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SOME INEXACT HYBRID PROXIMAL AUGMENTED LAGRANGIAN ALGORITHMS

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Some Inexact Hybrid Proximal Augmented Lagrangian Algorithms

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Abstract

In this work, we use recent results from Solodov-Svaiter's hybrid projection-proximal and extragradient-proximal methods [17, 16] to derive two similar algorithms to find a Karush-Kuhn-Tucker pair of a convex programming problem. These algorithms are variations of the proximal augmented Lagrangian. As main feature, both algorithms do not require increasing relative accuracy in the solution of the unconstrained subproblems. We also show that the convergence is Q-linear under strong second order assumptions.

Preliminary computational experiments are also presented.

Keywords: augmented Lagrangian, proximal methods, convex programming.
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1 Introduction

The proximal point algorithm [11, 10, 14, 9] and its connection to augmented Lagrangian algorithms for non-linear programming were established in [13]. In this article, Rockafellar showed that the augmented Lagrangian method introduced by Hestenes and Powell [5, 12] can be viewed as the proximal point algorithm applied to solve the dual of a non-linear programming problem. He also introduced the *proximal augmented Lagrangian*, based on the the proximal algorithm used to find a saddle point of the Lagrangian function, which corresponds to Karush-Kuhn-Tucker (KKT) pairs for the non-linear problem. The convergence of the algorithm was proved considering summable relative errors in the unconstrained minimization needed at each step. Moreover, under extra second order conditions, the rate of convergence was shown to be Q-linear.

In this work, based on Solodov-Svaiter's hybrid methods [17, 16], we derive two variations of Rockafellar's proximal augmented Lagrangian that share the convergence properties, but that require only constant relative accuracy. Some preliminary computational results are presented.

2 The Algorithm

2.1 Definitions and notation

Consider the problem

$$\begin{array}{ll} \min & f(x) \\ s.t. & g(x) \leq 0 \\ & x \in \mathbb{R}^n, \end{array} \qquad \text{(P)}$$

where $f : \mathbb{R}^n \mapsto (-\infty, \infty]$ and $g(\cdot)$ is composed of m components, such that each component $g_i : \mathbb{R}^n \mapsto (-\infty, \infty]$. We assume throughout this paper that $f(\cdot)$ and each component of $g(\cdot)$ are convex and closed, we also suppose that the relative interior of their effective domains intersect. Moreover, we assume that (P) and its dual have solutions.

The extended Lagrangian function of (P) is:

$$l(x,y) := \begin{cases} f(x) + \sum_{j=1}^{m} y_j g_j(x), & \text{if } y \in \mathbb{R}^m_+ \\ -\infty, & \text{otherwise.} \end{cases}$$

points of the Lagrangian are the KKT pairs for (P) and therefore the solutions of (P) and its dual.

Associated to the Lagrangian, $l(\cdot,\cdot)$, we have the maximal monotone operator $T_l : (x,y) \mapsto \{(u,v) \mid (u,-v) \in \partial l(x,y)\}$, *i.e.*:

$$T_l(x,y) \doteq \begin{bmatrix} \partial f(x) + \sum_{j=1}^{m} y_j \partial g_j(x) \\ -g(x) + \mathbf{N}_{\mathbf{R}_+^m}(y) \end{bmatrix},$$

where

$$\mathbf{N}_{\mathbf{R}_+^m}(y)_j = \begin{cases} 0, & \text{if } y_j > 0; \\ (-\infty, 0], & \text{if } y_j = 0; \\ \emptyset, & \text{if } y_j < 0. \end{cases}$$

Considering the convexity assumptions, it should be clear that the saddle points of $l(\cdot,\cdot)$ correspond to the zeroes of $T_l(\cdot,\cdot)$.

In [13], to find a zero of $T_l(\cdot,\cdot)$, one generates a sequence $\{x^k, y^k\}$ where x^{k+1} is an approximate unconstrained minimum of the function

$$\phi_k(x) \doteq f(x) + P\big(g(x), y^k, c^k\big) + \frac{1}{2c^k} \, \| x - x^k \|^2, \tag{1}$$

$$P(w,y,c) \doteq \frac{1}{2c} \sum_{j=1}^{m} \left[(y_j + c w_j)_+^2 - (y_j)^2 \right].^* \tag{2}$$

Then, one defines $y^{k+1} \doteq \nabla_w P\big(g(x^{k+1}), y^k, c^k\big)$.[2]

An approximate solution to the minimization of $\phi_k(\cdot)$ is considered acceptable if

$$\operatorname{dist}\left(0, \partial \phi_k(x^{k+1})\right) \; \leq \epsilon^k / c^k, \quad \sum_{i=0}^{\infty} \epsilon^k < \infty; \tag{A}$$

or

$$\operatorname{dist}\left(0, \partial \phi_k(x^{k+1})\right) \; \leq (\delta^k / c^k) \, \| (x^{k+1}, y^{k+1}) - (x^k, y^k) \|, \quad \sum_{i=0}^{\infty} \delta^k < \infty. \tag{B}$$

This error tolerance comes directly from the errors bounds demanded by the proximal point algorithm in [14]. In particular, an exact solution, x^{k+1} of the

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*As usual, a_+ denotes $\max\{0, a\}$. We shall use a_- for the corresponding minimum operation and we also allow these operations to be applied componentwise to vectors.

[2] Observe that $P(w,y,c)$ is Lipschitz continuously differentiable with respect to w, but it is not twice differentiable at the points at $(-1/c)y_i$.

minimization of $\phi_k(\cdot)$ and the corresponding $y^{k+1}(= \nabla_w P(g(x^{k+1}), y^k, c^k))$ are the solution of the exact proximal step.

Rockafellar used the criterion (A) to ensure convergence and (B) to prove Q-linear convergence rate.

2.2 The Hybrid algorithms

In [17, 16], Solodov and Svaiter introduce two variations of the proximal point algorithm to compute zeroes of maximal monotone operators. Their main feature is a less stringent acceptance criterion. To achieve this, a step is done on the direction of an image of the maximal monotone operator calculated at an approximate solution of the proximal step.

If we apply these algorithms to find a zero of $T_l(\cdot,\cdot)$, we have:

Hybrid Proximal Point Algorithms

1. *Initialization: Let* $(x^0, y^0) \in \mathbb{R}^n \times \mathbb{R}^m_+$ *and* $\sigma \in [0, 1)$.
2. *Iteration: Given* (x^k, y^k) *and* $c^k > 0$.

   (a) *Inner step: Find* $(\widetilde{x}^k, \widetilde{y}^k)$ *and* $\widetilde{v}^k \in T_l(\widetilde{x}^k, \widetilde{y}^k)$, *such that*

   $$\| \widetilde{v}^k + 1/c^k ((\widetilde{x}^k, \widetilde{y}^k) - (x^k, y^k)) \| \leq \sigma 1/c^k \, \| (\widetilde{x}^k, \widetilde{y}^k) - (x^k, y^k) \| \, . \quad (3)$$

   (b) *Extragradient step: If* $\widetilde{v}^k = 0$, *or* $(\widetilde{x}^k, \widetilde{y}^k) = (x^k, y^k)$, *Stop. Other wise, make a step in the direction* $\widetilde{v}^k$. *The step size must be one of the following, which characterizes each method:*

   - Projection-Proximal Method[3]

   $$(x^{k+1}, y^{k+1}) \doteq (x^k, y^k) - \frac{\langle \widetilde{v}^k, (x^k, y^k) - (\widetilde{x}^k, \widetilde{y}^k) \rangle}{\| \widetilde{v}^k \|^2} \widetilde{v}^k. \quad (4)$$

   - Extragradient-Proximal Method

   $$(x^{k+1}, y^{k+1}) \doteq (x^k, y^k) - c^k \widetilde{v}^k. \quad (5)$$

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[3] In the projection-proximal method we could use a weaker stopping criterion:

$$\| \widetilde{v}^k + 1/c^k ((\widetilde{x}^k, \widetilde{y}^k) - (x^k, y^k)) \| \leq \sigma \max\{ \| \widetilde{v}^k \|, 1/c^k \, \| (\widetilde{x}^k, \widetilde{y}^k) - (x^k, y^k) \| \}.$$

For more details see [17, 7].

■

Let us recall, from last section, that the exact minimization of $\phi_k(\cdot)$, gives a solution of the proximal step, *i.e.* a pair $(\widetilde{x}^k, \widetilde{y}^k)$ that has a $\widetilde{v}^k \in T_l(\widetilde{x}^k, \widetilde{y}^k)$ such that:

$$\widetilde{v}^k + 1/c^k\big((\widetilde{x}^k, \widetilde{y}^k) - (x^k, y^k)\big) = 0.$$

Then, it is natural to use an approximate solution of this minimization problem to perform the inner step above. The main difficulty here is how to compute a good element in $T_l(\cdot, \cdot)$ that permits to test the inner acceptance criterion, (3), and to perform the extragradient step above. This will be handled by the next two simple results:

Lemma 1. *Let $\widetilde{x} \in \mathbb{R}^n$ and $y \in \mathbb{R}^m_+$ and $c > 0$. Define*

$$\widetilde{y} \doteq \nabla_w P(g(\widetilde{x}), y, c) \quad \text{and} \quad \widetilde{\nu} \doteq \big(y + cg(\widetilde{x})\big)_-,$$

then

$$\widetilde{\nu} \in N_{\mathbb{R}^m_+}(\widetilde{y}),$$

and, for any $\widetilde{\gamma} \in \partial_x l(\widetilde{x}, \widetilde{y})$,

$$\widetilde{v} \doteq \begin{bmatrix} \widetilde{\gamma} \\ -g(\widetilde{x}) + \frac{1}{c}\widetilde{\nu} \end{bmatrix} \in T_l(\widetilde{x}, \widetilde{y}).$$

Proof. The result follows trivially, observing that $\nabla_w P(g(\widetilde{x}), y, c) = \big(y + cg(\widetilde{x})\big)_+$. □

The reason for choosing $\widetilde{\nu}$ among the elements of $N_{\mathbb{R}^m_+}(\widetilde{y})$ becomes clear as:

Proposition 1. *Let $\widetilde{x}^k \in \mathbb{R}^n$ and $y^k \in \mathbb{R}^m_+$ and $c^k > 0$. Defining $\widetilde{y}^k$, $\widetilde{\nu}^k$, $\widetilde{\gamma}^k$ and $\widetilde{v}^k$ as above it follows that:*

$$r^k \doteq \widetilde{v}^k + 1/c^k\big((\widetilde{x}^k, \widetilde{y}^k) - (x, y)\big) \in \begin{bmatrix} \partial\phi_k(\widetilde{x}^k) \\ 0 \end{bmatrix} \tag{6}$$

Proof. Clearly, $\partial\phi_k(\widetilde{x}^k) = \partial_x l(\widetilde{x}, \widetilde{y}) + 1/c^k(\widetilde{x}^k - x^k)$.

On the other hand

$$\begin{aligned} -g(\widetilde{x}^k) &+ (1/c^k)\widetilde{\nu}^k + 1/c^k(\widetilde{y}^k - y^k) = \\ &= -g(\widetilde{x}^k) + 1/c^k\left[\big(y^k + c^k g(\widetilde{x}^k)\big)_- + \big(y^k + c^k g(\widetilde{x}^k)\big)_+ - y^k\right] \\ &= -g(\widetilde{x}^k) + 1/c^k\big(y^k + c^k g(\widetilde{x}^k) - y^k\}\big) \\ &= 0. \end{aligned}$$

This completes the proof. □

Now, it is easy to present an implementable form of the hybrid algorithms. Since both algorithms are very similar we will focus the following presentation on the hybrid extragradient-proximal algorithm, which is simpler and performed slightly better in our experiments.[4]

Inexact Hybrid Extragradient-Proximal Augmented Lagrangian Method

1. *Initialization: Let* $(x^0, y^0) \in \mathbb{R}^n \times \mathbb{R}^m_+$ *and* $\sigma \in [0, 1)$.

2. *Iteration: Given* (x^k, y^k) *and* $c^k > 0$, *define* $P(\cdot, y^k, c^k)$ *as in equation* (2). *Define also*

$$\varphi_k(x) \doteq f(x) + P(g(x), y^k, c^k),$$
$$\phi_k(x) \doteq \varphi_k(x) + \frac{1}{2c^k} \|x - x^k\|^2 .$$

   (a) *Inner optimization: Find* $\tilde{x}^k$ *approximate solution of the unconstrained minimization of* $\phi(\cdot)$ *such that*

$$\|\nabla\phi_k(\tilde{x}^k)\| \leq \sigma(1/c^k)\,\|\tilde{z}^k\| \tag{7}$$

   where

$$\tilde{v}^k \doteq \begin{bmatrix} \nabla\varphi_k(\tilde{x}^k) \\ (1/c^k)(y^k - \tilde{y}^k) \end{bmatrix}, \quad \tilde{z}^k \doteq \begin{bmatrix} \tilde{x}^k - x^k \\ \tilde{y}^k - y^k \end{bmatrix}$$

   with

$$\tilde{y}^k \doteq \nabla_w P(g(\tilde{x}^k), y^k, c^k)$$

   (b) *Extragradient Step: If* $\tilde{v}^k = 0$, *or* $\tilde{z}^k = 0$, *Stop. Otherwise,*

$$(x^{k+1}, y^{k+1}) \doteq (x^k, y^k) - c^k\tilde{v}^k.$$

■

The main convergence theorems are:

[4]Moreover, to simplify the presentation we will suppose that the objective function and the constraints are differentiable. From the previous results, the adaptation to non-differentiable case should be straight forward.

Theorem 1. *If the problem* (P) *and its dual have solutions and the sequence of penalization parameters,* $\{c^k\}$, *is bounded away from zero; then the sequence generated by the inexact hybrid extragradient-proximal augmented Lagrangian converge to a pair of solution of these problems (a KKT pair).*

Proof. This is a corollary of the convergence of the hybrid extragradient-proximal point [16, Theorem 3.1]. □

Theorem 2. *Under the assumptions of theorem 1, if* $T_l^{-1}(\cdot,\cdot)$ *is Lipschitz continuous at the origin then the the convergence rate is at least Q-linear.*

Proof. The result follows from the convergence rate of the hybrid extragradient-proximal algorithm [16, Theorem 3.2]. □

Note that the Lipschitz continuity of $T_l^{-1}(\cdot,\cdot)$ can be guaranteed under strong second order conditions, as shown [13, pp. 102–103].

It is important to stress that, to the authors knowledge, this is the *first* convergence result of an optimization method similar to the augmented Lagrangian algorithm that does not require increasing relative accuracy, *i.e.*, the σ is held constant during the whole process. All the convergence results so far, asked the relative accuracy to decrease to zero [1, 2, 13].

3 Computational Experiments

In this section, we present some preliminary computational results to validate the applicability of the above algorithm. We also compare it to two different algorithms:

1. The ordinary Proximal augmented Lagrangian method, with the stringent error acceptance criterion (A), presented in the beginning of Section 2.[5]

2. The ordinary augmented Lagrangian with errors, as presented in [1, Chapter 5], usually implemented in practice. We remind that in [13], Rockafellar showed that this algorithm can be seen as the proximal point method applied to the dual of (P). We present the algorithm for the sake of completeness:

   Augmented Lagrangian with Inexact Minimization

[5] The method described in [13], that does not depend on the extragradient step.

(a) *Initialization: Let* $(x^0, y^0) \in \mathbb{R}^n \times \mathbb{R}^m_+$ *and* $\{\epsilon^k\}$ *a sequence converging to zero.*

(b) *Iteration: Given* (x^k, y^k) *and* $c^k > 0$*, define* $P(\cdot, y^k, c^k)$ *as in equation (2), and define* $\varphi_k(\cdot)$ *as in the hybrid algorithm:*

$$\varphi_k(x) \doteq f(x) + P(g(x), y^k, c^k).$$

i. *Inner optimization: Find an approximate solution of the unconstrained optimization of* $\varphi_k(\cdot)$*,* x^{k+1}*, such that*

$$\| \nabla \varphi_k(x^{k+1}) \| \leq \epsilon^k / c^k \; \| \nabla_w P(g(x^{k+1}), y^k, c^k) - y^k \| .$$

ii. *Multiplier update: Define* $y^{k+1} \doteq \nabla_w P(g(x^{k+1}), y^k, c^k)$.

■

3.1 Implementation details

We have implemented the methods in Fortran 90. Since they are very similar the results should not be influenced by implementation details. To solve the unconstrained optimizations problems, we used the LBFGS-B code from Byrd *et al* [3] which is freely available at the OTC site.[6]

Moreover, we did not try to fine tune the parameters of the algorithms to achieve better performance in *each* problem. The parameters were chosen to be robust, *i.e.* to guarantee convergence to all the problems tested. Some parameters that must be described are:

1. **Initialization**: The initial primal-dual pair was chosen randomly in $[-2, 2] \times [0, 2]$. These values are in the order of magnitude of the solutions.

2. **Stopping criterion**: Since the solutions to all problems are known, we decided to use as stopping criterion ϵ-optimality and ϵ-feasibility. Formally, let x^k be an approximate solution and f^* be the optimal value. The point x^k is accepted if:

$$|f(x) - f^*| \leq \max(\epsilon_1, \epsilon_2 |f^*|);$$
$$g_i(x) \leq \epsilon_3, \; \forall i = 1, \dots, m.$$

Where $\epsilon_1 = 5.0\text{E}{-5}$ and $\epsilon_2 = \epsilon_3 = 1.0\text{E}{-4}$.

[6]The source code is freely available in the site `http://www.ece.northwestern.edu/OTC/OTCsoftware.htm`.

3. **Update of the penalty parameter c^k:** We have decided to keep c^k fixed. Otherwise a slower method could force c^k to increase faster, hiding its deficiency.

4. **Stopping criterion for the inner step in the hybrid method:** Instead of using the original acceptance criterion given in (7), we decided to use a simpler threshold,

$$\|\nabla\phi_k(\tilde{x}^k)\| \leq \sigma 1/c^k \, \|\tilde{x}^k - x^k\|,$$

   that is faster to compute. The σ chosen was 0.9. If we had used the original acceptance test, smaller values for σ would be better.

5. **The error control sequence $\{\epsilon^k\}$** For the ordinary augmented Lagrangian, we have used

$$\epsilon^k \doteq \frac{1}{1+k/5},$$

   which is based on the harmonic sequence. This sequence was chosen as it goes slowly to zero and worked well in our tests.

   For the proximal augmented Lagrangian, we have used the same sequence squared.[7]

The code was run on a PC class computer based on the AMD K6-2 300 MHz CPU and with 128MB of main memory. Linux was the operating system. The compiler used was the Intel Fortran Compiler, version 5.0.1. Finally, each problem was solved a thousand times to minimize start up effects and to randomize the starting point.

3.2 The Tests Problems

The following convex test problems were used:

- From Hock and Schittkowski collection [6, 15]: problems 21, 28, 35, 51, 76, 215, 218, 224, 262, 268, 284, 315, 384;

- From Lasdon [8]: problems 1 and 3;

These are all small scale problems, with up to 15 variables and 10 constraints that are clearly convex.

[7] The error sequence of the proximal augmented Lagrangian method must be summable.

3.3 Computational results

The table below presents the processing time in seconds used to solve each test problem a thousand times. We also put on evidence is the number of unconstrained minimizations used by each method to find an approximate solution to the constrained problem.[8] This value will be used to better explain the methods behavior.

Problem	Aug. Lagrangian Time	Aug. Lagrangian #Min.	Prox. Lagrangian Time	Prox. Lagrangian #Min.	Hybrid Time	Hybrid #Min.
S21	1.11	4065	2.02	9186	1.94	9260
S215	2.48	16049	4.78	24106	3.71	24653
S218	1.41	4723	2.76	15523	3.46	17004
S224	9.07	34757	8.86	28169	7.80	28785
S262	8.38	9796	13.70	33515	19.20	48263
S268	18.80	3819	16.60	6875	17.30	8164
S28	4.68	3505	5.06	10925	3.78	8181
S284	11.80	8079	15.60	22222	10.80	14677
S315	1.82	6795	3.25	12365	2.65	12403
S35	2.56	7440	4.55	11881	3.79	12113
S384	43.80	38737	71.80	37598	38.40	37611
S51	6.16	15991	8.14	15017	6.63	15462
S76	6.49	14248	12.80	24164	9.83	27405
Lasdon1	2.61	12451	2.50	10743	2.36	11397
Lasdon3	14.40	29682	18.60	28650	16.10	28706

Table 1: Performance results.

The results confirm that the looser acceptance criterion followed by the extra-gradient step has a positive effect on the computational time. Actually, considering the mean behavior in all problems, the hybrid version of the proximal augmented Lagrangian used only 87% of the time used by the version with summable errors, without increasing the number of unconstrained minimizations.[9]

On the other hand, when compared to the augmented Lagrangian without a primal regularization, the Hybrid method is still slower. These seems to be

[8] The column "#Min.".

[9] This mean behavior is the geometric mean of the ratios of the times in both methods.

a consequence of an increase in the number of unconstrained minimizations required by the methods using the primal regularization. Actually, the mean time used by the hybrid method is 25% bigger than the one used by the ordinary augmented Lagrangian method and the number of unconstrained minimizations increased 67%. Hence, although less work is done at each minimization due to the new acceptance criterion, the higher number of unconstrained minimizations is still a bottleneck.

This last observation raises the question of whether it is possible to use the hybrid algorithm to solve directly the dual problem, deriving an augmented Lagrangian method with a better error criterion. Unfortunately, this is not a straightforward extension of the ideas presented in this paper. The error criterion and the extragradient step would need an element in the subgradient of the dual function, which requires a *full* unconstrained minimization to compute. This should be subject of further investigation.

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