Handbook of the 6th World Congress and School on Universal Logic

June 16–26, 2018
Vichy, France

UNILOG'2018
https://www.uni-log.org/vichy2018

Vichy University Campus
France

Edited by
Jean-Yves Beziau, Arthur Buchsbaum and Christophe Rey,
with the assistance of Alvaro Altair and Yanis Ayari.

The notion of filter pair was introduced in [1], for creating and analyzing general *finitary* propositional logics and their translation morphisms, expanding the work initiated in [4], that is restricted to the setting of algebraizable logics [2].

Considering the special case of filter pairs $\langle G, i\rangle$ where the functor $G = Co_K$ is given by congruences relative to a class of algebra K, we give criteria when the associated logic is protoalgebraic, equivalential, algebraizable, truth-equational, self-extentional or Lindenbaum algebraizable, just analyzing the relation between Leibniz operator, Suszko operator and Frege operator with the adjoint of i, improving our previous results.

We adjust the notion of filter pair in such a way that we can treat κ-compact logics, for each regular cardinal κ: The corresponding new notion is called κ-filter pair. We show that any κ-filter pair gives rise to a κ-logic and that every κ-logic comes from a κ-filter pair. Taking adequate notions of morphisms, we show that the category of κ-logics and translation morphisms is (isomorphic to) a full reflective subcategory of the category of κ-filter pairs. We use the notion of κ-filter pair to show that logics always admit natural extensions, providing an(other) answer to a question of Cintula and Noguera [3]. We further point out how κ-filter pairs allow to extend standard notions from finitary logics to arbitrary logics, e.g. those of being algebraizable, protoalgebraizable, equivalential or truth-equational.

References

1. P. Arndt, R. Jansana, H.L. Mariano & D.C. Pinto, "Filter functors in logic and applications to categorial analysis of meta-logic properties", to be published, 2016.
2. W.J. Blok & D. Pigozzi, *Algebraizable Logics*, Memoirs of the American Mathematical Society, vol. **77(396)**, 1989, doi:10.1090/memo/0396.
3. P. Cintula & C. Noguera, "A note on natural extensions in abstract algebraic logic", *Studia Logica*, vol. **103(4)**, 2015, pp. 815–823.
4. H.L. Mariano & D.C. Pinto, "Algebraizable Logics and a functorial encoding of its morphisms", *Logic Journal of the IGPL**, vol. **25(4)**, 2017, pp. 524–561.

Beyond the categorial forms of the Axiom of Choice

Andreas B.M. Brunner, Darllan Conceição Pinto
& Samuel G. da Silva
Federal University of Bahia, Brazil

*Interest Group in Pure and Applied Logics

andreas@dcc.ufba.br, darllan@ufba.br, samuel@ufba.br

Hugo Luiz Mariano
Institute of Mathematics and Statistics,
University of São Paulo, Brazil
hugomar@ime.usp.br

In the sequel of [1], besides to work on new categorial forms of axiom of choice, we explore categorial forms of statements on partial ordered sets such that are equivalent to the axiom of choice, namely Zorn's Lemma, Hausdorff Maximal Principle and the Principle of Cofinality. This categorial forms are different from categorial set-forms of axiom of choice defined in [1]. There, the authors state that a statement φ is a *categorial Set-Form of the Axiom of Choice* if the Axiom of Choice for Sets is equivalent to the statement $\varphi_{\mathbf{Set}}$, since φ declares properties of objects, morphisms and/or constructions in a category, and the relativization of φ with respect to the category C is denoted by φ_C. In the present case, the categorial form is a statement φ such that $\varphi_{(P,\leq)}$ ($(P,\leq)$ is viewed as a category) for any poset $(P,\leq)$ is equivalent to axiom of choice for sets.

In [2], the authors have introduced a notion of categorial Zorn's Lemma that is: "*in a category C, if every filtered diagram has an inductive limit, then C has a quasi terminal object*". We realized that an inductive limit does not translate precisely the notion of "upper bound". So, we introduce another categorial Zorn's Lemma: "*if every filtered diagram has a cocone in C, then it has an almost maximal object*". If $C = (P,\leq)$ is a poset viewed as a category, both categorial notions coincide and are equivalent to Zorn's Lemma on C (this means that the Zorn's Lemma can be considered for any poset $(P,\leq)$ but one concludes that it has maximal element if any chain has upper bound in$(P,\leq)$).

We also introduce the categorial Hausdorff Maximal Principle, that is: "*the category of filtered subcategories of C has a quasi terminal (almost maximal) object*". A property P on a locally finitely presentable category C has finite character if, for any object c of C has the property P, then c_i has the property P, for all $i \in I$, where c_i is a finitely presentable object, $\{c_i\}_{i\in I}$ is a directed diagram with $c = \operatorname{colim}_{i\in I} c_i$. The categorial Teichmüller-Tuchey Principle is considered as: "*For every locally finitely presentable category C and every property P of finite character, there exists a quasi terminal (almost maximal) object with the property P*".

References

1. A. Brunner, H.L. Mariano & S.G. da Silva, "Categorial forms of the ax-

iom of choice", *Logic Journal of the IGPL**, vol. **25(4)**, 2017, pp. 408–430, doi:10.1093/jigpal/jzx020.
2. M. Kashiwara & P. Schapira, *Categories and Sheaves*, Springer, 2006.

Boole-Weyl Algebras in a Categorical Context

Rafael Diaz
School of Mathematics,
National University of Colombia, Medellin, Colombia
ragadiaz@gmail.com

We review the construction of the Boole-Weyl algebras, which are the analogue of the algebra of algebraic differential operators on the affine plane over the field with two elements, a proceed to study these algebras from a categorical viewpoint, namely we show that they correspond to the endomorphisms of certain objects in the category of finite dimensional vector spaces over the field with two elements, and study the quantum-like structural properties of this category. This talk is based on [1].

Reference

1. R. Diaz, "Quantum Boolean Algebras", arXiv:1011.5215, Cornell University Library, 2015.

Logical rules are fractions

Dominique Duval
Laboratoire Jean Kuntzmann,
University Grenoble Alpes, Grenoble, France
Dominique.Duval@univ-grenoble-alpes.fr

Key: *A logical rule* $\frac{H}{C}$ *is indeed a fraction, but its numerator is* C *and its denominator is* H*.*

The link between inference rules as $\frac{H}{C}$ where H is the hypothesis and C the conclusion, and fractions as $\frac{N}{D}$ where N is the numerator and D the denominator, goes through the categorical notion of fraction. Categories of fractions were introduced by Gabriel and Zisman in [5] as a tool for homotopy theory. The link with logic, using limit sketches, was studied in [3,4].

*Interest Group in Pure and Applied Logics