nº 33

Homogeneous ℓ-hereditary algebras with maximum spectra.

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Outubro 1981

HOMOGENEOUS ℓ-HEREDITARY ALGEBRAS WITH MAXIMUM SPECTRA

by H. Merklen

1. Introduction.

A <u>point</u> in an abelian category <u>C</u> is an object X (or the isomorphic class of X) such that End(X)=D is a division ring. If $\dim(X)_D$ is finite, we say that X is a finite point. A Krull-Schmidt, pre-additive category <u>C</u> has <u>maximum spectrum</u> (abrev. MS) if all its indecomposable objects are finite points. If R is a ring, we say that R has MS (on the left) if mod R has MS.

Let Λ be an artin algebra. We have shown in [10] that, if Λ is hereditary, Λ has MS if and only if it is of finite representation type (abrev. f.r.t.).

It follows from the results of Drodz in [7] that, given a finite, partially ordered set (abrev. poset) I and a field K, then the category of I filtered K-vector spaces has MS with all division rings D isomorphic to K if and only if I is of f.r.t..

R. Bautista introduced in [4] the concept of what is now called an ℓ-hereditary artin algebra: the artin algebra Λ is ℓ-hereditary if and only if every non-zero morphism between indecomposable projectives of mod Λ is a monomorphism. The main theorem in [4] asserts that an

ℓ-hereditary algebra is of f.r.t. if and only if, for every indecomposable M in mod Λ, there is an n such that $(DTr)^n M$ is projective. Hence, ℓ-hereditary algebras of f.r.t. have MS.

In this paper we prove that if an ℓ-hereditary algebra has MS then it is of f.r.t., assuming that the algebra is homogeneous.

Let Λ be an indecomposable, ℓ-hereditary artin algebra. The argument in [2], p.406, is applicable here and shows that the center, K, of Λ is a field. Following Bautista in [4] we say that Λ is F-homogeneous if, for each indecomposable projective P in mod Λ, $\mathrm{End}(P) \cong F^{op}$ and, for all pairs of indecomposable projectives P, Q in mod Λ, $\mathrm{Hom}(P,Q) = (P,Q)$ is isomorphic to 0 or to F^{op} as F^{op}-bimodule.

The representation theory of homogeneous ℓ-hereditary algebras is closely related to the theory of linear representations of posets. Let I be a poset and F a finite dimensional division algebra over its center, K. $F^{\underline{I}}$, the category of (left) F-linear representations of I is the category of (covariant) functors from the category I to the category mod F of finite dimensional left vector spaces over F. In other words, an object of $F^{\underline{I}}$ is a family of (finite dimensional) F-vector spaces indexed by I, $(V_i)_{i \in I}$, together with a family of one linear map $V_i \longrightarrow V_j$ for each pair (i,j) such that $i < j$, such that all triangles

$$\begin{array}{ccc} V_i & \longrightarrow & V_k \\ & \searrow \quad \nearrow & \\ & V_j & \end{array}$$

corresponding to vertices $i < j < k$, are commutative. (This family is extended to all pairs (i,j) by choosing the map

1_{V_i} for the pairs (i,i) and the map 0 for pairs (i,j) with $i>j$. A morphism of ${}_F\underline{I}$ from the object V to the object W is a family of F-linear maps $a_i:V_i \longrightarrow W_i$ $(i\in I)$ which renders all squares of the following form commutative.

$$\begin{array}{ccc} V_i & \longrightarrow & V_j \\ {\scriptstyle a_i}\downarrow & & \downarrow{\scriptstyle a_j} \\ W_i & \longrightarrow & W_j \end{array}$$

It is well known that, given F, each poset I defines an ℓ-hereditary algebra, F-homogeneous, $\Lambda = \Lambda_{I,F}$ with the property that ${}_F\underline{I}$ is equivalent to mod Λ (see [4],Cor.1.4). Let $A_{I,F}$ be the incidence algebra of I over F (i.e. an F-vector space with basis $(e_{ij})_{i\leqslant j}$ with multiplication induced by $(ae_{ij})(be_{kr}) = ab\,\delta_{jk}e_{ir}$). Then $\Lambda_{I,F} = A_{I^{op},F}$. We observe that, if op is the usual functor from a category to the opposite and if D is the usual duality between mod F and mod F^{op}, then $V \longmapsto D\cdot V\cdot op$ defines, in the obvious way, a duality from ${}_F\underline{I}$ to ${}_{F^{op}}\underline{I}^{op}$. This reflects the fact that $(A_{I^{op},F})^{op} \cong A_{I,F^{op}}$.

On the other hand, there is a way of associating a poset $I=I_\Lambda$ to an homogeneous, indecomposable, basic, ℓ-hereditary artin algebra Λ. I is taken to be a set in one-to-one correspondence with the family of projective indecomposables in mod Λ : $(P_i)_{i\in I}$.

and the ordering is defined by $i \leqslant j$ if and only if Hom $(P_i, P_j) = (P_i, P_j) \neq 0$. It follows from [4], Prop.3.2 that if Λ is of f.r.t. then $\operatorname{mod} \Lambda \approx {}_F\underline{I}_\Lambda$ where $F \cong (P,P)^{op}$ for some indecomposable projective P in $\operatorname{mod} \Lambda$. In other words, $\Lambda \cong \Lambda_{(I_\Lambda, F)}$ and, clearly, $I \cong I_{(\Lambda_{I,F})}$.

In section 2 we show that Bautista's proof can be adapted to prove the above result under the assumption that Λ has MS (see Th.2).

In section 3 we study the representations of posets and show that an algebra of the form $\Lambda_{I,F}$ has MS if and only if it is of f.r.t..(see Th.3). These results imply the following theorem.

THEOREM 1. Let Λ be an F-homogeneous, ℓ-hereditary artin algebra. Then, Λ has maximum spectrum if and only if it is of finite representation type. In this case, for each indecomposable M in $\operatorname{mod} \Lambda$ we have $\operatorname{End}_\Lambda(M) \cong F^{op}$.

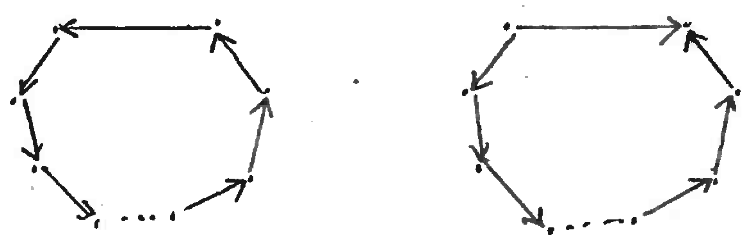
2. ℓ-hereditary algebras and posets.

We begin this section by recalling some terminology and notation applicable to posets.

With I we denote a finite partially ordered set (poset). The elements of I are called points or vertices. A pair of points (x,y) is an arrow of I (notation: $x \rightarrow y$) if $x < y$ and there is no z such that $x < z < y$. A path in I is a sequence of arrows: (x_1,x_2), $(x_2,x_3)\ldots(x_{n-2},x_{n-1})(x_{n-1},x_n)$ (written as a product) where $n \geqslant 2$. It is said that the path goes from x_1 to x_n, or that x_1 is the origin and x_n the end point of the path. Sometimes we will denote the path by the sequence of the corresponding points: $x_1,\ldots,x_n$, or by a diagram $x_1 \rightarrow x_2 \rightarrow \ldots \rightarrow x_n$. Paths may be multiplied in the obvious way. It is easy to see that $x < y$ in I if and only if there is a path going from x to y.

We will denote by α the function that to each path γ associates the origin of γ, and with β the function that to γ associates the end point of γ. Two paths γ_1, γ_2 are said to be equivalent (notation: $\gamma_1 \sim \gamma_2$) when $\alpha(\gamma_1)=\alpha(\gamma_2)$ and $\beta(\gamma_1)=\beta(\gamma_2)$.

A quiver is a pair (I,A) of finite sets I, A, whose elements are called points (vertices) and arrows, respectively, together with two applications $\alpha, \beta : A \rightarrow I$. If a is an arrow, $\alpha(a)$ is the origin and $\beta(a)$ the end point of a. We will consider only quivers with no multiple arrows, that is, given two arrows $a,b \in A$ if $\alpha(a)=\alpha(b)$ and $\beta(b)=\beta(a)$ then $a=b$. Also, by a reason which we make clear below, we will assume that the quivers under consideration never have subquivers of the forms:



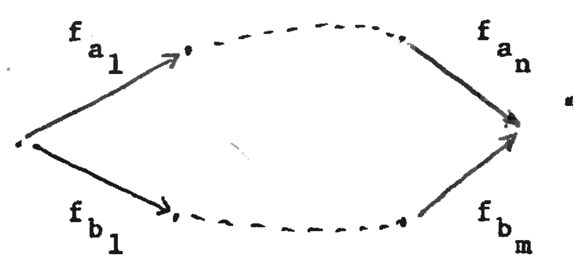
Then, each poset I defines a quiver Q_I which has the same vertices and arrows as I. Conversely, each quiver Q defines a poset I_Q with the same vertices as Q and with the order defined by $x < y$ if and only if there is a path in Q going from x to y. We have $I_{(Q_I)} = I$ and $Q_{(I_Q)} = Q$, so that there is a one-to-one correspondence between quivers and posets. If there is no possibility of misunderstanding, we will represent corresponding quivers and posets by the same letter, say I, or by the diagram of the quiver.

Let I be a quiver or a poset. A connection from the point x to the point y is a sequence of vertices of the form $x=x_1,\dots,x_n=y$ $(n \geq 1)$ such that, for each $i < n$, either there is a path from x_i to x_{i+1} or a path from x_{i+1} to x_i. This is an equivalence relation on the set of vertices and, if it has just one equivalence class, we say that I is connected.

Two vertices x, y, of a quiver or poset I, are called neighbors when either there is an arrow from x to y or an arrow from y to x.

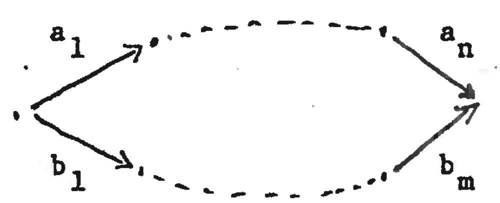
Let I be a poset and Q the corresponding quiver. Given a division ring F, the F-linear representations of I and Q are closely related. Let us recall that a representation of Q over F is a family of (finitely dimensional) F-vector spaces indexed by the set of vertices of Q, $(V_i)_{i\in I}$, together with a family of linear maps $V_i \longrightarrow V_j$ for each pair (i,j) such that there is an arrow from i to j. The morphisms between representations of Q are defined as for the case of posets. It follows that ${}_F\underline{I}$ is a category of representations of Q with commutative conditions.

More precisely, each representation of I, $(f_a)_{a\in A}$ (A= set of arrows of I) makes commutative all diagrams



(†)

corresponding to the subquivers of Q of the form



(††)

Conversely, each representation $(f_a)_{a\in Q}$ which makes commutative every diagram (†) corresponding to a full subquiver of Q of the form (††) is a representation of I. Clearly this defines a functor from ${}_F\underline{I}$ to the category of representations of Q with those commutative conditions which is the identity on morphisms.

For the rest of this section we fix the following notations:

Λ is an indecomposable, basic, artin algebra and $\underline{r}$ is the radical of Λ. $P_1,\ldots,P_n$ are the indecomposable projectives in $\mathrm{mod}\,\Lambda$ and $\underline{P}$ is the full subcategory defined by them. S_i denotes the simple $P_i/\underline{r}P_i$. We recall that $\mathrm{mod}(\underline{P})$, the category of finitely presented, contravariant functors from $\underline{P}$ to Ab is equivalent to $\mathrm{mod}\,\Lambda$ (see [1], Props. 2.5 and 2.7 or [4], Prop. 1.2).

We assume now furthermore that Λ is an F-homogeneous, ℓ-hereditary, algebra and call K the center of Λ. The letter I will denote the poset I_Λ or the corresponding quiver.

<u>Lemma 1</u>. If Λ has MS, then I is the opposite of the ordinary quiver, Γ, of Λ.

Proof. We recall the definition of the ordinary quiver Γ. One writes $\underline{r}/\underline{r}^2$ as a direct sum of indecomposable F_i-F_j-bimodules ${}_iM_j$ (where $F_1,\ldots,F_n$ are the ring components of $\Lambda/\underline{r}$; they are isomorphic to F as K-division algebras).

Γ has $1,2,\ldots,n$ as vertices and there is an arrow $i \leftarrow j$ if and only if ${}_iM_j \neq 0$.

Identifying Λ with $\mathrm{End}(P_1 \oplus \ldots \oplus P_n)^{op}$, we have

$$\Lambda = \begin{bmatrix} F_1 & (P_1,P_2) & \cdots & (P_1,P_n) \\ (P_2,P_1) & F_2 & \cdots & (P_2,P_n) \\ \vdots & & & \\ (P_n,P_1) & (P_n,P_2) & \cdots & F_n \end{bmatrix}$$

$$\underline{r} = \begin{bmatrix} 0 & (P_1,P_2) & \cdots & (P_1,P_n) \\ (P_2,P_1) & 0 & \cdots & (P_2,P_n) \\ \vdots & & & \\ (P_n,P_1) & (P_n,P_2) & \cdots & 0 \end{bmatrix}$$

If Λ has MS, then $\Lambda/\underline{r}^2$ has MS and it follows from [10], Cor.1, that $\Lambda/\underline{r}^2$ is of f.r.t. and, therefore, each (P_i,P_j) is a simple F_i-F_j-bimodule such that

$$\dim_{F_i}(P_i,P_j).\dim(P_i,P_j)_{F_j} \leqslant 3.$$

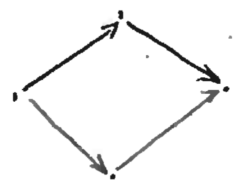
Since Λ is homogeneous, each $(P_i.P_j)$ is isomorphic, as a bimodule, to F and it follows that each product of the form:

$$(P_{i_1},P_{i_2}).(P_{i_2},P_{i_3})\dots(P_{i_{k-1}},P_{i_k})$$

with $i_r \neq i_{r+1}$ $(r=1,\dots,k-1)$ is either 0 or equal to (P_{i_1},P_{i_k}). This means that ${}_{i_1}M_{i_k}$ is 0 or (P_{i_1},P_{i_k}) depending on that there is or not a product of the above form, with $k \geqslant 2$, which is different from 0. In other words, ${}_{i_1}M_{i_k} \neq 0$ if and only if $i_1 \rightarrow i_k$ in I. This completes the proof.

In the sequel, fg $(f \in (P_i,P_j),\ g \in (P_j,P_k))$ indicates product within Λ and $g \circ f = fg$ indicates composition of maps.

Lemma 2. If Λ is not hereditary, I has a subposet of the form

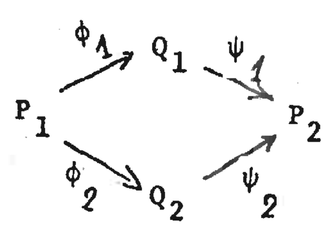


Proof.(cf. [4],Prop.1.10) If Λ is not hereditary, there is a P_2 in $\underline{P}$ such that $\underline{r}P_2$ is not projective. Let us consider a minimal projective resolution for $\underline{r}P_2$:

$$\bigoplus_j R_j \xrightarrow{\ \phi\ } \bigoplus_i Q_i \xrightarrow{\ \psi\ } \underline{r}P_2 \to 0$$

$$(R_j,\ Q_i \in \underline{P})$$

Since $\underline{r}P_2$ is not projective, there is an R_j, which we call P_1, which is non zero. Let $\phi_i : P_1 \to Q_i$ be the components of $\phi|_{P_1}$, and ψ_i the restriction of ψ to Q_i. Since each ψ_i is non zero and since Λ^* is ℓ-hereditary, there are at least two indices, say 1, 2, such that ϕ_1, ϕ_2 are non zero. Hence, we have the diagram:

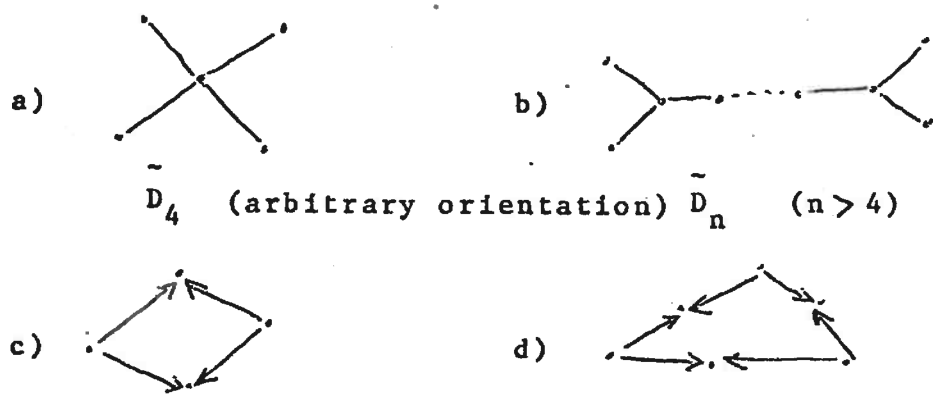


If it were $Q_1 \cong Q_2$, since $(Q_2,P_2) = (Q_2,Q_1)\,\psi_1$ we see that $\mathrm{Im}(\psi_1) = \mathrm{Im}(\psi_2)$ contradicting the fact that the ψ_i define a projective cover of $\underline{r}P_2$. This completes the proof.

Lemma 3. Let $\{P_{i_1},\ldots,P_{i_t}\}$ be a subset of $\underline{P}$ and let $\Gamma = \mathrm{End}(\bigoplus_k P_{i_k})^{op}$. Then, if Λ has MS, Γ has MS.

Proof. (see [4], Prop.1.8 , or [1],Prop.3.1 or Prop.5.1).

Lemma 4. Let Λ have MS. Then I does not have full posets of the following forms:



a) $\tilde{D}_4$ (arbitrary orientation) b) $\tilde{D}_n$ $(n > 4)$

c) d)

Proof. Let J be a subposet of I with one of the forms a),...,d). Let $\Gamma = \mathrm{End}(\bigoplus_{j\in J} P_j)^{op}$, an ℓ-hereditary algebra with quiver J. By lemma 3, Γ has MS, but by lemma 2 Γ is hereditary and, hence, of infinite representation type. This is a contradiction to the theorem in [10].

Let $i,j \in I$ be such that $i < j$ and let $e_{ij} \in (P_i,P_j)$, $e_{ij} \neq 0$. Since Λ is F-homogeneous, $(P_i,P_j)=(P_i,P_i)e_{ij}=e_{ij}(P_j,P_j)$, that is $(P_i,P_j) = Fe_{ij} = e_{ij}F$. Hence, e_{ij} defines a ring isomorphism

$$\sigma_{ij}:(P_i,P_i) \longrightarrow (P_j,P_j)$$

determined by

$$ae_{ij} = e_{ij}\sigma_{ij}(a):$$

On the other hand, we have, for each i, an isomorphism $\phi_i : F \to (P_i, P_i)$ defined by $\phi_i(a) = ae_{ii}$ $(e_{ii} = 1_{P_i})$.

If Λ is the ℓ-hereditary algebra associated to a poset, or, more particularly, if Λ is $\Lambda_{I,F}$ we have a natural choice for the e_{ij} above, with special properties. If the e_{ij} form the defining basis for $\Lambda_{I,F}$, then the following two conditions are satisfied:

1) for $i \leqslant j \leqslant k$, $e_{ij}e_{jk} = e_{ik}$;

2) for $i \leqslant j$, $\sigma_{ij} \circ \phi_i = \phi_j$.

This motivates the following definition.

Definition 1.(cf. [4], Def.1.6) A coordinate system for $\underline{P}$ is a family of non zero morphisms $e_{ij} \in (P_i, P_j)$, one for each pair (i,j) such that $(P_i, P_j) \neq 0$, satisfying

$$e_{ij}e_{jk} = e_{ik} \qquad (\text{whenever } (P_i, P_j) \neq 0 \neq (P_j, P_k)).$$

An orientation of the coordinate system (e_{ij}) is a family of ring isomorphisms $\phi_i : F \to (P_i, P_i)$ such that

$$\sigma_{ij} \cdot \phi_i = \phi_j \qquad (\text{whenever } (P_i, P_j) \neq 0)$$

where the isomorphisms σ_{ij} are defined by the e_{ij} in the form indicated above.

Lemma 5. In order that $\operatorname{mod} \Lambda \approx {}_F\underline{I}^{op}$ it is necessary and sufficient that $\underline{P}$ has a orientable coordinate system.

Proof. See [4], Prop.1.7.

The following lemma will be used to prove Theorem 2 as an application of lemma 5 above. The main objective is to define a good coordinate system $(e_{ij})_{i\leqslant j}$ in $\underline{P}$. The value e_{ij} may be obtained by the formula

$$e_{ij} = e_{x_1x_2}e_{x_2x_3}\cdots e_{x_{r-1}x_r} \qquad (\dagger)$$

where $x_1 \longrightarrow x_2 \longrightarrow \cdots \longrightarrow x_r = \rho$ is any path from $i=x_1$ to $j=x_r$. So, we are going to construct the coordinate system starting from a convenient set of arrows. To facilitate the exposition we are going to use the following notation: given the path $\rho = x_1 \longrightarrow x_2 \longrightarrow \cdots \longrightarrow x_r$ if $e_{x_ix_{i+1}}$ is already defined for the arrows $x_i \longrightarrow x_{i+1}$ then we define e_ρ by the formula $(\dagger)$.

We denote by $z_1,\ldots,z_m$ the maximal points of I and we consider the following assertions for some $u\in\{1,\ldots,m\}$ and some vertex z such that $z\leqslant z_u$ but $z\not\leqslant z_v$ for any $v<u$.

(A) Let D_z be the family of arrows $x \longrightarrow y$ such that either $y<z$ or $y\leqslant z_v$ for some $v<u$. There is a family of non zero elements $(e_{xy})_{x\to y\in D_z}$, $e_{xy}\in(P_x,P_y)$, such that, if ρ_1,ρ_2 are two equivalent paths ending in y (y as before) then $e_{\rho_1}=e_{\rho_2}$.

(B) Let $\hat{D}_z$ be the family of arrows $x \longrightarrow y$ such that either $y\leqslant z$ or $y\leqslant z_v$ for some $v<u$. There is a family of non zero elements $(e_{xy})_{x\longrightarrow y\in\hat{D}_z}$, $e_{xy}\in(P_x,P_y)$, such that, if ρ_1,ρ_2 are two equivalent parths ending in y (y as before) then $e_{\rho_1}=e_{\rho_2}$.

Lemma 6. If Λ has MS, (A) $\Longrightarrow$ (B). (Cf. [4],Prop.3.1.)

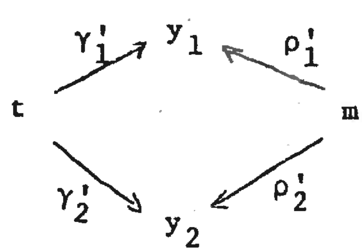
Proof. We know from lemma 4 that z has at most three neighbors.

1^{st} case) There are three equivalent paths, $\gamma_1,\gamma_2,\gamma_3$, beginning at some vertex t and ending at z:

$$\gamma_1 = \gamma_1'(y_1 \longrightarrow z), \quad \gamma_2 = \gamma_2'(y_2 \longrightarrow z), \quad \gamma_3 = \gamma_3'(y_3 \longrightarrow z)$$

with the y_i $(i=1,2,3)$ different.

It is always possible to choose $e_{y_i z}$ $(i=1,2,3)$ in such a way that $e_{\gamma_1}=e_{\gamma_2}=e_{\gamma_3}$. Having done this, let ρ_1,ρ_2 be two equivalent paths of the form: $\rho_1 = \rho_1'(y_1' \longrightarrow z)$, $\rho_2 = \rho_2'(y_2' \longrightarrow z)$. We have $y_i' \in \{y_1,y_2,y_3\}$ $(i=1,2)$. In case $y_1' = y_2'$ we have clearly $e_{\rho_1}=e_{\rho_2}$. In case $y_1' \neq y_2'$, we can assume $y_1' = y_1$, $y_2' = y_2$. We have a subposet of I of the following form:



$(m = \alpha(\rho_1))$

By lemma 4, we know this poset cannot be full. Since y_1, y_2 are not comparable because they are both neighbors of z, we must have one of the relations $t \leq m$, $m < t$. If $t \leq m$, we have $\gamma_1' = \delta\rho_1'$, $\gamma_2' = \delta\,\rho_2'$, where δ is some path from t to m. Then, by (A),

$$e_{\gamma_1'} = e_\delta \cdot e_{\rho_1'}\,, \quad e_{\gamma_2'} = e_\delta \cdot e_{\rho_2'}\,, \quad \text{which imply:}$$

$$e_\delta \cdot e_{\rho_1} = e_\delta \cdot e_{\rho_2} \quad \text{and} \quad e_{\rho_1} = e_{\rho_2}.$$

A similar argument is used if there is a path from m to t. Hence, the proof is complete in this aase.

2^{nd} case) The vertex z has three neighbors: y_1, y_2, y_3 and there are four paths ending in z according to the following dèscription:

$$\gamma_1 = \gamma_1'(y_1 \to z)$$

$$\alpha(\gamma_1) = \alpha(\gamma_2) = t$$

$$\gamma_2 = \gamma_2'(y_2 \to z)$$

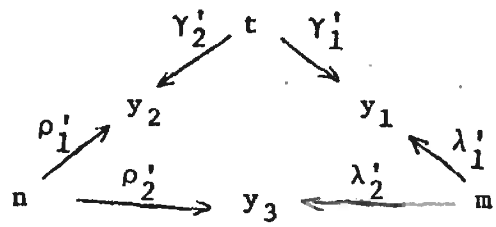
$$\lambda_1 = \lambda_1'(y_1 \to z)$$

$$\alpha(\lambda_1) = \alpha(\lambda_2) = m$$

$$\lambda_2 = \lambda_2'(y_3 \to z)$$

It is easy to see that it is possible to choose $e_{y_i z}$ $(i=1,2,3)$ in such a way that $e_{\gamma_1} = e_{\gamma_2}$ and $e_{\lambda_1} = e_{\lambda_2}$. Having done this, let ρ_1, ρ_2 be two equivalent paths ending at z. Clearly, $\rho_1 = \rho_1'(y_i \to z)$ where i is one of the numbers 1,2,3, and similarly for ρ_2. If one os these paths goes through y_1, we can assume it is ρ_1: $\rho_1 = \rho_1'(y_1 \to z)$. If ρ_2 has the form $\rho_2'(y_1 \to z)$, we have $e_{\rho_1} = e_{\rho_2}$. If ρ_2 has the form $\rho_2'(y_2 \to z)$ we repeat the argument in the proof of the 1^{st} case for the paths γ_1', γ_2', ρ_1', ρ_2'. If we have instead $\rho_2 = \rho_2'(y_3 \to z)$, we repeat this argument for the paths λ_1', λ_2', ρ_1', ρ_2'.

If none of the paths ρ_1, ρ_2 goes through y_1, we can assume they have the forms $\rho_1 = \rho_1'(y_2 \to z)$, $\rho_2 = \rho_2'(y_3 \to z)$, because if they go both through the same vertex y_2 or y_3 the proof is immediate. Therefore, we have a subposet of I of the following form.



It follows from lemma 4 that this subposet cannot be full. Since there are no relations among the y_i (because they are all neighbours of z) we see easily that there is at least one relation among m,n,t. If, for instance, we have $n \leqslant t$ then we fall into the first case because we have the three paths γ_1, γ_2 and $\delta\rho_2$ (where δ is a path from t to n) going from t to z and passing through y_1, y_2, y_3.

General case. We can assume that there are two equivalent paths $\gamma_1 = \gamma_1'(y_1 \longrightarrow z)$, $\gamma_2 = \gamma_2'(y_2 \longrightarrow z)$, $y_1 \neq y_2$. We choose $e_{y_1 z}$, $e_{y_2 z}$ in such a way that $e_{\gamma_1} = e_{\gamma_2}$.

Given two equivalent paths ρ_1, ρ_2 ending at z we have two possibilities. If one of them goes through a third neighbor of z, y_3, we fall into the second case. If ρ_1 goes through y_1 and ρ_2 through y_2 we can repeat the argument in the 1^{st} case.

The proof is complete.

THEOREM 2. Let Λ be an ℓ-hereditary, F-homogeneous, indecomposable, basic, artin algebra. Then the category $\text{mod}\,\Lambda$ is equivalent to the category ${}_F\underline{I}^{op}$.

Proof. According to lemma 5, we have to show that $\underline{P}$ has an orientable coordinate system. We show first that $\underline{P}$ has a coordinate system.

Let Φ be the family of all objects $(I',(e'_{xy})_{x \leqslant y})$ where I' is a subposet of I and (e'_{xy}) is a coordinate system for I'. We define an order relation on by saying that $(I',(e'_{xy})) \leqslant (I'',(e''_{xy}))$ if and only if I' is a subposet of I'' and $e''_{xy} = e'_{xy}$ for each arrow $x \longrightarrow y$ of I'. It is clear that Φ is inductive. Let $(I',(e'_{xy}))$ be a maximal element of Φ. If $I' \neq I$, let u be the first index such that $x \leqslant z_v \ (v < u) \implies x \in I'$ and let z be a minimal element in the set of vertices which are $\leqslant z_u$ but are not in I'. Then, applying lemma 6 we get a contradiction to the maximality of I'.

Now we show that a coordinate system $(e_{xy})_{x \leqslant y}$ has an orientation. We remark that, since Λ is indecomposable, I is connected. If ρ is any path going from x to y, we will write σ_ρ for the isomorphism σ_{xy} obtained by means of the coordinate system. We are going to associate an isomorphism $\sigma_\delta : (P_x, P_x) \longrightarrow (P_y, P_y)$ to each connection δ from x to y, and show that, as a matter of fact, it depends on the pair (x,y) but not on the connection δ. This has already been done above for the case δ is just one path. The connection corresponds to one of the following subposets:

$$x \overset{\rho_1}{\longleftarrow} \overset{\rho_2}{\longrightarrow} \overset{\rho_3}{\longleftarrow} \cdots \overset{\rho_n}{\longrightarrow} y$$

$$x \overset{\rho_1}{\longleftarrow} \overset{\rho_2}{\longrightarrow} \overset{\rho_3}{\longleftarrow} \cdots \overset{\rho_n}{\longleftarrow} y$$

$$x \overset{\rho_1}{\longrightarrow} \overset{\rho_2}{\longleftarrow} \overset{\rho_3}{\longrightarrow} \cdots \overset{\rho_n}{\longrightarrow} y$$

$$x \overset{\rho_1}{\longrightarrow} \overset{\rho_2}{\longleftarrow} \overset{\rho_3}{\longrightarrow} \cdots \overset{\rho_n}{\longleftarrow} y$$

and we define, respectively:

$$\sigma_\delta = \begin{cases} \sigma_{\rho_n} \circ \cdots \circ \sigma_{\rho_2} \circ \sigma_{\rho_1}^{-1} \\ \sigma_{\rho_n}^{-1} \circ \cdots \circ \sigma_{\rho_2} \circ \sigma_{\rho_1}^{-1} \\ \sigma_{\rho_n} \circ \cdots \circ \sigma_{\rho_2}^{-1} \circ \sigma_{\rho_1} \\ \sigma_{\rho_n}^{-1} \circ \cdots \circ \sigma_{\rho_2}^{-1} \circ \sigma_{\rho_1} \end{cases}$$

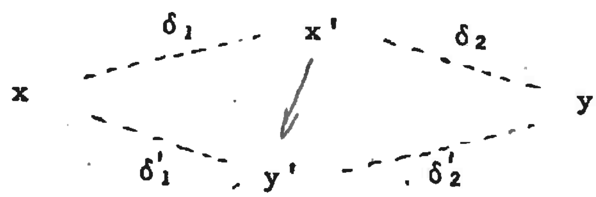
The proof that σ_δ does not depend on the connections δ, δ' from x to y is done by induction on $n+m$, where n is the number of paths of δ and m is the number of paths of δ'. The claim is true if $n+m \leqslant 2$.

In the general case, we observe that the subposet corresponding to the "union" of δ and δ' cannot be full. For it defines, by lemma 2, a hereditary algebra of infinite type, which leads to a contradiction with lemma 3 because of the main theorem in [10]. It follows easily that one of the vertices in δ,δ', say x', must be related to one of the vertices in δ,δ', say y'. We assume, without loss of generality, that $x' \leqslant y'$ and call λ a path from x' to y'.

If x',x' are vertices of one of the connections, say δ, we may write it in the form $\delta = \delta_1\lambda\delta_2$ and, applying the induction hypothesis twice, we obtain:

$$\sigma_{\delta} = \sigma_{\delta_2}\,\sigma_{\lambda}\,\sigma_{\delta_1} = \sigma_{\delta'}$$

If x' is in δ and y' is in δ' we have a sub-poset of the following form.



Applying the induction hypothesis we obtain:

$$\sigma_{\delta'_1} = \sigma_{\lambda}\circ\sigma_{\delta_1} \qquad \sigma_{\delta_2} = \sigma_{\delta'_2}\circ\sigma_{\lambda}$$

$$\sigma_{\delta'} = \sigma_{\delta'_2}\circ\sigma_{\delta'_1} = \sigma_{\delta'_2}\circ\sigma_{\lambda}\circ\sigma_{\lambda}^{-1}\circ\sigma_{\delta'_1} = \sigma_{\delta_2}\circ\sigma_{\delta_1} = \sigma_{\delta}$$

add this completes the proof of our claim.

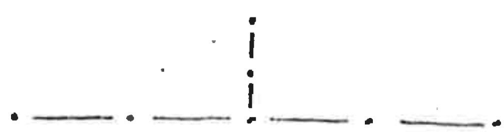
Finally, we can define the orientation of our coordinate system in the following form. First, we fix a vertex x_o in I and identify F with (P_{x_o}, P_{x_o}). Given any vertex y, we define ϕ_y as being equal to σ_{δ}, where δ is a connection from x_o to y. It follows directly, from this definition and the fact that σ_{δ} does not depend on the connection, that if $(P_i, P_j) \neq 0$, then $\sigma_{ij}\circ\phi_i = \phi_j$. This completes the proof of the theorem, because of lemma 5.

3. Posets with maximum spectrum.

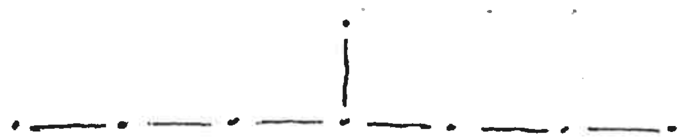
The categories of representations of a finite poset I over a field K were studied by Drodz, Kleiner, Nazarova, Roiter and others, and M. Loupias gave in [8] a complete solution for the problem of classifying posets I from the point of view of the representation type of $K\underline{I}$.

<u>Definition 2</u>. A finite, connected poset I is <u>crucial</u> if I or I^{op} belons in the following list. (Here the presence of an edge instead of an arrow means that the orientation may be chosen arbitrarily.)

$\tilde{E}_6$

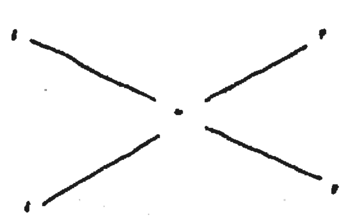


$\tilde{E}_7$

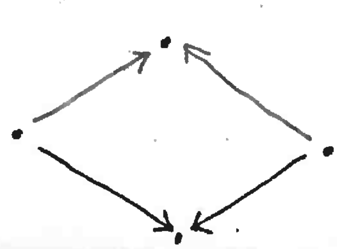


$\tilde{E}_8$

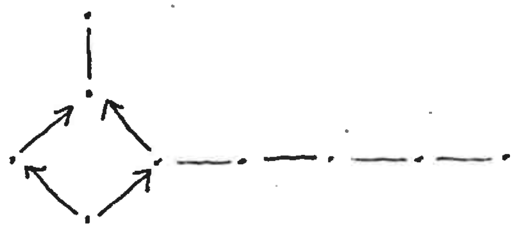
$\tilde{D}_4$



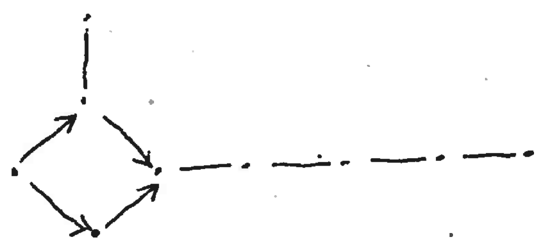
$\tilde{A}_3$



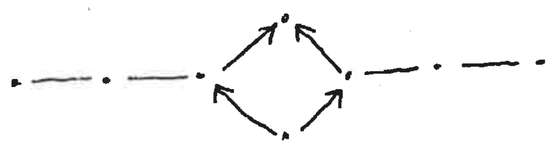
R_1



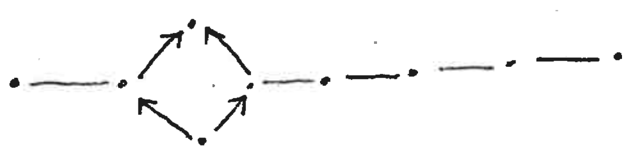
R_2



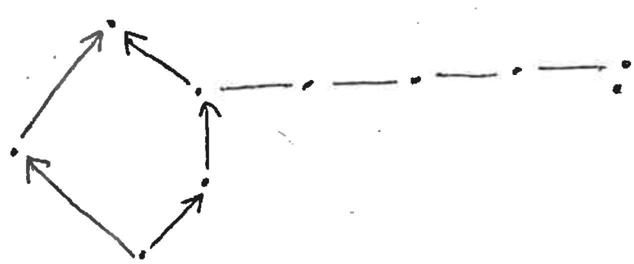
R_3



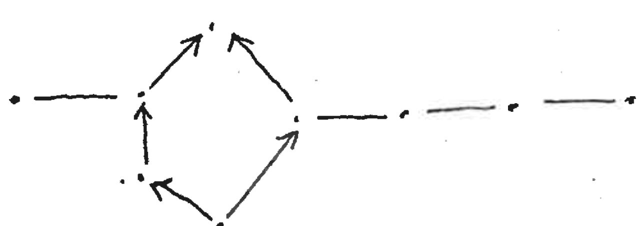
R_4



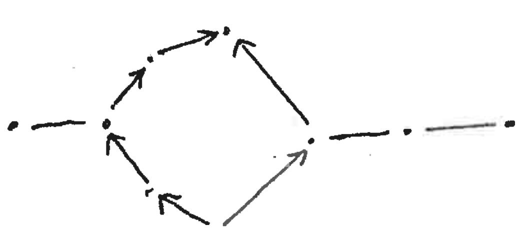
R_5



R_6



R_7



Definition 3. A surjection of posets $f:I \longrightarrow I'$ is called a contraction if $f^{-1}(x)$ is connected for every $x \in I'$.

Proposition 1. Given a division ring F and a contraction of posets $f:I \longrightarrow I'$, let G be the induced functor from ${}_F\underline{I}'$ into ${}_F\underline{I}$. Then, for every pair of objects M, N of ${}_F\underline{I}'$ we have

$$\mathrm{Hom}(M,N) \cong \mathrm{Hom}(G(M),G(N))$$

Proof. See [4], Prop.3.4.

Corollary 1. If, in the situation of Prop.1, ${}_F\underline{I}$ is of finite type, then ${}_F\underline{I}'$ is of finite type.

Corollary 2. If, in the situation of Prop.1, the category ${}_F\underline{I}$ has MS, then ${}_F\underline{I}'$ has MS.

Each of the following diagrams indicate the form of a representation of some quiver or poset I over a division ring F with finite dimension over its center K.

1) $$\begin{array}{ccccccccccccc} X_2 & \xrightarrow{\alpha_2} & X_1 & \xleftarrow{\alpha_1} & V & \xleftarrow{\beta_1} & Y_1 & \xleftarrow{\beta_2} & Y_2 & \xleftarrow{\beta_3} & Y_3 & \xleftarrow{\beta_4} & Y_4 & \xleftarrow{\beta_5} & Y_5 \\ & & & & \uparrow{\scriptstyle \gamma_1} & & & & & & & & & & \\ & & & & Z_1 & & & & & & & & & & \end{array}$$

(β_1, γ_1 injective; $\mathrm{Im}(\beta_1) + \mathrm{Im}(\gamma_1) = V$)

2) $$\begin{array}{ccccccccccccc} X_2 & \xrightarrow{\alpha_2} & X_1 & \xrightarrow{\alpha_1} & V & \xrightarrow{\beta_1} & Y_1 & \xleftarrow{\beta_2} & Y_2 & \xleftarrow{\beta_3} & Y_3 & \xleftarrow{\beta_4} & Y_4 & \xleftarrow{\beta_5} & Y_5 \\ & & & & \uparrow{\scriptstyle \gamma_1} & & & & & & & & & & \\ & & & & Z_1 & & & & & & & & & & \end{array}$$

(α_1, α_2, γ_1, $\beta_1\alpha_1$ injective; $\mathrm{Im}(\alpha_1) + \mathrm{Im}(\gamma_1) = V$)

$$3)\quad \begin{array}{ccccccccccccc} & & & & & & Z_1 & & & & & & \\ & & & & & & \uparrow{\scriptstyle \gamma_1} & & & & & & \\ X_3 & \xrightarrow{\alpha_3} & X_2 & \xrightarrow{\alpha_2} & X_1 & \xrightarrow{\alpha_1} & V & \xleftarrow{\beta_1} & Y_1 & \xleftarrow{\beta_2} & Y_2 & \xleftarrow{\beta_3} & Y_3 \end{array}$$

(α_1, α_2, α_3, β_1, $\gamma_1\alpha_1\alpha_2$ injective; $\mathrm{Im}(\alpha_1)+\mathrm{Im}(\beta_1)=V$)

$$4)\quad \begin{array}{ccccccccccccccc} & & & & Z_1 & & & & & & & & & & \\ & & & & \uparrow{\scriptstyle \gamma_1} & & & & & & & & & & \\ X_2 & \xrightarrow{\alpha_2} & X_1 & \xrightarrow{\alpha_1} & V & \xleftarrow{\beta_1} & Y_1 & \xleftarrow{\beta_2} & Y_2 & \xleftarrow{\beta_3} & Y_3 & \xleftarrow{\beta_4} & Y_4 & \xleftarrow{\beta_5} & Y_5 \end{array}$$

(α_1, β_1 injective; $\mathrm{Im}(\alpha_1)+\mathrm{Im}(\beta_1) = V$)

$$5)\quad \begin{array}{ccccccccccccccc} X_2 & \xrightarrow{\alpha_2} & X_1 & \xleftarrow{\alpha_1} & V & \xleftarrow{\beta_1} & Y_1 & \xleftarrow{\beta_2} & Y_2 & \xleftarrow{\beta_3} & Y_3 & \xleftarrow{\beta_4} & Y_4 & \xleftarrow{\beta_5} & Y_5 \\ & & & & \uparrow{\scriptstyle \gamma_1} & & & & & & & & & & \\ & & & & Z_1 & & & & & & & & & & \end{array}$$

(β_1, γ_1, $\alpha_1\beta_1$, β_2, β_3, β_4, β_5 injective; $\mathrm{Im}(\beta_1)+\mathrm{Im}(\gamma_1)=V$)

Lemma 7. For each of the forms 1),...,5) above, let I be the corresponding poset. Then, for each case, there is an indecomposable M in ${}_F\underline{I}$ such that $\mathrm{End}(M)$ is not a division ring.

Proof. We use the results and tables of [6] and, by this reason, we keep the notations and terminology of that paper. We are going to show, case by case, that some sequence of partial Coxeter functors takes one of the serial categories $\mathcal{R}^{(t)}$ (see [6], Th.3.5) defined in the tables for $\tilde{E}_7$ and $\tilde{E}_8$ (see [6],pp.48,49) into representations of the desired form, and preserving the endomorphism rings. It is enough to check the form 1),...,5) for the simple objects of $\mathcal{R}^{(t)}$.

<u>Form 1)</u>. Apply to E''_o, E''_1, in the tables for $\tilde{E}_8$, the Coxeter functors $S^-_{b_1}$, $S^-_{b_2}$, $S^-_{b_1}$, in this order.

<u>Form 2)</u>. Apply to E_o, E_1, E_2, E_3, E_4, in the tables for $\tilde{E}_8$, the Coxeter functors $S^-_{a_1}$, $S^-_{a_2}$, $S^-_{a_3}$, $S^-_{a_4}$, $S^-_{a_5}$, in this order. We get:

$$\begin{array}{ccccccccccccccccc}
 & & & & & & F & & & & & & & & & & \\
 & & & & & & \downarrow & & & & & & & & & & \\
E_o & \longmapsto & 0 & \to & F & \to & F & \to & F & \leftarrow & F & \leftarrow & 0 & \leftarrow & 0 & \leftarrow & 0
\end{array}$$

$$\begin{array}{ccccccccccccccccc}
 & & & & & & (1,1)F & & & & & & & & & & \\
 & & & & & & \downarrow & & & & & & & & & & \\
E_1 & \longmapsto & 0\times F & \to & 0\times F & \to & F\times F & \to & 0\times F & \leftarrow & 0 & \leftarrow & 0 & \leftarrow & 0 & \leftarrow & 0
\end{array}$$

$$\begin{array}{ccccccccccccccccc}
 & & & & & & 0 & & & & & & & & & & \\
 & & & & & & \downarrow & & & & & & & & & & \\
E_2 & \longmapsto & 0 & \to & F & \to & F & \to & F & \leftarrow & F & \leftarrow & F & \leftarrow & F & \leftarrow & F
\end{array}$$

$$\begin{array}{ccccccccccccccccc}
 & & & & & & F & & & & & & & & & & \\
 & & & & & & \downarrow & & & & & & & & & & \\
E_3 & \longmapsto & 0 & \to & 0 & \to & F & \to & F & \leftarrow & F & \leftarrow & F & \leftarrow & F & \leftarrow & 0
\end{array}$$

$$\begin{array}{ccccccccccccccccc}
 & & & & & & 0 & & & & & & & & & & \\
 & & & & & & \downarrow & & & & & & & & & & \\
E_4 & \longmapsto & F & \to & F & \to & F & \to & F & \leftarrow & F & \leftarrow & F & \leftarrow & 0 & \leftarrow & 0
\end{array}$$

<u>Form 3)</u>. Apply to E''_o, E''_1, in the tables for $\tilde{E}_7$, the Coxeter functor S^-_c. We get:

$$\begin{array}{ccccccccccccccc}
 & & & & & & & & F & & & & & & \\
 & & & & & & & & \uparrow\gamma_1 & & & & & & \\
E''_o & \longmapsto & 0 & \to & F\times 0 & \to & F\times 0 & \to & F\times F & \leftarrow & F\times F & \leftarrow & 0\times F & \leftarrow & 0\times F
\end{array}$$

where $\gamma_1 : (a,b) \longmapsto a-b$

$$\begin{array}{ccccccccccccccc}
 & & & & & & & & F & & & & & & \\
 & & & & & & & & \uparrow\gamma_1 & & & & & & \\
E''_1 & \longmapsto & F\times 0 & \to & F\times 0 & \to & F\times F & \to & F\times F & \leftarrow & 0\times F & \leftarrow & 0\times F & \leftarrow & 0
\end{array}$$

where $\gamma_1 : (a,b) \longmapsto a-b$.

Form 4). Apply to E''_o, E''_1, in the tables for $\tilde{E}_8$, the Coxeter functor S^-_c.

Form 5). Apply to E''_o, E''_1, in the tables for $\tilde{E}_8$, the Coxeter functors $S^-_{b_1}$, $S^-_{b_2}$, in this order. We get:

$$E''_o \longmapsto \begin{array}{cccccccccccc} 0\times F\times 0 & & & (1,1,0)F+(0,1,1)F & & & & & & & & 0 \\ \downarrow & & \swarrow & & & & & & & & & \downarrow \\ F\times F\times 0 & \leftarrow & F\times F\times F & \leftarrow & F\times F\times 0 & \leftarrow & F\times F\times 0 & \leftarrow & F\times 0\times 0 & \leftarrow & & F\times 0\times 0 \end{array}$$

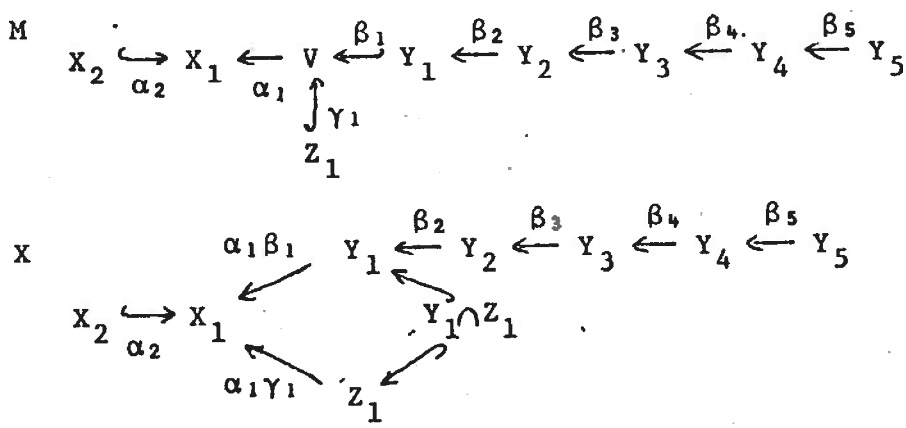
$$E''_1 \longmapsto \begin{array}{cccccccccccc} 0\times F\times F & & & (1,1,1)F & & & & & & & & F\times 0\times 0 \\ \downarrow & & \swarrow & & & & & & & & & \downarrow \\ F\times F\times F & \leftarrow & F\times F\times F & \leftarrow & F\times F\times F & \leftarrow & F\times F\times 0 & \leftarrow & F\times F\times 0 & \leftarrow & & F\times 0\times 0 \end{array}$$

Proposition 2. Let I be a poset and F a division algebra of finite dimension over its center, K. If I is crucial, $F^{\underline{I}}$ has an indecomposable representation, X, such that $\mathrm{End}(X)$ is not a division ring.

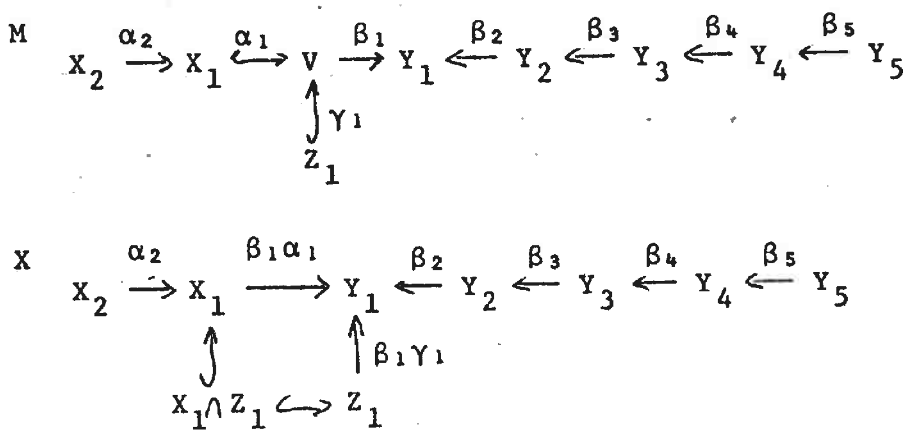
Proof. We remark first that, since there is a duality between the categories $F^{\underline{I}}$ and $F^{op}{}^{\underline{I}^{op}}$, we can limit the proof to diagrams of the forms shown in Def. 2. Secondly, by the results in [4], Section 4, we can choose the orientations which are not indicated in any way we wish. Thirdly, we can limit the proof to the diagrams of forms $R_1,\ldots,R_7$, because the other crucial diagrams correspond to hereditary algebras of infinite representation type.

For $R_1,\ldots,R_7$ we choose the orientation shown below. In each case we start with a representation M whose existence follows from lemma 7 and associate to it a representation X for the corresponding poset R_i. It will be clear from the definition of X that $\mathrm{End}(X) \cong \mathrm{End}(M)$.

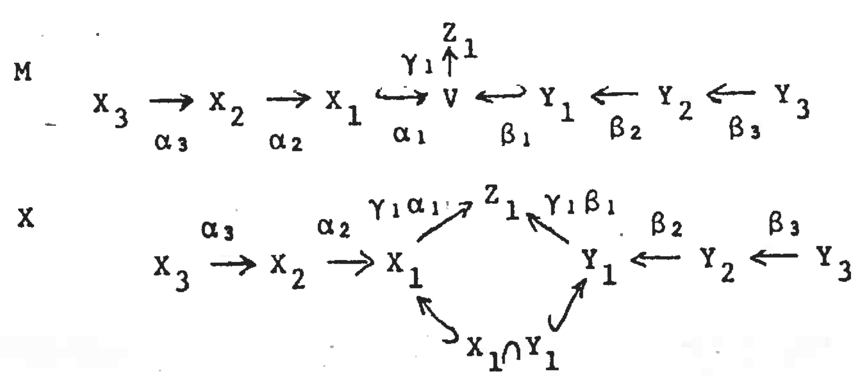
Case of R_1).



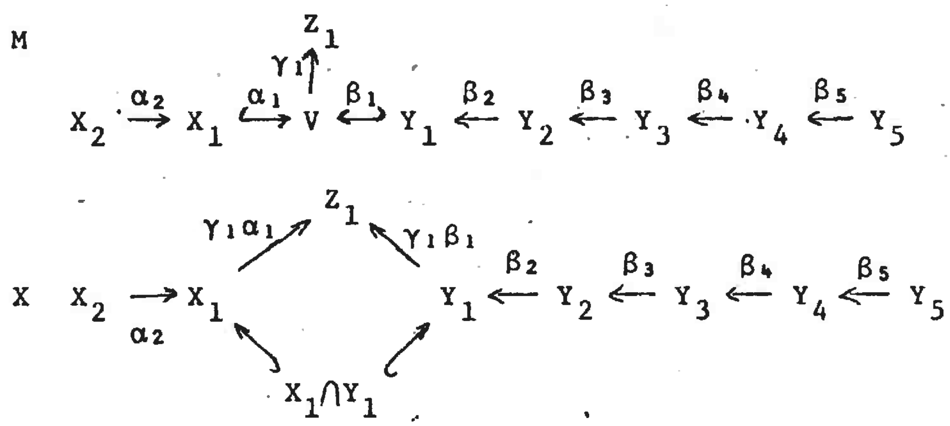
Case of R_2).



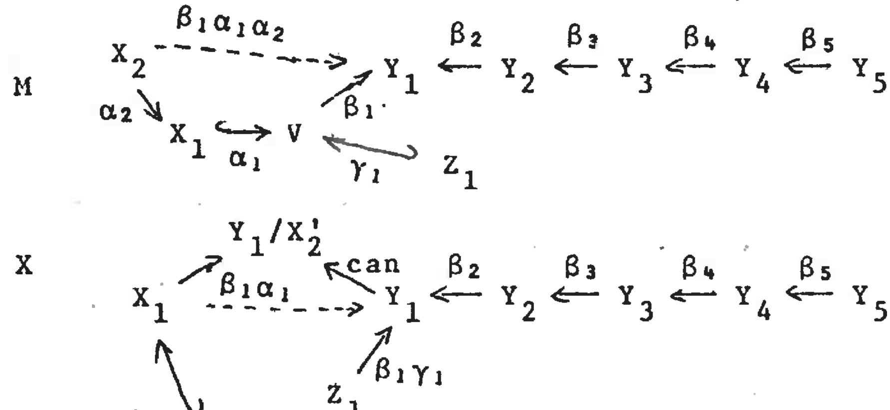
Case of R_3).



Case of R_4).

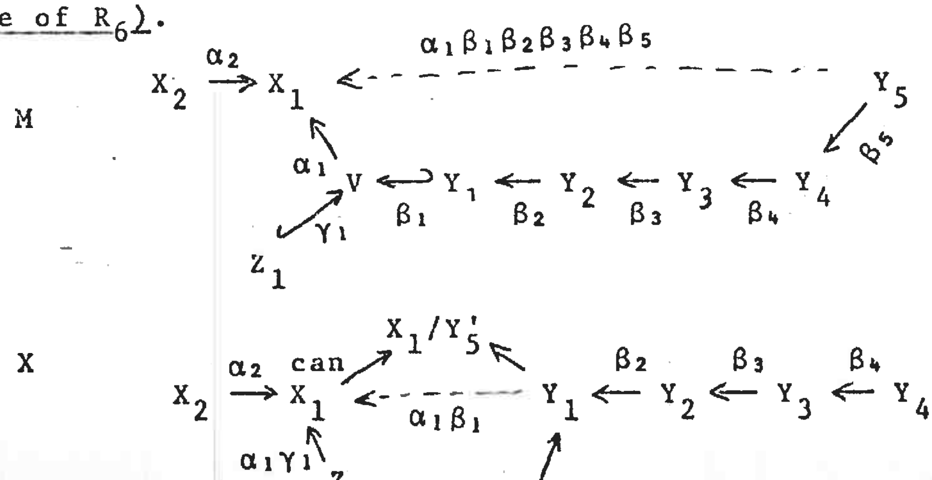


Case of R_5).



$X'_2 = \beta_1\alpha_1\alpha_2(X_2)$

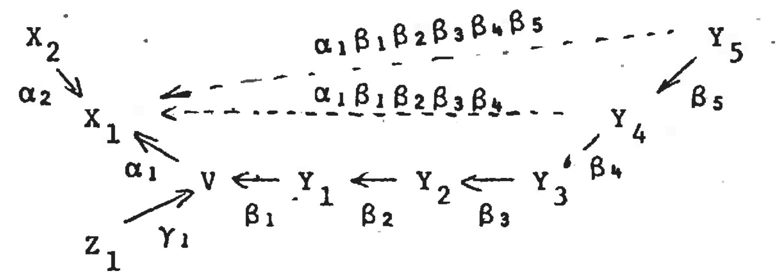
Case of R_6).



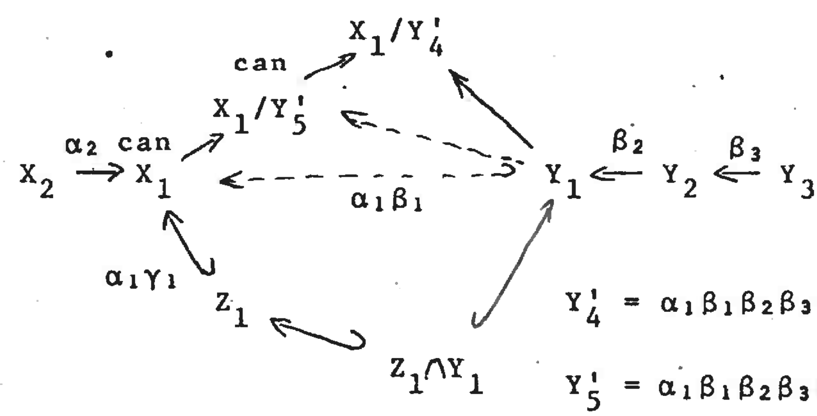
$Y'_5 = \alpha_1\beta_1\beta_2\beta_3\beta_4\beta_5(Y_5)$

Case of R_7).

M



X



$Y'_4 = \alpha_1\beta_1\beta_2\beta_3\beta_4(Y_4)$

$Y'_5 = \alpha_1\beta_1\beta_2\beta_3\beta_4\beta_5(Y_5)$

It follows from Prop.2, from Cor.2 to Prop.1, and from lemma 3, that if ${}_F\underline{I}$ has MS then I has neither a contraction or a subposet which is crucial. M. Loupias called these type of posets <u>kind</u> (see [8] and [9], Th. 1.5), and he showed that the kind posets are exactly those posets such that ${}_K\underline{I}$ is of f.r.t. for every field K. A list of the kind posets may be found also in [13].

<u>Lemma 8</u>. Let I be a poset such that ${}_F\underline{I}$ has MS. Then, for every M indecomposable in ${}_F\underline{I}$ we have $\mathrm{End}(M) \cong F^{op}$.

Proof. C.M.Ringel has shown in [11], 1, Ex.2 that the finite points M in a category of R-modules are in one-to-one correspondence with the epimorphisms (in the category of rings) $R \longrightarrow A$ where A is a simple artinian ring.

Obviously we can prove the lemma for the equivalent mod $\Lambda_{I,F}$ instead of ${}_F\underline{I}$. Clearly, $\Lambda_{I,F} = F \otimes_K \Lambda_{I,K}$ where K is the center of F.

By the remark above, I is kind and ${}_K\underline{I}$ (or $\Lambda_{I,K}$) is of f.r.t.. Hence, by the main theorem in [4], $\Lambda_{I,K}$ has MS and, moreover, for each indecomposable N in mod $\Lambda_{I,K}$, $\mathrm{End}(N) \cong K$.

Let $\phi : \Lambda_{I,F} \longrightarrow A$ be an epimorphism in the category of rings, where A is a simple artinian ring. Identifying F with $\phi(F)$ we may write $A = F \otimes_K A'$, where A' is the centralizer of F in A (see [5], §4). Then $\phi = 1 \otimes \phi'$, where, as is easily seen, $\phi' : \Lambda_{I,K} \longrightarrow A'$ is an epimorphism and A' is a simple K-algebra. Therefore, by Ringel's remark, A' must be of the form $\mathbb{M}_n(K)$ (= full matrix ring of type n×n with coefficients in K). It follows that $A = \mathbb{M}_n(F)$ and this means that $\mathrm{End}(M) \cong F^{op}$.

THEOREM 3. Let I be a finite, connected poset. Then, for a division ring F with finite dimension over its center, ${}_F\underline{I}$ has MS if and only if it is of finite representation type. This happens only if I is kind and, when it happens, for each indecomposable M in ${}_F\underline{I}$ we have $\mathrm{End}(M) \cong F^{op}$.

Proof. As we remarked above, it follows from Bautista's theorem that if ${}_F\underline{I}$ is f.r.t. and if M is an indecomposable representation, then $\mathrm{End}(M) \cong F^{op}$ and ${}_F\underline{I}$ has MS.

Conversely, let us assume that ${}_F\underline{I}$ has MS and let L be a splitting field for F. We observe that the algebra $A = L \otimes_K \Lambda_{I,F} \cong L \otimes F \otimes \Lambda_{I,K} \cong \mathbb{M}_r(L) \otimes \Lambda_{I,K}$ $\cong \mathbb{M}_r(\Lambda_{I,L})$ is Morita equivalent to $\Lambda_{I,L}$.

Given an indecomposable M in ${}_F\underline{I}$ we have, by lemma 8, that

$$\mathrm{End}_A(L \otimes_K M) \cong L \otimes_K F^{op} \cong \mathbb{M}_r(L)^{op}$$

which implies that $L \otimes_K M$ decomposes in mod A as a direct sum of r indecomposables. But, since A is Morita equivalent to $\Lambda_{I,L}$, A is of f.r.t. (see the remark before lemma 8). Since r is fixed, because $r^2 = \dim_K F$, this means that $\{\dim_K M \mid M$ indecomposable in ${}_F\underline{I}\}$ is a bounded set. Then, by Roiter's theorem on Brauer-Thrall I (see [12]) it follows that ${}_F\underline{I}$ is of f.r.t., and the proof is complete.

Remark. It is clear that Th.1 follows from Ths. 2 and 3. It is clear, also, that Th.1 holds for artin algebras Λ which are stably equivalent to ℓ-hereditary algebras. Moreover, if Λ is stably equivalent to an ℓ-hereditary artin algebra and if End(M) is a division ring for every indecomposable, non projective M in mod Λ, then Λ is of finite representation type. (See [1], p.246-247.)

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