_n(q^n(y))\| &= \left\| \sum_{j=0}^{n} \frac{n!}{j!(n-j)!} \overset{\vee}{P}_n (q^{m_1}(y))^j (q^n_{m_1}(y))^{n-j} \right\| \\ &\leq (n+1) \frac{n!}{j_{n,m_1}!(n-j_{n,m_1})!} \left\| \overset{\vee}{P}_n (q^{m_1}(y))^{j_{n,m_1}} (q^n_{m_1}(y))^{n-j_{n,m_1}} \right\| \end{aligned}$$

For some $j_{n,m_1} \in \mathbb{N}$ with $0 \leq j_{n,m_1} \leq n$, again using the Leibniz formula

$$\begin{aligned} &\frac{n!}{j_{n,m_1}!(n-j_{n,m_1})!} \left\| \overset{\vee}{P}_n (q^{m_1}(y))^{j_{n,m_1}} (q^n_{m_1}(y))^{n-j_{n,m_1}} \right\| = \\ &= (n+1) \frac{n!}{j_{n,m_1}!(n-j_{n,m_1})!} \left\| \sum_{j=0}^{n-j_{n,m_1}} \frac{(n-j_{n,m_1})!}{j(n-j_{n,m_1}-j)!} \overset{\vee}{P}_n \left((q^{m_1}(y))^{j_{n,m_1}} (q^{m_2}_{m_1}(y))^j (q^n_{m_2}(y))^{n-j_{n,m_1}-j} \right) \right\| \\ &\leq (n+1)(n-j_{n,m_1}+1) \frac{n!}{j_{n,m_1}!j_{n,m_2}!(n-j_{n,m_1}-j_{n,m_2})!} \times \\ &\quad \times \left\| \overset{\vee}{P}_n (q^{m_1}(y))^{j_{n,m_1}} (q^{m_2}_{m_1}(y))^{j_{n,m_2}} (q^n_{m_2}(y))^{n-j_{n,m_1}-j_{n,m_2}} \right\| \\ &= (d_{n,0}+1)(d_{n,1}+1) \frac{n!}{j_{n,m_1}!j_{n,m_2}!d_{n,2}!} \left\| \overset{\vee}{P}_n \left((q^{m_1}(y))^{j_{n,m_1}} (q^{m_2}_{m_1}(y))^{j_{n,m_2}} (q^n_{m_2}(y))^{d_{n,2}} \right) \right\| \end{aligned}$$

where $d_{n,0} := n$, e $d_{n,1} := n - j_{n,m_1}$, e $d_{n,2} := n - j_{n,m_1} - j_{n,m_2}$. Proceeding inductively we obtain

$$\|P_n(q^n(y))\| \leq \Pi_{s=0}^k (d_{n,s}+1) \frac{n!}{\Pi_{s=1}^k j_{n,m_s}! d_{n,k}!} \left\| \overset{\vee}{P}_n \left(\Pi_{s=1}^k \left(q^{m_s}_{m_{s-1}}(y) \right)^{j_{n,m_s}} \cdot (q^n_{m_k}(y))^{d_{n,k}} \right) \right\|$$

where $0 \leq \sum_{s=1}^r j_{n,m_s} \leq n$, e $d_{n,r} = n - \sum_{s=0}^r j_{n,m_s}$; $(m_0 = 0; j_{n,m_0} := 0)$. And by the inequalities of Cauchy we also have

$$\begin{aligned} \|P_n(q^n(y))\| &\leq \Pi_{s=0}^k (d_{n,s}+1) \frac{n!}{\Pi_{s=1}^k j_{n,m_s}! d_{n,k}!} \left\| \overset{\vee}{P}_n \left(\Pi_{s=1}^k \left(q^{m_s}_{m_{s-1}}(y) \right)^{j_{n,m_s}} \cdot (q^n_{m_k}(y))^{d_{n,k}} \right) \right\| \\ &\leq \Pi_{s=0}^k (d_{n,s}+1) \sup_{|\theta_s|=1} \left\| P_n \left(\sum_{s=1}^k \theta_s q^{m_s}_{m_{s-1}}(y) + \theta_{k+1} \cdot (q^n_{m_k}(y)) \right) \right\| \end{aligned}$$

These inequalities will be used extensively in this article without further commentaries.

Let $H_{bk}(E,F)$ be the space of the entire functions which are bounded in weakly compact sets of E, and $H_b(E,F)$ the space of the entire functions which are bounded in the bounded sets of E. The following result is well know [6].

Lemma 2.1. *Let (P_n) be a sequence of n-homogeneous polynomials from E into F. Then*

(1) $f = \sum_n P_n \in H(E,F)$ *if and only if for every compact subset $K \subset E$ we have* $\limsup_n \|P_n\|_K^{1/n} = 0$.
(2) $f \in H_{bk}(E,F)$ *if and only if for every weakly compact subset $W \subset E$, we have* $\limsup_n \|P_n\|_W^{1/n} = 0$.
(3) $f = \sum_n P_n \in H_b(E,F)$ *if and only if* $\limsup \|P_n\|_{S(E)}^{1/n} = 0$.

Lemma 2.2. *Let $(P_n)_{n\geq 1}$ be a sequence of d_n-homogeneous polynomials from E into F with $d_n \leq n$ for every n, such that for every weakly compact subset $W \subset E$ the* $\limsup \|P_n\|_W^{1/n} = 0$. *Let*

(1) $(j_{n,1}), (j_{n,2}), \ldots, (j_{n,i}) \subset \mathbb{N}$ *be sequences such that* $0 \leq \sum_{k=1}^i j_{n,k} \leq d_n$ *for every* $n \in \mathbb{N}$,
(2) $(y_{n,1})(y_{n,2})\ldots(y_{n,i}) \subset E$ *be sequences weakly convergent to* $y_k \in E$, $k = 1, 2, .., i$,

(3) (Q_n) *be a sequence of homogeneous polynomials with degree* $\deg(Q_n) := d_{n,i} := d_n - \sum_{k=0}^{i} j_{n,k}$ $(j_{n,0} := 0)$ *defined by*

$$Q_n(x) = \Pi_{k=0}^{i}(d_{n,k}+1)\frac{d_n!}{\Pi_{k=1}^{i} j_{n,k}!.d_{n,i}!}\overset{\vee}{P}_n\left(\Pi_{k=1}^{i}(y_{n,k}-y_k)^{j_{n,k}}x^{d_{n,i}}\right)$$

Then for every weakly compact set $W \subset E$ *we have that* $\limsup \|Q_n\|_W^{1/n} = 0$.

Proof. Let $W \subset E$ a weakly compact set. Using the theorem of Eberleim Smuliam it is not difficult to verify that the subset

$$\Theta \otimes W = \left\{\sum_{k=1}^{i}\theta_k(y_{n,k}-y_k)+\theta_{i+1}x : n \geq 1, |\theta_k| = 1, k = 1,2,..i+1, x \in W\right\}$$

is relatively weakly compact. Let $x \in W$ arbitrary. We observe that $d_{n,k} \leq n$ implies $(d_{n,k}+1)^{1/n} \leq (d_n+1)^{1/n}$ and by the inequalities of Cauchy we have that

$$\begin{aligned} \|Q_n(x)\|^{1/n} &= \Pi_{k=0}^{i}(d_{n,k}+1)^{1/n}\left\|\frac{d_n!}{\Pi_{k=1}^{i} j_{n,k}!.d_{n,i}!}\overset{\vee}{P}_n\left(\Pi_{k=1}^{i}(y_{n,k}-y_k)^{j_{n,k}}x^{d_{n,i}}\right)\right\|^{1/n} \\ &\leq (d_n+1)^{i/n}\sup_{|\theta_k|=1}\left\|\overset{\vee}{P}_n\left(\sum_{k=1}^{i}\theta_k(y_{n,k}-y_k)+\theta_{i+1}x\right)\right\|^{1/n} \\ &\leq (d_n+1)^{i/n}\|P_n\|_{\Theta\otimes W}^{1/n} \leq (d_n+1)^{i/n}\|P_n\|_{\overline{\Theta\otimes W}}^{1/n} \end{aligned}$$

where $\overline{\Theta\otimes W}$ is a the closure of $\Theta\otimes W$ in the weak topology of E. Being that $x \in W$ arbitrary it follows that $\|Q_n\|_W^{1/n} \leq (d_n+1)^{i/n}\|P_n\|_{\overline{\Theta\otimes W}}^{1/n}$. By hypothesis we have $\limsup_n \|P_n\|_{\overline{\Theta\otimes W}}^{1/n} = 0$. The affirmation follows. □

3. The main result and consequences.

In 1983 Dineen showed that if $E = c_0$, the space of the null sequences, then

$$H_{bk}(c_0) = H_b(c_0)$$

We will extend this result of Dineen, obtained for c_0, for Banach spaces with the U property and contains no subspace isomorphic to l_1. Firstly, we remember that a series $\sum_{i=1}^{\infty} y_i$ is called weakly unconditional Cauchy (wuC) if for every $\rho \in E'$ we have $\sum_{i=1}^{\infty}|\rho(x_i)| < \infty$. If the series $\sum_{i=1}^{\infty} y_i$ is wuC then for every increasing sequences of integer positive (n_k),(m_k) with $m_k < n_k$ the sequence $\left(\sum_{i=m_k}^{n_k} y_i\right)_k$ weakly converges to zero. Also, if (α_i) is a sequence of scalars with the $\lim \alpha_k = 0$ then $\sum_{k=1}^{\infty}\alpha_k y_k$ converges unconditionally in norm (to see [9],[3]).

A Banach space E has U property if for every weak Cauchy sequence $(x_n) \subset E$ there exists a series wuC, $\sum y_i$, so that a sequence $(x_n - \sum_{i=1}^{n} y_i)_{n\geq 1}$ converges to zero in a weak topology. This property was introduced by Pelczynski in [12].

Lemma 3.1. *Let* (P_n) *be a sequence of* d_n*-homogeneous polynomials from* E *into* F, *with* $d_n \leq n$ *for every* n *and* $\sum_{i\geq 1} y_i$ *a serie* wuC; *such that for every weakly compact set* $W \subset E$ *the* $\limsup_n \|P_n\|_W^{1/n} = 0$, *and*

$$\limsup_n \left\|P_n\left(\sum_{s=1}^{n} y_s\right)\right\|^{1/n} > 2\rho > 0$$

Then, there exist a positive integer m *and a sequence* $(j_{n,m})_{n\geq 1} \subset \mathbb{N}$ *with* $0 \leq j_{n,m} \leq d_n$ *for every* n, *such that* i)

$$\rho \leq (d_n+1)^{1/n}\left(\left(\begin{array}{c} d_n \\ j_{n,m} \end{array}\right)\left\|\overset{\vee}{P}_n\left(\sum_{s=1}^{m} y_s\right)^{j_{n,m}}\left(\sum_{s=m+1}^{n} y_s\right)^{d_n-j_{n,m}}\right\|\right)^{1/n}$$

ii)

$$0<\liminf_n \frac{j_{n,m}}{n}<1.$$

Proof. Taking a subsequence of (P_n) if it is necessary we can suppose that $\|P_n\left(\sum_{i=1}^{n} y_i\right)\| > \rho$ for every $n \geq 1$. For each $m \in \mathbb{N}$ we have

$$\text{(1)}\qquad \rho \leq \left\|P_n\left(\sum_{s=1}^{n} y_s\right)\right\| \leq (d_n+1)\frac{d_n!}{j_{n,m}!\,(d_n-j_{n,m})!}\left\|\overset{\vee}{P}_n\left(\sum_{s=1}^{m} y_s\right)^{j_{n,m}}\left(\sum_{s=m+1}^{n} y_s\right)^{d_n-j_{n,m}}\right\|$$

for some $j_{n,m} \in \mathbb{N}$ with $0 \leq j_{n,m} \leq d_n$. We will show that there exists an m such that

$$\text{(2)}\qquad 0<\liminf_n \frac{j_{n,m}}{n}.$$

Fact: If $\liminf_n \frac{j_{n,m}}{n}=0$ for every m, then given m there exist $n_m > m$ such that

$$\frac{j_{n_m,m}}{n_m}<\frac{1}{m}$$

Let

$$\alpha_m=\begin{cases} e^{-\sqrt{\frac{n_m}{j_{n_m,m}}}} & if \quad j_{n_m,m}\neq 0 \\ 1/m & if \quad j_{n_m,m}=0 \end{cases}$$

then $\lim \alpha_m = 0$, and $\lim_m (\alpha_m)^{\frac{j_{n_m,m}}{n_m}} = \lim_m e^{-\sqrt{\frac{j_{n_m,m}}{n_m}}} = 1$.

Since the sequence $\left(\sum_{s=m+1}^{n_m} y_s\right)_m$ converges weakly to zero and $\lim_m \alpha_m\left(\sum_{i=1}^{m} y_i\right)=0$, the lemma 2.2 implies that

$$\limsup\,(d_{n_m}+1)^{\frac{1}{n_m}}\left(\frac{d_{n_m}!}{j_{n_m,m}!\,(d_{n_m}-j_{n_m,m})!}\left\|\overset{\vee}{P}_{n_m}\left(\alpha_m\sum_{s=1}^{m} y_s\right)^{j_{n_m,m}}\left(\sum_{s=m+1}^{n_m} y_s\right)^{d_{n_m}-j_{n_m,m}}\right\|\right)^{\frac{1}{n_m}}=0$$

On the other hand, by the inequality 1 we have that

$$(d_{n_m}+1)^{\frac{1}{n_m}}\left(\frac{d_{n_m}!}{j_{n_m,m}!\,(d_{n_m}-j_{n_m,m})!}\left\|\overset{\vee}{P}_{n_m}\left(\alpha_m\sum_{s=1}^{m} y_s\right)^{j_{n_m,m}}\left(\sum_{s=m+1}^{n_m} y_s\right)^{d_{n_m}-j_{n_m,m}}\right\|\right)^{\frac{1}{n_m}}$$

$$=\ (\alpha_m)^{\frac{j_{n_m,m}}{n_m}}(d_{n_m}+1)^{\frac{1}{n_m}}\left(\frac{d_{n_m}!}{j_{n_m,m}!\,(d_{n_m}-j_{n_m,m})!}\left\|\overset{\vee}{P}_{n_m}\left(\sum_{s=1}^{m} y_s\right)^{j_{n_m,m}}\left(\sum_{s=m+1}^{n_m} y_s\right)^{d_{n_m}-j_{n_m,m}}\right\|\right)^{\frac{1}{n_m}}$$

$$\geq\ (\alpha_m)^{\frac{j_{n_m,m}}{n_m}}\rho$$

and $\limsup_m (\alpha_m)^{\frac{j_{n_m,m}}{n_m}}\rho = \rho > 0$. It is a contradiction.

We fix m_1, satisfying the inequality 2. We will show that $\liminf_n \frac{j_{n,m_1}}{n} < 1$. The fact, if $\liminf_n \frac{j_{n,m_1}}{n} = 1$ then given p there exists n_p such that

$$1-\frac{1}{p}<\frac{j_{n_p,m_1}}{n_p}$$

Therefore $0 \leq \lim_p \frac{d_{n_p}-j_{n_p,m_1}}{n_p} \leq \lim_p \frac{n_p-j_{n_p,m_1}}{n_p} = \lim_p \left(1-\frac{j_{n_p,m_1}}{n_p}\right) = 0$. Let

$$\alpha_p := \begin{cases} e^{-\sqrt{\frac{n_p}{d_{n_p}-j_{n_p,m_1}}}} & if \quad d_{n_p}-j_{n_p,m_1} \neq 0 \\ 1/p & if \quad d_{n_p}-j_{n_p,m_1} = 0 \end{cases}$$

then $\lim_p \alpha_p = 0$ and $\lim_p \alpha_p^{\frac{d_{n_p}-j_{n_p,m_1}}{n_p}} = 1$. Now we have $\lim_p \alpha_p \left(\sum_{s=m_1+1}^{n_p} y_s\right) = 0$ and as $\sum_{s=1}^{m_1} y_s$ is a fixed vector of E, the lemma 2.2 implies that

$$\limsup_p \left(d_{n_p}+1\right)^{\frac{1}{n_p}} \left(\frac{d_{n_p}!}{j_{n_p,m_1}!\left(d_{n_p}-j_{n_p,m_1}\right)!}\left\|\overset{\vee}{P}_{n_p}\left(\sum_{s=1}^{m_1} y_s\right)^{j_{n_p,m_1}}\left(\alpha_p \sum_{s=m_1+1}^{n_p} y_s\right)^{d_{n_p}-j_{n_p,m_1}}\right\|\right)^{\frac{1}{n_p}} = 0.$$

On the other hand

$$\begin{aligned} &\left(d_{n_p}+1\right)^{\frac{1}{n_p}} \left(\frac{d_{n_p}!}{j_{n_p,m_1}!\left(d_{n_p}-j_{n_p,m_1}\right)!}\left\|\overset{\vee}{P}_{n_p}\left(\sum_{s=1}^{m_1} y_s\right)^{j_{n_p,m_1}}\left(\alpha_p \sum_{s=m_1+1}^{n_p} y_s\right)^{d_{n_p}-j_{n_p,m_1}}\right\|\right)^{\frac{1}{n_p}} \\ = \; &\alpha_p^{\frac{d_{n_p}-j_{n_p,m_1}}{n_p}} \left(d_{n_p}+1\right)^{\frac{1}{n_p}} \left(\frac{d_{n_p}!}{j_{n_p,m_1}!\left(d_{n_p}-j_{n_p,m_1}\right)!}\left\|\overset{\vee}{P}_{n_p}\left(\sum_{s=1}^{m_1} y_i\right)^{j_{n_p,m_1}}\left(\sum_{s=m_1+1}^{n_p} y_s\right)^{d_{n_p}-j_{n_p,m_1}}\right\|\right)^{\frac{1}{n_p}} \\ \geq \; &\alpha_p^{\frac{d_{n_p}-j_{n_p,m_1}}{n_p}} \rho \end{aligned}$$

and $\limsup_p \alpha_p^{\frac{d_{n_p}-j_{n_p,m_1}}{n_p}} \rho = \rho$. It is a contradiction. □

Lemma 3.2. *Let E be a Banach space and (P_n) be a sequence of polynomials from E into F, with $n = \deg(P_n)$. If for every weakly compact subset $W \subset E$ the* $\limsup \|P_n\|_W^{1/n} = 0$*; then for every weakly unconditional Cauchy series $\sum y_i$, we have*

$$\limsup_n \left\|P_n\left(\sum_{s=1}^{n} y_s\right)\right\|^{1/n} = 0.$$

Proof. The proof will be made constructing subsequences $\left(P_{n_{i_k(t)}}\right)_t$ $(k = 1,2,...)$ of the sequence of polynomials (P_n), satisfying certain properties. We will use these properties and the diagonalization process, taking the sequence $\left(P_{n_{i_k(k)}}\right)$, in order to obtain a contradiction.

We suppose that $\limsup_n \|P_n(\sum_{i=1}^{n} y_i)\|^{1/n} = 2\rho > 0$. Then, taking a subsequence of (P_n) if it is necessary we can suppose that

$$\left\|P_n\left(\sum_{s=1}^{n} y_s\right)\right\| > \rho$$

for every $n \geq 1$.

The process that follows is, in essence, the same established by Dineen in [6]: Since $\lim_n n/(\ln(n+1)) = +\infty$, then given $i \in \mathbb{N}$ there exists a $n_i \geq i$ such that $n_i/(\ln(n_i+1)) \geq i$. Where we obtain $1/(n_i+1) \geq 1/e^{n_i/i}$ and therefore

$$\frac{1}{(n_i+1)^{i/n_i}} \geq \frac{1}{e} \tag{3}$$

we observe that if $d_{i-1} < d_{i-1} < ... < d_1 < n_i = d_0$ and $i < n_i$, then $\Pi_{s=0}^{i-1}(d_s+1) \leq (n_i+1)^i$. Therefore

$$\frac{1}{\Pi_{s=0}^{i}(d_s+1)^{i/n_i}} \geq \frac{1}{(n_i+1)^{i/n_i}} \geq \frac{1}{e} \tag{4}$$

and

$$\left\| P_{n_i}\left(\sum_{s=1}^{n_i} y_s\right)\right\| \geq \rho$$

We will show the existence of:

i) A sequence $(m_i)_{i\geq 1}$ of strictly increasing integer numbers,
ii) Strictly increasing sequences $(i_k(t))_t$, $(k = 1, 2, ...)$ with $(i_{k+1}(t)) \subset (i_k(t))$ and $i_k(k) \geq k$;
iii) Sequences $\left(j_{n_{i_k(t)}, m_k}\right)_t$ of integer numbers, $(k = 1, 2, ...)$ such that

$$\begin{aligned} 0 &\leq \sum_{s=1}^{k} j_{n_{i_k(t)}, m_s} \leq n_{i_k(t)}, \ \forall k \geq 1 \\ 0 &< \lim_t \frac{j_{n_{i_k(t)}, m_k}}{n_{i_k(t)}} = \delta_k < 1 \qquad \forall k \\ \frac{j_{n_{i_k(t)}, m_k}}{n_{i_k(t)}} &< 2\delta_s \qquad \forall k \geq s; \forall t \end{aligned}$$

and

iv)

$$\rho < \Pi_{s=0}^{k}\left(d_{n_{i_k(t)}, m_s} + 1\right)^{\frac{1}{n_{i_{k+1}(t)}}} \times$$

$$\left(\frac{n_{i_{k+1}(t)}!}{\Pi_{s=1}^{k+1} j_{n_{i_{k+1}(t)}, m_s}! d_{n_{i_{k+1}(t)}, m_{k+1}}!}\left\| \overset{\vee}{P}_{n_{i_{k+1}(t)}} \Pi_{s=1}^{k+1}\left(q_{m_{s-1}}^{m_s}(y)\right)^{j_{n_{i_{k+1}(t)} m_s}}\left(q_{m_{k+1}}^{n_{i_{k+1}(t)}}(y)\right)^{d_{n_{i_{k+1}(t)}, m_{k+1}}}\right\|\right)^{\frac{1}{n_{i_{k+1}(t)}}}$$

where

$$d_{n_{i_{k+1}}(t), m_{k+1}} := n_{i_{k+1}(t)} - \sum_{s=1}^{k+1} j_{n_{i_{k+1}(t)}, m_s}$$

The proof will be made by induction in k. By the lemma 3.1 there exists a positive integer m_1 so that $0 < \delta_1 := \liminf_n j_{n,m_1}/n < 1$ and

$$\rho < (n+1)\frac{n!}{j_{n,m_1}!(n - j_{n,m_1})!}\left\| \overset{\vee}{P}_n\left(\sum_{s=1}^{m_1} y_s\right)^{j_{n,m_1}}\left(\sum_{s=m_1+1}^{n} y_s\right)^{n-j_{n,m_1}}\right\|$$

We choose a sequence $(i_1(t))_t$ strictly increasing, such that

$$\begin{aligned} \lim_t \frac{j_{n_{i_1(t)}, m_1}}{n_{i_1(t)}} &= \delta_1 \\ \frac{j_{n_{i_1(t)}, m_1}}{n_{i_1(t)}} &< 2\delta_1, \forall t \end{aligned}$$

Thus for $k = 1$, the claim is true. We suppose the existence of the sequences $(i_s(t))$, $\left(j_{i_s(t), m_s}\right)$ and the integer m_s , for $s = 1, 2, ...k + 1$, satisfying conditions i),ii), iii) and iv) we will show the existence of a integer m_{k+2} and the sequences $(i_{k+2}(t))$, $\left(j_{i_{k+2}(t), m_{k+2}}\right)$. We consider the polynomial $Q_{d_{n_{i_{k+1}(t)}, m_{k+1}}} : E \to F$ defined by

$$Q_{d_{n_{i_{k+1}(t)}, m_{k+1}}}(x) \ : = \Pi_{s=0}^{k+1}\left(d_{n_{i_{k+1}(t)}, m_s} + 1\right) \times$$

$$\frac{n_{i_{k+1}(t)}!}{\Pi_{s=1}^{k+1} j_{n_{i_{k+1}(t)}, m_s}! d_{n_{i_{k+1}(t)}, m_{k+1}}!} \overset{\vee}{P}_{n_{i_{k+1}(t)}} \Pi_{s=1}^{k+1}\left(q_{m_{s-1}}^{m_s}(y)\right)^{j_{n_{i_{k+1}(t)} m_s}}(x)^{d_{n_{i_{k+1}(t)}, m_{k+1}}}$$

We have that $\deg Q_{d_{n_{i_{k+1}(t)},m_{k+1}}} = d_{n_{i_{k+1}(t)},m_{k+1}} \le n_{i_{k+1}(t)}$ for every t and being $q^{m_s}_{m_s-1}(y)$, $s = 1, \ldots k+1$ fixed vectors then lemma 2.2 implies that for every weakly compact subset $W \subset E$ we have

$$\limsup_t \left\| Q_{d_{n_{i_{k+1}(t)},m_{k+1}}} \right\|_W^{1/n_{i_{k+1}(t)}} = 0$$

Since $\sum_{s=m_{k+1}+1}^{\infty} y_s$ is wuC, lemma 3.1 implies then that there exist a integer positive number $m_{k+2} > m_{k+1}$, a sequence $\left(j_{n_{i_{k+1}(t)},m_{k+2}}\right)$ with $0 \le j_{n_{i_{k+1}(t)},m_{k+2}} \le d_{n_{i_{k+1}(t)},m_{k+1}}$ for every t, such that

$$0 < \liminf_t \frac{j_{n_{i_{k+1}(t)},m_{k+2}}}{n_{i_{k+1}(t)}} = \delta_{k+2} < 1$$

and

$$\begin{aligned}
\rho &< \Pi_{s=0}^{k}\left(d_{n_{i_{k+1}(t)},m_s}+1\right)^{\frac{1}{n_{i_{k+1}(t)}}} \times \left(d_{n_{k+1}(t),m_{k+1}}+1\right)^{\frac{1}{n_{i_{k+1}(t)}}} \\
&\left(\frac{n_{i_{k+1}(t)}!}{\Pi_{s=1}^{k} j_{n_{i_{k+1}(t)},m_s}! d_{n_{i_{k+1}(t)},m_{k+1}}!} \frac{d_{n_{i_{k+1}(t)},m_{k+1}}!}{\left(d_{n_{i_{k+1}(t)},m_{k+1}} - j_{n_{i_{k+1}(t)},m_{k+2}}\right)! j_{n_{i_{k+1}(t)},m_{k+2}}!}\right)^{\frac{1}{n_{i_{k+1}(t)}}} \times \\
&\left\|\left(\overset{\vee}{P}_{n_{i_{k+1}(t)}} \Pi_{s=1}^{k+2}\left(q^{m_s}_{m_s-1}(y)\right)^{j_{n_{i_k}(t),m_s}} \left(q^{n_{i_{k+1}(t)}}_{m_{k+2}}(y)\right)^{d_{n_{i_{k+1}(t)},m_{k+1}} - j_{n_{i_{k+1}(t)},m_{k+2}}}\right)\right\|^{\frac{1}{n_{i_{k+1}(t)}}} \\
&= \Pi_{s=0}^{k+1}\left(d_{n_{i_{k+1}(t)},m_s}+1\right)^{\frac{1}{n_{i_{k+1}(t)}}} \times \\
&\left(\frac{n_{i_{k+1}(t)}!}{\Pi_{s=1}^{k+2} j_{n_{i_{k+1}(t)},m_s}!} \frac{1}{\left(d_{n_{i_{k+1}(t)},m_{k+1}} - j_{n_{i_{k+1}(t)},m_{k+2}}\right)!}\right)^{\frac{1}{n_{i_{k+1}(t)}}} \\
&\left\|\left(\overset{\vee}{P}_{n_{i_{k+1}(t)}} \Pi_{s=1}^{k+2}\left(q^{m_s}_{m_s-1}(y)\right)^{j_{n_{i_{k+1}(t)},m_s}} \left(q^{n_{i_{k+1}(t)}}_{m_{k+2}}(y)\right)^{d_{n_{i_{k+1}(t)},m_{k+1}} - j_{n_{i_{k+1}(t)},m_{k+2}}}\right)\right\|^{\frac{1}{n_{i_{k+1}(t)}}}
\end{aligned}$$

We choose a subsequence $(i_{k+2}(t))$ of $(i_{k+1}(t))$ such that,

$$\begin{aligned}
i_{k+2}(k+2) &\ge k+2 \\
\lim_t \frac{j_{n_{i_{k+2}(t)},m_{k+2}}}{n_{i_{k+2}(t)}} &= \liminf_t \frac{j_{n_{i_{k+1}(t)},m_{k+2}}}{n_{i_{k+1}(t)}} = \delta_{k+2} \\
\frac{j_{n_{i_{k+2}(t)},m_{k+2}}}{n_{i_{k+2}(t)}} &< 2\delta_{k+2} \qquad \forall k \ge s, \forall t
\end{aligned}$$

and we define $d_{n_{i_{k+2}(t)},m_{k+2}} := d_{n_{i_{k+1}(t)},m_{k+1}} - j_{n_{i_{k+1}(t)},m_{k+2}}$. The affirmation is shown.
We observe that given $(i_k(t)) \subset (i_s(t))$ for $k \ge s$ then

$$\begin{aligned}
\lim_t \frac{j_{n_{i_k(t)},m_s}}{n_{i_k(t)}} &= \lim_t \frac{j_{n_s(t),m_s}}{n_{i_s(t)}} = \delta_s \\
\frac{j_{n_{i_k(t)},m_s}}{n_{i_k(t)}} &< 2\delta_s \qquad \forall k \ge s, \forall t
\end{aligned}$$

Now, we will show that the sequence (δ_k) is absolutely summing, in fact, since $0 \le \sum_{s=1}^{r} j_{n_{i_k(t)},m_s} \le n_{i_k(t)}$ for every t, and $k \ge r$; then we have that

$$\sum_{s=1}^{r} \delta_s = \sum_{s=1}^{r} \lim_t \frac{j_{n_{i_k(t)},m_s}}{n_{i_k(t)}} = \lim_t \sum_{s=1}^{r} \frac{j_{n_{i_k(t)},m_s}}{n_{i_k(t)}} \le 1$$

Hence $\sum_{i=1}^{r} \delta_i \le 1$. Given r arbitrary, we obtain that $\sum_{k=1}^{\infty} \delta_k \le 1$.

Let $(\alpha_i) \in c_0$ such that

$$\Pi_{i=1}^{\infty} \alpha_i^{\delta_i} = 2 \tag{5}$$

Since $\frac{j_{n_{i_k(t)},m_s}}{n_{i_k(t)}} < 2\delta_s$, for every $k \geq s$, then

$$\Pi_{s=1}^{k} \alpha_s^{\frac{j_{n_{i_k(k)},m_s}}{n_{i_k(k)}}} > \Pi_{s=1}^{k} \alpha_s^{2\delta_s}$$

and therefore

$$\begin{aligned}
&\left(\frac{n_{i_k}(k)!}{\Pi_{s=1}^{k} j_{n_{i_k(k)},m_s}! d_{n_{i_k(k)},m_k}!} \left\| \overset{\vee}{P}_{n_{i_k(k)}} \Pi_{s=1}^{k} \left(\alpha_s q_{m_{s-1}}^{m_s}(y) \right)^{j_{n_{i_k(k)},m_s}} \left(q_{m_k}^{n_{i_k(k)}}(y) \right)^{d_{n_{i_k(k)},m_k}} \right\| \right)^{\frac{1}{n_{i_k(k)}}} \\
= \; & \Pi_{s=1}^{k} \alpha_s^{\frac{j_{n_{i_k(k)},m_s}}{n_{i_k(k)}}} \left(\frac{n_{i_k}(k)!}{\Pi_{s=1}^{k} j_{n_{i_k(k)},m_s}! d_{n_{i_k(k)},m_k}!} \left\| \overset{\vee}{P}_{n_{i_k(k)}} \Pi_{s=1}^{k} \left(q_{m_{s-1}}^{m_s}(y) \right)^{j_{n_{i_k(k)},m_s}} \left(q_{m_k}^{n_{i_k(k)}}(y) \right)^{d_{n_{i_k(k)},m_k}} \right\| \right)^{\frac{1}{n_{i_k(k)}}} \\
\geq \; & \Pi_{s=1}^{k} \alpha_s^{2\delta_s} \frac{\rho}{\Pi_{s=0}^{k-1} \left(d_{n_{i_k(k)},m_s} + 1 \right)^{1/n_{i_k(k)}}} \\
\geq \; & \frac{\rho}{e} \Pi_{s=1}^{k} \alpha_s^{2\delta_s}.
\end{aligned}$$

and $\lim \frac{\rho}{e} \Pi_{s=1}^{k} \alpha_s^{2\delta_s} = 4\rho/e$.

On the other hand, the set

$$W = \left\{ \alpha_1 \theta_1 q^{m_1}(y) + \alpha_2 \theta_2 q_{m_1}^{m_2}(y) + \ldots + \alpha_k \theta_k q_{m_{k-1}}^{m_k}(y) + \theta_{k+1} \cdot q_{m_k}^{n_{i_k(k)}}(y) : |\theta_i| = 1, k \geq 1 \right\}$$

is relatively weakly compact, since the series

$$\alpha_1 \left(\sum_{k=1}^{m_1} y_i \right) + \alpha_2 \left(\sum_{k=m_1+1}^{m_2} y_i \right) + \ldots + \alpha_k \left(\sum_{k=m_{k-1}+1}^{m_k} y_i \right) + \ldots$$

is unconditionally convergent in norm and the sequence $\left(q_{m_k}^{n_{i_k(k)}}(y) \right)_k = \left(\sum_{s=m_k+1}^{n_{i_k(k)}} y_s \right)_k$ converges weakly to zero. Thus

$$\begin{aligned}
&\left(\frac{n_{i_k}(k)!}{\Pi_{s=1}^{k} j_{n_{i_k(k)},m_s}! d_{n_{i_k(k)},m_k}!} \left\| \overset{\vee}{P}_{n_{i_k(k)}} \Pi_{s=1}^{k} \left(\alpha_s q_{m_{s-1}}^{m_s}(y) \right)^{j_{n_{i_k(k)} m_s}} \left(q_{m_k}^{n_{i_k(k)}}(y) \right)^{d_{n_{i_k(k)},k}} \right\| \right)^{1/n_{i_k(k)}} \\
\leq \; & \left\| P_{n_{i_k(k)}} \left(\alpha_1 \theta_1 q^{m_1}(y) + \alpha_2 \theta_2 q_{m_1}^{m_2}(y) + \ldots + \alpha_k \theta_r q_{m_{k-1}}^{m_k}(y) + \theta_{k+1} \cdot q_{m_k}^{n_{i_k(k)}}(y) \right) \right\|^{1/n_{i_k(k)}} \\
\leq \; & \left\| P_{n_{i_k(k)}} \right\|_{\overline{W}}^{1/n_{i_k(k)}}
\end{aligned}$$

where $\overline{W}$ is the closure of W, in the weak topology. By the hypothesis of the lemma we have that $\limsup_k \left\| P_{n_{i_k(k)}} \right\|_{\overline{W}}^{1/n_{i_k(k)}} = 0$. It is a contradiction. □

The following theorem generalizes the result of Dineen obtained for c_0.[6]

Theorem 3.3. *Let E be a Banach space. If E has U property and contains no subspace isomorphic to l_1 then*

$$H_{bk}(E,F) = H_b(E,F)$$

Proof. It is obvious that $H_b(E,F) \subset H_{bk}(E,F)$. Let $f = \sum_{n=0}^{\infty} P_n \in H_{bk}(E,F)$. We suppose that $f \notin H_b(E,F)$ then $\limsup \|P_n\|_{S(E)}^{1/n} = \rho > 0$. Taking subsequences, if it is necessary, we can suppose that $\|P_n\|_{S(E)}^{1/n} \geq \rho$ for every $n \geq 1$. Therefore, by continuity, there exists a sequence $(x_n) \subset S(E)$, such that

$$\|P_n(x_n)\|^{1/n} > \rho/2$$

or $\left\|P_n\left(\frac{2}{\rho}x_n\right)\right\| > 1$ for every $n \geq 1$. As E contains no subspace isomorphic to l_1 and the sequence $(z_n) := \left(\frac{2}{\rho}x_n\right)$ is bounding, then by the Rosenthal theorem of l_1, there exists a Cauchy weak subsequence $(z_{n_i}) := \left(\frac{2}{\rho}x_{n_i}\right)$. Since E has a U property then there exists a series weakly unconditional Cauchy $\sum_{k=1}^{\infty} y_k$ such that $(z_{n_i} - \sum_{k=1}^{n_i} y_k)_k$ converges to zero in weak topology. Now, for each n_i we have by the Leibniz formula

$$P_{n_i}(z_{n_i}) = \sum_{j=0}^{n_i} \frac{n_i!}{j!\,(n_i - j)!} P_{n_i}\left(\left(z_{n_i} - \sum_{k=1}^{n_i} y_k\right)^j \left(\sum_{k=1}^{n_i} y_k\right)^{n_i - j}\right),$$

thus given n_i there exist a j_{n_i} with $0 \leq j_{n_i} \leq n_i$ for every i such that

$$1 \leq \|P_{n_i}(z_{n_i})\| \leq (n_i + 1)\frac{n_i!}{j_{n_i}!\,(n_i - j_{n_i})!}\left\|P_{n_i}\left(\left(z_{n_i} - \sum_{k=1}^{n_i} y_k\right)^{j_{n_i}} \left(\sum_{k=1}^{n_i} y_k\right)^{n_i - j_{n_i}}\right)\right\| \tag{6}$$

We will define the polynomial $Q_{n_i} : E \to F$ making

$$Q_{n_i}(x) = (n_i + 1)\frac{n_i!}{j_{n_i}!\,(n_i - j_{n_i})!} P_{n_i}\left(\left(z_{n_i} - \sum_{k=1}^{n_i} y_k\right)^{j_{n_i}} (x)^{n_i - j_{n_i}}\right)$$

$i = 1, 2, \ldots$ Then by lemma 2.2 we have

$$0 = \limsup_i \|Q_{n_i}\|_W^{1/n_i}$$

for every weakly compact subset $W \subset E$. By lemma 3.2 this implies that

$$\begin{aligned} 0 &= \limsup_i \left\|Q_{n_i}\left(\sum_{k=1}^{n_i} y_k\right)\right\|^{1/n_i} \\ &= \limsup_i \left\|(n_i + 1)\frac{n_i!}{j_{n_i}!\,(n_i - j_{n_i})!} P_{n_i}\left(\left(z_{n_i} - \sum_{k=1}^{n_i} y_k\right)^{j_{n_i}} \left(\sum_{k=1}^{n_i} y_k\right)^{n_i - j_{n_i}}\right)\right\|^{1/n_i} \end{aligned}$$

and this contradicts 6. □

We denote $H_w(E,F)$ the space of the entire functions which are weakly continuous in the bounded sets of E; that means $f \in H_w(E,F)$ if for each bounded set $M \subset E$, the restriction $f_{/M}$ is weakly continuous. Similarly we define the space $H_{wu}(E,F)$ of the entire functions which are weakly uniformly continuous in the bounded sets of E. Aron, Herves and Valdivia in [1] raised the following question: Does is every weakly continuous function in the bounded sets of E, weakly uniformly continuous? In other word, $H_w(E,F) = H_{wu}(E,F)$ for every Banach space E? Clearly we have the following inclusion $H_{wu}(E,F) \subset H_w(E,F)$. In [1] it is shown that the equality of these spaces is equivalent to showing the inclusion $H_w(E,F) \subset H_b(E,F)$. Since all weakly continuous functions in the bounded sets of a Banach space is bounded in the weakly compacted sets of E, the theorem 3.3 implies that when E is a Banach space with U property contains no subspace isomorphic to l_1 then $H_{wu}(E,F) \subset H_w(E,F)$. Thus for these spaces the l_1 problem has an affirmative answer. The answer is also affirmative for every F subspace of a space with U property, since this property is hereditary. Examples of this space are the spaces with shrinking and unconditional basis([15]) and more generally, all Banach spaces that are a M-ideal in their bidual, since these spaces have the U property [7, Thm. 3.8] and contain no subspace isomorphic to l_1, because these are Asplund spaces. [7, Thm.3.1]

Let $H_{wsc}(E,F)$ the space of the entire functions which applies weakly convergent sequences of E in convergent sequences of F. Clearly $H_{wu}(E,F) \subset H_w(E,F) \subset H_{wsc}(E,F)$.

Corollary 3.4. *Let E be a Banach space. If E has U property and contains no subspace isomorphic to l_1 then*

$$H_w(E,F) = H_{wsc}(E,F) = H_{wu}(E,F)$$

Proof. By [1, Prop. 3.3] $H_w(E,F) = H_{wsc}(E,F)$ and by the commentary of the theorem 3.3 above $H_w(E,F) = H_{wu}(E,F)$. □

We will remember that a Banach space E has the Dunford Pettis property and contains no subspace isomorphic to l_1, if and only if, E' is a Schur space.

Corollary 3.5. *Let E be a Banach space. If E has U property and E' is a Schur space then*

$$H_{wu}(E) = H_w(E) = H_b(E) = H_{bk}(E)$$

Proof. Since E' is Schur then E has the Dunford Pettis property, and contains no subspace isomorphic to l_1 by [11, Prop. 4] we have $H_{bk}(E) = H_w(E)$. Since E has the U property, the theorem 3.3 implies $H_{bk}(E) = H_b(E)$, and by corollary 3.4 $H_w(E) = H_{wu}(E)$. The corollary follows. □

Examples of spaces that satisfy the conditions of corollary 3.5 are Banach spaces that are a M-ideal in their bi-dual and also some spaces $C(K)$ [4].

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