NUMERICAL ANALYSIS OF A MULTI-MATERIAL DAM VIA IGABEM AND THE SUBREGIONS TECHNIQUE

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1. INTRODUCTION

The Boundary Element Method (BEM) is a widely used numerical technique that involves boundary discretization and solving boundary integral equations. Traditionally, Lagrangian polynomials are used to discretize both the geometry and fields under analysis. However, recent research has shown that non-uniform rational B-splines (NURBS) can provide a more precise description of complex geometries, giving rise to a new method known as Isogeometric Boundary Element Method (IGABEM). With IGABEM, the boundary is discretized using the same basis functions as the geometry, resulting in a more accurate representation of the geometry, providing then a more precise analysis [1].

Thus, the BEM has a wide range of applications in several engineering areas, and great advances have been developed in the geotechnics field [2, 3]. It is known that dams are one of the most impressive geotechnical constructions. Many modern dams are constructed using a combination of materials to take advantage of the beneficial properties of each one [4]. In this context, as the BEM provides a very good precision of the stress field evaluation [5, 6], the use of this method allows an adequate estimative of the adopted constitutive model, predicting more accurately the real behavior of the structures. For this, there are techniques such as the sub-regions approach, that easily allow the consideration of non-homogenous materials [7].

This study uses the Isogeometric Boundary Element Method (IGABEM) with the sub-regions technique to conduct an elastic analysis of a multi-material dam. The dam design is inspired by the Itaipu Dam [8] in Foz do Iguaçu, Paraná, Brazil, and includes clay, a transition material, and rockfill for reinforcement. The parameters and dimensions that are of public knowledge are used for numerical modeling. Overall, this study serves as an important application of the BEM and sub-regions technique in the analysis of multi-material dams.

2. THE BOUNDARY ELEMENT METHOD

The integral formulation of the elasticity problem can be obtained through Betti's reciprocity theorem. For sake of simplicity, more details of the mathematical development are presented in [7, 9]. By applying the strain-displacement relations in Betti's reciprocity theorem, by integrating it by parts, and by considering the fundamental problem, the Somigliana Identity is obtained after integrating the Dirac Delta function. Equation 1 shows the integral equation of the elasticity problem, already considering the limit analysis to take the inner point to the contour to solve the boundary value problem, as follows:

$$\frac{1}{2}\mathbf{C}\mathbf{u}(\mathbf{s})+\int_{\Gamma}\mathbf{P}^{*}(\mathbf{s},\mathbf{f})\mathbf{u}(\mathbf{f})d\Gamma=\int_{\Gamma}\mathbf{U}^{*}(\mathbf{s},\mathbf{f})\mathbf{p}(\mathbf{f})d\Gamma+\int_{\Omega}\mathbf{U}^{*}(\mathbf{s},\mathbf{f})\mathbf{b}(\mathbf{f})d\Omega \quad (1)$$

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where **C** is the free-term; $\mathbf{P}^*(\mathbf{s},\mathbf{f})$ and $\mathbf{U}^*(\mathbf{s},\mathbf{f})$ are the traction and the displacement fundamental solution, respectively; $\mathbf{u}(\mathbf{s})$ and $\mathbf{p}(\mathbf{f})$ are the displacement and traction boundary solutions of the real problem, respectively; and $\mathbf{b}(\mathbf{f})$ refers to the domain term [10]. Thus, a set of matrices can be assembled relating all displacement and all the tractions components:

$$\mathbf{Hu} = \mathbf{Gt} + \mathbf{b} \tag{2}$$

where the square matrix **H** contains all the integrals of the $\mathbf{P}^*$(s,f) kernel and **G** of the $\mathbf{U}^*$(s,f) one; **u**, **t** and **b** are the vectors that contain the nodal displacements, the tractions and the independent vector of the domain forces obtained through the Galerkin vector [7], respectively. After prescribing the boundary conditions, Eq. (2) is rearranged according to the following system:

$$\mathbf{Ax} = \mathbf{f} \tag{3}$$

where **x** corresponds to the vector of unknown degrees of freedom and **A** is a full and non-symmetric matrix.

2.2 Subregion Technique

Non-homogenous problems are problems where more than one region must be considered. Hence, consider a region V assembled by two different materials, where V_1 and V_2 are two different subregions which have boundaries S_1 and S_2 and are connected by an interface S_I [7]. The nodal displacements and tractions at the external boundary S_1 is defined by $\mathbf{U}^1$ and $\mathbf{T}^1$, respectively; similarly, $\mathbf{U}^2$ and $\mathbf{T}^2$ are defined for boundary S_2 and $\boldsymbol{U}_I^1$, $\boldsymbol{U}_I^2$, $\mathbf{T}_\mathrm{I}^1$and $\mathbf{T}_\mathrm{I}^2$ are displacements and tractions at the interface S_I. Thus, according to [7], the system of equations for the subregion V_1 can be written as:

$$\begin{bmatrix}\mathbf{H}^1 & \mathbf{H}_\mathrm{I}^1\end{bmatrix}\begin{Bmatrix}\mathbf{U}^1\\ \mathbf{U}_\mathrm{I}^1\end{Bmatrix}=\begin{bmatrix}\mathbf{G}^1 & \mathbf{G}_\mathrm{I}^1\end{bmatrix}\begin{Bmatrix}\mathbf{T}^1\\ \mathbf{T}_\mathrm{I}^1\end{Bmatrix} \tag{4}$$

A similar equation is written for V_2. The compatibility and the equilibrium conditions at the interface S_I are:

$$\begin{aligned}\mathbf{U}_\mathrm{I}^1&=+\mathbf{U}_\mathrm{I}^2\equiv\mathbf{U}_\mathrm{I}\\ \mathbf{T}_\mathrm{I}^1&=-\mathbf{T}_\mathrm{I}^2\equiv\mathbf{T}_\mathrm{I}\end{aligned} \tag{5}$$

Applying the relations shown in Eq. (4), the following expression is obtained:

$$\begin{bmatrix}\mathbf{H}^1 & \mathbf{H}_I^1 & 0 & 0\\ 0 & 0 & \mathbf{H}^2 & \mathbf{H}_I^2\\ 0 & 1 & 0 & -1\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\mathbf{U}^1\\ \mathbf{U}_I^1\\ \mathbf{U}^2\\ \mathbf{U}_I^2\end{Bmatrix}=\begin{bmatrix}\mathbf{G}^1 & \mathbf{G}_I^1 & 0 & 0\\ 0 & 0 & \mathbf{G}^2 & \mathbf{G}_I^2\\ 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}\begin{Bmatrix}\mathbf{T}^1\\ \mathbf{T}_I^1\\ \mathbf{T}^2\\ \mathbf{T}_I^2\end{Bmatrix} \tag{6}$$

3. NUMERICAL EXAMPLE

The dam illustrated in Fig. 1 is based on section K of the Itaipu Binational Dam [8] and is composed of three materials: clay, transition, and rockfill. The dam's dimensions, including a height of 70m and a crest weight of 12m, are the same as those of Itaipu's section K. However, the other dimensions are adopted due to the unavailability of public information.

To discretize the model, 18 linear NURBS are used, with additional control points inserted via knot insertion technique [10], without modifying the order of each NURB. Hence, a total of 545 control points is used for the analysis. A plane strain state is considered.

The adopted materials characteristics are: transition - E=250MPa, $v = 0{,}375$ and γ=18kN/m^3; clay - E=50MPa, v=0,35 and γ=15kN/m³; and rockfill - E=35GPa, v=0,2 and γ=20kN/m³. Note that the body force of each domain is herein considered via the Galerkin vector approach [7].

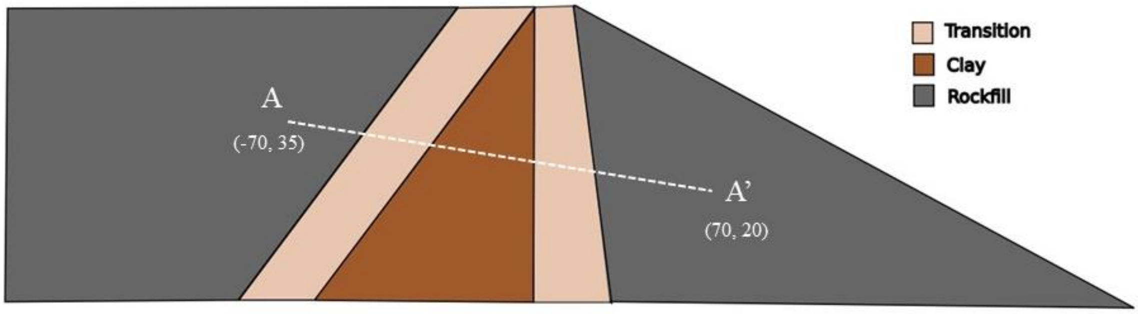

Figure 1 - Dam composed of three different materials: transition, clay, and rockfill.

Figure 2 presents the obtained deformation section via IGABEM and the subregions technique. A comparison with the Finite Element software ANSYS is also carried out. The results show agreement between the two numerical approaches. Furthermore, the internal path *A-A'* shown in Fig. 1 is chosen for the internal stress evaluation. A coordinate *s* that represents the path length is introduced, assuming the value *s=0m* at the beginning and *s=140,8m* at its end. The comparative results are presented in Fig. 3.

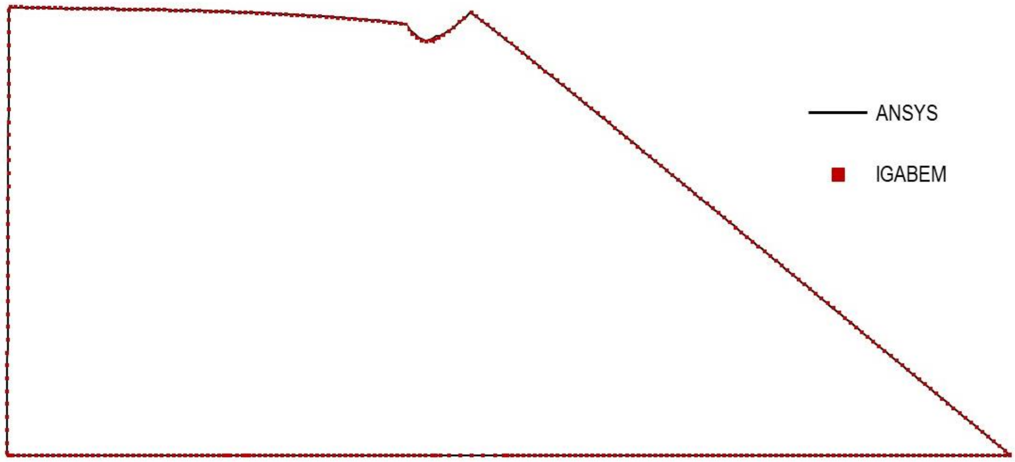

Figure 2 – Deformed Configuration via FEM versus IGABEM.

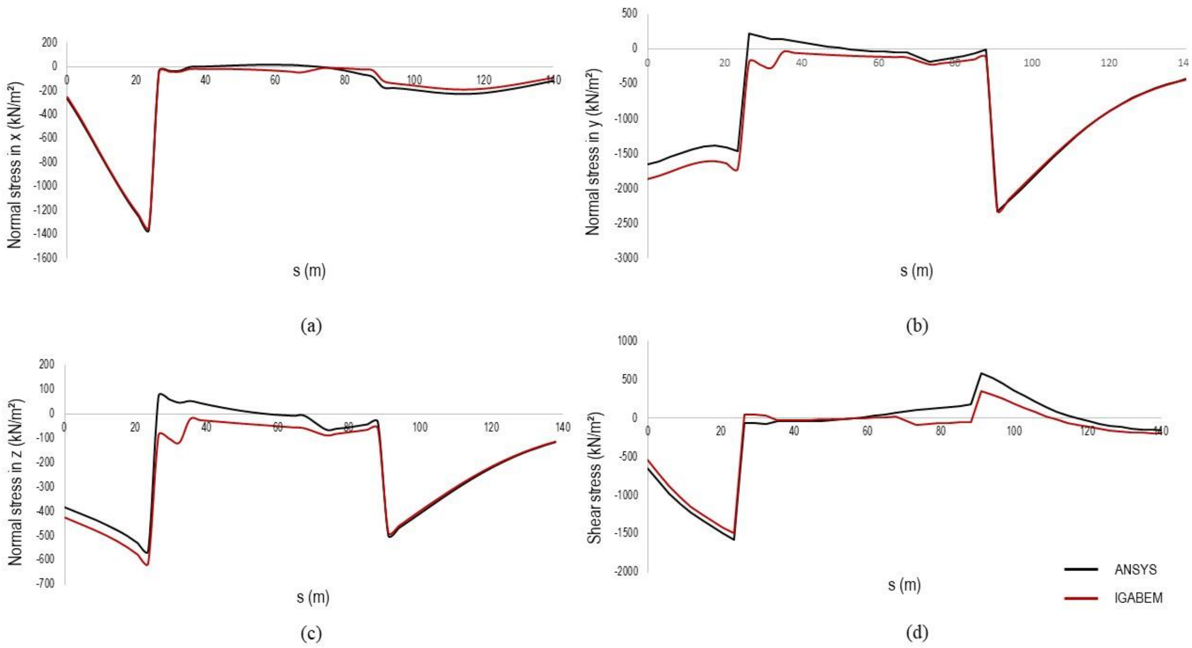

Figure 3 – (a) Normal stresses in x (b) Normal stresses in y (c) Normal stresses in z and (d) Shear stresses.

The maximum and minimum displacements on the boundary in the *x* direction are approximately *0.015m* and *-0.02m*, respectively, while in the *y* direction they are around *0.002m* and *-0.05m*. Furthermore, it is important to observe that the regions with the maximum displacements coincide precisely with the areas primarily composed of clay material, which has the lowest elasticity modulus among the constituents. Therefore, this complex example based on a real-life structure employs a variety of techniques in a single simulation, such as h-refinement, subregions, and Galerkin vector to include the domain term. As demonstrated, it shows a very satisfactory agreement with the FEM analysis via ANSYS, validating the code implementation.

The techniques outlined and applied herein allow further studies in cases where different materials need to be considered. Furthermore, in scenarios where the influence of body forces is crucial for analysis, the Galerkin vector strategy is a very suitable tool, as it transforms a domain integral into a boundary one, facilitating the analysis.

REFERENCES

[1] SIMPSON, R. N. et al. A two-dimensional isogeometric boundary element method for elastostatic analysis. Computer Methods in Applied Mechanics and Engineering, Elsevier, v. 209, p. 87–100, 2012.
[2] Procházka, P. (1992). The BEM applied to optimization and contact problems in geotechnics. *Acta Montana IGt CSAS*, 21-32.
[3] Panji, M., & Ansari, B. (2017). Modeling pressure pipe embedded in two-layer soil by a half-plane BEM. *Computers and Geotechnics*, 360-367.
[4] CHEN, S.-C.; LIN, T.-W.; CHEN, C.-Y. Modeling of natural dam failure modes and downstream riverbed morphological changes with different dam materials in a flume test. Engineering Geology, Elsevier, v. 188, p. 148–158, 2015.
[5] RAVEENDRA, S.; CRUSE, T. BEM analysis of problems of fracture mechanics. *Developments in Boundary Element Methods: Industrial applications*, CRC Press, p. 182, 1989.

[6] LEONEL, E. D.; VENTURINI, W. S.; CHATEAUNEUF, A. A BEM model applied to failure analysis of multi-fractured structures. *Engineering Failure Analysis*, Elsevier, v. 18, n. 6, p. 1538–1549, 2011.

[7] ALIABADI, M. H. The boundary element method, volume 2: applications in solids and structures. [S.l.]: John Wiley & Sons, 2002. v. 2.

[8] ITAIPU. Principais características técnicas. Av. Tancredo Neves, 6702 - CEP 85866-900 - Foz do Iguaçu - PR, 2023. Available in < https://www.itaipu.gov.br/sites/default/files/af_df/2202_023_revista_aspectos_tecnicos_bizuarios_018.pdf>, accessed in March 2023.

[9] Brebbia CA, Dominguez J, "Boundary elements: An introductory course". McGraw Hill, Southampton, UK, 1989.

[10] PIEGL, L.; TILLER, W. The NURBS book. [S.l.: s.n.]: Springer Science & Business Media, 1996.

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