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Minimal Polynomial Identities of Baric Algebras

Luiz Antonio Peresi

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Luiz Antonio Peresi
Departamento de Matemática
Universidade de São Paulo
C. P. 20570 (Ag. Iguatemi), São Paulo-SP, 01498-970 Brazil

1. Introduction.

A baric algebra is a pair (A,ω) consisting of a nonassociative algebra A over a field F and a nonzero algebra homomorphism $\omega : A \to F$. Classes of commutative baric algebras have been defined throught the literature by equations of the form

$$x^{\{n\}} - \omega(x)^{n-m}x^{(m)} = 0$$

where $1 \leq m < n$ and $x^{\{n\}}$, $x^{(m)}$ denote powers of x with some distribution of parentheses. If a commutative algebra satisfies such an equation then it satisfies the identity

$$(x^{\{n\}}, y, x^{(m)}) = 0$$

where the homomorphism ω does not appear. Here and so on (a,b,c) is the associator $ab.c - a.bc$, we use the term identity to mean polynomial identity and the term equation to mean any kind of identity.

Thus an interesting problem is to find identities of lowest degree not implied by the commutative law that hold for all algebras in such a class of commutative baric algebras. We call this type of identities minimal identities. In general such a class of algebras is not a variety of algebras since it is not closed by the operation of taking subalgebras. A more specific problem is then to determine the minimal variety of algebras containing the class, i. e., the variety defined by all minimal identities and commutativity.

This kind of problem has been studied by Costa [5], Alcalde-Burgueño-Mallol [6] and Correa-Hentzel-Peresi [7]. In this last paper we described a general method for 'processing identities' and then applied it to find the minimal variety that contains the class of Bernstein algebras, i. e., the class defined by equations $xy - yx = 0$ and $(x^2)^2 - \omega(x)^2x^2 = 0$. It was written with more emphasis in the method itself than applications. In the present paper we reverse this situation by applying the method to find minimal identities for nine classes of commutative baric algebras. For six of these classes we determine the minimal variety. In order to make the paper more understandable we include the full argument for the classes defined by $x^2 - \omega(x)x = 0$ and $x^3 - \omega(x)^2 x = 0$. For the other classes we just state the results.

All algebras considered in the paper are commutative algebras over a field F. The characteristic of F is zero or greater than the degree of the identities considered.

2. Degree two.

Let (A, ω) be a baric algebra satisfying $x^2 - \omega(x)x = 0$. As clear A satisfies the Jordan identity $(x^2, y, x) = 0$. On the other hand if we apply our method to search for identities of degree three we find none. Thus the minimal identities have degree four. To find all of them we proceed as follows.

Linearizing $x^2 - \omega(x)x = 0$ we obtain

$$f(a,b) = 2ab - \omega(a)b - \omega(b)a = 0.$$

Using this equation we obtain the eight possible equations of degree four:

$$(1) \qquad \begin{array}{lll} f(a,b)c.d = 0, & f(a,b).cd = 0, & \omega(d)f(a,b)c = 0, \\ \omega(cd)f(a,b) = 0, & f(ac,b)d = 0, & \omega(d)f(ac,b) = 0, \\ f(ac,bd) = 0, & f(ac.d,b) = 0. & \end{array}$$

These equations are expressed in terms of five association types. They are:

$$\begin{array}{lll} T_1.\ \omega(R)RR.R, & T_2.\ \omega(RR)RR, & T_3.\ \omega(RRR)R, \\ T_4.\ (RR.R)R, & T_5.\ RR.RR. & \end{array}$$

(Here R has no meaning. It is just a convenient way to say how the parentheses and ω appear.) Any degree four equation satisfied by A is a consequence of equations (1) and commutativity. Since minimal identities are not consequence of the commutative law we need to know the equations it implies. We do this by applying commutativity to the association types. We obtain:

$$
\begin{aligned}
&\omega(a)bc.d - \omega(a)cb.d = 0, &\qquad& \omega(ab)cd - \omega(ba)cd = 0,\\
&\omega(ab)cd - \omega(ab)dc = 0, && \omega(abc)d - \omega(bac)d = 0,\\
&\omega(abc)d - \omega(cab)d = 0, && (ab.c)d - (ba.c)d = 0,\\
&ab.cd - ba.cd = 0, && ab.cd - cd.ab = 0.
\end{aligned}
\tag{2}
$$

The minimal identities of A are the identities involving only types T_4 and T_5 implied by equations (1) that are not consequence of equations (2).

Now, instead of working directly with the equations (1) and (2) we represent them by matrices and then perform calculations with these matrices. This technique was introduced by Hentzel [2] and was slightly modified in [7].

Let S_n denote the symmetric group and FS_n the corresponding group algebra. Clifton [3] gives an algorithm which associates to $\pi \in S_n$ a matrix A_π. We use the map $\pi \to A_\pi$ to construct an isomorphism between FS_n and a certain direct sum of matrix algebras. We call the direct summands (irreducible) representations although they are not representations in the usual sense for $n > 4$. For $n = 4$ we have five representations and the isomorphism is given by

$$
\begin{aligned}
(12) &\mapsto [1] \oplus \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \oplus \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} \oplus \begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \oplus [-1].\\
(1234) &\mapsto [1] \oplus \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \oplus [-1].
\end{aligned}
\tag{3}
$$

Considering how the positions of the variables $a,\ b,\ c,\ d$ are changed in the terms of an equation we can represent it by an element of the direct sum $FS_4 \oplus \dots \oplus FS_4$ (here the number of summands is the number of association types, i. e., five). For instance, the equation $f(a,b)c.d = 0$ expands to

$$2(ab.c)d - \omega(a)bc.d - \omega(b)ac.d = 0$$

and is represent by

$$\begin{array}{cc} T_1 & T_4 \\ -I-(12) & \oplus\, 2I \end{array}$$

Now using (3) we represent this equation by matrices. Representation number 2 for example gives

$$\begin{array}{ccc} T_1 & & T_4 \\ \begin{bmatrix} -2 & 0 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} & \oplus & \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \end{array}$$

It is crucial to keep the different association types separarated.

It is too much work to calculate by hand all these matrices. So we use a computer program named CRUNCH. The imput is a set of equations involving r different types. For each representation the program calculates a $m \times r$ block matrix of $k \times k$ matrices where m is the number of equations and k is the representation degree. The output is the row canonical form of this block matrix. The row canonical form obtained from the set of equations (2) is given in table I and that obtained when we consider together the sets (1) and (2) is given in table II. Since our interest is concentrated on association types T_4 and T_5 we have written only part of these matrices. Comparing these two tables we see that there are five new stairstep ones. They correspond to the minimal identities we are looking for.

Finally we write the minimal identities in polynomial form. Since representation 1 is the identity representation the identity given by it is

$$(x^2, x, x) = 0. \tag{4}$$

To find the identities given by the other representations we use the process described in [7]. As an example we obtain the identity appearing in representation 2. It is given by

$$T_4$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The standard tableaus for this representation are

$$\tau_1 = \begin{matrix} 1 & 2 & 3 \\ 4 & & \end{matrix} \qquad \tau_2 = \begin{matrix} 1 & 2 & 4 \\ 3 & & \end{matrix} \qquad \tau_3 = \begin{matrix} 1 & 3 & 4 \\ 2 & & \end{matrix}$$

Since the nonzero entry of the matrix appears in row 3 we consider the horizontal permutations I, (13), (14), (34), (134), (143) and the vertical permutations I, (12) of τ_3. Let

$$H = \{I + (13) + (14) + (34) + (134) + (143)\}\{I - (12)\}.$$

Since the position of the nonzero entry is $(3, 2)$ we have to multiply H by the permutation that maps τ_3 to τ_2, i.e., (23). The identity is then given by type T_4 and $(12)H$ and is

(5) $$(a, c, b)d + (b, cd, a) + (a, d, b)c = 0.$$

Representations 3 and 4 yield the identities

(6) $$(b, c, a)d + (a, d, b)c = 0,$$

(7) $$ac.bd - ad.bc = 0,$$

(8) $$(a, cd, b) + (b, ad, c) + (c, bd, a) = 0.$$

To see that (6) implies (8) it is enough to add identities $(d, a, b)c+(b, c, d)a = 0$, $(d, b, c)a+(c, a, d)b = 0$ and $(d, c, a)b + (a, b, d)c = 0$. We claim that an algebra satisfies (4) and (5) if and only if it is a Jordan algebra. For a linearization of (4) gives $2(yx, x, x) + (x^2, y, x) + (x^2, x, y) = 0$ and (5) with $a = y$, $b = c = d = x$ gives $2(y, x, x)x + (x, x^2, y) = 0$. Subtracting these two identities we obtain the Jordan identity $(x^2, y, x) = 0$. On the other hand, clearly $(x^2, y, x) = 0$ implies $(x^2, x, x) = 0$. Linearizing $(x^2, b, x) = 0$ we obtain $2(ax, b, x) + (x^2, b, a) = 0$. Interchanging a and b and subtracting we get $2(a, x, b)x + (b, x^2, a) = 0$ and from this identity we obtain (5) by linearization.

Since it is necessary to put commutativity, the Jordan law, identities (6) and (7) together to obtain the matrix in table II no further reduction is possible and we have

Theorem 1. *The minimal variety of algebras containing the class of baric algebras given by* $x^2 = \omega(x)x$ *is defined by identities* $ac.bd = ad.bc$, $(a,c,b)d = (a,d,b)c$ *and* $(x^2,y,x) = 0$.

Remark. In [5] Costa used a different argument to prove that the minimal variety when we consider the equation $x^2 = \omega(x)x$ is defined by identity

$$(a,b,cd) + (d,c,ab) + (b,c,d)a + (c,b,a)d = 0.$$

Using the representation technique it is possible to verify that an algebra satisfies this identity if and only if it satisfies the identities given in theorem 1.

3. Degree three.

As shown by Walcher [4] baric algebras satisfying $x^3 = \omega(x)x^2$ are Jordan algebras. Now we have

Theorem 2. *Jordan algebras form the minimal variety of algebras which contains the class of baric algebras defined by* $x^3 = \omega(x)x^2$.

Although our method of attacking the problem is general, for some classes of baric algebras the problem is not doable. The number of stairstep ones that give the minimal identities is too high. It is too much work to write down all these identities in polynomial form. In these cases we do not find the minimal variety containing the class. But we do find the degree n of a minimal identity and construct a set of identities which generate all minimal identities of type $[n-1,1]$. We say that an equation has type $[n-1,1]$ when it is expressed in the variables x and y and in each term the degree of x is $n-1$ and the degree of y is 1.

We find this situation when considering the class defined by the equation $x^3 - \omega(x)^2 x = 0$. The minimal identities have degree five and there are many of them. Clearly $(x^3,y,x) = 0$ is an identity and since its degree is five it is minimal. To find a set of generators for minimal identities of type $[4,1]$ we proceed as follows. Let

$$f(x) = x^3 - \omega(x)^2 x, \quad p(y,x) = 2yx.x + x^2 y - 2\omega(xy)x - \omega(x^2)y,$$

$g(a,b,c)$ the linearized form of $f(x)$ and $q(y,a,b)$ the linearized form of $p(y,x)$. Any algebra in this class satisfies the following equations:

$$
\begin{array}{lllll}
 & f(x)x.y = 0, & f(x)y.x = 0, & f(x).xy = 0, & \omega(x)f(x)y = 0, \\
 & \omega(y)f(x)x = 0, & \omega(xy)f(x) = 0, & g(x^2,x,x)y = 0, & \omega(y)g(x^2,x,x) = 0, \\
(9) & p(y,x)x.x = 0, & p(y,x)x^2 = 0, & \omega(x)p(y,x)x = 0, & \omega(x)^2 p(y,x) = 0, \\
 & q(y,x^2,x)x = 0, & \omega(x)q(y,x^2,x) = 0, & q(yx,x,x)x = 0, & \omega(x)q(yx,x,x) = 0, \\
 & q(yx.x,x,x) = 0, & q(yx^2,x,x) = 0, & q(yx,x^2,x) = 0, & q(y,x^2,x^2) = 0, \\
 & q(y,x^3,x) = 0. & & &
\end{array}
$$

These are all the equations of type $[4,1]$ that are consequence of $f(x) = 0$. They are expressed in terms of the following types:

$$
\begin{array}{llllll}
\omega(y)x^4, & \omega(x)(xy.x)x, & \omega(x)x^2y.x, & \omega(x)x^3y, & \omega(xy)x^3, & \omega(x^2)xy.x, \\
\omega(x^2)x^2y, & \omega(x^2y)x^2, & \omega(x^3)xy, & \omega(x^3y)x, & \omega(x^4)y, & \omega(y)x^2x^2, \\
\omega(x)xy.x^2, & (xy.x)x.x, & (x^2y.x)x, & x^3y.x, & x^4y, & (xy.x)x^2, \\
x^2y.x^2, & x^3.xy, & (xy.x^2)x, & x^2x^2.y. & &
\end{array}
$$

Now we consider (9) as a system of equations where the indeterminates are these types. Reducing the matrix of the system to row canonical form we obtain

$$
\begin{array}{ccccccccc}
2 & & & & & 1 & -3 & -1 & 1 \\
 & 1 & & & -1 & & & & \\
 & & 1 & & & & -1 & & \\
 & & & 1 & -1 & -2 & 2 & 1 & -1
\end{array}
$$

The columns of this matrix correspond to the last nine types. We write these identities in polynomial form:

$$2(xy.x)x.x + x^2y.x^2 - 3x^3.yx - (xy.x^2)x + x^2x^2.y = 0, \tag{10}$$

$$(x^2y,x,x) + (x,x,y)x^2 = 0, \tag{11}$$

$$(x^3,y,x) = 0, \tag{12}$$

$$(x^2,x,x)y + (x^2,yx,x) + 2(x^2,x,yx) + 2(y,x,x)x^2 = 0. \tag{13}$$

From the linearized form of identity (12) we obtain

(12') $$2(yx.x, x, x) + (x^2y, x, x) + (x^3, x, y) = 0.$$

If we subtract from (12') the identities (11) and (13) we get identity (10). Thus (10) is a consequence of (11), (12) and (13). Processing each one of the identities (11), (12) and (13), and looking to the (irreducible) representation two it is possible to see that they are linearly independent, i.e., no one is a consequence of the other two. Therefore, we may state

Theorem 3. *Let A be a baric algebra satisfying $x^3 - \omega(x)^2 x = 0$. Then A satisfies the identities (11), (12) and (13). Furthermore any minimal identity of type $[4,1]$ is a consequence of these identities.*

4. Degree four.

We have the following results:

Theorem 4.

(i) For the equation $x^2x^2 = \omega(x)x^3$ the minimal variety is defined by

$$(x^2x^2, x, x) + 2(x, x^2, x^3) = 0,$$

$$3(x, x, x^2)x^2 + 2(x^3, x^2, x) + (x^3, x, x^2) = 0.$$

(ii) For the equation $x^2x^2 = \omega(x)^2x^2$ the minimal variety is defined by

$$(x^2x^2, y, x) - 2(x^2, y, x)x^2 = 0,$$

$$(y, x^2x^2, x) + 2(x^2, x, yx^2) + 2y(x^2, x^2, x) + 2(x, yx.x, x^2) = 0.$$

(iii) For the equation $x^4 = \omega(x)x^3$ the minimal variety is defined by

$$(x^3, x, x^2) + (x^3, x, x)x = 0,$$

$$3(yx.x^2, x, x) + (x^2x^2, x, y) + 2(x, x, (yx.x)x)+$$
$$(y, x, x^4) + (x, x, y)x^3 + (x, yx, x^3) + 2(x, y, x^3)x = 0.$$

(iv) For the equation $x^4 = \omega(x)^2 x^2$ *the minimal variety is defined by*

$$(y,x,x)x^3 + (y,x,x)x^2.x + (x^2,y,x^3) + (x,x,x^3y) + (x,x,x^2y)x = 0,$$

$$\begin{aligned}&2(x,x,yx)x^2 + 2(x,x^3,yx) + (y,x^2x^2,x) + (y,x^3,x)x+\\&2(x,x,y)x^2.x + 2(x,x,yx.x)x + (y,x^3,x^2) + (x,x,x^3)y+\\&2(x^2,y,x)x.x + (x,x,x^2y.x) = 0.\end{aligned}$$

Theorem 5.

(i) For the equation $x^2x^2 = \omega(x)^3 x$ *the minimal identities have degree 6 and those of type [5,1] are given by*

$$(x^2x^2,y,x) = 0,$$

$$\begin{aligned}&2(x^2,x,(x,x,y)) + 2(yx.x,x,x^2) + 2(yx,x,x^3)+\\&(x,yx.x,x^2) + (x^2,x^3,y) = 0,\end{aligned}$$

$$\begin{aligned}&(x^2y,x^2,x) + (y,x^2,x^3) + 2(x,x,y)x^3 + 4(x,x^2,yx.x)+\\&2(x,x,x^2).yx + 2(x^3,yx,x) + (y,x^2,x)x^2 = 0,\end{aligned}$$

$$\begin{aligned}&10(yx,x^2,x)x + 3(x^2,x^3,y) + 6(yx,x,x^3) + 5(x,x^2y,x^2)+\\&2(yx.x,x^2,x) + 6(x^2,x,x^2y) = 0.\end{aligned}$$

(ii) For the equation $x^4 = \omega(x)^3 x$ *the minimal identities have degree 6 and those of type [5,1] are given by*

$$(x^4,y,x) = 0,$$

$$2(y,x,x)x^3 + 2(y,x,x)x^2.x + 4((y,x,x)x.x)x + 2(x,x^4,y) = 0,$$

$$\begin{aligned}&(x,y,x^2x^2) + 2(x,y,x^2)x^2 + 2(x^2,y,x^3) + 2(x^3,x^2,y) + 2((y,x,x)x.x)x+\\&4(x^3,yx,x) + 3(y,x,x)x^2.x + 3(x,x^2,yx.x) + 3(x^3,x,x)y + (y,x,x)x^3+\\&(x,x^2,y)x.x + 2(yx.x^2,x,x) + (y,x,x^2x^2) + (x^2,x,x).yx = 0,\end{aligned}$$

$$\begin{aligned}&2(y,x,x)x^3 + (x,x^2x^2,y) + 3(x,x^2y,x^2) + 3(x^2y.x,x,x)+\\&2(x,y,x^4) + 2(yx,x^3,x) + 10((y,x,x)x.x)x + (x^2,x^3,y)+\\&2(y,x^2,x)x.x + 3(x,x^4,y) + (x,x,x^3y) + 2(y,x^3,x)x = 0.\end{aligned}$$

Table I. Equations (2).

Representation 1

The rows contain only zeros.

Representation 2

$$\begin{array}{cccccc} \multicolumn{3}{c}{T_4} & \multicolumn{3}{c}{T_5} \\ 1 & 1 & 2 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{array}$$

Representation 3

$$\begin{array}{cccc} \multicolumn{2}{c}{T_4} & \multicolumn{2}{c}{T_5} \\ 1 & 2 & & \\ & & 1 & 2 \end{array}$$

Representation 4

$$\begin{array}{cccccc} \multicolumn{3}{c}{T_4} & \multicolumn{3}{c}{T_5} \\ 1 & & -2 & & & \\ & 1 & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{array}$$

Representation 5

$$\begin{array}{cc} T_4 & T_5 \\ 1 & \\ & 1 \end{array}$$

Table II. Equations (1) and (2).

Representation 1

$$\begin{array}{cc} T_4 & T_5 \\ 1 & -1 \end{array}$$

Representation 2

$$\begin{array}{cccccc} \multicolumn{3}{c}{T_4} & \multicolumn{3}{c}{T_5} \\ 1 & & 2 & & & \\ & 1 & & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{array}$$

Representation 3

$$\begin{array}{cccc} \multicolumn{2}{c}{T_4} & \multicolumn{2}{c}{T_5} \\ 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{array}$$

Representation 4

$$\begin{array}{cccccc} \multicolumn{3}{c}{T_4} & \multicolumn{3}{c}{T_5} \\ 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{array}$$

Representation 5

$$\begin{array}{cc} T_4 & T_5 \\ 1 & \\ & 1 \end{array}$$

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